NAME: ______________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

END-OF-YEAR EXAMINATION 2014
SECONDARY 3 EXPRESS

ADDITIONAL MATHEMATICS   4047

Date: 7October 2014   **Duration: 2 hours 30 minutes**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Setters: MdmHaliza and Mrs Lim Chee Chin

This question paper consists of 8 printed pagesincluding the cover page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

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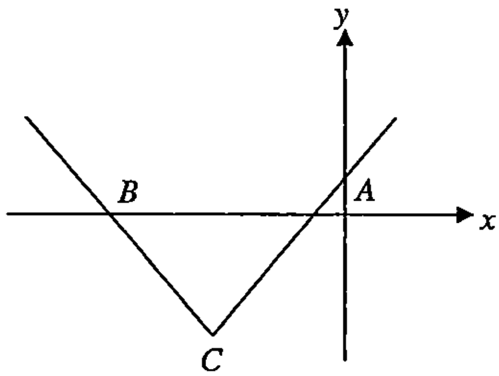
1 Given that $a+b\sqrt{2}=\left(\frac{7}{3+\sqrt{2}}\right)^2$, where a and b are integers, find, without using a calculator, the value of a and of b. [3]

2 The diagram shows part of the graph of $y=\left|\frac{x}{2}+3\right|-2$. Find the coordinates of the points A, B and C. [3]



3 Given that $\tan A=-\frac{3}{4}$ and $\cos B=-\frac{1}{\sqrt{5}}$, where A and B are in the same quadrant.

Without using a calculator, find the value of

(i) $\cos A \operatorname{cosec} B$, [2]

(ii) $\cot A+\tan B$. [2]

4 Express $\frac{14+7x-3x^2}{x^2(x+2)}$ in partial fractions. [4]

5 **(a)** Show that the line $y=4x+m$ meets the curve $y=mx^2-2x-6$ for all real values of x. [3]

(b) Find the range of values of k for which the graph of $y=x^2+(3-k)x+4$ lies entirely above the x-axis. [3]

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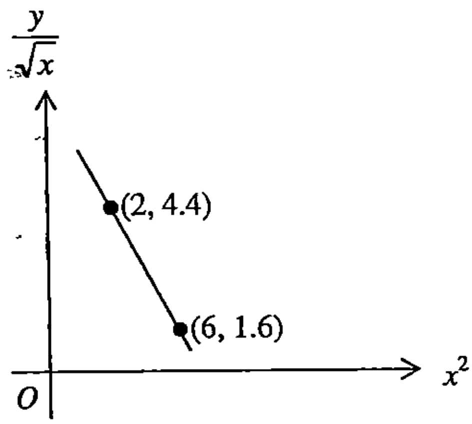


The variables x and y are related in such a way that when $\frac{y}{\sqrt{x}}$ is plotted against x^2, astraight line is obtained which passes through the points (2, 4.4) and (6, 1.6). Find

(i) an expression for y in terms of x, [3]

(ii) the value of y when $x = 4$. [2]

7 The roots of the quadratic equation $2x^2 - 6x + 3 = 0$ are α and β.

(i) State the value of $\alpha + \beta$ and of $\alpha\beta$. [2]

(ii) Find the quadratic equation in x whose roots are $\frac{3}{\beta^3}$ and $\frac{3}{\alpha^3}$. [5]

8(a) Given that $6x^3 - 16x^2 + 10x - 4 = A(x-1)^2(2x-1) + B(x+3)(x-1) + C$ for all values of x, find the value of A, B and C. [3]

(b) The expression $ax^3 + 2x^2 + bx + 6$ is divisible by $x+2$ and leaves a remainder of 3 when divided by $x-1$. Find the valueof a and of b. [4]

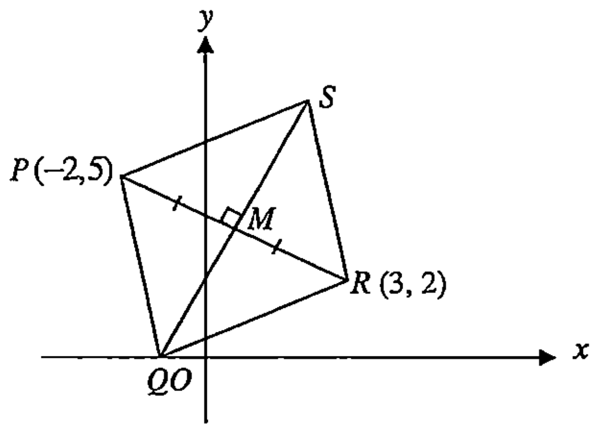
9(a)(i) Write in ascending powers of x, the first three terms in the expansion of $(1+ax)^8$. [1]

