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Answer all the questions.

- 1 A curve has the equation $y = 2x^3 \ln x$, where $x > 0$.

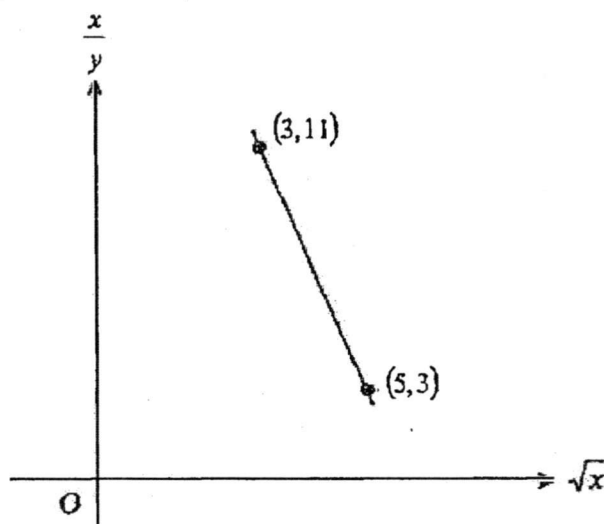
(i) Find $\frac{dy}{dx}$. [2]

(ii) Show that x -coordinate of the turning point is $\frac{1}{\sqrt[3]{e}}$ and determine whether the turning point is a maximum or a minimum. [4]

- 2 (a) Find the range of values of p for which the expression $(p+6)x^2 - 8x + p$ is always positive for all real values of x . [4]

(b) Show that the line $y = \frac{x}{k} + \frac{k}{4}$ is a tangent to the curve $y^2 = x$ for all real values of k . [3]

- 3 The diagram shows part of a straight line graph drawn to represent the equation $y = \frac{x}{b\sqrt{x-a}}$, where a and b are constants.



Given that the line passes through (3, 11) and (5, 3), find the values of a and of b . [4]

- 4 (a) Using an appropriate substitution, or otherwise,
solve $(\sqrt{9})^{2x} - 3^{x+2} = 6(3^x) - 54$. [6]

- (b) Without using a calculator, find the value of 10^x , given
that $2^{2x+5} \times 5^{x-2} = 5^{2x} \times 8^{x+1}$. [3]

- 5 (a) (i) State the values between which the principal value of $\cos^{-1} x$ must lie. [1]
(ii) Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ in radians. [1]

- (b) It is given that $\cos A = \frac{4}{5}$ where $270^\circ < A < 360^\circ$

Without the use of calculator, find the exact value of each of the following.

- (i) $\cot 2A$. [3]

- (ii) $\sin \frac{1}{2}A$. [2]

- 6 (a) Evaluate $\int_1^9 \left(\sqrt{x} + 2 - \frac{4}{\sqrt{x}} \right) dx$. [3]

- (b) The gradient of a curve is $\frac{12}{(4x-1)^2}$. Given that the curve passes through the
points $\left(\frac{1}{2}, 5\right)$ and $(-2, k)$, find the value of k . [4]

Questions 7 to 11 must be handed in separately from Questions 1 to 6.

Begin your answer to question 6 on a fresh sheet of paper.

- 7 It is given that $3x + 4y = k$ and $(6x - 20)^2 + (4y + 3)^2 = 200$.
- (i) If $k = 8$, find the solutions of these simultaneous equations. [4]
 - (ii) If $k = -10$, show that there are no solutions without solving the equations. [2]
 - (iii) Explain why there cannot be more than 2 solutions for all values of k . [2]
- 8 Express $\frac{2x^3 - 5x^2 - 11x + 44}{x^2 - x - 6}$ in partial fractions. [5]
- 9 (a) Find the equation of the normal to the curve $y = \sin 4x - 3 \cos 2x$ at the point where $x = \frac{\pi}{12}$. [7]
- (b) Singapore has two high tides and two low tides a day. The tidal movement on East Coast beach during a particular day can be modelled by the curve $y = \sin x$. If the four tides occur at 5 am, 11 am, 5 pm and 11 pm respectively, at what time will the flow of the water onto East Coast beach be the fastest? [1]
- 10 (a) Given that the coefficient of x^2 in the expansion of $(1 - 3x)^2(1 - kx)^8$ is 117, find the two possible values of the constant k . [5]
- (b) Find the term independent of x in the expansion of $\left(\frac{1}{x} - \frac{x^3}{4}\right)^{24}$. [4]

- 11 A curve has the equation $y = \left(\frac{x}{2} + 1\right)^2 - 4$.
- (i) Explain why the lowest point on the curve has the coordinates $(-2, -4)$. [2]
 - (ii) Find the x -coordinates of the points at which the curve intersects the x -axis. [2]
 - (iii) Sketch the graph of $y = \left(\frac{x}{2} + 1\right)^2 - 4$, indicating clearly the coordinates of the turning point and the points where the curve meets the axes. [3]
 - (iv) State the set of the values of k for which the line $y = k$ intersects the curve
 - (a) at 2 distinct points, [2]
 - (b) at 4 distinct points. [1]

End of Paper I

Answer all the questions.

- 1 A particular species of fish living in a fish farm is being studied. After t years, its population P is given by $P = 300(2 + 5e^{-kt})$, where k is a constant.
- (a) Find the initial population of the fish in the farm. [1]
- The population of the fish in the farm after 3 years is predicted to be 2400.
- (b) Find the value of k . [2]
- The fish farm owner has to replenish the supply of fish in the farm when the population drops below 1000.
- (c) Using the k value obtained in part (b), determine, with working, whether the fish farm owner needs to replenish the fish supply after 5 years. [2]
- 2 Given that $f(x) = 3x^4 + x^3 - mx^2 - nx + 36$,
- (a) find the values of m and n when $(x^2 - 9)$ is a factor of $f(x)$, [4]
- (b) hence solve the equation $f(x) = 0$. [3]
- 3 The quadratic equation $2x^2 + 4x - 7 = 0$ has roots α and β .
- (a) State the values of $(\alpha + \beta)$ and $\alpha\beta$. [2]
- (b) Find the value of $\alpha^2 + \beta^2$. [2]
- (c) Hence, form a quadratic equation whose roots are $\frac{2}{\alpha^2}$ and $\frac{2}{\beta^2}$. [3]
- 4 (a) Solve the equation $\log_2(x-3) - 6\log_{x-3}(2) = 1$. [5]
- (b) Given that $w = \log_5 a$, find, in terms of w ,
- (i) $\log_5 \frac{5}{a}$, [1]
- (ii) $(\log_5 a)^4$ [1]
- (iii) $\log_5 125a^2$ [2]

