Name	Register Number	Class

MACPHERSON SECONDARY SCHOOL

ADDITIONAL MATHEMATICS

Paper 1 **4038/01**

October 2012 2 hours 30 minutes

END-OF-YEAR EXAMINATION

SECONDARY THREE EXPRESS

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Show all your workings on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
The total marks for this paper is 100.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use	
Total	**100**

Mathematical Formulae

1. Algebra

Quadratic Equation

For the equation $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. Trigonometry

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$
$$\sin A+\sin B=2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$
$$\sin A-\sin B=2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$
$$\cos A+\cos B=2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$
$$\cos A-\cos B=-2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

1 The line $3x-2y=10$ meets the curve $2y^2-7y+x=0$ at the points A and B. Find the coordinates of A and B. [4]

2 (a) Find the inverse of the matrix $\begin{pmatrix} 5 & -2 \\ -3 & 2 \end{pmatrix}$. [1]

(b) Hence, solve the simultaneous equations [3]

$$5x-2y=17$$
$$-3x+2y=-11$$

3 Find the numerical value of $\dfrac{243\times 3^{n-2}}{3^n-3^{n-1}}$. [4]

4 (a) Given that $(k-1)\sqrt{7}=(k+1)\sqrt{5}$, without using a calculator, find k in the form of $a+b\sqrt{c}$, where a, b and c are integers. [4]

(b) Hence, evaluate $k+\dfrac{1}{k}$. [3]

5 Solve the following equations:

(a) $\ln x^3+2\ln x^2-5\ln x+\ln\sqrt{x}=5$ [3]

(b) $\log_2 4x=1-\log_4 x$ [3]

6 (a) The expression $f(x)=2x^3+ax^2+bx-6$ is divisible by $(x+2)$ but leaves a remainder of -5 when divided by $(2x-1)$. Find the values of a and b. Find the remainder when $f(x)$ is divided by x^2+x-2. [6]

(b) Express $\dfrac{7x-12}{(x+2)(3x^2+1)}$ in partial fractions. [5]

7 (a) Find the range of values of x for which $(2x-3)^2-(x-1)^2<5$. [3]

(b) Find the range of values of p for which the equation $4x^2-2px+p+3=0$ has no real roots. [3]

8 Given that the roots of a quadratic equation $3x^2 + 7x + 3 = 0$ are α and β, form a quadratic equation which has roots $\frac{1}{2\alpha}$ and $\frac{1}{2\beta}$. [5]

9 Sketch the graph of $y = |3x - 7|$. [2]

Find

(a) the range of y corresponding to the domain $0 \le x \le 4$, [1]

(b) the range of values of x for which $y > 4$. [1]

10 Sketch the graph $y = |2x^2 + x - 3|$ for $-3 \le x \le 3$. [6]

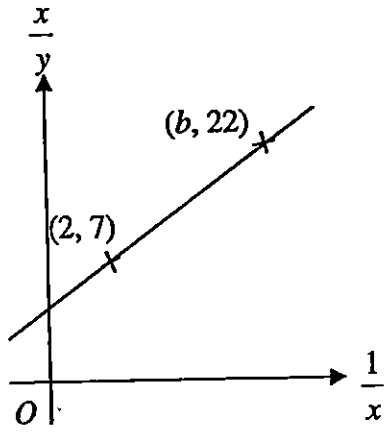
Hence, solve the inequality $|2x^2 + x - 3| \le 3$.

11 **(a)** Find the first four terms in the expansion of $(1 - x)^6$ in ascending powers of x. Using this expansion, or otherwise, find the first four terms in the expansion of $\left(1 - \frac{x^2}{2}\right)^6$. [3]

(b) Find, in ascending powers of x, the first three terms in the expansion of $(2 - 3x)^5$. Use the expansion to find the value of $(1.997)^5$, correct to 2 decimal places. [4]

(c) Find the term independent of x in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$. [4]

12 The diagram shows a straight line graph representing the curve $x^2 - 2xy + ay = 0$. The coordinates of two points are shown on the line. Calculate the values of a and b. [3]



13 The table shows experimental values of two variables x and y, related by the equation $y = ab^{\frac{x}{2}}$.

x	1	2	3	4	5
y	6.57	10.79	17.75	29.16	47.92

Express the equation in a form suitable for drawing a straight line graph. Draw this graph for the given data and use it to estimate [2]

(a) the values of a and b, [2]

(b) the value of x when $y = 25$. [1]

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14 Without using a calculator, evaluate

(a) $\sin 225^\circ$ [2]

(b) $\tan(-240^\circ)$ [2]

(c) $\cos \frac{5\pi}{3}$ [2]

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15 Sketch, on the same diagram, the graphs of $y = 3\sin x + 1$ and $y = 2\cos 2x$ for the interval $0^\circ \le x \le 360^\circ$. Hence, for this interval, state the number of solutions of the equations [4]

(a) $3\sin x + 1 = 2\cos 2x$ [1]

(b) $|3\sin x + 1| = 2\cos 2x$ [1]

(c) $|3\sin x + 1| = 2|\cos 2x|$ [1]

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16 Find all the angles for the interval $0 \le \theta \le 2\pi$ which satisfy the equation $2(\cos\theta + 5\sin\theta) = 3\sin\theta$. [4]

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17 Prove the identity $\frac{3\sin x - 2\cos^2 x}{\sin^2 x + 2\sin x} = 2 - \operatorname{cosec} x$. [3]

