BEATTY SECONDARY SCHOOL

END OF YEAR EXAMINATION 2014

SUBJECT : Additional Mathematics **LEVEL : Secondary 3 Express**

PAPER : 4047/ 01 **DURATION : 2 hours**

SETTER : Mrs Samsol **DATE : 9 October 2014**

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This question paper consists of 5 printed pages.

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A rectangle of length $(10+3\sqrt{3})$ cm has an area of $(61+11\sqrt{3})\text{ cm}^2$. Find the breadth of the rectangle in the form $(a+b\sqrt{3})$ cm, where a and b are constants. [3]

2 Given that $\sin A=\frac{3}{\sqrt{13}}$ and that $\sin A$ and $\cos A$ have opposite signs, find without using a calculator

(i) $\sec A$, [2]

(ii) $\cot\ (-A)$. [2]

3 (i) Sketch the graph of $y=3+|5-2x|$. [2]

(ii) Determine the number of intersections of the line $y=2x+c$ with $y=3+|5-2x|$ when $c>0$, justifying your answer. [1]

4 Express $\frac{3x^3-16x^2-2x+12}{(x-1)(x-5)}$ in partial fractions. [5]

5 The function f is defined by $\mathrm{f}(x) = 4\sin 3x-1$.

(i) State the period of f. [1]

(ii) Sketch the graph of $y = 4\sin 3x-1$ for $0^\circ \le x \le 180^\circ$. [2]

6 (i) Find the value of p and of q for which $2x^2-5x-3$ is a factor of $2x^3+px^2+32x+q$. [4]

(ii) Using the values of p and q found in part (i), find the remainder when $2x^3+px^2+32x+q$ is divided by $x+2$. [1]

7 (i) Sketch, on the same axes, the graphs of $y^2=8x$ and $y=x^{-\frac{1}{4}}$. [2]

(ii) Find the coordinates of the point of intersection of these graphs. [3]

8 The roots of the quadratic equation $2x^2-6x+5=0$ are α and β.

Find a quadratic equation whose roots are α^3-1 and β^3-1. [5]

9 A circle C_1 has a diameter AB where A is the point $(-2,3)$ and B is the point $(6,-5)$.

(i) Find the coordinates of the centre and the radius of C_1. [2]

(ii) Find the equation of the circle. [1]

The equation of a second circle C_2 with radius $\sqrt{15}$ units is $x^2+y^2-10x+8y+c=0$.

(iii) Find the value of c. [2]

(iv) Determine whether the point $(4,-2)$ lies inside or outside C_2. [1]

10 A curve has the equation $y=3x^2+cx+9+c$, where c is a constant.

(i) Find the range of values of c for which the curve lies completely above the x-axis. [3]

(ii) In the case where $c=2$, find the values of m for which the line $y=mx+8$ is a tangent to the curve. [3]

11 ***Answer the whole of this question on a single sheet of graph paper.***

The table shows some experimental values of x and y.

x	1.0	1.5	2.0	2.5	3.2
y	3.5	8.4	15.1	24	40.0

The variables x and y are related by the equation $ay=x+bx^2$, where a and b are constants.

(i) Using a scale of 2 cm to represent 0.5 unit on the x-axis and 1 cm to represent 1 unit on the $\frac{y}{x}$-axis, plot $\frac{y}{x}$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate

(a) the value of a and of b, [2]

(b) the value of y when $y=6x$. [1]

12 **(a)** The coefficient of x^2 in the expansion of $(3+kx)(1+5x)^8$ is 2020.
Find the value of the constant k. [3]

(b) In the expansion of $(3+2x)^n$, the coefficient of x^2 is 16 times the constant term.
Given that n is positive, find the value of n. [3]

13 **(a)** Show that $(1-\cos x)\left(1+\dfrac{1}{\cos x}\right)=\sin x \tan x$. [3]

