1 Find the value of x for which $\frac{3^{2x-1}}{\sqrt{9^x}} = \frac{1}{27}$. [2]

2 Find the range of values of k for which $y = (k+2)x^2 - 12x + 2(k-1)$ cuts the x-axis at two distinct points and has a minimum point. [4]

3 The expression $x^3 + ax^2 + bx - 6$ is divisible by $(x+3)$ but leaves a remainder of -8 when divided by $(x-1)$.

(i) Find the value of a and of b. [4]

(ii) Factorise the expression completely. [2]

4 **(i)** Given that $(3-\sqrt{7})x = 3+\sqrt{7}$, express x in the form $a+b\sqrt{7}$ where a and b are constants. [3]

(ii) Hence, evaluate $x + \frac{1}{x}$ without using a calculator. [3]

5 **(i)** Solve the simultaneous equations

$$x = 1+2y,$$

$$x = 6xy - 2y - 3.$$ [5]

(ii) Explain the geometrical meaning of the solutions. [1]

6 Given that $\frac{4x^3+7x^2-13x-2}{x^2+2x-3} = ax+b+\frac{x+c}{x^2+2x-3}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using the values of a, b and c obtained in part (i),

(ii) express $\frac{4x^3+7x^2-13x-2}{x^2+2x-3}$ in partial fractions. [3]

[Turn over

7 The roots of the equation $2x^2 - 4x + 3 = 0$ are α and β.

(i) Find in the form $ax^2 + bx + c = 0$, where a, b and c are integers, the equation whose roots are $\alpha\beta^2$ and $\alpha^2\beta$. [5]

(ii) Find the exact value of $\alpha^3 + \beta^3$ without finding α and β. [3]

8 By using the substitution $u = 2^x$ or otherwise,

(i) find the values of x such that $2^{2x+1} - 7 \times 2^x + 3 = 0$, [5]

(ii) prove that $2^{x+3} - 2^x$, is exactly divisible by 7. [2]

9 Given that $y = mx + c$ is a tangent to the curve $y^2 = 10x$, find the relationship between m and c. [4]

10 (a) Given that $\dfrac{1}{\log_a ab} + \dfrac{1}{\log_b ab} = a - b$, express b in terms of a. [3]

(b) Solve the equation $\lg(20 + 5x) - \lg(10 - x) = 1$. [3]

11 (a) Sketch the graph of $y = \log_3 x$ [2]

(b) It is observed that the population of a city is growing in such a way that its population, P, satisfies the formula $P = 500\,000\, e^{kt}$, where k is a constant and t is the time in years from beginning of 1990.

(i)If the population of the city was 700 000 in the beginning of 2005, find the value of k. [1]

(ii)Find the population in the beginning of 2010, giving your answer correct to the nearest 10 000. [2]

(iii)Find the year in which the population will first reach 1.5 million. [2]

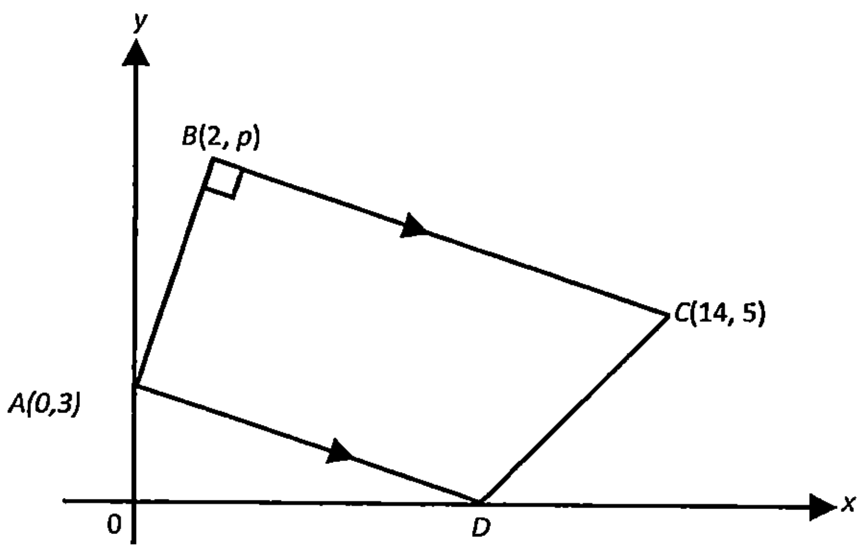
12 A piece of wire, 360 cm long, is used to make the twelve edges of a rectangular box in which the length is twice the breadth.