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18 Given that $5\sin x - 12\cos x = R\sin(x - \alpha)$, where $R > 0$ and α is acute, find

(a) the values of R and α, [2]

(b) the maximum value of $5\sin x - 12\cos x + 2$ and the corresponding value of x for $0^\circ \le x \le 360^\circ$. [2]

End of Paper 1

1 $(5, \frac{5}{2}), (\frac{34}{9}, \frac{2}{3})$

2a $\frac{1}{4}\begin{pmatrix} 2 & 2 \\ 3 & 5 \end{pmatrix}$

2b $x = 3, y = -1$

3 $\frac{81}{2}$

4a $k = 6 + \sqrt{35}$

4b 12

5a $x = 7.39$

5b $x = 0.630$

6a $a = 5, b = -1$, remainder $= 0$

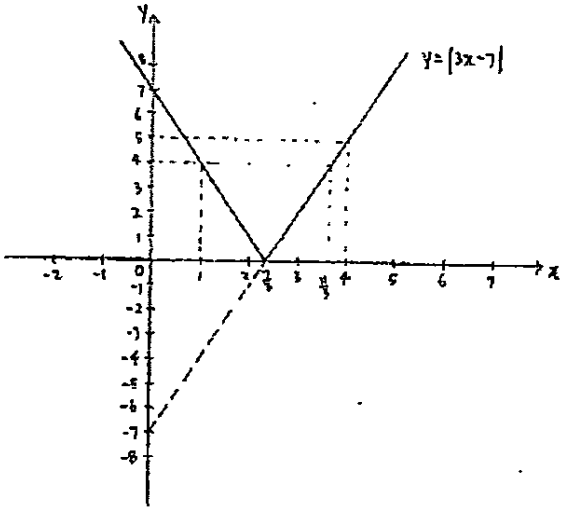
6b $-\frac{2}{x+2} + \frac{6x-5}{3x^2+1}$

7a $\frac{1}{3} < x < 3$

7b $-2 < p < 6$

8 $12x^2 + 14x + 3 = 0$

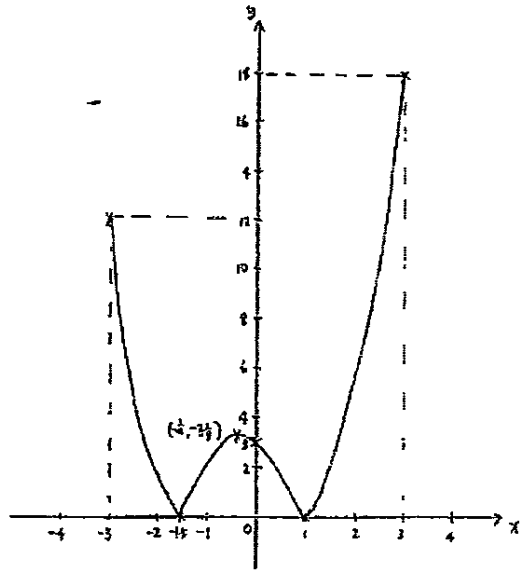
9



9a $0 \le y \le 7$

9b $x < 1$ or $x > \frac{11}{3}$

10



$-2 \le x \le -\frac{1}{2}$ or $0 \le x \le \frac{3}{2}$

11a $1 - 6x + 15x^2 - 20x^3 + \ldots$

$1 - 3x^2 + \frac{15}{4}x^4 - \frac{5}{2}x^6 + \ldots$

11b $32 - 240x + 720x^2 + \ldots$

31.76

11c 7920

12 $a = -\frac{5}{2}, b = 8$

13 $\lg y = \left(\frac{1}{2}\lg b\right)x + \lg a$

13a $a = 3.98, b = 2.70$

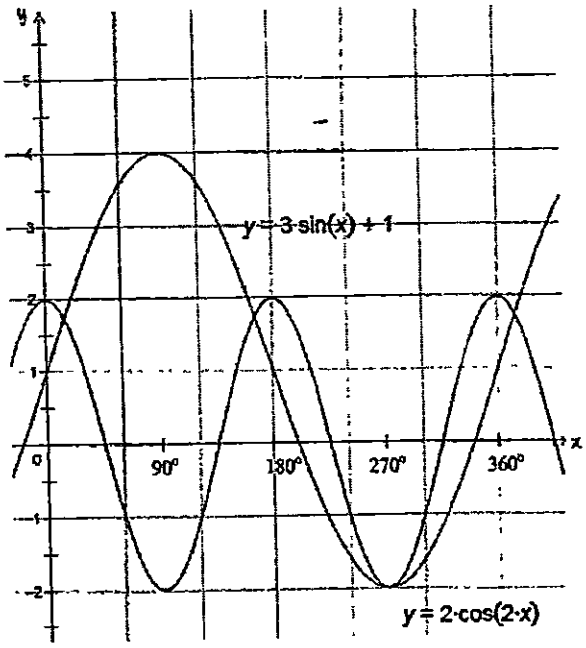
13b $x = 3.7$

14a $-\frac{\sqrt{2}}{2}$

14b $-\sqrt{3}$

14c $\frac{1}{2}$

15



15a 3 solutions
15b 4 solutions
15c 5 solutions

16 2.86 rad, 6.00 rad

18a $13\sin(x - 67.4^\circ)$
18b max value = 15 when $x = 157.4^\circ$