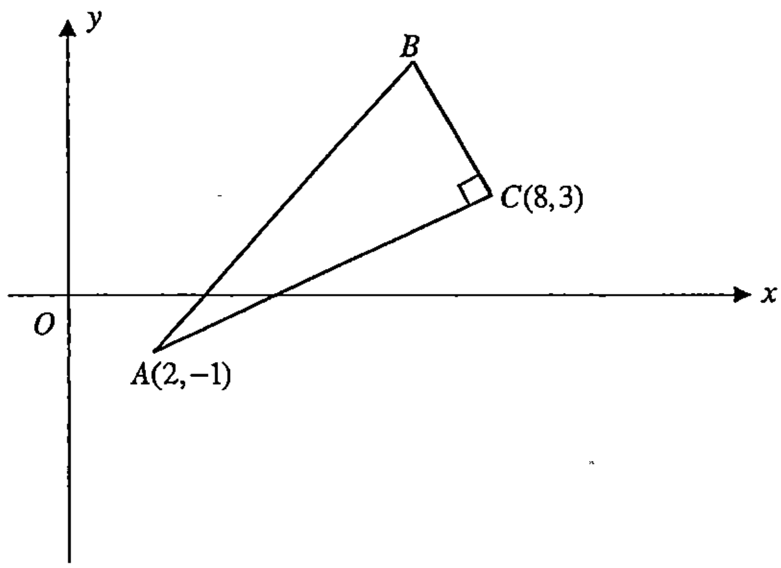
(b) Solve the equation $2\sec^2 x+9\tan x-20=0$ for $0^\circ \le x \le 360^\circ$. [4]

14 Solve the equation

(i) $4^x = 24+2^{x+1}$, [3]

(ii) $\log_3 x^2 + 5 = \log_x 27$. [4]

15 **Solutions to this question by accurate drawing will not be accepted.**



The diagram, which is not drawn to scale, shows a triangle ABC in which angle $ACB=90^\circ$ and the points A and C are $(2,-1)$ and $(8,3)$ respectively. Given that AB is parallel to the line $4y=7x+8$, find

(i) the coordinates of B. [5]

The point D is such that $ABCD$ is a parallelogram.

(ii) Find the coordinates of D. [2]

(iii) Hence or otherwise find the area of the parallelogram $ABCD$. [2]

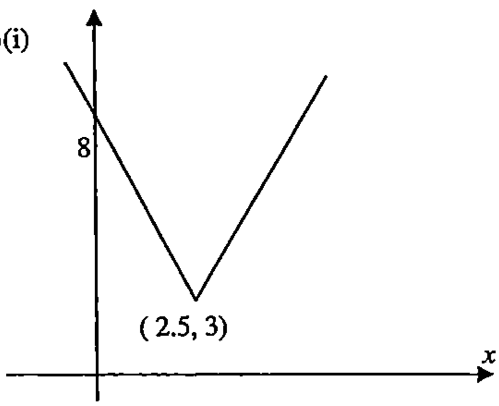
End of Paper

Answer Key

1 $(7-\sqrt{3})$

2(i) $-\dfrac{\sqrt{13}}{2}$ (ii) $\dfrac{2}{3}$

3(i)

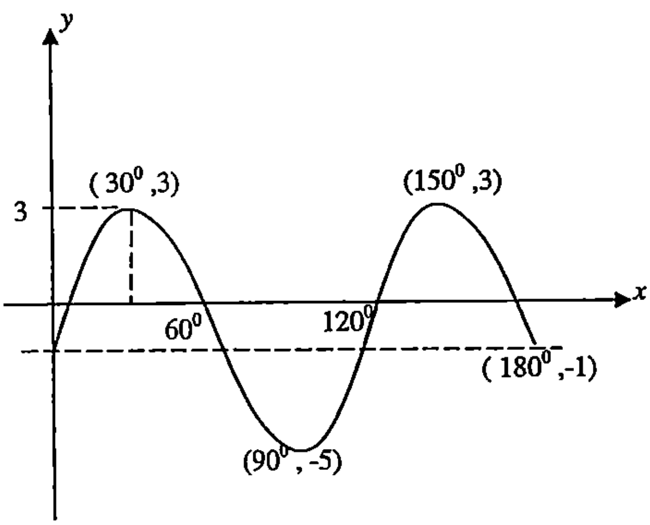


3(ii) Line is parallel to right hand arm of graph and since c > 0,
it intersects left hand arm only, therefore there is 1 intersection.