(i) Denoting the breadth of the box by x cm and the height by h cm, express h in terms of x and show that the volume of the box, V cm^3, is given by

$$V = 180x^2 - 6x^3$$ [3]

(ii) Determine, by solving a cubic equation, the dimensions of the box so that the volume is 24000 cm^3. [4]

13



The diagram shows a trapezium $ABCD$. The coordinates of A, B and C are (0, 3), (2, p) and (14, 5) respectively. AB is perpendicular to BC and AD is parallel to BC.

(i) Show that $p = 9$. [2]

(ii) Find the equation of the perpendicular bisector of AB. [4]

(iii) Find the coordinates of D. [2]

(iv) Calculate the area of trapezium$ABCD$. [2]

3Exp AM (4047) - Marking Scheme

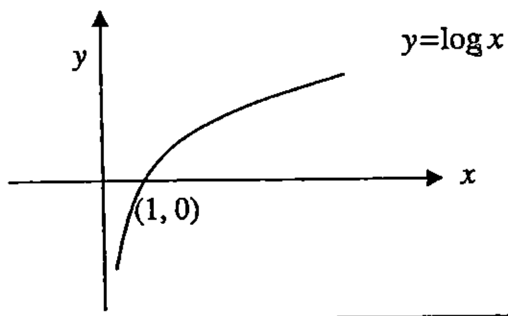
Question	Solution	Marks
1	$\dfrac{3^{2x-1}}{3^x}=\dfrac{1}{3^3}$ $3^{x-1}=3^{-3}$ $x-1=-3$ $x=-2$	M1A1[2]
2	$(-12)^2-4(k+2)(2k-2)\square\ 0$ $144-8k^2-8k+16\square\ 0$ $-8k^2-8k+160\square\ 0$ $k^2+k-20\square\ 0$ $(k+5)(k-4)\square\ 0$ $-5\square\ k\square\ 4$ $k+2\square\ 0$ (minimum point) $k\square\ -2$ $\therefore -2\square\ k\square\ 4$	M1 M1A1 A1 [4]
3	(i) Let $f(x)=x^3+ax^2+bx-6$ $f(-3)=0\Rightarrow(-3)^3+a(-3)^2+b(-3)-6=0$ $-27+9a-3b-6=0$ $3a-b=11$ ——— (1) $f(1)=-8\Rightarrow 1+a+b-6=-8$ $a+b=-3$ ——— (2) $(1)+(2)\Rightarrow 4a=8$ $a=2$ $b=-5$	M1 M1 A2 [4]

	(ii) $$\begin{array}{r} x^2-x-2 \\ x+3\overline{)x^3+2x^2-5x-6} \\ \underline{x^3+3x^2} \\ -x^2-5x \\ \underline{-x^2-3x} \\ -2x-6 \\ \underline{-2x-6} \\ 0 \end{array}$$ $$x^3+2x^2-5x-6=(x+3)(x^2-x-2)$$ $$=(x+3)(x-2)(x+1)$$	M1 A1 [2]
4	(i) $x=\frac{3+\sqrt{7}}{3-\sqrt{7}}$ $=\frac{(3+\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$ $=\frac{9+6\sqrt{7}+7}{3^2-(\sqrt{7})^2}$ $=\frac{16+6\sqrt{7}}{2}$ $=8+3\sqrt{7}$ (ii) $x+\frac{1}{x}=8+3\sqrt{7}+\frac{1}{8+3\sqrt{7}}$ $=8+3\sqrt{7}+\frac{8-3\sqrt{7}}{8^2-(3\sqrt{7})^2}$ $=8+3\sqrt{7}+\frac{8-3\sqrt{7}}{64-63}$ $=8+3\sqrt{7}+8-3\sqrt{7}$ $=16$	B1 M1 A1 [3] M1 M1A1[3]

