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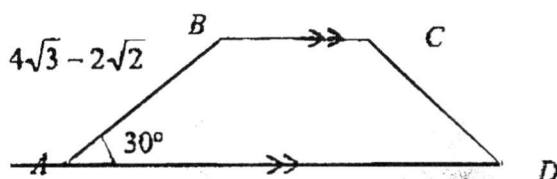
Answer all the questions

- 1 The equation of a curve is $y = 2x^2 - 6x + k$, where k is a constant.
- (i) In the case when $k = -20$, find the set of values of x for which $y < 0$. [2]
- (ii) In the case when $k = 10$, show that the line $y + 2x = 8$ is a tangent to the curve. [3]
- 2 (i) Given that $u = 2^x$, express $4^x - 2^{x+1} = 3$ as an equation in u . [2]
- (ii) Hence find the value of x , correct to 2 decimal places. [3]
- (iii) Explain why the equation $4^x - 2^{x+1} = k$ has no solution if $k < -1$. [2]
- 3 The equation $3x^2 - x + 5 = 0$ has roots α and β .
- (i) Find the value of $\alpha^3 + \beta^3$. [5]
- (ii) Find a quadratic equation with integer coefficient whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [3]
- 4 (i) Express $\frac{1 - \sin x \cos x + 2 \cos^2 x}{\sin^2 x}$ as a quadratic expression in $\cot x$. [3]
- (ii) Hence, using (i) solve the equation $1 + 2 \cos^2 x = \sin x(3 \sin x + \cos x)$ for $0^\circ < x < 360^\circ$. [4]

- 5 A freshly baked bread, with an initial temperature of 75°C , is left to cool on the rack. The temperature, $T^{\circ}\text{C}$ of the bread, t minutes after it has been placed on the rack is given by $T = 27 + ke^{-nt}$, where k and n are positive constants.

- (i) Calculate the value of k . [1]
- (ii) If the bread has cooled down by 20°C after 10 minutes, find the value of n . [2]
- (iii) Find the least time taken, to the nearest minute, for the bread to have a temperature of less than 30°C . [3]
- (iv) Explain with the sketch for $T = 27 + ke^{-nt}$, why the temperature of the bread can never reach 27°C . [1]

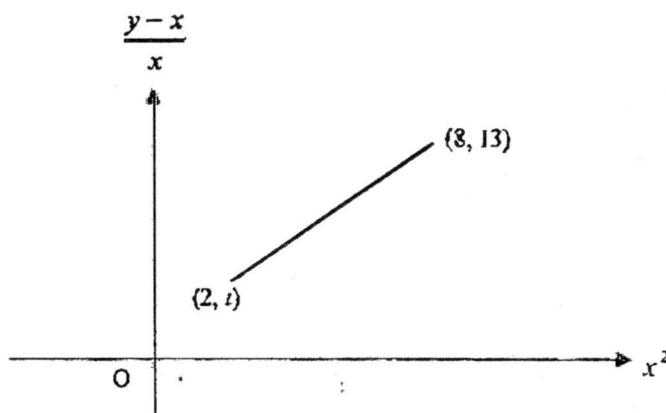
6



The diagram shows a trapezium $ABCD$ in which side $AB = 4\sqrt{3} - 2\sqrt{2}$ cm and angle $BAD = 30^{\circ}$. Given that the length of AD is twice the length of BC and that area of the trapezium is $25 + 5\sqrt{6}$ cm^2 , find without the use of a calculator, the length of AD in the form $a\sqrt{2} + b\sqrt{3}$.

[4]

- 7 The diagram shows part of a straight line, passing through $(2, t)$ and $(8, 13)$, drawn to represent the equation $3y = 4x^3 + ax$, where a and t are constants. Find the value of a and t .



[4]

- 8 (a) Given that the first 3 terms in the expansion of $(a-x)(1+3x)^n$ in ascending powers of x is

$$2 + 29x + bx^2 + \dots$$

Find the values of the constants a , b and n .

[5]

- (b) (i) Write down the general term in the binomial expansion of

$$\left(x^2 + \frac{1}{2x^3}\right)^{10}.$$

[1]

- (ii) Write down the power of x in this general term.

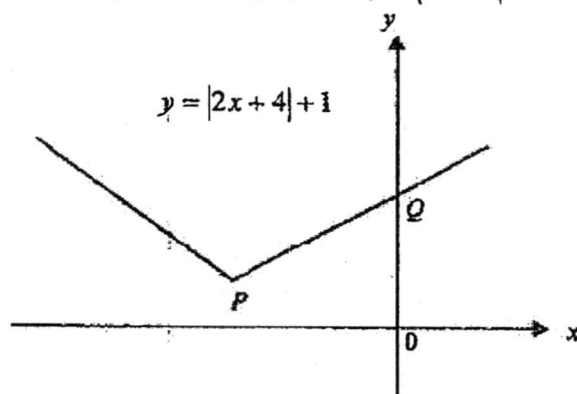
[1]

- (iii) Hence, or otherwise, determine the coefficient of x^{-15} in the binomial expansion of $\left(x^2 + \frac{1}{2x^3}\right)^{10}$.

[2]

- 9 A curve has equation $y = \ln\left(\frac{3x-1}{5-2x}\right)$. The normal to the curve at the point (a, b) is parallel to the line $13y + 5x - 26 = 0$. Given that where $a > 1$, find the values of a and b . [5]

- 10 The diagram shows part of the graph of $y = |2x + 4| + 1$.