(ii) Given that the first three terms in the expansion of $(b+2x)(1+ax)^8$ are $5 - 38x + cx^2$, find the values of a, b and c. [4]

(b) Find the term independent of x in the expansion of $\left(2x^3 + \frac{1}{3x^2}\right)^{10}$. [3]

10 **Solutions to this question by accurate drawing will not be accepted.**

The diagram (not drawn to scale), shows a rhombus $PQRS$ where P is $(-2,5)$, $Q$is on the x-axis and$R$ is (3, 2). The diagonal SQ bisects PR at right angle at M.



Find

(i) the equation of QS, [3]

(ii) the coordinates of Q and of S, [2]

(iii) the area of $PQRS$. [2]

11 Solve the following equations for $0° \le z \le 360°$.

(i) $\sec(2z + 15°) = \sqrt{3}$ [4]

(ii) $2\tan^2 z + 5\tan z = 0$ [4]

12 **(a)** Evaluate $\log_p 8 \times \log_{16} p$. [3]

(b) Solve the equation $2 + \log_3(3x - 7) = \log_3(2x - 3)$. [3]

(c) Given that $2^{2x+2} \times 3^{x-1} = 8^x \times 3^{2x}$, evaluate 6^x. [3]

13 **(a)** The cubic polynomial is such that the coefficient of x^3 is -1 and the roots of the equation $f(x) = 0$ are -2, 3 and k. Given that $f(x)$ has a remainder of 12 when divided by $x - 4$, find the value of k. [3]

(b) **(i)** Solve the equation $4x^3 - 7x^2 - 21x + 18 = 0$. [4]

(ii) Hence solve the equation $\frac{1}{2}x^3 - \frac{7}{4}x^2 = \frac{21}{2}x - 18$. [2]

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14 **Solutions to this question by accurate drawing will not be accepted.**

The equation of a circle, C_1,is $x^2 + y^2 - 6x - 4y + 5 = 0$. Given that the straight line $2x - y - 6 = 0$ intersects the circle at A and B and that the y-coordinate of B is negative, find

(i) the coordinates of A and of B, [4]

(ii) thecentre, M, and radius of the circle, [3]

(i) the coordinates of D, such that AD is the diameter of the circle. [2]

(iv) the equation of another circle, C_2, which is a reflection of C_1 in the line $y = -1$. [1]

15 The function*f* is defined, for $0 \le x \le 6\pi$, by $f(x) = 2\sin\left(\frac{x}{3}\right) + 1$.

(i) State the maximum and minimum value of $f(x)$. [2]

(ii) State the amplitude of f. [1]

(iii) State the period of f. [1]

(iv) Find the smallest value of x such that $f(x) = 0$. [2]

(v) Sketch the graph of $y = 2\sin\left(\frac{x}{3}\right) + 1$ for $0 \le x \le 6\pi$. [3]

(vi) On the same diagram, draw a suitable graph to solve $2\sin\left(\frac{x}{3}\right) + 1 = 3 - \frac{2x}{3\pi}$.

Hence,state the number of solutions for $0 \le x \le 6\pi$. [2]

~ End of Paper ~

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Answer Key

Sec 3 Express Additional Math EOY Exam 2014

1	a =11 and b = - 6
2	A is (0, 1), B is (–10, 0), C is (–6, –2)
3(i)	$-\frac{2\sqrt{5}}{5}$
(ii)	$-\frac{10}{3}$
4	$\frac{7}{x^2}-\frac{3}{x+2}$
5(a)	$4(m+3)^2 \geq 0$ For any values of m, $4(m+3)^2 \geq 0$ $D \geq 0$ ⇒roots are real and hence the line meets the curve.
(b)	$-1 < k < 7$
6(i)	$y=-0.7x^{\frac{5}{2}}+5.8x^{\frac{1}{2}}$
(ii)	$y=-10.8$
7(i)	$\alpha+\beta=3$ $\alpha\beta=\frac{3}{2}$
(ii)	$3x^2-36x+8=0 / x^2-12x+\frac{8}{3}=0$
8(a)	$A=3$, $B=-1$, $C=-4$
(b)	$a=4$, $b=-9$
9(a)(i)	$(1+ax)^8=1+8ax+28a^2x^2+....$
(ii)	$a=-1$, $b=5$, $c=124$
(b)	$4\frac{148}{243}$
10(i)	$y=\frac{5}{3}x+\frac{8}{3}$
(ii)	$\left(\frac{13}{5},7\right)$
(iii)	$23\frac{4}{5}\text{ unit}^2$
11(i)	$z=19.9°, 145.1°, 199.9°, 325.1°$
(ii)	$z=111.8°, 291.8°$
12(a)	$\frac{3}{4}$
(b)	$x=2\frac{2}{5}$ or 2.4