5(i)	$1+2y=6y(1+2y)-2y-3$	M1
	$1+2y=6y+12y^2-2y-3$ $12y^2+2y-4=0$ $6y^2+y-2=0$ $(3y+2)(2y-1)=0$	M1
	$y=-\dfrac{2}{3} \quad\quad y=\dfrac{1}{2}$ $\downarrow \quad\quad\quad \downarrow$ $x=-\dfrac{1}{3} \quad\quad x=2$	M1
	$\therefore x=2, y=\dfrac{1}{2}$ or $x=-\dfrac{1}{3}, y=-\dfrac{2}{3}$	A2 [5]
5(ii)	The line $x = 1 + 2y$ intersects the curve $x = 6xy - 2y - 3$ at the points $(2,\frac{1}{2})$ and $(-\frac{1}{3},-\frac{2}{3})$	B1 [1]
6(i)	$\begin{array}{r} 4x-1 \\ x^2+2x-3\overline{)4x^3+7x^2-13x-2} \\ \underline{4x^3+8x^2-12x} \\ -x^2-x-2 \\ \underline{-x^2-2x+3} \\ \underline{x-5} \end{array}$	M1
	$\dfrac{4x^3+7x^2-13x-2}{x^2+2x-3}=4x-1+\dfrac{x-5}{x^2+2x-3}$ $a=4, b=-1, c=-5$	A3 [4]

6(ii)	$x^2+2x-3=(x+3)(x-1)$ Let $\dfrac{x-5}{(x+3)(x-1)}=\dfrac{A}{x+3}+\dfrac{B}{x-1}$ $x-5=A(x-1)+B(x+3)$ $x=1$ $1-5=4B$ $B=-1$ $x=-3$ $-3-5=-4A$ $A=2$ $\therefore \dfrac{4x^3+7x^2-13x-2}{x^2+2x-3}=4x-1+\dfrac{2}{x+3}-\dfrac{1}{x-1}$	M1 M1 A1 [3]
7(i)	$\alpha+\beta=2 \qquad \alpha\beta=\dfrac{3}{2}$ $\alpha\beta^2+\alpha^2\beta=\alpha\beta(\alpha+\beta)$ $=2\times\dfrac{3}{2}$ $=3$ $(\alpha\beta^2)(\alpha^2\beta)=(\alpha\beta)^3=\left(\dfrac{3}{2}\right)^3=\dfrac{27}{8}$ $\therefore$ The equation is $x^2-3x+\dfrac{27}{8}=0$ $8x^2-24x+27=0$	B1 B1 M1 M1 A1 [5]
7(ii)	$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)\left[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta\right]$ $=(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$ $=2\left[2^2-3\times\dfrac{3}{2}\right]$ $=2\left(4-\dfrac{9}{2}\right)$ $=-1$	B1 M1 A1[3]

8(i)	$2^{2x+1}-7\times2^x+3=0$ $2^{2x}\times2^1-7\times2^x+3=0$ Let $u=2^x$ $2u^2-7u+3=0$ $(2u-1)(u-3)=0$ $u=\frac{1}{2}$ or $u=3$ $2^x=2^{-1}$ $2^x=3$ $x=-1$ $x\lg2=\lg3$ $x=\frac{\lg3}{\lg2}=1.58$ $x=-1, 1.58$	M1A1 M1 A2 [5]
8(ii)	$2^{x+3}-2^x=2^x\times2^3-2^x$ $=2^x(2^3-1)$ $=2^x\times7$ $\therefore 2^{x+3}-2^x$ is exactly divisible by 7	M1 A1 [2]
9	$(mx+c)^2=10x$ $m^2x^2+2mxc+c^2-10x=0$ $m^2x^2+(2mc-10)x+c^2=0$ $(2mc-10)^2-4m^2c^2=0$ $4m^2c^2-40mc+100-4m^2c^2=0$ $40mc=100$ $mc=\frac{5}{2}$	M1 M1 M1A1[4]
10(a)	$\frac{1}{\log_a ab}+\frac{1}{\log_b ab}=a-b$ $\log_{ab}a+\log_{ab}b=a-b$ $\log_{ab}ab=a-b$ $1=a-b$ $b=a-1$	B1 B1 A1