- (i) Find the coordinates of P and of Q .

[3]

A line of gradient m passes through the point $(0, 3)$.

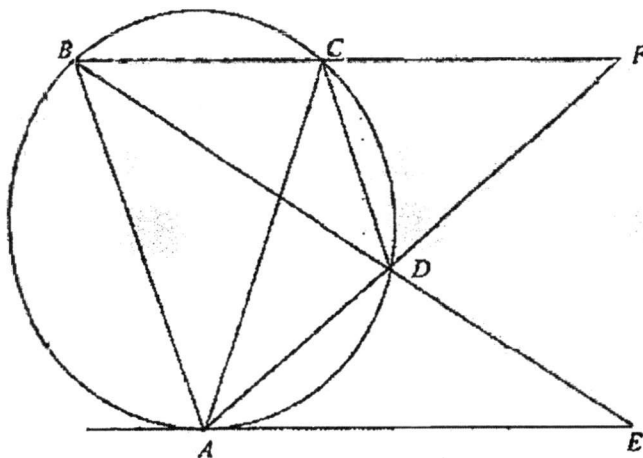
- (ii) In the case where $m = -1$, find the x -coordinates of the points of intersection between the line and the graph of $y = |2x + 4| + 1$

[3]

- (iii) Determine the set of values of m for which the line intersects the graph of $y = |2x + 4| + 1$ at two points.

[2]

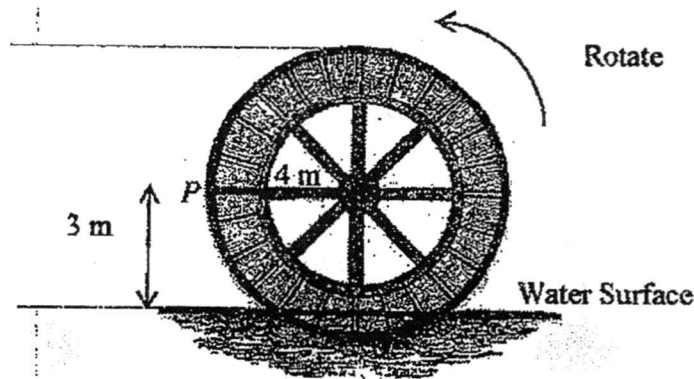
- 11 A particle starts from rest and moves in a straight line, so that t seconds after leaving O , its velocity, v m/s is given by $v = 28 - 2e^{\frac{-t}{3}}$.
- Calculate the initial acceleration of the particle. [2]
 - Calculate, to 2 decimal places, the displacement of the particle from O when $t = 10$. [4]
 - Determine, with explanation, whether the particle will return to O . [1]
- 12 In the diagram below, the line AE is tangent to the circle at A . The line EB is the angle bisector of angle ABC and cuts the circle at D . The chord AC is the angle bisector of angle BAD and cuts the circle at C . The chords BC and AD are produced to meet at F . The line segments AD and FD are equal.



- Prove that angle $DAC =$ angle DAE . [3]
- Prove that triangles ADE and BAE are similar. [2]
- Prove that C is a mid-point of BF . [2]
- Hence, using (ii) and (iii), show that $AF \times AE = 4CD \times DE$. [2]

Answer all the questions

- 1 (a) Given that $\lg(3y+2) - 4x^2 = 2$, express y in terms of x . [3]
- (b) Solve the equation $2\log_4(8-2x) - \log_2(x-2) = 3 - \log_2(1+x)$ [5]
- 2 The diagram shows a water wheel which rotates at 3 revolutions per minute in an anticlockwise direction. At the start of the revolution, a point P on the rim of the wheel is at the height of 3 m above the surface of the water. The radius of the water wheel is 4 m.



The height, h m, of point P above the water surface is given as $h = a \sin\left(\frac{\pi}{b}t\right) + c$, where t is the time in seconds.

- (i) State the values of a , b and c . [3]
- (ii) Find the time, t , where point P first emerge from the water. [3]
- 3 (i) Show that $\frac{d}{dx}[e^{2x}(2x+1)] = e^{2x}(4x+4)$. [2]
- (ii) Hence, or otherwise, evaluate $\int_0^1 2xe^{2x} dx$. [4]

- 4 The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 4 and the constant term is -3. When $x+1$ is a factor of $f(x)$, the quadratic factor is $px^2 + qx + r$. It is given that $f(x)$ leaves a remainder of -20 when divided by $x-1$.

(i) Find the values of p , q and r . [3]

(ii) Solve $f(x) = 0$. [2]

(iii) Hence solve the equation $f(-x) = 0$. [1]

5 It is given that $\frac{x^3 - x^2 - 4x + 1}{x^2 - 4} = ax + b + \frac{c}{x^2 - 4}$

(i) Find the values of a , b and c . [3]

(ii) Hence, using partial fractions and the values of a , b and c obtained in part (i),

find $\int \frac{x^3 - x^2 - 4x + 1}{x^2 - 4} dx$. [6]

- 6 (a) The curves $y = a(\sqrt{x})$ and $y = \frac{2a}{k}\left(\frac{1}{x^2}\right)$ meets at the point (1,5) where a and k are constants.