4 $\dfrac{3x^3-16x^2-2x+12}{(x-1)(x-5)}=3x+2+\dfrac{3}{4(x-1)}-\dfrac{23}{4(x-5)}$

5 (i) Period = 120^0

(ii)

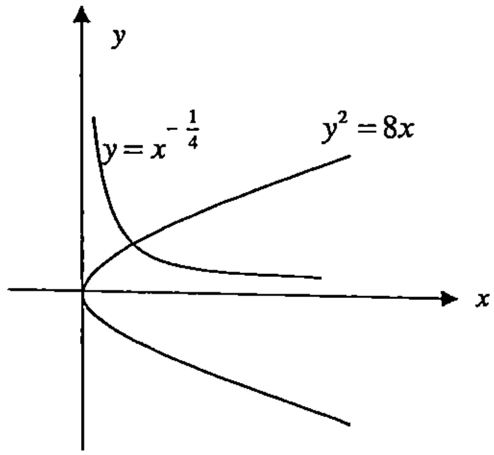


6(i) $p=-19$

$q=21$

6(ii) -135

7(i)



7(ii) $\left(\frac{1}{4},\sqrt{2}\right)$

8 $x^2-\frac{5}{2}x+\frac{97}{8}=0$ or $8x^2-20x+97=0$

9(i) centre = $(2,-1)$, radius $=4\sqrt{2}$

(ii) $(x-2)^2+(y+1)^2=32$

(iii) $c=26$

(iv) $\sqrt{5}<\sqrt{15}$

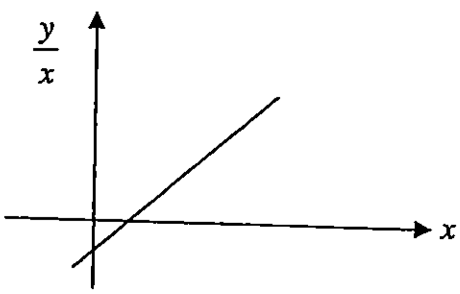
The point lies inside the second circle.

10(i) $-6<c<18$

(ii) $m=8$ or -4

11 (i)

x	1	1.5	2	2.5	3.2
$\frac{y}{x}$	3.5	5.6	7.55	9.6	12.5



(ii)(a) a = -1.67 to -1.25 , b = -5.0 to -6.7

(b) y = 9.6 to 9.9

12(a) $k=-2$

(b) $n=9$

13(a) $(1-\cos x)\left(1+\frac{1}{\cos x}\right)$

$=(1-\cos x)\left(\frac{\cos x+1}{\cos x}\right)$

$=\left(\frac{1-\cos^2 x}{\cos x}\right)$

$=\frac{\sin^2 x}{\cos x}$

$=\sin x\tan x$

13(b) $x=56.3^\circ, 236.3^\circ$ or $x=99.5^\circ, 279.5^\circ$

14(i) $x=2.58$

(ii) $x=\sqrt{3}$ or $x=\frac{1}{27}$

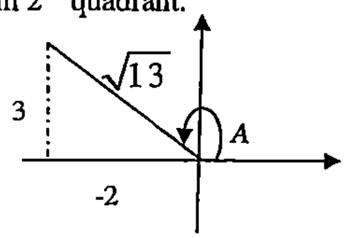
15(i) $B=(6,6)$

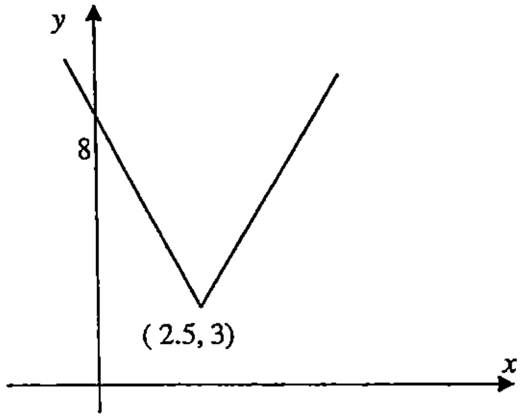
(ii) $D=(4,-4)$

(iii) 26 square units

Beatty Secondary School
End of Year Exam 2014
Sec 3E Additional Mathematics (4047/01)