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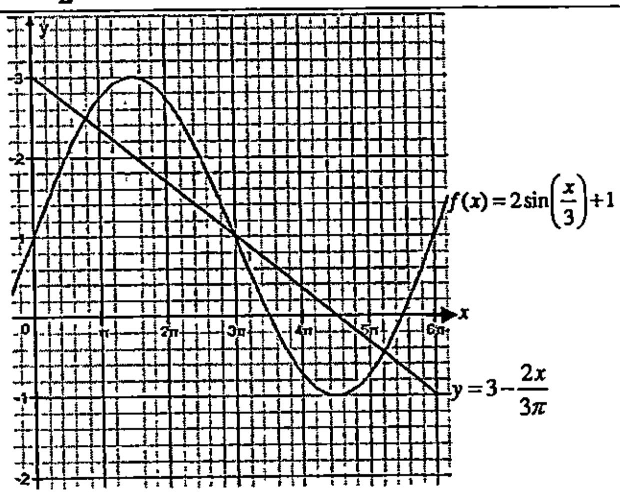
(c)	$6^x = \frac{4}{3}$
13(a)	$k = 6$
(b)(i)	$x = -2,\ \frac{3}{4},\ 3$
(ii)	$x = -4,\ \frac{3}{2},\ 6$
14(i)	A is $(5, 4)$ and B is $(2.6, -0.8)$
(ii)	$2\sqrt{2}$
(iii)	$(1, 0)$
(iv)	$(x-3)^2 + (y+4)^2 = 8$
15(i)	Max. $= 3$ Min. $= -1$
(ii)	Amplitude of $f = 2$
(iii)	Period of $f = 6\pi$
(iv)	$x = \frac{7}{2}\pi$
(v)	
(vi)	3 solutions

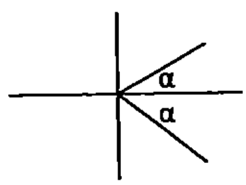
1	$(a+b\sqrt{2})=\left(\frac{7}{3+\sqrt{2}}\right)^2$	
	$=\frac{49}{11+6\sqrt{2}}$	M1-square
	$=\frac{49}{11+6\sqrt{2}}\times\frac{11-6\sqrt{2}}{11-6\sqrt{2}}$	M1-rationalise denominator
	$=\frac{539-294\sqrt{2}}{121-36(2)}$	
	$=\frac{539-294\sqrt{2}}{49}$	
	$=11-6\sqrt{2}$	
	a=11 and b = - 6	A1
2	To find coordinate of A, let $x=0$	
	A is (0, 1)	B1
	To find coordinate of B, let $y=0$	
	$0=\left\lvert\frac{x}{2}+3\right\rvert-2$	
	$\frac{x}{2}+3=2$ or $\frac{x}{2}+3=-2$	
	B is (–10, 0)	B1
	To find coordinate of C, let $\frac{x}{2}+3=0$	
	C is (–6, –2)	B1
3(i)	$\cos A \operatorname{cosec} B$	
	$=\cos A\times\frac{1}{\sin B}$	
	$=-\frac{4}{5}\times\frac{\sqrt{5}}{2}$	[M1]
	$=-\frac{-2\sqrt{5}}{5}$	[A1]