(i) Find the value of a and of k . [2]

(ii) On the same axes, sketch the two curves, for $x > 0$. [2]

- (b) Given that $y = (2x + \tan x)^2$ and that $\frac{dy}{dx} = a\pi + b\sqrt{3}$ when $x = \frac{\pi}{3}$, find the value of a and of b . [4]

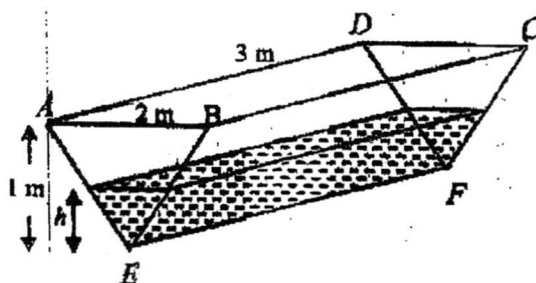
- 7 A curve has the equation $y = f(x)$, where $f(x) = \frac{3(x-1)}{5x+3}$ for $x > 0$.

- (i) Find an expression for $f'(x)$. [2]
- (ii) Explain why the curve has no stationary points. [1]
- (iii) Show with full workings, determine whether the gradient function of the curve is an increasing or decreasing function for $x > 0$. [2]

- 8 The points $(1, 10)$ and $(7, 10)$ are on the circumference of a circle whose centre, C , lies above the x -axis. The line $y = 1$ is tangent to the circle.

- (i) Find the coordinates of C . [3]
- (ii) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [2]
- (iii) Find the equations of the tangents to the circle parallel to the y -axis. [2]

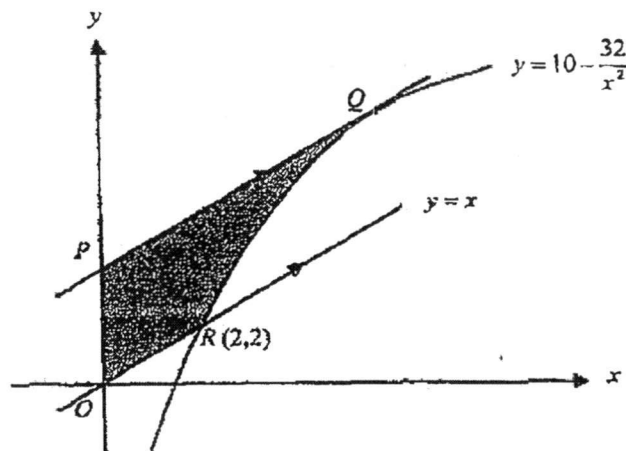
9



The above diagram shows a trough of 3m long and 1m deep with $ABCD$ horizontal. Its cross section is an isosceles triangle of base 2m with its vertex downwards. The empty trough is filled with water at the rate of $0.03 \text{ m}^3/\text{s}$.

- (i) If the depth of the water at time t seconds is h m, show that the volume of water is $3h^2 \text{ m}^3$. [1]
- (ii) Hence, find the rate at which the water level is rising after the water has been running for 25s. [4]

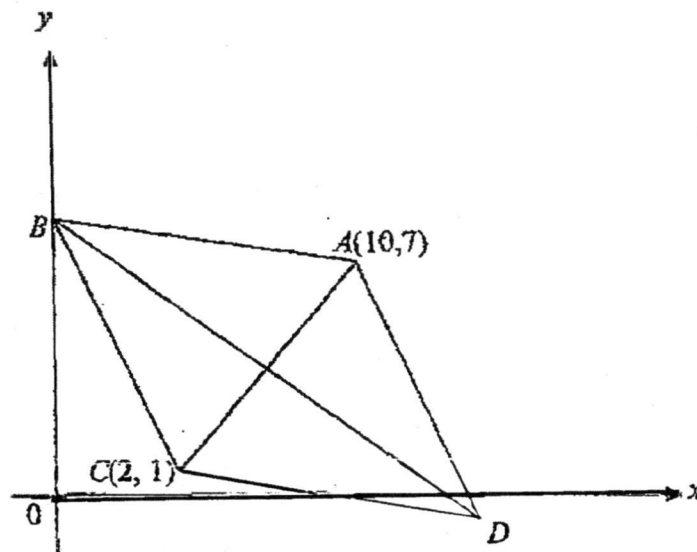
10



The diagram shows part of the curve and two parallel lines OR and PQ . The line OR intersects the curve at the point $R(2, 2)$ and the line PQ is a tangent to the curve at the point Q .

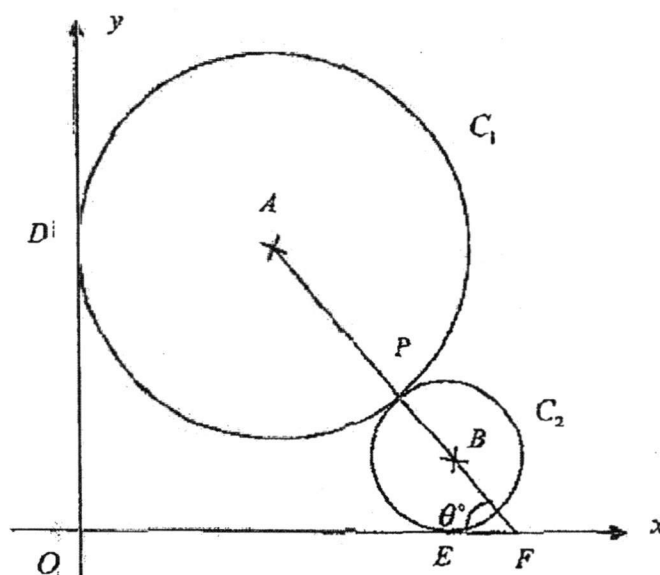
- (i) Find the coordinates of P and of Q . [4]
- (ii) Find the area of the shaded region $OPQR$. [6]

- 11 The diagram shows a rhombus $ABCD$ where $A(10, 7)$ and $C(2, 1)$. B is a point on the y -axis and is equidistant from A and C .