5 (a) (i) Prove that $\frac{1 + \cos x}{1 - \cos x} = \cot^2 \frac{x}{2}$. [3]

(ii) Hence prove that $\cot^2 15^\circ = 7 + 4\sqrt{3}$. [3]

(b) Solve the equation $\frac{4 \sec x}{1 + \sec^2 x} = -1$ for $0 \leq x \leq 360^\circ$, giving your answers correct to 2 decimal places. [5]

6 The area of a triangle is $(3 + \sqrt{15}) \text{ cm}^2$. $\angle 5$

(a) In the case ~~whereby the triangle~~ is a right angle triangle with height $(\sqrt{5} - \sqrt{3}) \text{ cm}$, find, without using a calculator, the length of the base of this triangle in the form $(a\sqrt{5} + b\sqrt{3}) \text{ cm}$. [4]

(b) In the case whereby the triangle is an equilateral triangle with the length of each side $w \text{ cm}$, find the value of (w^2) , giving your answer in the form $a(\sqrt{3} + \sqrt{5})$. [4]

Begin Question 7 on a fresh sheet of Answer Paper.

7 A curve has the equation $y = x(3 - x)^3$. Points A and B are the two stationary points on the curve.

(a) Find the coordinates of points A and B . [4]

(b) Determine the nature of these 2 stationary points on the curve. [4]

(c) (i) Find the values of x for which y is increasing. [1]

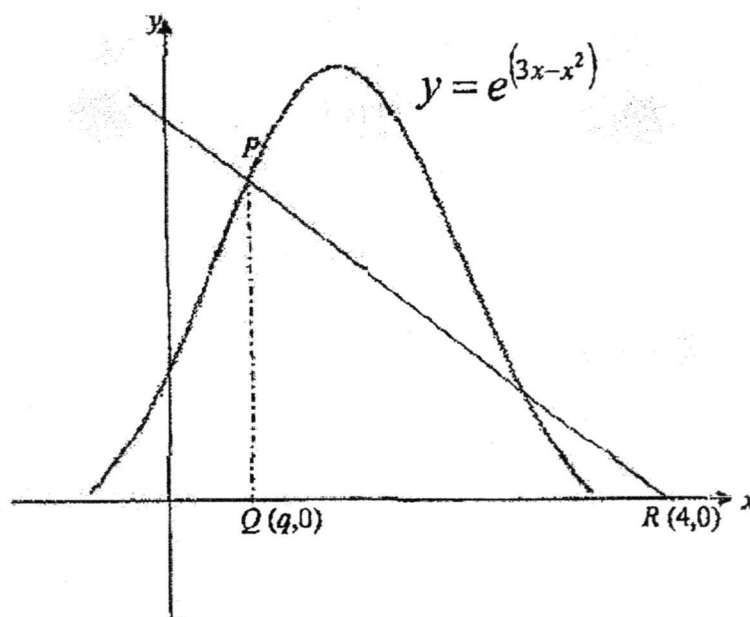
(ii) Find the values of x for which y is decreasing. [1]

- 8 Answer the whole of this question on the graph paper provided.

The table below shows the experimental values of x and y which are related by the equation $y = b^{a-x}$. One value of y has been recorded wrongly.

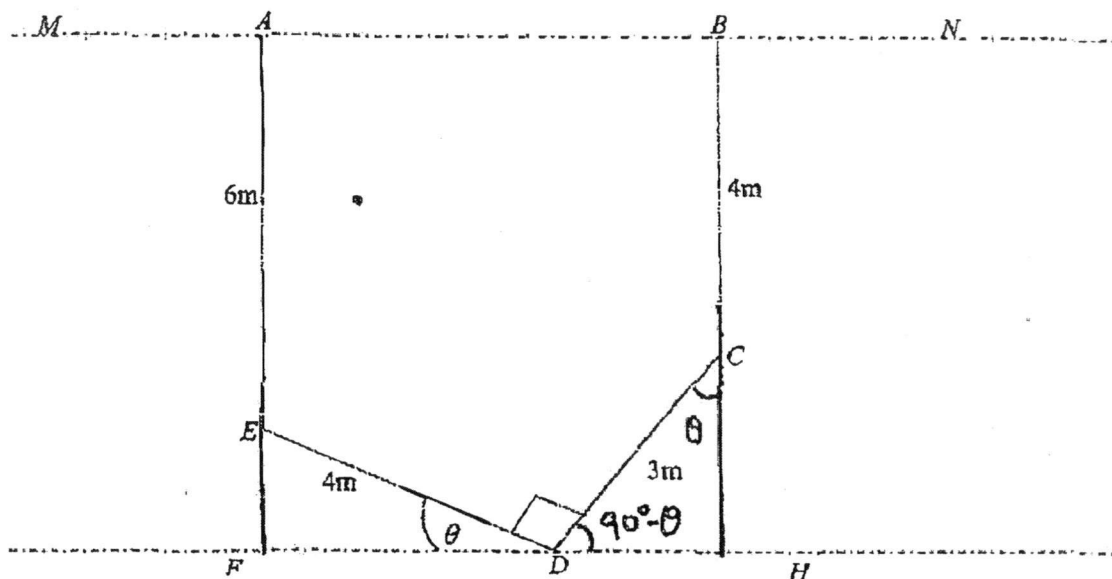
x	1	1.5	2	2.5	3
y	4	2.21	2	1.41	1

- (a) Plot $\lg y$ against x and draw a straight line graph. [4]
 (b) Use your graph to estimate the values of a and b . [4]
 (c) Determine which value of y is incorrect and estimate the correct value of y . [2]
- 9 The diagram below shows the graph of $y = e^{(3x-x^2)}$. Points Q and R lie on the x -axis such that their coordinates are $(q, 0)$ and $(4, 0)$ respectively. P is a point on the curve such that PQ is parallel to the y -axis.



- (a) Express the coordinates of P in terms of q . [1]
 (b) Show that the area of triangle PQR , $A = \left(2 - \frac{q}{2}\right)e^{(3q-q^2)}$. [2]
 (c) If q is decreasing at a rate of 2 units per second, find the rate at which the area of triangle PQR is changing at the instant when $\boxed{q=2}$. [4]

10



In the diagram above, $AE = 6$ m, $BC = DE = 4$ m and $CD = 3$ m, $\angle CDE = 90^\circ$
 $\angle EDF = \theta$, AE and BC are both perpendicular to the line MN . MN is parallel to FH .