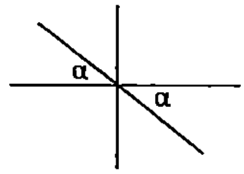
(ii)	$\cot A+\tan B$ $=\frac{1}{\tan A}+\tan B$ $=-\frac{4}{3}-2$ $=-\frac{10}{3}$	 [M1] [A1]
4	Let $\frac{14+7x-3x^2}{x^2(x+2)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2}$ $=\frac{Ax(x+2)+B(x+2)+Cx^2}{x(x+2)}$ By comparing numerator, $14+7x-3x^2=Ax(x+2)+B(x+2)$ $+Cx^2$ Sub $x=-2$, $C=-3$ Sub $x=0$, $B=7$ Sub $x=1$, $A=0$ $A=0, B=7, C=-3$ $\frac{14+7x-3x^2}{x^2(x+2)}=\frac{7}{x^2}-\frac{3}{x+2}$ Comparing coefficient of Constant: $2B=14$ $B=7$ x : $2A+B=7$	B1-correct partial fractions M1(by sub/comparing) A1 B1
5(a)	$y=4x+m$ --(1) $y=mx^2-2x-6$ --(2) sub (1) into (2): $4x+m=mx^2-2x-6$ $0=mx^2-6x+(-6-m)$ $D=b^2-4ac$ $=(-6)^2-4(m)(-6-m)$ $=4(9+6m+m^2)$ $=4(m+3)^2\geq 0$ For any values of m, $4(m+3)^2\geq 0$ $D\geq 0$ $\Rightarrow$roots are real and hence the line meets the curve.	 M1-form equation M1-sub values AG1
(b)	Since the graph is always positive, there are no real roots. $D<0$ $b^2-4ac<0$ $(3-k)^2-4(1)(4)<0$ $k^2-6k-7<0$ $(k+1)(k-7)<0$ $-1<k<7$	 M1- after substitution M1- factorisation A1

6(i)	Gradient of line $= \frac{4.4 - 1.6}{2 - 6}$ $= -0.7$ Eqn. of line: $\frac{y}{\sqrt{x}} = -0.7x^2 + c$ Sub. (2, 4.4): $4.4 = -0.7(2) + c$ or $\frac{y}{\sqrt{x}} - 4.4 = -0.7(x^2 - 2)$ $c = 5.8$ $\therefore$ eqn. of line is $\frac{y}{\sqrt{x}} = -0.7x^2 + 5.8$ $\Rightarrow y = -0.7x^{\frac{5}{2}} + 5.8x^{\frac{1}{2}}$	[M1] [M1] [A1]
(ii)	When $x = 4$, $y = -0.7\left(4^{\frac{5}{2}}\right) + 5.8\left(4^{\frac{1}{2}}\right)$ $= -22.4 + 11.6$ $= -10.8$	[M1] [A1]
7(i)	$\alpha + \beta = 3$ $\alpha\beta = \frac{3}{2}$	B1 B1
(ii)	Roots: $\frac{3}{\beta^3}$ and $\frac{3}{\alpha^3}$ Sum of roots $= \frac{3}{\beta^3} + \frac{3}{\alpha^3}$ $= \frac{3(\alpha^3 + \beta^3)}{(\alpha\beta)^3}$ $= \frac{3\left[(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)\right]}{(\alpha\beta)^3}$ $= \frac{3(\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]}{(\alpha\beta)^3}$ $= \frac{3(3)\left[(3)^2 - 3\left(\frac{3}{2}\right)\right]}{\left(\frac{3}{2}\right)^3}$ $= 12$	M1-sum of roots formula A1

	Product of root $=\left(\frac{3}{\beta^3}\right)\left(\frac{3}{\alpha^3}\right)$ $=\frac{9}{(\alpha\beta)^3}$ $=\frac{8}{3}$ Equation: $3x^2-36x+8=0$ / $x^2-12x+\frac{8}{3}=0$	M1 A1 B1/ FT1
8(a)	$6x^3-16x^2+10x-4=A(x-1)^2(2x-1)+B(x+3)(x-1)+C$ When $x=1$, $C=6-16+10-4$ $=-4$ When $x=\frac{1}{2}$, $B(\frac{1}{2}+3)(\frac{1}{2}-1)-4=6\left(\frac{1}{2}\right)^3-16\left(\frac{1}{2}\right)^2+10\left(\frac{1}{2}\right)-4$ $B=-1$ When $x=0$, $A(-1)^2(-1)+-1(3)(-1)-4=-4$ $A=3$	B1 B1 B1
(b)	Let $f(x)=ax^3+2x^2+bx+6$ When divisible by $(x+2)$, $f(-2)=0$ $-8a+8-2b+6=0$ $4a+b=7$ --- (1) When divided by $(x-1)$, $f(1)=3$ $a+2+b+6=3$ $a+b=-5$ --- (2) (1) − (2), $3a=12$ $a=4$ $b=-9$	M1(equate to zero) M1(equate to 3) M1 A1 (for both answers)
9a(i)	$(1+ax)^8=1+\binom{8}{1}(ax)+\binom{8}{2}(ax)^2+....$ $=1+8ax+28a^2x^2+....$	B1
(ii)	Given $(b+2x)(1+ax)^8=5-38x+cx^2+...$ $(b+2x)(1+ax)^8=(b+2x)(1+8ax+28a^2x^2+...)$ $=b+(8ab+2)x+(28a^2b+16a)x^2+.....$ Comparing,	M1-expansion

