BEATTYSECONDARY SCHOOL
MID-YEAREXAMINATION 2014

SUBJECT : Additional Mathematics **LEVEL : Sec 3Express**

PAPER : 4047 / 1 **DURATION : 2 hours**

SETTER : MdmRashimaSidik **DATE : 15 May 2014**

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Give non exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80.**

This paper consists of 3 printed pages (including this cover page)

1 The line $2y-1=x$ intersects the curve $x^2+y^2=19-xy$ at the points A and B. Find thecoordinates of A and of B. [4]

2 (a) Find the smallest integer value of a for which $a(x^2+5x)-3x+4a$ is always positive forall real values of x. [5]

(b) Show that the equation $x^2+bx=3(3-b)$ has real roots for all real values of x. [3]

(c) Find the range of values of c for which the line $y+x+c=0$ intersects the curve $y=12x^2+3cx-1$ at two distinct points. [5]

3 (a)Express $y=-2x^2-12x-9$ in the form $y=a(x-h)^2+k$. [2]

(b) Hence sketch the curve $y=-2x^2-12x-9$, labeling clearly the coordinates of the turning point and the y-intercept. [2]

4(a) Without using a calculator, find the value of p such that

$$\left(\frac{2}{\sqrt{75}}-\frac{\sqrt{147}}{3}\right)\div\sqrt{2}=p\sqrt{6}.$$ [4]

(b) Express $(3+\sqrt{7})^2-\frac{15}{2-\sqrt{7}}$ in the form $q+r\sqrt{7}$, where q and r are integers to be determined. [4]

5 (a)Given that $3^{x+2}\times2^{x-2}=3^{2x-1}\times4^x$, evaluate 6^x. [3]

(b)Using an appropriate substitution, solve $3^{2x}+81^{\frac{x+2}{4}}=36$. [4]

6Express $\frac{8x^3-7x^2-11x+16}{(2x+1)(x-1)^2}$ in partial fractions. [6]

7 The function $f(x)=2x^3+7x^2+ax+b$, where a and b are constants, has a factor of $x-2$ and leaves a remainder of 15 when divided by $x+3$.

(a) Find the value of a and of b. [4]

(b) Solve the equation $f(x)=0$. [3]

(c)Hence solve $2y^6+7y^4+ay^2+b=0$. [2]

8 The roots of the equation $x^2-4x+2=0$ are α and β. Find

(a) the value of $\alpha^2+\beta^2$. [2]

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(ii) Hence, estimate the value of $3.25\times(1.15)^6$. [3]

(b) In the expansion of $(1+2x)^n$ in ascending powers of x, the coefficient of the third term is 60. Find thevalue of n, where n is a positive integer. [4]

~End of Paper~

Answer Key

1 $A = \left(-3\frac{4}{7}, -1\frac{2}{7}\right)$, $B = (3,2)$

2(a) 1

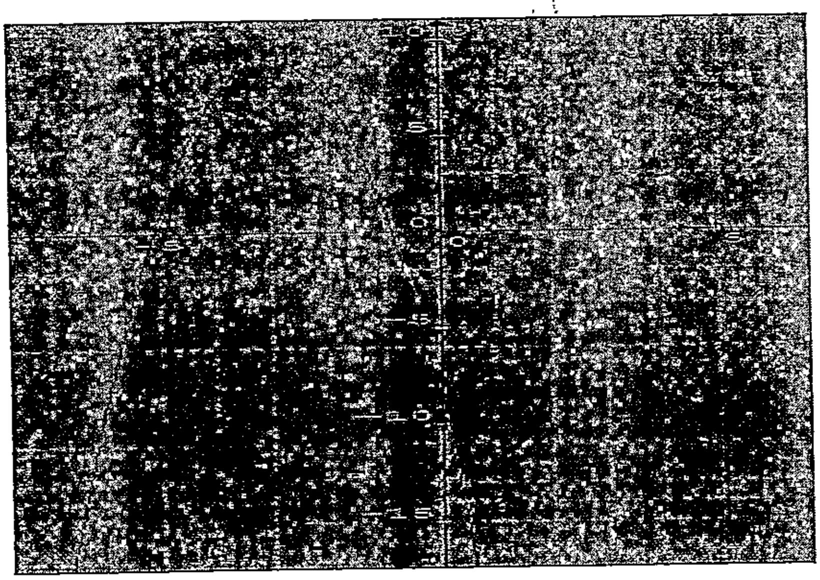
(b) $x^2 + bx - 9 + 3b = 0$

$$
\begin{aligned}
D &= b^2 - 4(-9+3b) \\
&= b^2 - 12b + 36 \\
&= (b-6)^2 \\
&\geq 0
\end{aligned}
$$

Since $D \geq 0$, the equation has real roots for all values of x.

