	Class	Register Number
Name :		

DUNEARN

DUNEARN SECONDARY SCHOOL
Mid Year Examination 2014
Additional Mathematics Paper (4047)

Secondary 3 Express

Friday **16 May 2014** **0800 – 1000 hrs** **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write your name, class and register number in the spaces provided on the answer paper.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The use of an electronic calculator is expected, where appropriate.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
You are reminded of the need for clear presentation in your answers.

For Examiner's Use									
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Parent's Signature:				**Total Score:**			**/ 80**		

Setter: Ms Yvonne Tan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}.$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}.$$

Formulae for $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A,$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions.

1 In 1990, Nancy deposited \$4000 in a mutual trust fund which pays an interest of 8.75% per annum, compounded half yearly. The amount \$$y$ at the end of n years is $y = 4000(1.04375)^{2n}$. Find the year in which the amount in the fund first reached \$20 000. [4]

2 (a) Solve the equation $(2^{3-4x})(4^{x+4}) = \frac{1}{2}$. [3]

(b) By using substitution or otherwise, find the values of x such that $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} = 4$. [3]

3 (a) Given that the roots of the quadratic equation $2x^2 + bx + c = 0$ are p and 2, show that $b = \frac{8-2p^2}{p-2}$. [2]

(b) Find the range of values of x for which $5 - x \le 2x^2 - 4x < 16$. [4]

4 It is given that the roots of the equation $2x^2 - 4x + 8 = 0$ are α and β.

(i) Calculate the value of $3\alpha^2 + 3\beta^2$. [4]

(ii) Construct a quadratic equation whose roots are $2\alpha + 1$ and $2\beta + 1$. [3]

5 It is given that $\frac{P(x)}{Q(x)} = \frac{2x^3 - 3x^2 - 4x + 5}{x^2 - x - 2}$.

(i) Explain why $\frac{P(x)}{Q(x)}$ is an improper fraction. [1]

(ii) The expression for $\frac{P(x)}{Q(x)}$ when written in proper fraction is

$ax + b + \frac{-x+3}{x^2 - x - 2}$. Find the values of a and of b. [3]

(iii) Hence, express $\frac{P(x)}{Q(x)}$ in partial fractions. [4]

6 (a) Solve the equation $\sqrt{10 + x^2 - \sqrt{3x+4}} - 3 = x$, giving your answers correct to 2 decimal places. [4]

(b) Express $\frac{2\sqrt{3} - 1}{2\sqrt{3} - 2} + \frac{4}{\sqrt{2}}$ in the form $p + q\sqrt{2} + r\sqrt{3}$, where p, q and r are rational numbers. [4]

7 (a) Given that $5x^3+18x^2+7x+4 \equiv (Ax-2)(x+B)(x+1)+C$, for all values of x, find the values of A, B and C. [3]

(b) Given that $f(x)=3x^5-11x^3+30x^2+39=(x-1)(x+3)Q(x)+ax+b$ for all values of x and that $Q(x)$ is a polynomial,

(i) find the values of a and of b, [4]

(ii) find the remainder when $f(x)+2$ is divided by x^2+2x-3. [2]

8 (a) Given that $x=3^k$, express $\log_x 9+\log_{\sqrt{3}} x$ in terms of k. [4]

(b) Solve without the use of a calculator $(2x)^{\lg 2}=(5x)^{\lg 5}$. [6]

9 (a) Find the range of values of a for which $ax^2-6x-2<0$ for all values of x. [3]

(b) Show that $x(3-x)=1-k(x-2)$ has two real and distinct roots for all values of k. [4]

(c) Find the range of values of k for which the line $y=kx+2$ will not meet the curve $x^2+y^2=2$. [4]

10 (a) Solve the equation $3|x+1|=4x+11$. [3]

(b) The equation of the curve is given by $y=9\left(x-\frac{1}{3}\right)^2-16$.

(i) State the coordinates of the turning point of the curve. [1]

(ii) Sketch the curve $y=\left|9\left(x-\frac{1}{3}\right)^2-16\right|$. [4]

(iii) Find and justify in words, or with an aid of a diagram, the number of solutions to the equation $mx+15=\left|9\left(x-\frac{1}{3}\right)^2-16\right|$ where

(a) $m=-9$, [2]

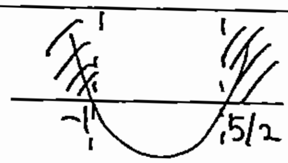
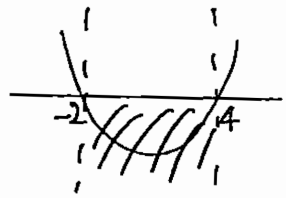
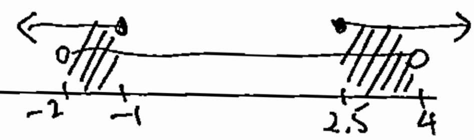
(b) $-9<m<0$. [1]