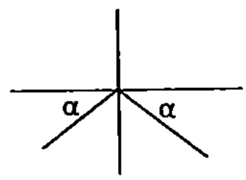
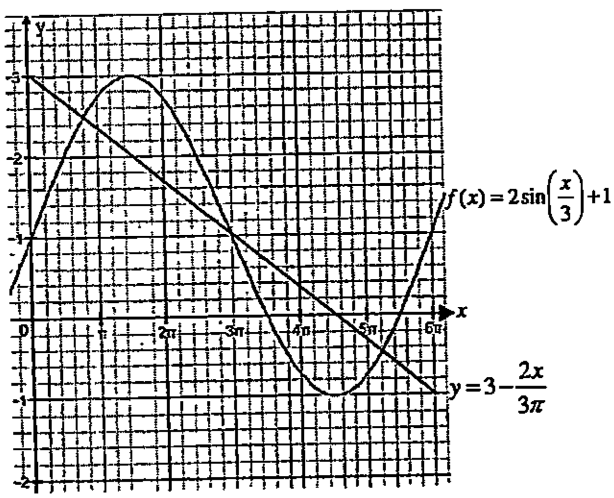
	$b=5$, $a=-1$ $c=124$	A1 A1 A1/FT 1
(b)	$\left(2x^3+\frac{1}{3x^2}\right)^{10}$ $T_{r+1}=\binom{10}{r}(2)^{10-r}\left(\frac{1}{3}\right)^r x^{30-5r}$ For independent term, $30-5r=0$ $r=6$ Coefficient of independent term $=\binom{10}{6}(2)^{10-6}\left(\frac{1}{3}\right)^6$ $=4\frac{148}{243}$	M1 M1 A1
10(i)	Gradient of $PR=\frac{5-2}{-2-3}$ $=-\frac{3}{5}$ $\therefore$ gradient of QS is $\frac{5}{3}$. Coordinates of M are $\left(\frac{-2+3}{2},\frac{5+2}{2}\right)$ $=\left(\frac{1}{2},\frac{7}{2}\right)$ $\therefore$ Eqn. of QS is $y-\frac{7}{2}=\frac{5}{3}\left(x-\frac{1}{2}\right)$ $y=\frac{5}{3}x+\frac{8}{3}$	[M1](gradient of QS) [M1](mid-pt.) [A1]

(ii)	Sub. $y=0$ into $y=\frac{5}{3}x+\frac{8}{3}$, $0=\frac{5}{3}x+\frac{8}{3}$ $-5x=8$ $x=-\frac{8}{5}$ $\therefore$ Coordinates of Q are $\left(-\frac{8}{5},0\right)$. Let S be (x_1, y_1). Since $PQRS$ is a rhombus, mid-point of PR = mid-point of QS $\left(\frac{1}{2},\frac{7}{2}\right)=\left(\frac{x_1+\left(-\frac{8}{5}\right)}{2},\frac{y_1+0}{2}\right)$ $\frac{x_1+\left(-\frac{8}{5}\right)}{2}=\frac{1}{2}\Rightarrow x_1=\frac{13}{5}$ $\frac{y_1}{2}=\frac{7}{2}\Rightarrow y_1=7$ $\therefore$ Coordinates of S are $\left(\frac{13}{5},7\right)$.	[B1]ft [B1]ft
(iii)	Area of $PQRS=\frac{1}{2}\begin{vmatrix}-2 & -\frac{8}{5} & 3 & \frac{13}{5} & -2\\ 5 & 0 & 2 & 7 & 5\end{vmatrix}$ $=\frac{1}{2}\left(30\frac{4}{5}-\left(-16\frac{4}{5}\right)\right)$ $=23\frac{4}{5}\text{ unit}^2$	[M1] [A1]
11(i)	$\sec(2z+15°)=\sqrt{3}$ $\cos(2z+15°)=\frac{1}{\sqrt{3}}$  Basic $\angle$, $\alpha=54.74°$ Since $0°\le z\le 360°$, $\therefore 15°\le 2z+15°\le 735°$. $2z+15°=54.74, 305.26, 414.74, 665.26$ $z=19.9°, 145.1°, 199.9°, 325.1°$ (to 1 dec. pl.)	[M1](change to cos) [M1] [A1,A1](1m for each pair of angles)