(c) True for all values of c except $c = \frac{7}{3}$

3(a) $-2(x+3)^2 + 9$

(b) 

4(a) $\frac{-11}{10}$

(b) $q = 26$, $r = 11$

5(a) $\frac{27}{4}$

(b) $x = 1$

6 $$\frac{8x^3-7x^2-11x+16}{(2x+1)(x-1)^2}=4+\frac{25}{3(2x+1)}-\frac{5}{3(x-1)}+\frac{2}{(x-1)^2}$$

7(a) $a = -10, b = -24$

(b) $x=2,-4,-\frac{3}{2}$

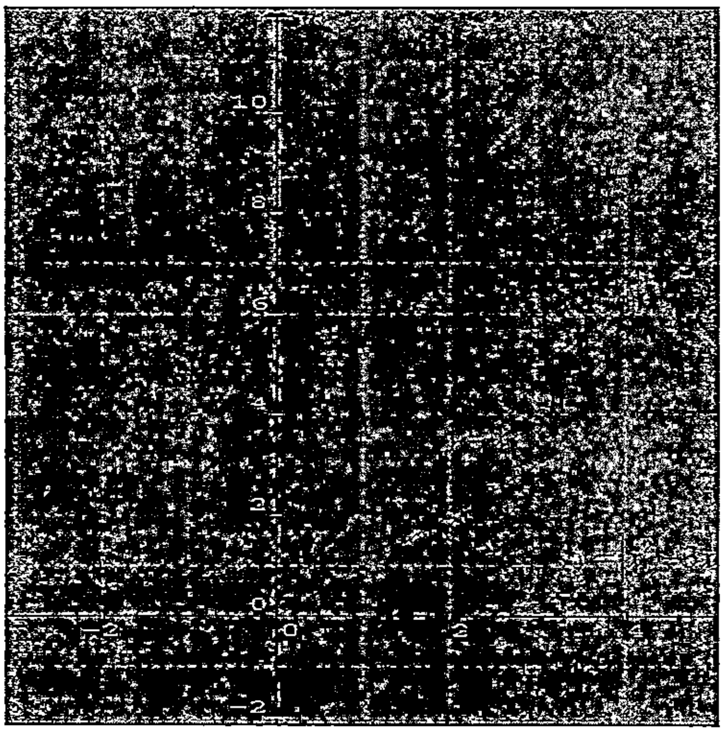
(c) $y=\pm\sqrt{2}$

8(a) 12

(b) $x^2-40x+8=0$

9(a) $x=\frac{1}{9}$

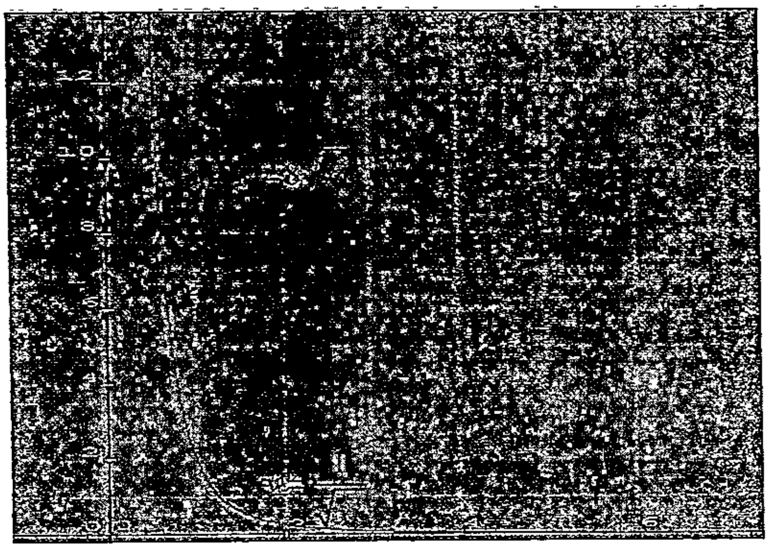
(b)



Range of y: $0 \le y \le 10$

(c) Number of solutions = 3

10 (a)