Mark Scheme

1	Breadth $= \dfrac{61+11\sqrt{3}}{10+3\sqrt{3}}$	
	$= \dfrac{61+11\sqrt{3}}{10+3\sqrt{3}} \times \dfrac{10-3\sqrt{3}}{10-3\sqrt{3}}$	M1(rationalization)
	$= \dfrac{610-183\sqrt{3}+110\sqrt{3}-99}{100-27}$	M1(expansion)
	$= \dfrac{511-73\sqrt{3}}{73}$	
	$= (7-\sqrt{3})$ cm	A1
2	$\sin A = \dfrac{3}{\sqrt{13}}$	
	A is in 2[nd] quadrant. 	M1 (correct quadrant and evaluation of adjacent side)
	(i) $\sec A = \dfrac{1}{\cos A}$	
	$= \dfrac{1}{(-\frac{2}{\sqrt{13}})} = -\dfrac{\sqrt{13}}{2}$	A1
	(ii) $\cot(-A) = -\dfrac{1}{\tan A}$	
	$= -\dfrac{1}{(-\frac{3}{2})}$	M1 (for value of tan A)
	$= \dfrac{2}{3}$	A1

<table>
<tr>
<td>3</td>
<td>(i)


(ii) Line is parallel to right hand arm of graph and since $c > 0$, it intersects left hand arm only, therefore there is 1 intersection</td>
<td>G1 : V shaped graph
G1 : y-intercept and vertex shown

B1</td>
</tr>
<tr>
<td>4</td>
<td>

$$\frac{3x^3-16x^2-2x+12}{(x-1)(x-5)}$$

$$\begin{array}{r} 3x+2 \\ x^2-6x+5\overline{)3x^3-16x^2-2x+12} \\ 3x^3-18x^2+15x \\ +2x^2-17x+12 \\ +2x^2-12x+10 \\ -5x+2 \end{array}$$