(ii)	$2\tan^2 z + 5\tan z = 0$ $\tan z(2\tan z + 5) = 0$ $\tan z = 0$ $z = 0°, 180°, 360°$ or $\tan\ z = -\frac{5}{2}$  Basic $\angle, \alpha = 68.20°$ $z = 111.8°, 291.8°$ (to 1 dec. pl.)	[M1](factorise) [A1] [M1] [A1]
12(a)	$\log_p 8 \times \log_{16} p$ $= (\log_p 8)(\frac{1}{\log_p 16})$ $= (\log_p 2^3)(\frac{1}{\log_p 2^4})$ $= \frac{3}{4}$	M1 (change log to base p) M1(change to base 2) A1
(b)	$\log_3(\frac{3x-7}{2x-3}) = -2$ $\frac{3x-7}{2x-3} = (3)^{-2}$ $\frac{3x-7}{2x-3} = \frac{1}{9}$ $x = 2\frac{2}{5}$ or 2.4	M1 – quotient/product rule M1-express in indices/solve A1
(c)	$2^{2x+2} \times 3^{x-1} = 8^x \times 3^{2x}$ $2^{2x+2} \times 3^{x-1} = 2^{3x} \times 3^{2x}$ $2^{2x}.2^2 \times \frac{3^x}{3} = 2^{3x} \times 3^{2x}$ $\frac{2^{3x} \times 3^{2x}}{2^{2x} \times 3^x} = \frac{4}{3}$ $2^x \times 3^x = \frac{4}{3}$ $6^x = \frac{4}{3}$	M1-change to base of 2/3 and applying law of indices M1-grouping same base together A1

13(a)	$f(x) = -(x+2)(x-3)(x-k)$	B1
	$f(4) = 12$ $-(4+2)(4-3)(4-k) = 12$	M1
	$4-k = -2$ $k = 6$	A1
(b)(i)	$4x^3 - 7x^2 - 21x + 18 = 0$ Let $f(x) = 4x^3 - 7x^2 - 21x + 18$ By trial and error, $f(3) = 4(3)^3 - 7(3)^2 - 21(3) + 18$ $= 0$ $\therefore$ Since R=0, $(x-3)$ is a factor of $f(x)$.	B1-finding one factor
	$f(x) = (x-3)(4x^2 + 5x - 6)$ $= (x-3)(4x-3)(x+2)$	M1-quadratic factor/Factorisation
	When $f(x) = 0$ $(x-3)(4x-3)(x+2) = 0$	M1
	$x = -2\ ,\ \frac{3}{4}\ , 3$	A1
	OR	
	Let $f(x) = 4x^3 - 7x^2 - 21x + 18$ By trial and error, $f(3) = 4(3)^3 - 7(3)^2 - 21(3) + 18$ $= 0$ $\therefore$ Since R=0, $(x-3)$ is a factor of $f(x)$.	B1-finding one factor
	By trial and error, $f(-2) = 4(-2)^3 - 7(-2)^2 - 21(-2) + 18$ $= 0$ $\therefore$ Since R=0, $(x+2)$ is a factor of $f(x)$. By trial and error, $f\left(\frac{3}{4}\right) = 4\left(\frac{3}{4}\right)^3 - 7\left(\frac{3}{4}\right)^2 - 21\left(\frac{3}{4}\right) + 18$ $= 0$ $\therefore$ Since R=0, $(4x-3)$ is a factor of $f(x)$.	M1-finding 2 other factors
	When $f(x) = 0$ $(x-3)(4x-3)(x+2) = 0$	M1
	$x = -2\ ,\ \frac{3}{4}\ , 3$	A1
(ii)	$\frac{1}{2}x^3 - \frac{7}{4}x^2 = \frac{21}{2}x - 18$ $\frac{1}{2}x^3 - \frac{7}{4}x^2 - \frac{21}{2}x + 18 = 0$	