End of Paper

MYE/Ms YVT/2014

Solutions to 3E A Math 2014

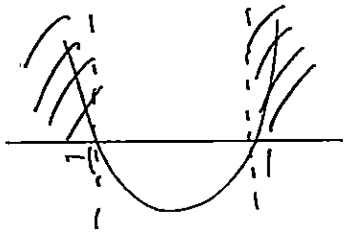
1		$20000=4000(1.04375)^{2n}$ $5=(1.04375)^{2n}$ $\lg 5=2n\lg(1.04375)$ $n=18.793$ $1990+18$ or $19=2008$ or 2009 Year 2008 or 2009	M1 – for simplifying both sides M1 – for lg on both sides *(award M1 lg on both sides without first step of simplification)* M1 – for correct value of n A1
Qn 1		Subtotal	4
2	(a)	$\left(2^{3-4x}\right)\left(4^{x+4}\right)=\frac{1}{2}$ $2^{3-4x+2x+8}=2^{-1}$ $-2x+11=-1$ $-2x=-12$ $x=6$	M1 – for simplifying indices to one term on the LHS (ok to leave as ½ on RHS) M1 – for comparing powers *(ecf allowed)* A1
2	(b)	$x^{\frac{2}{3}}-3x^{\frac{1}{3}}=4$ Let $u=x^{\frac{1}{3}}$ $u^2-3u-4=0$ $(u-4)(u+1)=0$ $u=4$ or $u=-1$ $x^{\frac{1}{3}}=4$ or $x^{\frac{1}{3}}=-1$ $x=64$ or $x=-1$	M1 – for quadratic expression M1 – for correct values of $x^{\frac{1}{3}}$ A1
Qn 2		Subtotal	7
3	(a)	$2p^2+bp+c=0$ --- (1) $8+2b+c=0$ --- (2) Sub $c=-8-2b$ into (1): $2p^2+bp-8-2b=0$ $bp-2b=8-2p^2$ $b(p-2)=8-2p^2$ $b=\frac{8-2p^2}{p-2}$ (shown) ***Or*** Sum of roots: $p+2=-\frac{b}{2}$ $b=-2p-4$ $b=\frac{(-2p-4)(p-2)}{p-2}=\frac{8-2p^2}{p-2}$	M1 – for obtaining equation (1) and (2) A1 ***Or*** M1 – for obtaining $b=-2p-4$ A1
3	(b)	$5-x\le 2x^2-4x<16$ $2x^2-3x-5\ge 0$ $(2x-5)(x+1)\ge 0$	M1 – for obtaining $2x^2-3x-5\ge 0$ and $2x^2-4x-16<0$

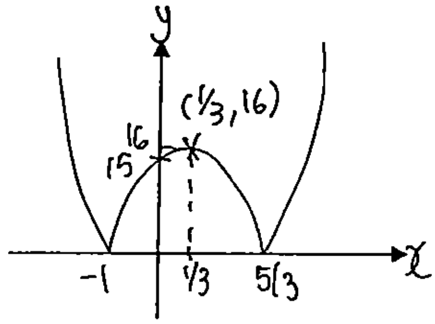
		 $x \le -1, x \ge 2.5$ $2x^2 - 4x - 16 < 0$ $x^2 - 2x - 8 < 0$ $(x-4)(x+2) < 0$  $-2 < x < 4$  $-2 < x \le -1$ or $2.5 \le x < 4$	M1 – for obtaining $x \le -1, x \ge 2.5$ M1 – for obtaining $-2 < x < 4$ A1
Qn 3		Subtotal	6
4	(i)	$2x^2 - 4x + 8 = 0$ $\alpha + \beta = 2$ $\alpha\beta = 4$ $3\alpha^2 + 3\beta^2$ $= 3(\alpha^2 + \beta^2)$ $= 3\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$ $= 3\left[2^2 - 2(4)\right]$ $= -12$	M1 – correct sum of roots M1 – correct product of roots M1 – correct expression of $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ A1
4	(ii)	Sum of roots $= 2\alpha + 1 + 2\beta + 1 = 2(\alpha+\beta) + 2 = 6$ Product of roots $= (2\alpha+1)(2\beta+1)$ $= 4\alpha\beta + 2\alpha + 2\beta + 1$ $= 4\alpha\beta + 2(\alpha+\beta) + 1$ $= 4(4) + 2(2) + 1 = 21$ $x^2 - 6x + 21 = 0$	M1 *(ecf allowed for incorrect values of $\alpha\beta$ and $\alpha+\beta$ from part i)* M1 *(ecf allowed for incorrect values of $\alpha\beta$ and $\alpha+\beta$ from part i)* A1
Qn 4		Subtotal	7
5	(i)	The degree in the numerator is greater than the denominator.	B1 – the use of "degree".