- (a) Show that $AB = 4\cos\theta + 3\sin\theta$. [2]
- (b) Express AB in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < 90^\circ$. [3]
- (c) Find the maximum perimeter \overline{P} of the figure and the corresponding value of θ . [4]

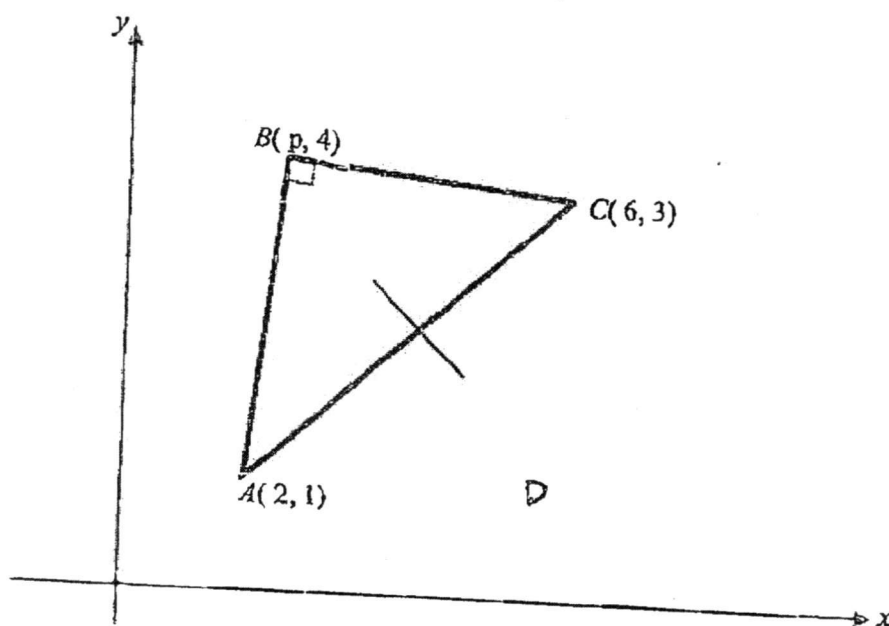
- 11 The function f is defined, for $x \geq 0$, by $f(x) = p\cos\left(\frac{x}{3}\right) - q$.

- (a) State the period of $f(x)$. [1]

Given that the maximum and minimum values of $f(x)$ are 1 and -5 respectively, find

- (b) the amplitude of f , [1]
- (c) the values of p and q , [2]
- (d) Using the values of p and q found in part (iii), sketch the graph of $f(x)$ for $0 \leq x \leq 3\pi$. [3]

- 12 Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a triangle ABC in which point A is $(2, 1)$, point B is $(p, 4)$ and point C is $(6, 3)$. The line AB is perpendicular to the line BC .

Find

- (a) the value of p , where $p < 4$, [3]
 (b) the equation of the perpendicular bisector of AC . [3]

The point D is such that $ABCD$ is a square. Find, using the value of p from part (i),

- (c) the coordinates of D , [2]
 (d) the area of $ABCD$. [2]

~ End of Paper 2 ~

Paper 1 Answer

Q1

SFC 4 EXP A-MATHS SAI PAPER 1 (2017)

Q21 (i) $y = 2x^3 \ln x, x > 0$

$$\frac{dy}{dx} = 2x^3 \left(\frac{1}{x}\right) + (\ln x)(6x^2)$$

does not accept $\ln x \times 6x^2$

$$= 2x^2 + 6x^2 \ln x$$

[B1] [B1]

(ii) when $\frac{dy}{dx} = 0, 2x^2 + 6x^2 \ln x = 0$ [B1]

Does not know
how to solve the
equation by factorisation
 $2x^2 + 6x^2 \ln x = 0$

$$2x^2(1 + 3\ln x) = 0$$

Since $x \neq 0, \therefore 1 + 3\ln x = 0$

$$\ln x = -\frac{1}{3}$$

$$\therefore x = e^{-\frac{1}{3}}$$

$$x = \frac{1}{\sqrt[3]{e}} \text{ (shown)}$$

[M1]

$$\frac{d^2y}{dx^2} = 4x + 6x^2 \left(\frac{1}{x}\right) + (\ln x)(12x)$$

$$= 10x + 12x \ln x$$
 [B1]

When $x = \frac{1}{\sqrt[3]{e}}, \frac{d^2y}{dx^2} = 10\left(\frac{1}{\sqrt[3]{e}}\right) + 12\left(\frac{1}{\sqrt[3]{e}}\right) \ln\left(\frac{1}{\sqrt[3]{e}}\right)$

$$= 4.299 > 0 \text{ (Min). [B1]}$$

\therefore the turning pt is a Minimum point.

$$Q2(a): b^2 - 4ac < 0 \text{ and } (p+6) > 0$$

$$(-8)^2 - 4(p+6)(p) < 0 \quad [M1] \quad p > -6 \quad [M1]$$

$$64 - 4p^2 - 24p < 0$$

$$4p^2 + 24p - 64 > 0$$

$$p^2 + 6p - 16 > 0$$

$$(p+8)(p-2) > 0$$

$$p < -8 \text{ or } p > 2 \quad [M1]$$

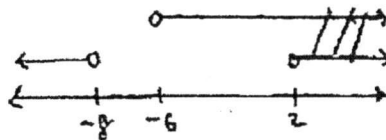
Mistake made:

(i) Used $b^2 - 4ac > 0$

(ii) not indicate

$$p+6 > 0$$

(iii) Did not write down the final answer



$$\text{Ans: } p > 2 \quad [A1]$$

Q2(b)

$$\left(\frac{x}{k} + \frac{k}{4}\right)\left(\frac{x}{k} + \frac{k}{4}\right) = x \quad [B1]$$

$$\frac{x^2}{k^2} + \frac{x}{2} + \frac{k^2}{16} = x$$

$$\frac{x^2}{k^2} - \frac{x}{2} + \frac{k^2}{16} = 0$$

$$16x^2 - 8k^2x + k^4 = 0$$

$$b^2 - 4ac = (-8k^2)^2 - 4(16)(k^4) \quad [B1]$$

$$= 64k^4 - 64k^4$$

$$= 0$$

Since $b^2 - 4ac = 0$ for all values of k ,

the line $y = \frac{x}{k} + \frac{k}{4}$ is a tangent to

the curve $y^2 = x$ (shown) } [M1]

Alternate solution:

$$y = \frac{y^2}{k} + \frac{k}{4}$$

$$4ky = 4y^2 + k^2$$

$$4y^2 - 4ky + k^2 = 0$$

$$(2y - k)^2 = 0$$

$$y = \frac{k}{2}$$

there is only one solution

$\therefore y = \frac{x}{k} + \frac{k}{4}$ is a tangent to the curve

Shideck did not present the working correctly.