(b) $x = \frac{1}{2}$

11(a)(i) $4 + \frac{19}{5}x + \frac{6}{5}x^2 + \ldots$

(ii) 7.525

(b) $n = 6$

AM_MYE_P1_2014_Solutions

1 The line $2y-1=x$ intersects the curve $x^2+y^2=19-xy$ at the points A and B.
Find thecoordinates of A and of B. [4]

$(2y-1)^2+y^2+(2y-1)y-19=0$ M1

$4y^2-4y+1+y^2+2y^2-y-19=0$

$7y^2-5y-18=0$

$(7y+9)(y-2)=0$ M1

$y=-1\frac{2}{7},\ y=2$

$x=2(-1\frac{2}{7})-1,\ x=2(2)-1$

$=-3\frac{4}{7},\qquad =3$

$A=(-3\frac{4}{7},-1\frac{2}{7}),\ B=(3,2)$ A2

2 (**a**)Find the smallest integer value of a for which $a(x^2+5x)-3x+4a$ is always positive forall real values of x. [5]

$a(x^2+5x)-3x+4a=ax^2+(5a-3)x+4a$ M1

Always positive→ D<0

$(5a-3)^2-4a(4a)<0$ M1

$25a^2-30a+9-16a^2<0$

$9a^2-30a+9<0$

$3a^2-10a+3<0$

$(3a-1)(a-3)<0$ M1

$\frac{1}{3}<a<3$ M1

Smallest integer = 1 A1

(b) Show that the equation $x^2+bx=3(3-b)$ has real roots for all real values of x.

[3]

$$x^2+bx-9+3b=0$$

$D=b^2-4(-9+3b)$ M1

$=b^2-12b+36$

$=(b-6)^2$ M1

≥ 0

Since $D\geq 0$, the equation has real roots for all values of x. A1

(c) Find the range of values of c for which the line $y+x+c=0$ intersects the curve $y=12x^2+3cx-1$ at two distinct points. [5]

$y=12x^2+3cx-1=-x-c$ M1

$12x^2+(3c+1)x-1+c=0$ M1

$D>0\rightarrow(3c+1)^2-4(12)(c-1)>0$ M1

$9c^2+6c+1-48c+48>0$

$9c^2-42c+49>0$

$(3c-7)^2>0$ M1

$c>\frac{7}{3}, c<\frac{7}{3}$ A1

Or True for all values of c except $c=\frac{7}{3}$

3 (a) Express $y=-2x^2-12x-9$ in the form $y=a(x-h)^2+k$. [2]

(b) Hence sketch the curve $y=-2x^2-12x-9$, labeling clearly the coordinates of the turning point and the y-intercept. [2]

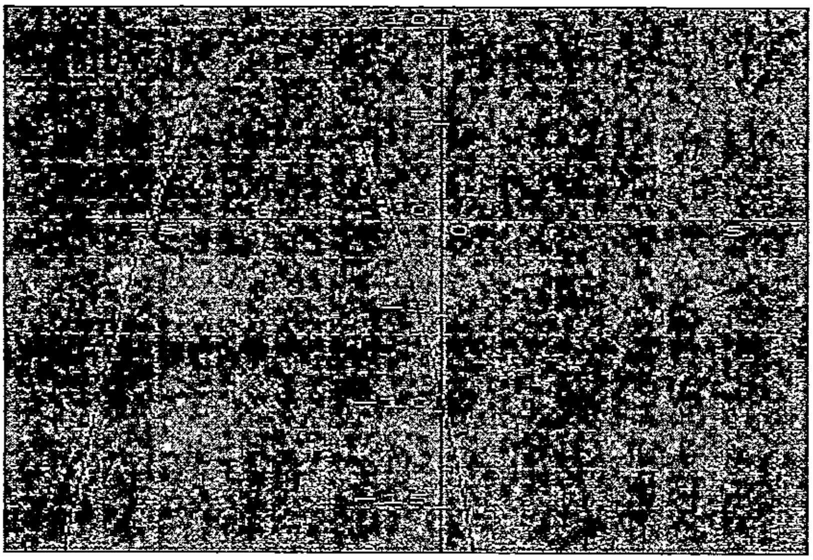
(a)

$y=-2x^2-12x-9$

$=-2(x^2+6x)-9$ M1

$=-2[(x+3)^2-9]-9$

$=-2(x+3)^2+9$ A1

(b) 

Shape of curveand
y-intercept G1

Label turning point G1

4 (a) Without using a calculator, find the value of p such that $\left(\frac{2}{\sqrt{75}}-\frac{\sqrt{147}}{3}\right)\div\sqrt{2}=p\sqrt{6}$.