5	**(ii)**	$\dfrac{2x^3-3x^2-4x+5}{x^2-x-2}$ $\begin{array}{r} 2x-1 \\ x^2-x-2\,/\,2x^3-3x^2-4x+5 \\ -\underline{(2x^3-2x^2-4x)} \\ -x^2+5 \\ -\underline{(-x^2+x+2)} \\ -x+3 \end{array}$ $a=2, b=-1$	B1 – long division is used and value of remainder is obtained from long division. B1 – for value of *a* B1 – for value of *b*
5	**(iii)**	$\dfrac{-x+3}{x^2-x-2}$ $=\dfrac{-x+3}{(x-2)(x+1)}=\dfrac{A}{x-2}+\dfrac{B}{x+1}$ $-x+3=A(x+1)+B(x-2)$ Sub $x=-1,\ B=-\dfrac{4}{3}$ Sub $x=2,\ A=\dfrac{1}{3}$ $\dfrac{P(x)}{Q(x)}=2x-1+\dfrac{1}{3(x-2)}-\dfrac{4}{3(x+1)}$	M1 – for splitting partial fraction correctly in terms of *A* and *B*. A1 – correct value for *A* A1 – correct value for *B* A1 – overall expression correctly written *(ecf allowed for wrongly calculated values of a and b from part ii but not for values of A and B)*
Qn 5		**Subtotal**	**8**
6	**(a)**	$\sqrt{10+x^2-\sqrt{3x+4}}=3+x$ $10+x^2-\sqrt{3x+4}=9+6x+x^2$ $(1-6x)^2=3x+4$ $36x^2-12x+1=3x+4$ $36x^2-15x-3=0$ $12x^2-5x-1=0$ $x=\dfrac{5\pm\sqrt{25+48}}{24}$ $=-0.15$ or 0.56	M1 – for 'removing outer square root' M1 – for 'removing inner square root' *(ecf allowed)* M1 – for obtaining quadratic equation and showing working in solving *(ecf allowed)* A1 – for both answers
6	**(b)**	$\dfrac{2\sqrt{3}-1}{2\sqrt{3}-2}+\dfrac{4}{\sqrt{2}}$ $=\dfrac{(2\sqrt{3}-1)(2\sqrt{3}+2)}{\left(2\sqrt{3}\right)^2-2^2}+\dfrac{4\sqrt{2}}{2}$	M1 – for rationalising first term corectly M1 – for rationalising second term correctly

		$=\dfrac{12+2\sqrt{3}-2}{8}+\dfrac{4\sqrt{2}}{2}$ $=\dfrac{5+\sqrt{3}}{4}+2\sqrt{2}$ $=1\dfrac{1}{4}+2\sqrt{2}+\dfrac{1}{4}\sqrt{3}$	M1 – for simplifying the first term by opening brackets in numerator and evaluating denominator *(ecf allowed)* A1 – correct form
Qn 6	**Subtotal**		**8**
7	**(a)**	$5x^3+18x^2+7x+4\equiv(Ax-2)(x+B)(x+1)+C,$ Comparing x^3 coefficient: $A=5$ Comparing constant: $4=-2B+C$ Comparing x coefficient: $7=AB-2-2B$ $5B-2-2B=7$ $3B=9$ $B=3$ $C=4+2(3)=10$	B1 (accept substitution method) B1 B1
7	**(bi)**	$3x^5-11x^3+30x^2+39=(x-1)(x+3)Q(x)+ax+b$ Sub $x=1$, $61=a+b$ --- (1) Sub $x=-3$, $-123=-3a+b$ --- (2) Sub (1) into (2), $-123=-3(61-b)+b$ $-123+183=3b+b$ $b=15$ $a=46$	M1 – for obtaining equation M1 – for obtaining equation M1 – for using substitution or elimination method *(ecf allowed)* A1 – for both answers
7	**(bii)**	Since $(x-1)(x+3)=x^2+2x-3$, Remainder $=46x+15+2$ $=46x+17$	M1 A1 or B2 *(ecf for values of a and b allowed for both marks)*
Qn 7	**Subtotal**		**9**
8	**(a)**	$x=3^k$ $\log_3 x=k$ $\log_x 9+\log_{\sqrt{3}} x$ $=\dfrac{\log_3 9}{\log_3 x}+\dfrac{\log_3 x}{\log_3\sqrt{3}}$ $=\dfrac{2\log_3 3}{\log_3 x}+\dfrac{\log_3 x}{\frac{1}{2}\log_3 3}$ $=\dfrac{2}{k}+\dfrac{k}{\frac{1}{2}}$ $=\dfrac{2}{k}+2k$	M1 – for conversion to log form M1 – for changing of log to base 3 for both terms M1 – for applying power law A1