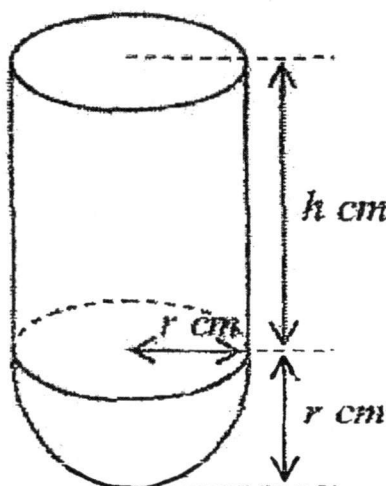
- (i) Find the coordinates of B and D . [4]
- (ii) Find the area of $ABCD$. [2]
- (iii) If point P lies on AC extended such that $PC : PA = 1 : 5$. Find the coordinates of P . [3]

- 12 In the diagram, two circles C_1 and C_2 whose centres are A and B respectively, touch each other at P . The radii of C_1 and C_2 are 4 units and 1 unit respectively. C_1 touches the y -axis at D and C_2 touches the x -axis at E . The line AB joining the centres of C_1 and C_2 meets the x -axis at F and $\angle BFO = \theta^\circ$.



- (i) Express OD and OE in terms of θ . [2]
- (ii) Hence, or otherwise, show that $DE^2 = 40 \cos \theta + 10 \sin \theta + 42$. [3]
- (iii) Express DE^2 in the form $R \cos(\theta - \alpha) + 42$ where $R > 0$ and α is acute. [3]
- (iii) Find the maximum value of DE and the value of θ at which the maximum value occurs. [3]

- 13 The diagram below shows an open container. It consists of a hemisphere of radius r cm, and a cylinder of radius r cm and height h cm. The hemisphere is fixed to the end of the cylinder and the volume of the container is 800 cm^3 .



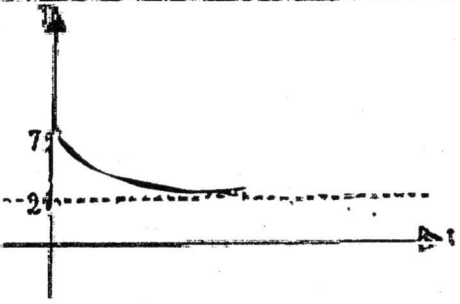
- (i) Show that $h = \frac{2400 - 2\pi r^3}{3\pi r^2}$. [2]

The container is made of some thin metal sheets. The cost of metal sheets for the cylindrical surface is \$1.50 per cm^2 and the cost of metal sheets for hemispherical surface is \$3.20 per cm^2 .

- (ii) Let $\$C$ be the cost of making the container. Show that $C = 4.4\pi r^2 + \frac{2400}{r}$. [3]
- (iii) Find the value of h and r such that the total cost of constructing the container is minimum. [5]

		Answers	
1	(i)	When $k = -20$	
		$y = 2x^2 - 6x - 20$	
		$2x^2 - 6x - 20 < 0$	
		$(x-5)(x+2) < 0$	
		$-2 < x < 5$	
	(ii)	When $k = 10$	
		$y = 2x^2 - 6x + 10$	
		$y = -2x + 8$	
		$2x^2 - 6x + 10 = -2x + 8$	
		$2x^2 - 4x + 2 = 0$	
		$x^2 - 2x + 1 = 0$	
		$b^2 - 4ac$	
		$= 4 - 4(1)(1)$	
		$= 0$ (shown)	
		The line is tangent to curve	
2	(i)	$(2^x)^2 - 2^x \times 2 = 3$	
		$u^2 - 2u - 3 = 0$	
	(ii)	$(u+1)(u-3) = 0$	
		$u = 3$ or $u = -1$	
		$2^x = 3$ or $2^x = -1$ (NA)	
		$x = \frac{\lg 3}{\lg 2}$	
		$x = 1.58$	
	(iii)	$u^2 - 2u - k = 0$	
		For no solution, $b^2 - 4ac < 0$	
		$(-2)^2 - 4(1)(-k) < 0$	
		$4 + 4k < 0$	
		$1 + k < 0$	
		$k < -1$	
		The equation has no solution if $k < -1$	

3		$\alpha + \beta = \frac{1}{3}, \quad \alpha\beta = \frac{5}{3}$	
		$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	
		$= \left(\frac{1}{3}\right)^2 - 2\left(\frac{5}{3}\right)$	
		$= -\frac{29}{9}$	
		$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$	
		$\alpha^3 + \beta^3 = \left(\frac{1}{3}\right)\left(-\frac{29}{9} - \frac{5}{3}\right)$	
		$= -\frac{44}{27}$	
(iii)		$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\beta^3 + \alpha^3}{\alpha^3\beta^3}$	
		$= -\frac{44}{27} + \left(\frac{5}{3}\right)^2$	
		$= -\frac{44}{125}$	
		$\left(\frac{1}{\alpha^3}\right)\left(\frac{1}{\beta^3}\right) = \frac{27}{125}$	
		$x^2 + \frac{44}{125}x + \frac{27}{125} = 0$	
		$125x^2 + 44x + 27 = 0$	
4	(i)	$\frac{1}{\sin^2 x} - \frac{\cos x}{\sin x} + \frac{2\cos^2 x}{\sin^2 x}$	
		$\frac{\cos^2 x - \cot x + 2\cot^2 x}{(\cot^2 x + 1) - \cot x + 2\cot^2 x}$	
		$\frac{3\cot^2 x - \cot x + 1}{3\cot^2 x - \cot x + 1}$	
	(ii)	$1 + 2\cos^2 x = 3\sin^2 x + \sin x \cos x$	
		$1 + 2\cos^2 x - \sin x \cos x = 3\sin^2 x$	
		$\frac{1 - \sin x \cos x + 2\cos^2 x}{\sin^2 x} = 3$	
		$3\cot^2 x - \cot x + 1 = 3$	
		$3\cot^2 x - \cot x - 2 = 0$	
		$(3\cot x + 2)(\cot x - 1) = 0$	