[4]

$$\left(\frac{2}{\sqrt{75}}-\frac{\sqrt{147}}{3}\right)\div\sqrt{2}=\left(\frac{2}{5\sqrt{3}}-\frac{7\sqrt{3}}{3}\right)\times\frac{1}{\sqrt{2}}$$ M1 simplify surd

$$=\frac{6-35(3)}{15\sqrt{3}}\times\frac{1}{\sqrt{2}}$$ M1combine into 1 fraction

$$=\frac{-99}{15\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$ M1 rationalise

$$=\frac{-11}{10}\sqrt{6}$$

$$p=\frac{-11}{10}$$ A1

(b) Express $(3+\sqrt{7})^2-\frac{15}{2-\sqrt{7}}$ in the form $q+r\sqrt{7}$, where q and r are integers to be determined. [4]

$$(3+\sqrt{7})^2-\frac{15}{2-\sqrt{7}}=9+6\sqrt{7}+7-\left(\frac{15}{2-\sqrt{7}}\times\frac{2+\sqrt{7}}{2+\sqrt{7}}\right)$$ M1

$$=16+6\sqrt{7}-\frac{30+15\sqrt{7}}{4-7}$$ M1

$$=16+6\sqrt{7}-(-10-5\sqrt{7})$$
$$=26+11\sqrt{7}$$

$q=26$, $r=11$ A2

5 (a) Given that $3^{x+2}\times 2^{x-2}=3^{2x-1}\times 4^{x}$, evaluate 6^x. [3]

$3^x(9)\times\frac{2^x}{4}=\frac{3^x.3^x}{3}\times 4^x$	M1		$9(3^x)\times\frac{2^x}{4}=\frac{3^{2x}}{3}\times 2^{2x}$	M1
$\frac{9}{4}(6^x)=\frac{36^x}{3}$	M1	or	$\frac{9}{4}(6^x)=\frac{6^{2x}}{3}$	M1
$\frac{36^x}{6^x}=\frac{9}{4}(3)$			$6^x=\frac{27}{4}$	A1
$6^x=\frac{27}{4}$	A1			

(b) Using an appropriate substitution, solve $3^{2x}+81^{\frac{x+2}{4}}=36$. [4]

$3^{2x}+81^{\frac{x+2}{4}}-36=0$

$(3^x)^2+3^{x+2}-36=0$ M1

Let $3^x=y$

$y^2+9y-36=0$ M1

$(y+12)(y-3)=0$

$3^x=-12,\ 3^x=3$ M1

nax $=1$ A1

6 Express $\frac{8x^3-7x^2-11x+16}{(2x+1)(x-1)^2}$ in partial fractions. [6]

$(2x+1)(x-1)^2=2x^3-3x^2+1$ M1

$$\begin{array}{r l}
 & 4 \\
2x^3-3x^2+1 \,\big) & 8x^3-7x^2-11x+16 \\
 & -(8x^3-12x^2+0x+4) \\ \hline
 & 5x^2-11x+12
\end{array}$$

M1 Long division

$$\frac{8x^3-7x^2-11x+16}{(2x+1)(x-1)^2}=4+\frac{5x^2-11x+12}{(2x+1)(x-1)^2}$$

$$=4+\frac{A}{2x+1}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}$$ M1

$5x^2-11x+12=A(x-1)^2+B(x-1)(2x+1)+C(2x+1)$ M1

Sub. $x = 1, 6 = 3C$
$C = 2$

Sub. $x = -0.5, 18.75 = 2.25A$
$A = \frac{25}{3}$
Sub. $x = 0, 12 = A - B + C$
$B = -\frac{5}{3}$

M1

$$\frac{8x^3 - 7x^2 - 11x + 16}{(2x+1)(x-1)^2} = 4 + \frac{25}{3(2x+1)} - \frac{5}{3(x-1)} + \frac{2}{(x-1)^2}$$ A1

7 The function $f(x) = 2x^3 + 7x^2 + ax + b$, where a and b are constants, has a factor of $x - 2$ and leaves a remainder of 15 when divided by $x + 3$.

(a) Find the value of a andof b. [4]

(b) Solve the equation $f(x) = 0$. [3]

(c)Hence solve $2y^6 + 7y^4 + ay^2 + b = 0$. [2]

(a)

$f(x) = 2x^3 + 7x^2 + ax + b$
$f(2) = 0$
$2(2)^3 + 7(2)^2 + a(2) + b = 0$ M1
$2a + b = -44 ----(1)$

$f(-3) = 15$
$2(-3)^3 + 7(-3)^2 + a(-3) + b = 15$ M1
$-3a + b = 6 ----(2)$

Sub b=6+3a into (1):

2a+6+3a=-44
$a = -10$ A1
$b = 6 + 3(-10) = -24$ A1

(b)

$f(-4)=2(-4)^3+7(-4)^2-10(-4)-24$

$=-128+112+40-24=0$ M1

$(x+4)$ is a factor of f(x).