$$\frac{3x^3-16x^2-2x+12}{(x-1)(x-5)}=3x+2+\frac{2-5x}{(x-1)(x-5)}$$

Let $\frac{2-5x}{(x-1)(x-5)}=\frac{A}{x-1}+\frac{B}{x-5}$

$2-5x=A(x-5)+B(x-1)$

Sub $x=1$, $-3=-4A$

$A=\frac{3}{4}$

Sub $x=5$, $-23=4B$

$B=-\frac{23}{4}$

$$\frac{3x^3-16x^2-2x+12}{(x-1)(x-5)}=3x+2+\frac{3}{4(x-1)}-\frac{23}{4(x-5)}$$

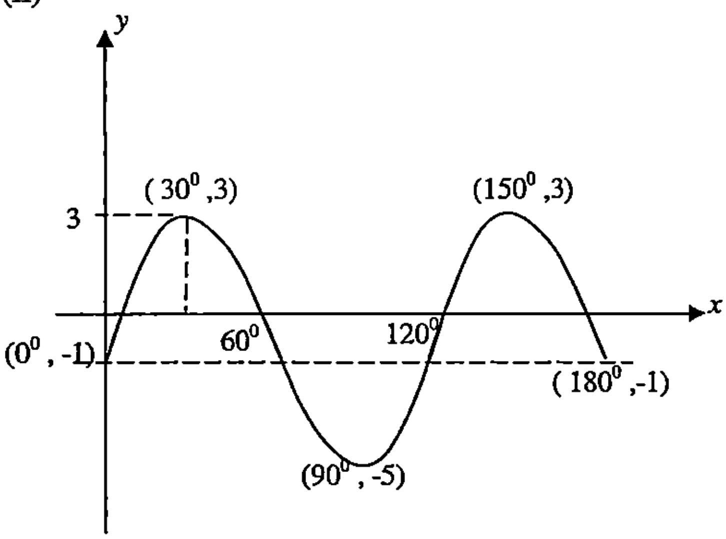
</td>
<td>M1

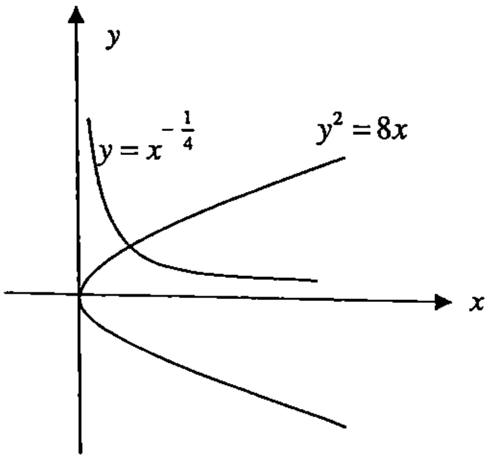
M1

A1

A1

A1</td>
</tr>
</table>

5	(i) Period = 120^0	B1
	(ii) 	G1 : correct shape G1 : maximum and minimum points are clearly indicated.
6	(i) $2x^2-5x-3=(2x+1))(x-3)$ f(3)=$54+9p+96+q=0$ $q=-150-9p$ --------- (1) f$(-\frac{1}{2})=-\frac{1}{4}+\frac{1}{4}p-16+q=0$ ------ (2) Sub (1) into (2) $-\frac{1}{4}+\frac{1}{4}p-16-150-9p=0$ $-\frac{35}{4}p=\frac{665}{4}$ $p=-19$ $q=21$	M1 M1 A1 A1
	(ii) Remainder = f (-2) $= 2(-2)^3-19(-2)^2+32(-2)+21$ $= -135$	B1

<table>
<tr><td>7</td><td>(i)


(ii) $y^2=8x$ -------- (1)

$y=x^{-\frac{1}{4}}$---------(2)

Sub (2) into (1)

$\left(x^{-\frac{1}{4}}\right)^2=8x$

$\left(x^{-\frac{1}{2}}\right)=8x$

Squaring both sides,

$\left(\frac{1}{x}\right)=64x^2$

$64x^3=1$

$x^3=\frac{1}{64}$

$x=\frac{1}{4}$

$y=\sqrt{2}$

Point of intersection is $\left(\frac{1}{4},\sqrt{2}\right)$.</td><td>G1 – graph of $y^2=8x$

G1 – graph of $y=x^{-\frac{1}{4}}$

M1

M1

A1</td></tr>
</table>

8	$2x^2-6x+5=0$	
	$\alpha+\beta=3$ and $\alpha\beta=\frac{5}{2}$	B1
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=3^2-2(\frac{5}{2})$ $=4$	
	$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ $=3(4-\frac{5}{2})$	M1
	$=\frac{9}{2}$ or $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $\alpha^3-1+\beta^3-1$ $=\alpha^3+\beta^3-2$ $=\frac{9}{2}-2$	
	$=\frac{5}{2}$	A1
	$(\alpha^3-1)(\beta^3-1)$ $=(\alpha\beta)^3-(\alpha^3+\beta^3)+1$ $=(\frac{5}{2})^3-(\frac{9}{2})+1$	A1
	$=\frac{97}{8}$	
	Equation is $x^2-\frac{5}{2}x+\frac{97}{8}=0$ or $8x^2-20x+97=0$	A1

9	(i) centre $=\left(\dfrac{-2+6}{2},\dfrac{3-5}{2}\right)$	
	$=(2,-1)$	B1
	Radius $=\sqrt{(6-2)^2+(-5+1)^2}$	
	$=4\sqrt{2}$ or 5.66 units	B1
	(ii) Equation is $(x-2)^2+(y+1)^2=32$	B1
	(iii) centre = $(5,-4)$	M1 or using completing of square
	$c=5^2+(-4)^2-(\sqrt{15})^2$	
	$c=26$	A1
	(iv) distance $=\sqrt{(5-4)^2+(-4+2)^2}$	
	$=\sqrt{5}<\sqrt{15}$ (radius of C_2)	
	The point lies inside the second circle.	B1
10	(i) $y=3x^2+cx+9+c$	
	$c^2-4(3)(9+c)<0$	M1
	$c^2-12c-108<0$	M1
	$(c-18)(c+6)<0$	A1
	$-6<c<18$	
	(ii) Sub $c=2$,	
	$y=3x^2+2x+11$--------(1)	
	$y=mx+8$ -------------(2)	
	Sub (2) into (1)	
	$3x^2+2x+11=mx+8$	
	$3x^2+(2-m)x+3=0$	M1
	$(2-m)^2-4(3)(3)=0$	
	$(2-m)^2=36$	M1
	$2-m=\pm6$	
	$m=2\pm6$	A1
	$m=8$ or -4	

<table>
<tr><td>11</td><td>

x	1	1.5	2	2.5	3.3
$\frac{y}{x}$	3.5	5.6	7.55	9.6	12.5

(ii)(a) a = -2 to -1.1 B1
b = -8 to -4.5 B1
(ii)(b) $y = 9.6$ to 9.9
</td><td>Table M1
G1 – correct scale for graph and points plotted
G1 – straight line</td></tr>
</table>