Q3

Q3:

$$y = \frac{x}{b\sqrt{x} - a}$$

$$b\sqrt{x} - a = \frac{x}{y}$$

$$\frac{x}{y} = b\sqrt{x} - a \quad [M1]$$

$$\text{let } Y = \frac{x}{y}, X = \sqrt{x}.$$

then grad = b and vertical intercept = $-a$.

$$\text{Grad} = b = \frac{11-3}{3-5} = -4 \quad [A1]$$

$$Y = -4X + c$$

When $X=5, Y=3$, we have,

$$3 = -4(5) + c$$

$$c = 23 \quad [M1]$$

$$\therefore -a = 23$$

$$\therefore a = -23 \quad [A1]$$

$$\text{OR } \begin{array}{l} \text{OR } \begin{array}{l} (3, 11) \\ X=3, Y=11 \\ 11 = -4(3) + c \\ c = 23 \end{array} \end{array}$$

Q 4

Q4(a) $(\sqrt{9})^{2x} - 3^{x+2} = 6(3^x) - 54$

$$(3^x)^2 - 9(3^x) = 6(3^x) - 54$$

$$(3^x)^2 - 15(3^x) + 54 = 0 \quad [M1]$$

let $u = 3^x$.

$$\therefore u^2 - 15u + 54 = 0$$

$$(u - 6)(u - 9) = 0 \quad [M1]$$

$$\therefore u = 6 \text{ or } u = 9 \quad [B1]$$

$$3^x = 6$$

$$3^x = 9$$

$$3^x = 3^2$$

$$x \lg 3 = \lg 6$$

$$\therefore x = \frac{\lg 6}{\lg 3} \quad [M1]$$

$$= 1.63 \text{ (3sf)} \quad [A1]$$

$$\Rightarrow x = 2 \quad [A1]$$

(1) Students gave answers $u = 6$ or $u = 9$

Did not solve the equations to find values of x .

(2) Did not know how to solve $3^x = 6$

4(b) $2^{2x+5} \times 5^{x-2} = 5^{2x} \times 8^{x+1}$

$$32(2^{2x}) \times \frac{5^x}{25} = 5^{2x} \times 8(2^{3x}) \quad [M1]$$

$$\frac{32}{8 \times 25} = \frac{5^{2x} \times 2^{3x}}{5^x \times 2^{2x}} \quad [M1]$$

$$5^x \times 2^x = \frac{4}{25}$$

$$\Rightarrow 10^x = \frac{4}{25} \quad [A1]$$

Students did not change 8^{x+1} to 2^{3x+3}

Q5

5(a) (i) let the principal value of $\cos^{-1} x$ be θ .

$$\therefore 0 \leq \theta \leq \pi \quad \& \quad [B1]$$

5(a) (ii) let $\cos^{-1}(\frac{1}{\sqrt{2}}) = \theta$, $0 \leq \theta \leq \pi$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}}$$

$$\text{basic } \angle \alpha = \frac{\pi}{4}$$

$$\therefore \theta = \pi - \frac{\pi}{4} \\ = \frac{3\pi}{4} \quad \& \quad [B1]$$

Most students did not know what principal value is.

Did not know that $\cos^{-1} x$ is an angle

5(b) (i) $\cos A = \frac{4}{5}$, $270^\circ < A < 360^\circ$.

$$\tan A = -\frac{3}{4} \quad [B1]$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \\ = \frac{2(-\frac{3}{4})}{1 - (-\frac{3}{4})^2} \\ = -\frac{24}{7}$$

[M1]

$$\therefore \cot 2A = -\frac{7}{24} \quad \& \quad [A1]$$

5(b) (ii) $\cos 2A = 1 - 2 \sin^2 A$

$$\cos A = 1 - 2 \sin^2 \frac{A}{2}$$

$$\sin^2 \frac{A}{2} = \frac{1 - \cos A}{2}$$

$$= \frac{1 - \frac{4}{5}}{2} \quad [M1]$$

$$= \frac{1}{10}$$

$$\therefore \sin \frac{1}{2} A = \frac{1}{\sqrt{10}} \quad [A1] \text{ or } -\frac{1}{\sqrt{10}} \quad [WA]$$



Did not know that

$$\sin A = -\frac{3}{5}$$

$$\tan A = -\frac{3}{4}$$

Most students tried to use

$$\sin 2A = 2 \sin A \cos A \text{ formula}$$

Q6

$$\begin{aligned}
 Q6(a) \quad & \int_1^9 \left(\sqrt{x} + 2 - \frac{4}{\sqrt{x}} \right) dx \\
 &= \int_1^9 \left(x^{\frac{1}{2}} + 2 - 4x^{-\frac{1}{2}} \right) dx \\
 &= \left[\frac{2x^{\frac{3}{2}}}{3} + 2x - 8x^{\frac{1}{2}} \right]_1^9 \quad [B1] \\
 [B1] \quad &= \left[\frac{2(9)^{\frac{3}{2}}}{3} + 2(9) - 8(9)^{\frac{1}{2}} \right] - \left[\frac{2}{3} + 2 - 8 \right] \\
 &= 12 - \left(-5\frac{1}{3} \right) \\
 &= 17\frac{1}{3} \quad [B1]
 \end{aligned}$$

$$6(b) \quad \frac{dy}{dx} = \frac{12}{(4x-1)^2} = 12(4x-1)^{-2}$$

$$\therefore y = \frac{12(4x-1)^{-1}}{(-1)(4)} + c = \frac{-3}{4x-1} + c$$

[B1]

When $x = \frac{1}{2}$, $y = 5$,

$$5 = \frac{-3}{4(\frac{1}{2})-1} + c$$

$$c = 8 \quad [B1]$$

$$\therefore y = \frac{-3}{4x-1} + 8$$

When $x = -2$,

$$k = \frac{-3}{4(-2)-1} + 8 \quad [M1]$$

$$k = 8\frac{1}{3} \quad [A1]$$

Some students tried to use

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

formula

$$3x + 4y = 8$$

$$4y = 8 - 3x \quad \text{--- (1)}$$

$$(6x - 20)^2 + (4y + 3)^2 = 200 \quad \text{--- (2)}$$

Subst. (1) into (2),

$$(6x - 20)^2 + (8 - 3x + 3)^2 = 200 \quad \text{[M1]}$$

$$36x^2 - 240x + 400 + (-3x + 11)^2 = 200$$

$$36x^2 - 240x + 400 + 9x^2 - 66x + 121 - 200 = 0$$

$$45x^2 - 306x + 321 = 0$$

$$\therefore x = \frac{(306) \pm \sqrt{(-306)^2 - 4(45)(321)}}{2(45)} \quad \text{[M1]}$$

$$= \frac{306 \pm \sqrt{35856}}{90}$$

$$= 5.50 \text{ (3sf)} \text{ or } 1.30 \text{ (3sf)} \quad \text{[A1]}$$

7(ii) If $k = -10$, then $4y = -10 - 3x$.

$$\therefore y = -2.13 \text{ (3sf)} \text{ or } 1.03 \text{ (2sf)} \quad \text{[A1]}$$

$$\therefore (6x - 20)^2 + (-10 - 3x + 3)^2 = 200$$

$$36x^2 - 240x + 400 + (-3x - 7)^2 = 200$$

$$36x^2 - 240x + 400 + 9x^2 + 42x + 49 - 200 = 0$$

$$45x^2 - 198x + 249 = 0 \quad \text{[B1]}$$

$$b^2 - 4ac = (-198)^2 - 4(45)(249)$$

$$= -5616 < 0 \quad \text{[B1]}$$

Since $b^2 - 4ac < 0 \Rightarrow$ no real roots \Rightarrow no sol^{ns}.