8	(b)	$(2x)^{\lg 2}=(5x)^{\lg 5}$ $\lg(2x)^{\lg 2}=\lg(5x)^{\lg 5}$ $(\lg 2)\lg 2x=(\lg 5)\lg 5x$ $(\lg 2)(\lg 2+\lg x)=(\lg 5)(\lg 5+\lg x)$ $(\lg 2)^2+\lg 2\lg x=(\lg 5)^2+\lg 5\lg x$ $\lg 2\lg x-\lg 5\lg x=(\lg 5)^2-(\lg 2)^2$ $\lg x(\lg 2-\lg 5)=(\lg 5)^2-(\lg 2)^2$ $\lg x=\dfrac{(\lg 5-\lg 2)(\lg 5+\lg 2)}{\lg 2-\lg 5}$ $\lg x=-(\lg 5+\lg 2)$ $\lg x=-\lg 10$ $x=\dfrac{1}{10}$	M1 – for lg both sides and apply power law M1 – for product law M1 – for placing terms with x on one side M1 – for $\lg x$ as the subject M1 – for applying a^2-b^2 A1
Qn 8		Subtotal	10
9	(a)	$ax^2-6x-2<0$ for all x $a<0$ Since there are no real roots, Discriminant <0 $(-6)^2-4a(-2)<0$ $36+8a<0$ $8a<-36$ $a<-4.5$	M1 – for stating $a<0$ M1 – for substituting correct values into discriminant A1
9	(b)	$3x-x^2+kx-2k-1=0$ $x^2+(-k-3)x+(2k+1)=0$ Discriminant $=(-k-3)^2-4(2k+1)$ $=k^2+6k+9-8k-4$ $=k^2-2k+5$ $=(k-1)^2-1+5$ $=(k-1)^2+4$ Since $(k-1)^2\geq 0$, $(k-1)^2+4>0$	*(Note: Award only maximum of 1 mark for substituing correct values into discriminant, if student writes the inequality sign at the start of question instead of showing)* M1 – for substituting correct values into discriminant M1 – for obtaining quadratic equation *(allow ecf)* M1 – for completing the square *(allow ecf)* A1 – with explanation
9	(c)	$y=kx+2$ --- (1) $x^2+y^2=2$ --- (2) $x^2+(kx+2)^2=2$ $x^2+k^2x^2+4kx+4-2=0$ $(1+k^2)x^2+4kx+2=0$ Discriminant <0	M1 – for expanding and writing equation into this form

		$(4k)^2-4(1+k^2)(2)<0$ $16k^2-8-8k^2<0$ $k^2-1<0$ $(k+1)(k-1)<0$  $-1<k<1$	M1 – for substituting correct values into discriminant, and $D<0$ M1 – for obtaining $k^2-1<0$ A1
Qn 9		Subtotal	11
10	(a)	$3\lvert x+1\rvert=4x+11$ $\lvert x+1\rvert=\dfrac{4x+11}{3}$ $x+1=\dfrac{4x+11}{3}$ or $x+1=\dfrac{-4x-11}{3}$ $3x+3=4x+11$ or $3x+3=-4x-11$ $x=-8$ (reject) or $x=-2$.	M1 A1 – for obtaining and rejecting -8 , A1 *(Note: Allow awarding of maximum of B1 for solving one side of the equation)*
10	(bi)	$\left(\dfrac{1}{3},-16\right)$	B1
10	(bii)	$y=\left\lvert 9\left(x-\dfrac{1}{3}\right)^2-16\right\rvert$ When $x=0$, $y=\left\lvert 9\left(0-\dfrac{1}{3}\right)^2-16\right\rvert=15$ When $y=0$, $0=\left\lvert 9\left(x-\dfrac{1}{3}\right)^2-16\right\rvert$ $\dfrac{16}{9}=\left(x-\dfrac{1}{3}\right)^2$ $\pm\dfrac{4}{3}=x-\dfrac{1}{3}$ $x=-1, x=\dfrac{5}{3}$	

			D1 – Correct x-intercepts D1 – Correct y-intercept *(Allow D1 if y-intercept is -15)* D1 – Correct turning point *(Allow D1 if the turning point is (1/3,-16))* D1 – Mod graph
10	**biii**	(a) $y=-9x+15$ When $y=0, x=5/3$ The straight line cuts the curve at $\left(\frac{5}{3},0\right)$ and $(0,15)$, and cuts the curve at a third point on the left side of the curve. Number of solutions = 3 (b) While the y-intercept remains at $(0,15)$, the straight line will be gentler in the negative direction and cuts at 4 points. Number of solutions = 4	M1 (or sketch line $y=-9x+15$ in bii) A1 A1
		Subtotal	11