		$\cot x = -\frac{2}{3}$ or $\cot x = 1$	
		$\tan x = -\frac{3}{2}$ or $\tan x = 1$	
		$x = 180^\circ - 56.3^\circ, 360^\circ - 56.3^\circ, 45^\circ, 225^\circ$	
		$x = 45^\circ, 123.7^\circ, 225^\circ, 303.7^\circ$ (1 d.p.)	
5	(i)	when $t = 0, T = 75$ $75 = 27 + k$ $k = 48$	
	(ii)	when $t = 10, T = 55$ $55 = 27 + 48e^{-10n}$ $28 = 48e^{-10n}$ $\frac{7}{12} = e^{-10n}$ $-10n = \ln \frac{7}{12}$ $n = 0.0539$ (3 s.f.)	
	(iii)	$27 + 48e^{-0.0539t} < 30$ $48e^{-0.0539t} < 3$ $e^{-0.0539t} < \frac{1}{16}$ $-0.0539t < \ln \frac{1}{16}$ $t > \frac{\ln \frac{1}{16}}{-0.0539}$ $t > 51.439$ Least time taken = 52 minutes	
	(iv)		
		The temperature of the bread can never reach 27°C	

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6	Let $AD = 2x \Rightarrow BC = x$	
	$\sin 30^\circ = \frac{h}{4\sqrt{3} - 2\sqrt{2}} \Rightarrow h = 2\sqrt{3} - \sqrt{2}$	
	$\frac{1}{2}(x + 2x)(2\sqrt{3} - \sqrt{2}) = 25 + 5\sqrt{6}$	
	$\frac{3x}{2} = \frac{25 + 5\sqrt{6}}{2\sqrt{3} - \sqrt{2}} \times \frac{2\sqrt{3} + \sqrt{2}}{2\sqrt{3} + \sqrt{2}}$	
	$x = \frac{2}{3} \times \frac{50\sqrt{3} + 25\sqrt{2} + 10\sqrt{18} + 5\sqrt{12}}{12 - 2}$	
	$x = \frac{2(50\sqrt{3} + 25\sqrt{2} + 30\sqrt{2} + 10\sqrt{3})}{30}$	
	$x = \frac{(60\sqrt{3} + 55\sqrt{2})}{15}$	
	$AD = \frac{22}{3}\sqrt{2} + 8\sqrt{3}$	

7	$y = \frac{4}{3}x^3 + \frac{a}{3}x$	
	$y' - x = \frac{4}{3}x^2 + \frac{a}{3}x - x$	
	$\frac{y-x}{x} = \frac{4}{3}x^2 + \frac{a}{3} - 1$ ---(1)	
	$\frac{y-x}{x} = \frac{13-t}{6}x^2 + C$ ---(2)	
	$\frac{13-t}{6} = \frac{4}{3}$	
	$t = 5$	
	Sub (8,13) into (2)	
	$13 = \frac{13-t}{6}(8) + C$	
	Sub $t = 5$	
	$13 = \frac{13-5}{6}(8) + C$	
	$C = \frac{7}{3}$	

		$\frac{a}{3} - 1 = \frac{7}{3}$	
		$a = 10$	
8	(a)	$(a-x) \left(1^n + \binom{n}{1}(1)(3x) + \binom{n}{2}(1)(3x)^2 + \dots \right)$	
		By comparing like terms	
		$a = 2$	
		$-x + 3anx = 29x$	
		$-1 + 6n = 29$	
		$6n = 30$	
		$n = 5$	
		$a \left(\frac{n(n-1)}{2} \right) (1)(9x^2) - \binom{n}{1}(1)(3x^2)$	
		Sub $a = 2, n = 5$	
		$2 \left(\frac{5(4)}{2} \right) (1)(9x^2) - \binom{5}{1}(1)(3x^2)$	
		$180x^2 - 15x^2 = bx^2$	
		$b = 165$	
	(b)(i)	$\binom{10}{r} (x^3)^{10-r} \left(\frac{1}{2x^2} \right)^r$	
	(ii)	x^{20-5r}	
	(iii)	$20 - 5r = -15$	
		$r = 7$	
		$\binom{10}{r} (2^{-r})$	
		$\binom{10}{7} (2^{-7})$	
		$\frac{15}{16}$	
9		$y = \ln(3x-1) - \ln(5-2x)$	
		$\frac{dy}{dx} = \frac{3}{3x-1} - \frac{2}{5-2x}$	
		$y = -\frac{5}{13}x + 2$	
		Gradient of normal $= -\frac{5}{13}$	