$2x^3+7x^2-10x-24=(x-2)(x+4)(2x+A)$ or by long division method M1

Compare coeff. ofconstant: $-24=-8A$

$A=3$

$\therefore f(x)=0 \to (x-2)(x+4)(2x+3)=0$

$x=2,-4,-\frac{3}{2}$ A1

Or

$$\begin{array}{r} 2x^2+11x+12 \\ x-2\overline{\big)\ 2x^3+7x^2-10x-24} \\ \underline{-(2x^3-4x^2)} \\ 11x^2-10x-24 \\ \underline{-(11x^2-22x)} \\ 12x-24 \\ \underline{-(12x-24)} \\ \underline{0} \end{array}$$ M1

$f(x)=(x-2)(2x^2+11x+12)$

$=(x-2)(2x+3)(x+4)$ M1

$\therefore f(x)=0 \to (x-2)(x+4)(2x+3)=0$

$x=2,-4,-\frac{3}{2}$ A1

(c) Using the substitution $x=y^2$,

$y^2=2,-4,-\frac{3}{2}$ M1

(na), (na)

$y=\pm\sqrt{2}$ A1

8 The roots of the equation $x^2-4x+2=0$ are α and β . Find

(a) the value of $\alpha^2+\beta^2$. [2]

(b) the quadratic equation in x whose roots are α^3 and β^3 . [3]

(a) $x^2-4x+2=0$

$\left.\begin{array}{l}\alpha+\beta=4\\ \alpha\beta=2\end{array}\right\}$ M1

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=4^2-2(2)$

$=12$ A1

(b)

$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$

$=4(12-2)$ M1

$=40$

$\alpha^3\beta^3=(\alpha\beta)(\alpha\beta)^2$

$=2(2)^2$ M1

$=8$

Equation is $x^2-40x+8=0$. A1

9 (a) Solve $|2x-5|-7x=4$. [3]

(b) Sketch the graph of $y=|x^2-3x|$ for $-2\le x\le 4$ and state the corresponding rangeof y. [4]

(c) By drawing a suitable line on the same axes, state the number of solutions to the equation $y=|x^2-3x|=x$ [2]

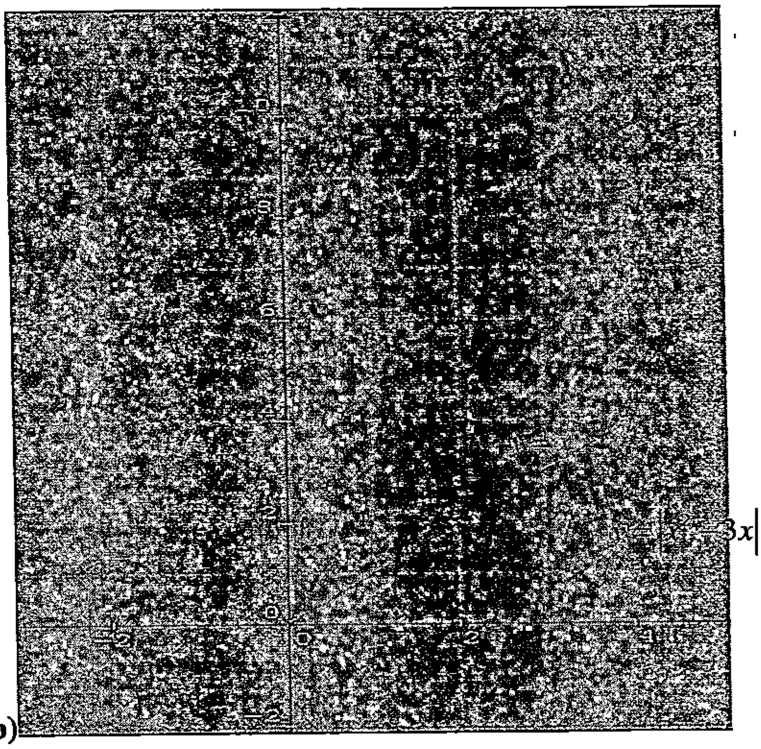
(a)