7(iii) Since $4y = k - 3x$,

$$(6x - 20)^2 + (k - 3x + 3)^2 = 200$$

* many students thought that $(6x - 20)^2 + (4y + 3)^2 = 200$ is a quadratic curve.

Addition of two quadratic expressions in this eqⁿ will result in a quadratic equation. [B1]

And the max. no. of roots in a quad eqⁿ is 2. [B1]

* many students did not show working to how they solved for x or y in the quad eqⁿ.
* many students forgot to solve for y.

Q28 :

$$\begin{array}{r}
 2x - 3 \\
 x^2 - x - 6 \overline{) 2x^3 - 5x^2 - 11x + 44} \\
 \underline{-(2x^3 - 2x^2 - 12x)} \\
 -3x^2 + x + 44 \\
 \underline{-(-3x^2 + 3x + 18)} \\
 -2x + 26
 \end{array}$$

* Many students did not find the quotient. They went straight into express the fractⁿ as partial fractions.

$$\therefore \frac{2x^3 - 5x^2 - 11x + 44}{x^2 - x - 6} = 2x - 3 + \frac{-2x + 26}{x^2 - x - 6} \quad [B1]$$

$$\frac{-2x + 26}{x^2 - x - 6} = \frac{-2x + 26}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} \quad [B1]$$

$$-2x + 26 = A(x+2) + B(x-3)$$

$$\text{When } x = -2, \quad -2(-2) + 26 = B(-2-3)$$

$$-5B = 30$$

$$B = -6 \quad [B1]$$

$$\text{When } x = 3, \quad -2(3) + 26 = A(3+2)$$

$$5A = 20$$

$$A = 4 \quad [B1]$$

$$\therefore \frac{2x^3 - 5x^2 - 11x + 44}{x^2 - x - 6} = 2x - 3 + \frac{4}{x-3} - \frac{6}{x+2} \quad [A1]$$

Q9

Q9(a) $y = \sin^4 x - 3 \cos 2x$

$$\frac{dy}{dx} = 4 \cos^3 x + 6 \sin 2x \quad [B1]$$

When $x = \frac{\pi}{12}$, $\frac{dy}{dx} = 4 \cos^3\left(\frac{\pi}{12}\right) + 6 \sin 2\left(\frac{\pi}{12}\right)$
 $= 2 + 6\left(\frac{1}{2}\right)$
 $= 5 \quad [B1]$

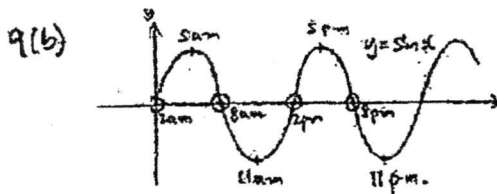
\therefore Grad of normal $= -\frac{1}{5} \quad [B1]$

When $x = \frac{\pi}{12}$, $y = \sin^4\left(\frac{\pi}{12}\right) - 3 \cos 2\left(\frac{\pi}{12}\right)$
 $= \frac{\sqrt{3}}{2} - 3\left(\frac{\sqrt{3}}{2}\right)$
 $= \frac{\sqrt{3} - 3\sqrt{3}}{2} = -\sqrt{3} \quad [B1]$

$\therefore y = -\frac{1}{5}x + c \quad -\sqrt{3} = -\frac{\pi}{60} + c$

When $x = \frac{\pi}{12}$, $y = -\sqrt{3}$, $c = \frac{\pi}{60} - \sqrt{3} \quad [B1]$

\therefore eqn of normal is $y = -\frac{1}{5}x + \frac{\pi}{60} - \sqrt{3} \quad [B1]$



From the graph, the timings of the fastest water flow are

2am, 8am, 2pm, 8pm. $[B1]$

several students cannot differentiate the trig terms.

* Students were found working on degree mode (calculator) when this is in radians.

* Students should learn to give answers in exact values.

Q10

$$Q10(a) \quad (1-3x)^2 (1-kx)^8$$

$$= (1-6x+9x^2) (1-8kx+28k^2x^2+\dots) \quad [B1]$$

$$\Rightarrow 28k^2 + 48k + 9 = 117 \quad [B1]$$

$$28k^2 + 48k - 108 = 0$$

$$7k^2 + 12k - 27 = 0$$

$$(7k-9)(k+3) = 0 \quad [M1]$$

$$\therefore k = \frac{9}{7} \text{ or } k = -3 \quad [A1]$$

[A1]

$$10(b) \quad T_{r+1} = {}^{24}C_r \left(\frac{1}{x}\right)^{24-r} \left(-\frac{x^3}{4}\right)^r \quad [M1]$$

$$= {}^{24}C_r \left(-\frac{1}{4}\right)^r x^{-24+r+3r}$$

$$= {}^{24}C_r \left(-\frac{1}{4}\right)^r x^{4r-24}$$

$$\Rightarrow 4r-24=0 \quad [A1]$$

$$4r=24$$

$$r=6 \quad [B1]$$

$$\therefore T_7 = {}^{24}C_6 \left(-\frac{1}{4}\right)^6 x^{4(6)-24}$$

$$= \frac{33649}{1024} \quad [A1]$$

* After finding $r=6$,
Many students claimed that
the 7th term was the answer.