		$\frac{13}{5}$	
		Gradient of tangent = $\frac{13}{5}$	
		$\frac{13}{5} = \frac{3}{3x-1} \div \frac{2}{5-2x}$	
		$\frac{13}{5} = \frac{3(5-2x) + 2(3x-1)}{17x-5-6x^2}$	
		$\frac{13}{5} = \frac{13}{17x-5-6x^2}$	
		$17x-5-6x^2 = 5$	
		$6x^2 - 17x + 10 = 0$	
		$(6x-5)(x-2) = 0$	
		$x = \frac{5}{6}$ or $x = 2$	
		since $x = a$	
		$a = \frac{5}{6}$ or $a = 2$	
		since $a > 1$,	
		$a = 2$	
		$b = \ln \frac{5}{1} \approx 1.61$	
10	(i)	At $P = (x, 1)$	
		$y = 2x+4 + 1$	
		$1 = 2x+4 + 1$	
		$ 2x+4 = 0$	
		$2x+4 = 0$	
		$x = -2$	
		$P = (-2, 1)$	
		At $Q = (0, y)$	
		$y = 4 + 1$	
		$y = 5$	
		$Q = (0, 5)$	
	(ii)	$ 2x+4 + 1 = -x+3$	
		$ 2x+4 = -x+2$	
		$2x+4 = -x+2$ or $2x+4 = x-2$	
		$x = -\frac{2}{3}$ or $x = -6$	

	(iii)	$\frac{3-1}{0-(-2)} = 1$ $\text{Max} = 0 - (-2)$ $\text{Min} = -2$ $-2 < m < 1$	
11	(i)	$a = \frac{2}{3}e^{-\frac{1}{3}}$ When $t = 0$, $a = \frac{2}{3}$	
	(ii)	$s = \int 28 - 2e^{-\frac{1}{3}t} dt$ $s = 28t + 6e^{-\frac{1}{3}t} + c$ $s = 0, t = 0$ $c = -6$ $s = 28t + 6e^{-\frac{1}{3}t} - 6$ $t = 10$ $s = 28t + 6e^{-\frac{10}{3}} - 6$ $s = 274.21$	
	(iii)	No. For $t \geq 0$, $2e^{-\frac{1}{3}t} \leq 2$ $v = 28 - 2e^{-\frac{1}{3}t} \leq 26$ $\therefore v > 0$	
12	(i)	Let $\angle DAC = a$ $\angle DBC = a$ (angles in the same segment) $\angle DBA = a$ (BD is angle bisector of $\angle ABC$) $\angle DAE = \angle DBA = a$ (tangent chord theorem) $\square \angle DAC = \angle DAE$ (proven)	
	(ii)	$\angle DEA = \angle BEA$ (common angle) $\angle DAE = \angle ABE = a$ (tangent chord theorem) $\triangle ADE$ and $\triangle BAE$ are similar (AA property)	
	(iii)	$\angle DCA = \angle DBA = a$ (angles in the same segment) $\angle DCA = \angle CAB = a$ $\angle DCA, \angle CAB$ are alternate angles $\therefore CD \parallel BA$ $CD \parallel BA$ & D is the midpoint of AF	

		$\therefore C$ is a midpoint of BF (By midpoint theorem)	
(iv)		$\triangle ADE$ and $\triangle BAE$ are similar	
		$\frac{AE}{BE} = \frac{AD}{AB} = \frac{DE}{AE}$	
		$AD \times CE = BC \times DE$	
		$\triangle GCD$ and $\triangle GBA$ are similar	
		$\frac{AD}{AB} = \frac{DE}{AE}$	
		$\frac{\frac{1}{2}AF}{2CD} = \frac{DE}{AE}$	
		$AF \times AE = 4CD \times DE$ (shown)	

Anglo-Chinese (Barker) Prelim 2017 4E Add Math P2 Answer

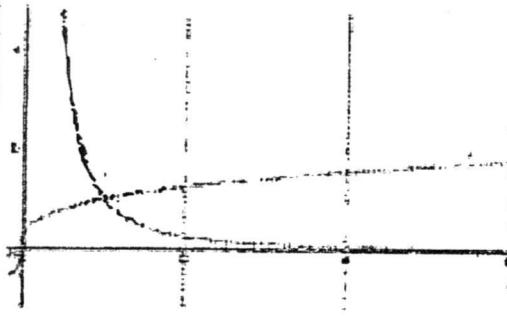
1(a)	$\lg(3y+2) - 4x^2 = 2$	
	$\lg(3y+2) = 4x^2 + 2$	
	$10^{2+4x^2} = 3y+2$	
	$3y = 10^{2+4x^2} - 2$	
	$y = \frac{1}{3}(10^{2+4x^2} - 2)$	
(b)	$2\log_2(8-2x) - \log_2(x-2) = 3 - \log_2(1+x)$	
	$2\left[\frac{\log_2(8-2x)}{\log_2 2^2}\right] - \log_2(x-2) = \log_2 2^3 - \log_2(1+x)$	
	$\log_2(8-2x) - \log_2(x-2) = \log_2 \frac{8}{(1+x)}$	
	$\log_2 \frac{8-2x}{x-2} = \log_2 \frac{8}{(1+x)}$	
	$\frac{8-2x}{x-2} = \frac{8}{(1+x)}$	
	$(8-2x)(1+x) = 8(x-2)$	
	$8+8x-2x-2x^2 = 8x-16$	
	$-2x^2-2x+24 = 0$	
	$x^2+x-12 = 0$	
	$(x-3)(x+4) = 0$	
	$x = 3 \quad x = -4(\text{NA})$	
	$\therefore x = 3$	