	$4\left(\frac{x^3}{8}\right)-7\left(\frac{x^2}{4}\right)-21\left(\frac{x}{2}\right)+18=0$ $4\left(\frac{x}{2}\right)^3-7\left(\frac{x}{2}\right)^2-21\left(\frac{x}{2}\right)+18=0$ Hence $\frac{x}{2}=-2\ ,\ \frac{3}{4}\ ,3$ $x=-4\ ,\ \frac{3}{2}\ \ ,\ 6$	M1 A1
14(i)	Sub. $y=2x-6$ into $x^2+y^2-6x-4y+5=0$, $x^2+(2x-6)^2-6x-4(2x-6)+5=0$ $x^2+4x^2-24x+36-6x-8x+24+5=0$ $5x^2-38x+65=0$ $x=\frac{-(-38)\pm\sqrt{(-38)^2-4(5)(65)}}{2(5)}$ $=\frac{38\pm\sqrt{144}}{10}$ $=5$ or 2.6 Sub. values of x into $y=2x-6$, $y=2(5)-6$ or $y=2(2.6)-6$ $=4$ $\quad$ $=-0.8$ $\therefore A$ is $(5,4)$ and B is $(2.6,-0.8)$	[M1] [M1] [A1, A1]
(ii)	$2g=-6\Rightarrow -g=3$ $2f=-4\Rightarrow -f=2$ $\therefore$ centre, M, of circle $=(-g,-f)=(3,2)$ Radius of circle $=\sqrt{g^2+f^2-c}$ $=\sqrt{(-3)^2+(-2)^2-5}$ $=\sqrt{8}$ $=2\sqrt{2}$	[B1] [M1] [A1]
(iii)	Let D be (x, y). Mid-point of AD is $\left(\frac{x+5}{2},\frac{y+4}{2}\right)=(3,2)$ $\frac{x+5}{2}=3\Rightarrow x=1$ $\frac{y+4}{2}=2\Rightarrow y=0$ $\therefore$ coordinates of D are $(1,0)$.	[M1] [A1]

(iv)	Eqn.of circle C_2 is $(x-3)^2+(y+4)^2=8$	[B1]ft
15(i)	Max. $f(x)=2(1)+1=3$ Min. $f(x)=2(-1)+1=-1$	[B1] [B1]
(ii)	Amplitude of $f=2$	[B1]
(iii)	Period of $f=6\pi$	[B1]
(iv)	$f(x)=2\sin\left(\frac{x}{3}\right)+1$ $2\sin\left(\frac{x}{3}\right)+1=0$ $\sin\left(\frac{x}{3}\right)=-\frac{1}{2}$ Basic $\angle, \alpha=\frac{\pi}{6}$ Smallest $\frac{x}{3}=\frac{7}{6}\pi$ Smallest $x=\frac{7}{2}\pi$  	[M1] [A1]
(v)	 	S1(sine curve) P1(max. 3 & min. -1) P1 (correct points (0,1), (3π,1) & (6π,1)
(vi)	3 solutions	[B1] Graph [B1]

Fairfield Methodist School (Secondary)
Mathematics Department
Markers' Report 2014

Subject : **Additional Mathematics**

Level / Stream : **3 (Exp)**

Classes: Sec 3A, 3B, 3C, 3D, 3E, 3F

Overall Percentage Pass : **45.6**
Overall Percentage Distinction : **13.5**

Remarks: Generally the candidates had time to complete all 15 questions. There was effort to try most of the questions and not leave blanks. Topics that need to be revised are Trigonometry, Linear Law, Coordinate Geometry, Logarithms and Indices.

Question Number	**Comments**
1	Generally well done. Common error $(3+\sqrt{2})^2 = 9+2$ or multiplied the denominator $11+6\sqrt{2}$ by $a+b\sqrt{2}$ instead of its conjugate surd.
2	Candidates were not observant about the positions of points A, B and C on the given diagram resulting in incorrect coordinates. Many could not find point C. Common errors Point A was mistaken as an x-intercept instead of the y-intercept. Point B was given as (-2, 0) which is the other unlabelled x-intercept. Point C was called the turning point instead of the vertex of the graph.
3	Poorly done. Many candidates could identify the correct quadrant for A and B but made mistakes with the signs or sides of the trigo ratios. This could be because many drew both angles A and B on the same diagram instead of on different diagrams. Despite the instruction 'without the use of calculator', some candidates calculated the values of angle A and B and used them to answer the questions.
4	Generally well done. Common error $\frac{14+7x-3x^2}{x^2(x+2)} = \frac{A}{x^2} + \frac{Bx+C}{x+2}$
5(a)	Many candidates did not address the question. They were able to form the quadratic equation but instead of **showing** that $b^2-4ac \geq 0$, they proceeded to find m by using this inequality.