* Many students thought

$$\left(\frac{1}{x} - \frac{x^3}{4}\right)^{24} = {}^{24}C_r \left(\frac{1}{x}\right)^{24-r} \left(-\frac{x^3}{4}\right)^r$$

Q11

11 (i) $y = \left(\frac{x}{2} + 1\right)^2 - 4$

When $\frac{x}{2} + 1 = 0$

$\frac{x}{2} = -1$

$x = -2$ [B1]

When $x = -2$, $y = \left(\frac{-2}{2} + 1\right)^2 - 4 = -4$

Since the coeff of x^2 is +ve, \therefore the lowest pt. of the graph is $(-2, -4)$. [B1]

11 (ii) When $y = 0$, $\left(\frac{x}{2} + 1\right)^2 - 4 = 0$

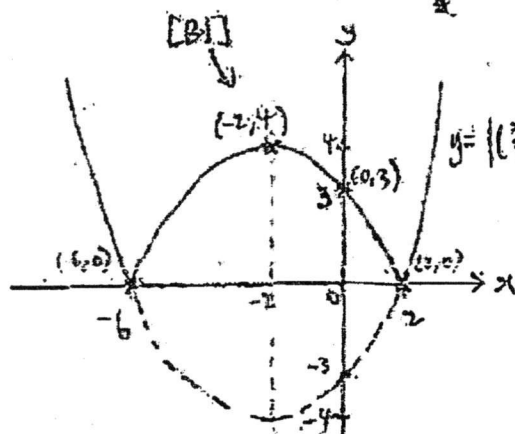
$\left(\frac{x}{2} + 1\right)^2 = 4$

$\frac{x}{2} + 1 = 2$ or -2 . [M1]

$\frac{x}{2} = 1$ or -3

$x = 2$ or -6 [A1]

11 (iii)



When $x = 0$, $y = \left(\frac{0}{2} + 1\right)^2 - 4$
 $y = -3$.

* Correct shape. [B1]

* Intercepts clearly indicated. [B1]

11 (iv)(a) $k = 0$ and $k > 4$. [B1]

11 (iv)(b) $0 < k < 4$. [B1]

Q1

Marking Scheme

$$(1) P = 300(2 + 5e^{-kt})$$

$$(a) \text{ When } t=0: P = 300[2+5] = 2100 \text{ [61]}$$

$$(b) 2400 = 300(2 + 5e^{-3t})$$

$$\frac{2400}{300} = 2 + 5e^{-3t}$$

$$8 = 2 + 5e^{-3t}$$

$$6 = 5e^{-3t} \quad (M1)$$

$$e^{-3t} = \frac{6}{5}$$

$$-3t = \ln\left(\frac{6}{5}\right)$$

$$t = -0.0608 \text{ [A1]}$$

$$(c) P = 300(2 + 5e^{0.061t})$$

$$\text{When } t=5: P = 300[2 + 5e^{0.061(5)}] \quad (M1)$$

$$= 2634.94 > 1000$$

hence no need to refresh. [A1]

Q2

2(b) Hence $f(x) = 3x^4 + x^3 - 31x^2 - 9x + 36$

$$\begin{array}{r} 3x^2 + x - 4 \\ x^2 - 9 \overline{) 3x^4 + x^3 - 31x^2 - 9x + 36} \\ \underline{-(3x^4 - 27x^2)} \\ x^3 - 4x^2 - 9x \\ \underline{-(x^3 - 9x)} \\ -4x^2 + 36 \\ \underline{-(-4x^2 + 36)} \\ 0 \end{array}$$

Hence $(x^2 - 9)(3x^2 + x - 4) = 0$ [M1]

$x^2 - 9 = 0$ or $3x^2 + x - 4 = 0$

$x = \pm 3$
[A1]

$(3x + 4)(x - 1) = 0$

$\therefore x = -\frac{4}{3}$ or $x = 1$ [A1], [A1].

3) $\alpha + \beta = -\frac{4}{2} = -2$ [01] $\alpha\beta = -\frac{7}{2}$ [01]

(b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ [M1]

$= (-2)^2 - 2(-\frac{7}{2}) = 4 + 7 = 11$ [A1]

(c) $\frac{2}{\alpha^2} + \frac{2}{\beta^2} = \frac{2\beta^2 + 2\alpha^2}{\alpha^2\beta^2}$

$(\frac{2}{\alpha^2})(\frac{2}{\beta^2}) = \frac{4}{(\alpha\beta)^2}$

$= \frac{2(\alpha^2 + \beta^2)}{(\alpha\beta)^2}$

$= \frac{4}{(-\frac{7}{2})^2}$

$= \frac{2(11)}{(-\frac{7}{2})^2} = \frac{22}{\frac{49}{4}} = \frac{88}{49}$ [A1]

$= \frac{4}{\frac{49}{4}}$

$= \frac{16}{49}$ [B1].

Hence $x^2 - (-\frac{88}{49})x + \frac{16}{49} = 0$

$49x^2 + 88x + 16 = 0$ [B1]

Q4

$$\log_2(x-3) - 6 \left[\frac{\log_2(x-3)}{\log_2(x-3)} \right] = 1 \quad \text{[M1]} \quad (\text{change of base})$$

$$\log_2(x-3) - \frac{6}{\log_2(x-3)} = 1$$

$$\text{Let } x = \log_2(x-3) : x - \frac{6}{x} = 1 \quad \text{[M1]}.$$

$$x^2 - 6 = x$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x=3 \quad \text{or} \quad x=-2$$

$$\log_2(x-3) = 3 \quad \log_2(x-3) = -2$$

$$x-3 = 2^3 \quad x-3 = 2^{-2} \quad \text{[M1]}$$

$$x = 5 \quad \text{[B1]} \quad \text{or} \quad x = 3\frac{1}{4} \quad \text{[B1]}$$

$$(b) (i) \log_5\left(\frac{5}{A}\right) = \log_5 5 - \log_5 A$$

$$= 1 - w \quad \text{[B1]}$$

$$(ii) (\log_5 A)^4 = w^4 \quad \text{[B1]}$$

$$(iii) \log_5 125a^2 = \log_5 125 + \log_5 a^2 \quad \text{[M1]}$$

$$= \log_5(5)^3 + 2\log_5 a$$

$$= 3 + 2w \quad \text{[A1]}.$$

Q5

$$5(a)(i) \text{ LHS: } \frac{1+\cos X}{1-\cos X} = \frac{1+[2\cos^2 \frac{X}{2}-1]}{1-[1-2\sin^2 \frac{X}{2}]} \quad [B1]$$

$$= \frac{2\cos^2 \frac{X}{2}}{2\sin^2 \frac{X}{2}} = \cot^2 \left(\frac{X}{2} \right) \quad [B1] \text{ (proven)}.$$

$$(ii) \text{ let } X=30^\circ: \frac{1+\cos 30^\circ}{1-\cos 30^\circ} = \cot^2(15^\circ)$$