2(i)	$a = -4$	
	$b = 10$	
	$c = 3$	
(ii)	$h = -4\sin\left(\frac{\pi}{10}t\right) + 3$	
	$h = 0$	
	$-4\sin\left(\frac{\pi}{10}t\right) + 3 = 0$	
	$\sin\left(\frac{\pi}{10}t\right) = \frac{3}{4}$	
	$\frac{\pi}{10}t = \sin^{-1}\frac{3}{4}$	
	$\frac{\pi}{10}t = 0.84806, \pi - 0.84806$	
	$t = \frac{10(\pi - 0.84806)}{\pi}$	
	$t = 7.30 \text{ (3sf)}$	

3(i)	$\frac{d}{dx}(e^{2x}(2x+1))$	
	$= 2e^{2x} + (2x+1)(2e^{2x})$	
	$= 2e^{2x}(2+4x+2)$	
	$= 2e^{2x}(4x+4)$ (shown)	
(ii)	$\int_0^1 4xe^{2x} + 4e^{2x} dx = [e^{2x}(2x+1)]$	
	$2 \int_0^1 2xe^{2x} dx = [e^{2x}(2x+1)]_0^1 - \int_0^1 4e^{2x} dx$	
	$\int_0^1 2xe^{2x} dx = \frac{1}{2} [e^{2x}(2x+1)]_0^1 - \frac{1}{2} \int_0^1 4e^{2x} dx$	
	$= \frac{1}{2} [e^{2x}(2x+1)]_0^1 - \frac{1}{2} [2e^{2x}]_0^1$	
	$= \frac{1}{2} [3e^2 - 1] - \frac{1}{2} [2e^2 - 2]$	
	$= \frac{1}{2} [3e^2 - 2e^2 - 1 + 2]$	
	$= \frac{1}{2} [e^2 + 1]$	
	$= 4.19$	

4(i)	$f(x) = (x+1)(px^2 + qx + r)$	
	$p = 4$	
	$r = -3$	
	$f(1) = -20$	
	$(2)(p+q+r) = -20$	
	$(2)(4+q-3) = -20$	
	$(2)(q+1) = -20$	
	$q = -11$	
(ii)	$f(x) = (x+1)(4x^2 - 11x - 3)$	
	$f(x) = 0$	
	$(x+1)(4x+1)(x-3) = 0$	
	$x = -1, -\frac{1}{4}, 3$	
(iii)	$(-x+1)(-4x+1)(-x-3) = 0$	
	$x = 1, \frac{1}{4}, -3$	

5(i)	By long division, $\frac{x^3 - x^2 - 4x + 1}{x^2 - 4} = x - 1 - \frac{3}{x^2 - 4}$	
	$a = 1, b = -1, c = -3$	
(ii)	$\frac{3}{x^2 - 4} = \frac{A}{x + 2} + \frac{B}{x - 2}$	
	$-3 = A(x - 2) + B(x + 2)$	
	let $x = -2, A = \frac{3}{4}$	
	let $x = 2, B = -\frac{3}{4}$	
	$\int x - 1 + \frac{3}{4(x + 2)} - \frac{3}{4(x - 2)} dx$	
	$= \frac{1}{2}x^2 - x + \frac{3}{4}\ln(x + 2) - \frac{3}{4}\ln(x - 2) + c$	

6a(i)	sub $x = 1, y = 5$ into $y = a\sqrt{x}$	
	$a = 5$	
	sub $x = 1, y = 5, a = 5$ into $y = \frac{2a}{kx^2}$	
	$k = 2$	
6a(ii)		
6b	$\frac{dy}{dx} = 2(2x + \tan x)(2 + \sec^2 x)$	
	When $x = \frac{\pi}{3}$	
	$\frac{dy}{dx} = 2\left(2\left(\frac{\pi}{3}\right) + \tan \frac{\pi}{3}\right)(2 + \sec^2 \frac{\pi}{3})$	
	$\frac{dy}{dx} = 2\left(\frac{2\pi}{3} + \sqrt{3}\right)\left(2 + \frac{1}{2}\right)$	
	$\frac{dy}{dx} = \left(\frac{4\pi}{3} + 2\sqrt{3}\right)(6)$	
	$\frac{dy}{dx} = 8\pi + 12\sqrt{3} \Rightarrow a = 8, b = 12$	

7(i)	$f'(x) = \frac{3(5x+3) - 3(x-1)(5)}{(5x+3)^2}$	
	$= \frac{9+15}{(5x+3)^2}$	
(ii)	$f'(x) > 0$ since $(5x+3)^2 > 0, x > 0$	
	$f'(x) \neq 0$, the curve has no stationary points	
(ii)	$f''(x) = -48(5x+3)^{-3}(5)$	
	$= \frac{-240}{(5x+3)^3}$	
	For $x > 0$, $f''(x) < 0$, the gradient function is a decreasing function.	