<table>
<tr><td>12</td><td>
(a) $(3+kx)(1+5x)^8$
$=(3+kx)(1+40x+700x^2+.....)$
$3(700)+(k)(40)=2020$
$40k=-80$
$k=-2$

(b) $(3+2x)^n$
$\frac{n(n-1)}{2}(3)^{n-2}(2)^2=16(3)^n$
$\frac{n(n-1)}{2}(\frac{1}{9})(4)=16$
$2n^2-2n-144=0$
$n^2-n-72=0$
$(n-9)(n+8)=0$
Since $n>0$, $n=9$
</td><td>
M1 (binomial expansion)
M1
A1

M1
M1
A1
</td></tr>
</table>

13	(a) $(1-\cos x)\left(1+\dfrac{1}{\cos x}\right)$ $=(1-\cos x)\left(\dfrac{\cos x+1}{\cos x}\right)$ $=\left(\dfrac{1-\cos^2 x}{\cos x}\right)$ $=\dfrac{\sin^2 x}{\cos x}$ $=\sin x\tan x$	M1 M1 A1
13	(b) $2\sec^2 x+9\tan x-20=0$ $2(1+\tan^2 x)+9\tan x-20=0$ $2\tan^2 x+9\tan x-18=0$ $(2\tan x-3)(\tan x+6)=0$ $\tan x=\dfrac{3}{2}$ or $\tan x=-6$ $\alpha_1=56.30^\circ$ or $\alpha_2=80.53^\circ$ $x=56.3^\circ, 236.3^\circ$ or $x=99.5^\circ, 279.5^\circ$	M1 M1 A1, A1
14	(i) $4^x=24+2^{x+1}$ Let $u=2^x$ $u^2=24+2u$ $u^2-2u-24=0$ $(u+4)(u-6)=0$ $u=-4$ or $u=6$ (reject) $2^x=6$ $x=\dfrac{\lg 6}{\lg 2}$ $x=2.58$ (to 3 sf)	M1 A1 A1

	(ii) $\log_3 x^2+5=\log_x 27$	
	$2\log_3 x+5=\dfrac{3}{\log_3 x}$	M1
	Let $y=\log_3 x$	
	$2\log_3 x+5=\dfrac{3}{\log_3 x}$	
	$2y+5=\dfrac{3}{y}$	M1
	$2y^2+5y-3=0$	
	$(2y-1)(y+3)=0$	
	$y=\dfrac{1}{2}$ or $y=-3$	A1
	$\log_3 x=\dfrac{1}{2}$ or $\log_3 x=-3$	
	$x=\sqrt{3}$ or $x=\dfrac{1}{27}$	A1
15	(i) $m_{AC}=\dfrac{3+1}{8-2}$	
	$=\dfrac{2}{3}$	
	$m_{BC}=-\dfrac{3}{2}$	
	Sub (8,3)	
	$3=-\dfrac{3}{2}(8)+c$	M1
	$c=15$	
	Equation of BC is $y=-\dfrac{3}{2}x+15$ ----------(1)	A1
	$m_{AB}=\dfrac{7}{4}$	
	Sub (2,-1)	
	$-1=\dfrac{7}{4}(2)+c$	
	$c=-\dfrac{9}{2}$	

	Equation of AB is $y=\frac{7}{4}x-\frac{9}{2}$ --------(2) Sub (1) into (2) $\frac{7}{4}x-\frac{9}{2}=-\frac{3}{2}x+15$ $\frac{13}{4}x=\frac{39}{2}$ $x=6$ $y=6$ $B=(6,6)$	A1 M1 A1
	(ii) $\left(\frac{2+8}{2},\frac{-1+3}{2}\right)=\left(\frac{x+6}{2},\frac{y+6}{2}\right)$ $x=4$ and $y=-4$ $D=(4,-4)$	M1 A1
	(iii) Area of parallelogram $=2\times\frac{1}{2}\begin{vmatrix}2 & 8 & 6 & 2\\ -1 & 3 & 6 & -1\end{vmatrix}$ $=6+48-6-(-8+18+12)$ $=26$ square units	 M1 [accept alternative methods] A1