Question Number	Comments
5(b)	Well done. <u>Common errors</u> $b^2-4ac>0$ instead of $b^2-4a<0$ (graph lies entirely above the x-axis). $(k+1)(k-7)<0 \Rightarrow k+1<0$ or $k-7<0$.
6(i)	<u>Common error</u> Substitutedthe coordinates into the linear equation as x and y instead of x^2 and $\frac{y}{\sqrt{x}}$.
6(ii)	Error in answers due to wrong equation in (i).
7(i)	Well done except for some students in 3A who confused sum and product of roots.
7(ii)	Poorly done. Many could not find the sum of the new roots. <u>Common errors</u> $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-2\alpha\beta+\beta^2)$ or $(\alpha+\beta)^3$. New equation was not equated to zero.
8(a)	Well done. Many candidates expanded the polynomial and compared the coefficients instead of solving by substitution. This was a very long and time consuming method and prone to careless mistakes.
8(b)	Well done. Some candidates tried to solve by long division although the dividend given was in terms of a and b.
9(a)(i)	Well done.
9(a)(ii)	There were many careless mistakes in the expansion of $(b+2x)(1+ax)^8=(b+2x)(1+8ax+28a^2x^2+...)$ with answers like $...+28abx^2+...$
9(b)	Many candidates did not give the **value** of the term independent of x but mentioned that 'the term the independent of x is the 7th term'. Some solved by expanding the binomial expression completely to get the term independent of $x$which is a time consuming method and prone to careless mistakes.
10(i)	Poorly done except for 3E and F. Did not show the use of the midpoint M to find the equation of QS.
10(ii)	Most candidates could find the coordinates of point Q but had difficulty finding point S, especially if they equated the sides of the rhombus,$QR=SR$ or $QP=SP$ resulting in 2 unknowns.

Question Number	Comments
	<u>Error</u> Lengths of the diagonals of a rhombus are equal, $PR=QS$.
10(iii)	Not all the candidates used the 'shoelace' method to find the area of the rhombus. <u>Common error</u> Area of rhombus = area of square = (length of side)^2
11(i)	Poorly done except for 3E and F. <u>Common error</u> $\sec(2z+15°)=\sqrt{3}$ $\frac{1}{\cos}(2z+15°)=\sqrt{3}$ or $\frac{1}{\sin}(2z+15°)=\sqrt{3}$ $2z+15°=\cos^{-1}\sqrt{3}$ $2z+15°=\sin^{-1}\sqrt{3}$
11(ii)	Poorly done except for 3E and F. Many candidates treated the trigo function as an algebraic term and factorised accordingly. <u>Common errors</u> $2\tan^2 z+5\tan z=0$ $\tan z(2\tan+5)=0$ or $\tan\ (2\tan z+5z)=0$ Many also did not solve for $\tan z=0$.
12(a)	Many candidates could change the base of the log functions but could not simplify further. <u>Common errors</u> $\frac{\log_p 8}{\log_p 16}=\frac{1}{2}$ or $\log_p 16=\log_p 8^2$
12(b)	Poorly done. <u>Common error</u> $\log_3(3x-7)=\frac{\log_3 3x}{\log_3 7}$
12(c)	Poorly done. Misuse of the laws of indices. <u>Common errors</u> $2^{2x+2}\times 3^{x-1}=2^{3x}\times 3^{2x}$ $2^{2x+2-3x}\times 3^{x-1-2x}=0$ or $2^{2x+2}\times 3^{x-1}=2^3\times 2^x\times 3^2\times 3^x$
13(a)	Poorly done. Many candidates did not make the coefficient of x^3 to be -1. Some formed cubic equation with 3 unknowns and attempted to solve simultaneously but were mainly unsuccessful:

Question Number	Comments
	$f(x) = -x^3 + ax^2 + bx + c$ Let $x = -2, 3$ and k.
13(b)(i)	Ok. Some candidates gave the calculator solutions for the cubic equation without showing any working.
13(b)(ii)	Poorly done. Most candidates could not relate the given equation to the previous equation to solve for x. Instead they used the calculator values.
14(i)	Well done.
14(ii)	Some candidates gave the centre as (-2g, -2f) – did not dive by 2.
14(iii)	Some candidates equated the radii of the circle, $AM = MD$, resulting in 2 unknowns and could not proceed.
14(iv)	Mostly blank or wrong equation of circle was given.
15(i)-(iii)	Well done except some candidates gave the period in degrees instead of radians.
15(iv)	Poorly done. Could not solve for x.
15(v)	Most candidates could sketch the sine curve and showed the maximum and minimum values correctly but the period was wrong.
15(vi)	Mostly not attempted.