8(i)		
	Midpoint = $\left(\frac{1+9}{2}, \frac{10+10}{2}\right)$	
	= (4, 10)	
	Let the centre be (4, y)	
	$(4-4)^2 + (y-1)^2 = (4-1)^2 + (y-10)^2$	
	$y^2 - 2y + 1 = 9 + y^2 - 20y + 100$	
	$18y = 108$	
	$y = 6$	
	Centre (4, 6)	
(ii)	Radius = 5 units	
	$(x-4)^2 + (y-6)^2 = 25$	
	$x^2 + y^2 - 8x - 12y + 27 = 0$	
(iv)	$x = -1$	
	$x = 9$	

9(i)	Let the base of the water be x	
	$\frac{x}{2} = \frac{h}{1}$	
	$x = 2h$	
	$V = \frac{1}{2}(2h)(h)(3)$	
	$V = 3h^2$ (shown)	
(ii)	$\frac{dV}{dt} = 0.03$	
	$\frac{dV}{dh} = 6h$	
	When $t = 25$	
	$V = 25 \times 0.03 = 0.75 \text{ m}^3$	
	$3h^2 = 0.75$	
	$h = 0.5$	
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$	
	$= \frac{1}{6h} \times 0.03$	
	$= \frac{1}{6(0.5)} \times 0.03$	
	$\frac{dh}{dt} = 0.01 \text{ m/s}$	

10(i)	$y = 10 - 32x^2$	
	$\frac{dy}{dx} = 64x$	
	$\frac{64}{x^3} = 1$	
	$x^3 = 64$	
	$x = 4$	
	When $x = 4, y = 8$	
	$Q = (4, 8)$	
	$P = (0, y)$	
	$\frac{4-0}{8-y} = 1$	
	$4 = 8 - y$	
	$y = 4$	
	$\therefore P = (0, 4)$	
	Area of trapezium = $\frac{1}{2}(4+8)(4) = 24$	
	Area of triangle = $\frac{1}{2}(2)(2) = 2$	
	Area of under curve = $\int_2^4 10 - 32x^{-2} dx$	
	$= \left[10x - \frac{32x^{-1}}{-1} \right]_2^4$	
	$= \left[10x + \frac{32}{x} \right]_2^4$	

	$= 48 - 36$	
	$= 12$	
	Shaded area $= 24 - 12 - 2$	
	$= 10$	

1(i)	Since D is equidistant from A and C \therefore DB is perpendicular bisector of AC.	
	Midpoint of AC = $\left(\frac{10+2}{2}, \frac{7+1}{2}\right)$	
	= (6,4)	
	Gradient of AC = $\frac{1-7}{2-10}$	
	= $\frac{3}{4}$	
	Gradient of BD = $-\frac{4}{3}$	
	Equation of BD, $y-4 = -\frac{4}{3}(x-6)$	
	$y = -\frac{4}{3}x + 12$	
	At the y-axis, $x = 0$	
	$y = 12$	
	$\therefore B(0,12)$	
	Midpoint of DB = Midpoint of AC = (6,4)	
	D(0,12), B(x,y)	
	$\left(\frac{0+x}{2}, \frac{12+y}{2}\right) = (6,4)$	
	$\therefore \frac{x}{2} = 6$ $\therefore \frac{12+y}{2} = 4$	
	$x = 12$ $y = -4$	
	$\therefore D(12,-4)$	

9(i)	Let the base of the water be x	
	$\frac{x}{2} = \frac{h}{1}$	
	$x = 2h$	
	$V = \frac{1}{2}(2h)(h)(3)$	
	$V = 3h^2$ (shown)	
(ii)	$\frac{dV}{dt} = 0.03$	
	$\frac{dV}{dh} = 6h$	
	When $t = 25$	
	$V = 25 \times 0.03 = 0.75 \text{ m}^3$	
	$3h^2 = 0.75$	
	$h = 0.5$	
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$	
	$= \frac{1}{6h} \times 0.03$	
	$= \frac{1}{6(0.5)} \times 0.03$	
	$\frac{dh}{dt} = 0.01 \text{ m/s}$	

10(i)	$y = 10 - 32x^{-2}$	
	$\frac{dy}{dx} = 64x^{-1}$	
	$\frac{64}{x^1} = 1$	
	$x^1 = 64$	
	$x = 4$	
	When $x = 4, y = 8$	
	$Q = (4, 8)$	
	$P = (0, y)$	
	$\frac{4-0}{8-y} = 1$	
	$4 = 8 - y$	
	$y = 4$	
	$\therefore P = (0, 4)$	
	Area of trapezium $= \frac{1}{2}(4+8)(4) = 24$	
	Area of triangle $= \frac{1}{2}(2)(2) = 2$	
	Area of under curve $= \int_2^4 10 - 32x^{-2} dx$	
	$= \left[10x - \frac{32x^{-1}}{-1} \right]_2^4$	
	$= \left[10x + \frac{32}{x} \right]_2^4$	

	$\approx 48 - 36$	
	$= 12$	
	Shaded area = $24 - 12 - 2$	
	$= 10$	

11(i)	Since D is equidistant from A and C \therefore DB is perpendicular bisector of AC.	
	Midpoint of AC = $\left(\frac{10+2}{2}, \frac{7+1}{2}\right)$	
	$= (6,4)$	
	Gradient of AC = $\frac{1-7}{2-10}$	
	$= \frac{3}{4}$	
	Gradient of BD = $-\frac{4}{3}$	
	Equation of BD, $y-4 = -\frac{4}{3}(x-6)$	
	$y = -\frac{4}{3}x + 12$	
	At the y-axis, $x = 0$	
	$y = 12$	
	$\therefore B(0,12)$	
	Midpoint of DB = Midpoint of A = $(6,4)$	
	D(0,12), B(x,y)	
	$\left(\frac{0+x}{2}, \frac{12+y}{2}\right) = (6,4)$	
	$\therefore \frac{x}{2} = 6$ $\therefore \frac{12+y}{2} = 4$	
	$x = 12$ $y = -4$	
	$\therefore D(12,-4)$	