$$\cot^2 15^\circ = \frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}$$

$$= \frac{2+\sqrt{3}}{2} \div \frac{2-\sqrt{3}}{2}$$

$$= \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \quad [M1]$$

$$= \frac{4+4\sqrt{3}+3}{1} = 7+4\sqrt{3}. \quad [A1].$$

$$(b) 4\sec X = -1 - \sec^2 X$$

$$\sec^2 X + 4\sec X + 1 = 0$$

$$\sec X = \frac{-4 \pm \sqrt{16-4}}{2} \quad [M1].$$

$$= \frac{-4 \pm \sqrt{12}}{2}$$

$$\sec X = -0.26795 \quad \text{or} \quad \sec X = -3.7321 \quad [M1].$$

$$\cos X = \frac{1}{-0.26795}$$

$$= -3.73 \quad (\text{NA})$$

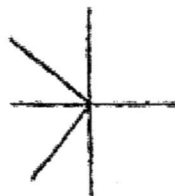
$$[A1]$$

$$\cos X = \frac{1}{-3.7321}$$

$$\cos X = -0.26795$$

$$\sec X = -3.73$$

$$\therefore X = 105.54^\circ \text{ or } 254.46^\circ$$



Q6

$$(b)(i) \frac{1}{2}(b)(\sqrt{5}-\sqrt{3}) = (3+\sqrt{15}) \text{ [B1]}.$$

$$b = \frac{2(3+\sqrt{15})}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \text{ [M1]}.$$

$$= \frac{(6+2\sqrt{15})(\sqrt{5}+\sqrt{3})}{5-3}$$

$$= \frac{6\sqrt{5}+6\sqrt{3}+2\sqrt{75}+2\sqrt{45}}{2} \text{ [M1]}$$

$$= 3\sqrt{5}+3\sqrt{3}+\sqrt{75}+\sqrt{45}$$

$$= 3\sqrt{5}+3\sqrt{3}+5\sqrt{3}+3\sqrt{5}$$

$$= 6\sqrt{5}+8\sqrt{3} \text{ [A1]}$$

$$(ii) \frac{1}{2}w^2(\sin 60^\circ) = 3+\sqrt{15} \text{ [B1]}.$$

$$\frac{1}{2}w^2\left(\frac{\sqrt{3}}{2}\right) = 3+\sqrt{15}$$

$$w^2\left(\frac{\sqrt{3}}{2}\right) = 6+2\sqrt{15}$$

$$w^2 = (6+2\sqrt{15})\left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{12+4\sqrt{15}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{ [M1]}.$$

$$= \frac{12\sqrt{3}+4\sqrt{45}}{3}$$

$$= \frac{12\sqrt{3}+4(3\sqrt{5})}{3} \text{ [M1]}.$$

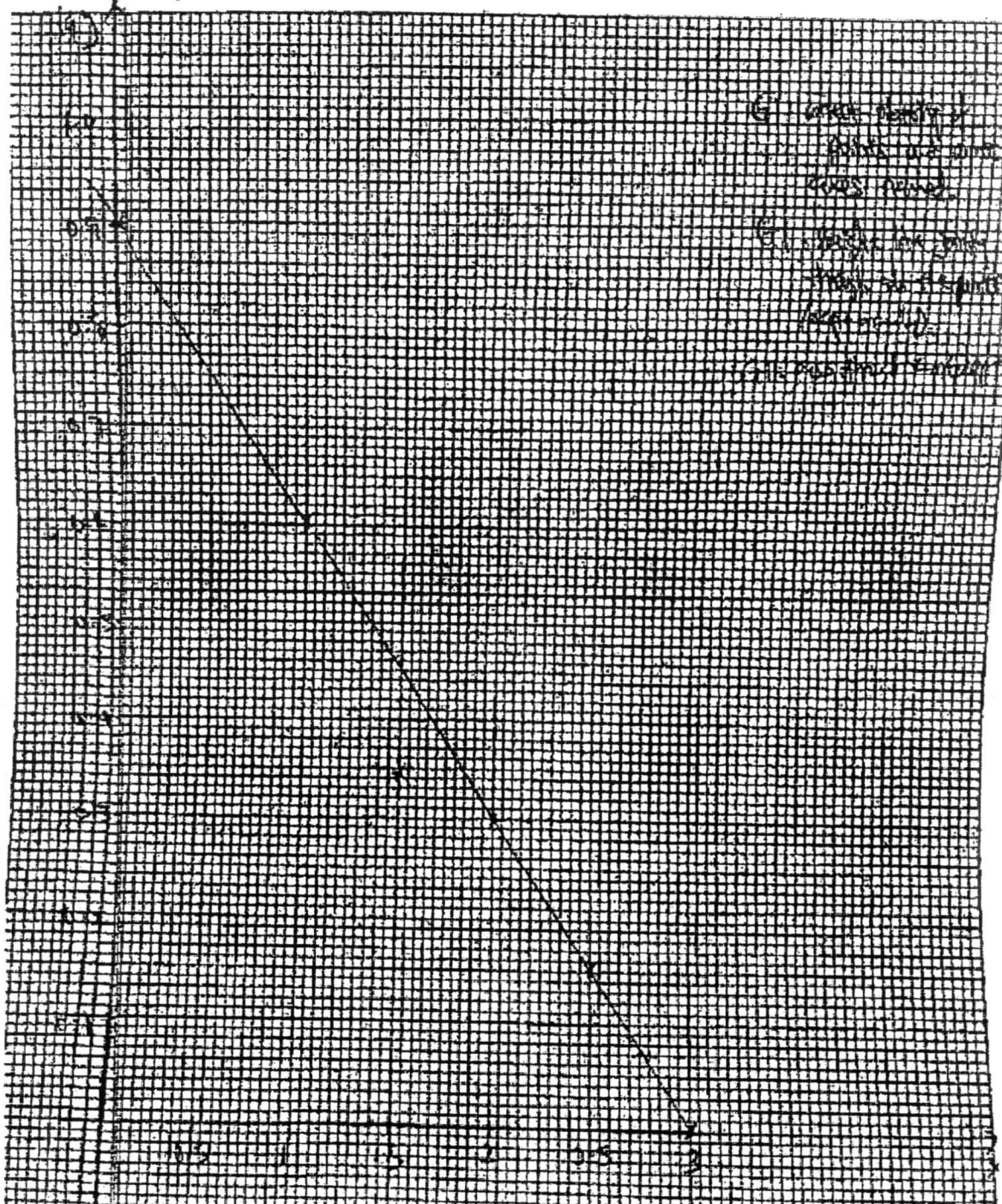
$$= \frac{12\sqrt{3}+12\sqrt{5}}{3}$$

$$= 4(\sqrt{3}+\sqrt{5}) \text{ [A1]}.$$

Q8

Subject: Paper:

Question No. (8)



- (a) When the value of the variable is 0, the value of the variable is 0.8.
- (b) When the value of the variable is 0.5, the value of the variable is 0.667.
- (c) When the value of the variable is 1, the value of the variable is 0.533.
- (d) When the value of the variable is 1.5, the value of the variable is 0.4.
- (e) When the value of the variable is 2, the value of the variable is 0.267.
- (f) When the value of the variable is 2.5, the value of the variable is 0.133.
- (g) When the value of the variable is 3, the value of the variable is 0.