$|2x-5|-7x=4$

$|2x-5|=4+7x$ M1

$2x-5=4+7x,\quad 2x-5=-4-7x$ M1

$x=-\frac{9}{5}(na),\qquad x=\frac{1}{9}$ A1

(b) 

x and y-intercepts G1

Turning point G1

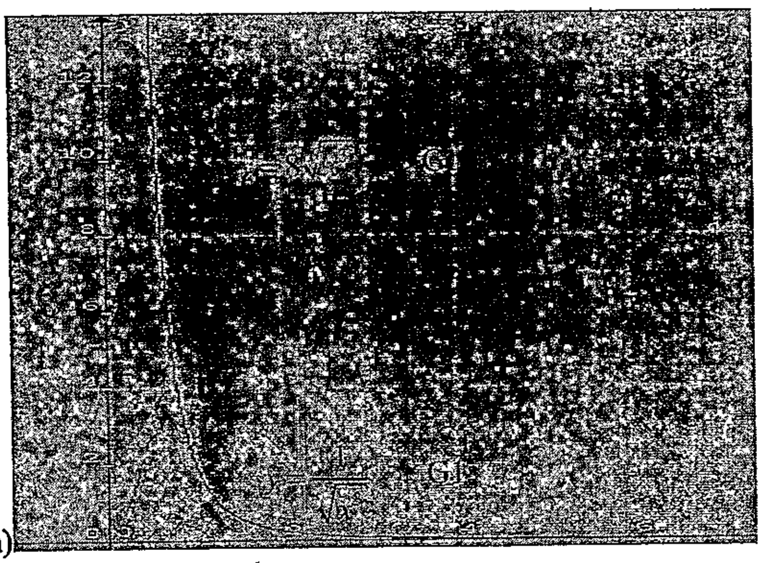
End points G1

Range of y: $0 \le y \le 10$ A1

(c) Number of solutions = 3 B1, Draw $y = x$ G1

10 **(a)** On the same axes, sketch the graphs of $y = \dfrac{1}{\sqrt{x^3}}$ and $y = 8\sqrt{x^3}$ for x>0. [2]

(b) Find the x-coordinate of the intersection point of the curves $y = \dfrac{1}{\sqrt{x^3}}$ and $y = 8\sqrt{x^3}$ in this interval. [2]

(a) 

(b)

$$x^{-\frac{3}{2}}=8x^{\frac{3}{2}}$$

$x^{-3}=8$ M1

$$\left(\frac{1}{x}\right)^3=2^3$$

$x=\frac{1}{2}$ A1

11 **(a) (i)** Obtain the binomial expansion of $(4-x)(1+\frac{1}{5}x)^6$, in ascending powers of x, asfaras the term in x^2. [4]

(ii) Hence, estimate the value of $3.25\times(1.15)^6$. [3]

(b) In the expansion of $(1+2x)^n$ in ascending powers of x, the coefficient of the third term is 60. Find thevalue of n, where n is a positive integer. [4]

(a)(i)

$$\left(1+\frac{1}{5}x\right)^6=1^6+\binom{6}{1}(1)^5\left(\frac{1}{5}x\right)^1+\binom{6}{2}(1)^4\left(\frac{1}{5}x\right)^2+\ldots$$

$=1+6\left(\frac{1}{5}x\right)+15\left(\frac{1}{25}x^2\right)+\ldots$ M1

$=1+\frac{6}{5}x+\frac{3}{5}x^2+\ldots$ M1

$(4-x)(1+\frac{1}{5}x)^6=(4-x)(1+\frac{6}{5}x+\frac{3}{5}x^2+\ldots)$ M1

$=4+\frac{24}{5}x+\frac{12}{5}x^2-x-\frac{6}{5}x^2+\ldots$

$=4+\frac{19}{5}x+\frac{6}{5}x^2+\ldots$ A1

(ii)

$4-x=3.25$

$x=0.75$ M1

$3.25\times(1.15)^6\approx4+\frac{19}{5}(0.75)+\frac{6}{5}(0.75)^2$ M1

$=7.525$ A1

(b)

3^{rd} term $= (2+1)^{th}$ term

$$= \binom{n}{2}(1)^{n-2}(2x)^2 \qquad \text{M1}$$

$$60 = \frac{n(n-1)}{2}(2)^2 \qquad \text{M1}$$

$$n^2 - n - 30 = 0$$

$$(n-6)(n+5) = 0 \qquad \text{M1}$$

$$n = 6, n = -5 \quad \text{(rejected)} \qquad \text{A1}$$