Q9

(i) Coord of $P = (2, e^{3-2^2})$ [B1]

(ii) $A = \frac{1}{2}(4-2)(e^{3-2^2})$ [B1, 31]

$$= (2 - \frac{2}{2})(e^{3-2^2})$$

(iii) $\frac{dy}{dx} = -2 \text{ units/sec}$

$$\begin{aligned} \frac{dA}{dt} &= \left(\frac{dA}{dx} \right) \left(\frac{dx}{dt} \right) \\ &= \left(-\frac{3}{2}e^2 \right) (-2) \text{ [M1]} \\ &= 3e^2 = 22.2 \text{ units}^2/\text{sec.} \end{aligned}$$

[A1]

$$\begin{aligned} \frac{dA}{dx} &= (2 - \frac{x}{2})(e^{3-x^2})^{3-2x} + \\ &\quad (e^{3-x^2})^{3-2x} \left(-\frac{1}{2} \right) \text{ [B1]} \end{aligned}$$

$$\begin{aligned} \text{When } x=2: \frac{dA}{dx} &= (2 - \frac{2}{2})(e^{3-4})^{3-4} + (e^{3-4})^{3-4} \left(-\frac{1}{2} \right) \\ &= -e^{-1} - \frac{1}{2}e^{-2} \\ &= -\frac{3}{2}e^{-1} \text{ [B1]} \\ &\quad (-1.08358) \end{aligned}$$

$$10(a) \angle CDH = 180^\circ - 90^\circ - \theta = 90^\circ - \theta$$

$$DF = 4 \cos \theta \quad DH = 3 \cos (90^\circ - \theta) \\ \text{[B1]} \quad \quad \quad = 3 \sin \theta \quad \text{[B1].}$$

$$\therefore AB = 4 \cos \theta + 3 \sin \theta$$

$$(b) R = \sqrt{3^2 + 4^2} = 5 \quad , \quad \alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ \\ \text{[B1]} \quad \quad \quad \text{[B1]}$$

$$\therefore AB = 5 \cos (\theta - 36.9^\circ) \\ \text{[B1].}$$

$$(c) \text{Max AB is when } \cos (\theta - 36.9^\circ) = 1 \quad \text{[B1]}$$

$$\therefore \text{max perimeter} = 6 + 4 + 3 + 4 + 5 \\ = 22 \text{ m. [B1].}$$

$$\text{When } \cos (\theta - 36.9^\circ) = 1$$

$$\theta - 36.9^\circ = 0^\circ, 360^\circ \text{ [M1]}$$

$$\therefore \theta = 36.9^\circ, 396.9^\circ$$

$$\text{[A1]. (NA).}$$



Q11

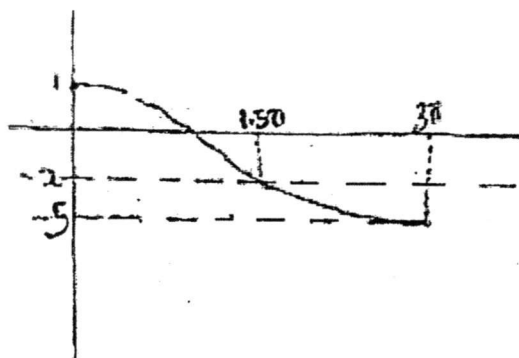
(11) $f(x) = p \cos\left(\frac{x}{3}\right) - q$

(i) Period = $\frac{2\pi}{\frac{1}{3}} = 6\pi$ [B1].

(ii) Amplitude = 3 [B1].

(iii) $p=3$, $q=2$. [B1, P1].

(iv)



$$f(x) = 3 \cos\left(\frac{x}{3}\right) - 2.$$

G1: correct shape

G1: correct y-intercept

G1: correct values of y at $x = 1.5\pi$
and $x = 3\pi$.

$$y = b^{a-x}$$

$$\lg y = \lg b^{a-x}$$

$$= (a-x) \lg b$$

$$= a \lg b - x \lg b$$

$$\therefore \lg y = (-\lg b)x + a \lg b$$

x	1	1.5	2	2.5	3
y	4	2.21	2	1.41	1
lg y	0.602	0.344	0.301	0.149	0

$$\text{Grad of graph} = \frac{0.35 - 0.05}{0.5 - 1.5} = -0.3 \quad [A1]$$

$$\text{Hence } -\lg b = -0.3 \quad \text{and } a \lg b = 0.9$$

$$b = 1.995 \approx 2.0 \quad [B1]$$

$$a = \frac{0.9}{\lg b} = 3 \quad [B1]$$

Incorrect value of y is $y = 2.21$ [B1]

$$\text{Correct value of } \lg y = 0.45$$

$$\text{hence correct } y = 2.82 \quad [B1]$$

$$\text{Grad of } AB = \frac{4-1}{p-2} = \frac{3}{p-2}$$

Since $AB \perp BC$: $\frac{1}{p-6} = \frac{p-2}{-3}$ (M1)

$$-3 = (p-6)(p-2)$$

$$-3 = p^2 - 8p + 12$$

$$p^2 - 8p + 15 = 0$$

$$(p-3)(p-5) = 0 \text{ (M1)}$$

$$\therefore p = 3 \text{ or } p = 5 \text{ (WA) (A1).}$$

(ii) Midpt $AC = \left(\frac{6+2}{2}, \frac{1+3}{2}\right)$
 $= (4, 2)$

$$\text{Grad of } AC = \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2} \text{ (B1)}$$

Grad of \perp bisector of $AC = -2$

$$\therefore y = -2x + c$$

at $(4, 2) \Rightarrow c = y + 2x \text{ (M1)}$
 $= 2 + 2(4)$
 $= 10$

hence $y = -2x + 10 \text{ (A1).}$

(iii) Midpt of $AC = \left(\frac{6+2}{2}, \frac{3+1}{2}\right) = (4, 2)$

Let D have a coord of (x, y) .

$$\frac{3+x}{2} = 4 \text{ and } \frac{1+y}{2} = 2 \text{ (M1)}$$

$$x = 5$$

$$y = 0$$

hence D is at $(5, 0)$

(iv) Area $= \frac{1}{2} \begin{vmatrix} 3 & 6 & 5 & 2 & 3 \\ 4 & 3 & 0 & 1 & 4 \end{vmatrix}$ (M1)

$$= \frac{1}{2} |22 - 42|$$

$$= \frac{1}{2} (20) = 10 \text{ units}^2 \text{ (A1).}$$