10(b)	$\lg(20+5x)-\lg(10-x)=1$ $\lg\left(\dfrac{20+5x}{10-x}\right)=\lg 10$ $\dfrac{20+5x}{10-x}=10$ $20+5x=100-10x$ $15x=80$ $x=\dfrac{16}{3}$	B1 M1 A1 [3]
11(a)		B210
11(b)	(b) (i) $700000=500000e^{15k}$ $\ln\dfrac{7}{5}=15k$ $k=0.022431482$ $k=0.0224$ (3 s.f.) (ii) population in the beginning of 2010 $=500000e^{0.022431482\times 20}$ $=783082.2525$ $=780000$ (correct to the nearest 10000) (iii) $1.5=0.5e^{kt}$ $3=e^{0.022431482t}$ $\ln 3=0.022431482t$ $t=\dfrac{\ln 3}{0.022431482}=48.976$ $\therefore$ The year is 2038 (2039 also accepted in this case as the value of t is very close to 49)	A1 [1] M1 A1 [2] M1 A1 [2]

12(i)	$4(2x+x+h)=360$ $3x+h=90$ $h=90-3x$ $V=(2x)(x)(90-3x)$ $=2x^2(90-3x)$ $=180x^2-6x^3$	B1 M1 A1[3]
12(ii)	$180x^2-6x^3=24000$ $-6x^3+180x^2-24000=0$ $x^3-30x^2+4000=0$ Let $f(x)=x^3-30x^2+4000$ $f(20)=20^3-30(20)^2+4000=0$ $(x-20)$ is a factor $\begin{array}{r} x^2-10x-200 \\ x-20\overline{)x^3-30x^2+0x+4000} \\ \underline{x^3-20x^2} \\ -10x^2+0x \\ \underline{-10x^2+200x} \\ -200x+4000 \\ \underline{-200x+4000} \end{array}$ $(x-20)(x^2-10x-200)=0$ $(x-20)(x-20)(x+10)=0$ $x=20, 20, -10$ (NA) $\therefore x=20$ Length = 40 cm, Breadth = 20 cm and Height = 30 cm	 M1 M1 A1 A1 [4]

13(i)	$\frac{5-p}{12}\times\frac{p-3}{2}=-1$ $(5-p)(p-3)=-24$ $5p-p^2-15+3p=-24$ $p^2-8p-9=0$ $(p+1)(p-9)=0$ $p=-1$ or $p=9$ (NA $\therefore P=9$	M1 A1[2]
13(ii)	A (0,3) and B (2,9) Midpoint of AB is (1,6) $m_{AB}=\frac{9-3}{2-0}=3$ Gradient of perpendicular bisector = $-\frac{1}{3}$ Equation of the perpendicular bisector is $y-6=-\frac{1}{3}(x-1)$ $3y-18=-x+1$ $x+3y=19$	B1 M1 M1 A1[4]
12(iii)	Equation of AD is $y=-\frac{1}{.3}x+3$ When $y=0$ $0=-\frac{1}{3}x+3$ $x=9$ $\therefore D(9,0)$	 M1 A1[2]
12(iv)	Area of ABCD = $\frac{1}{2}\begin{vmatrix}0 & 9 & 14 & 2 & 0\\ 3 & 0 & 5 & 9 & 3\end{vmatrix}$ $=\frac{1}{2}[(0+45+126+6)-(27+0+10+0)]$ $=70\ units^2$	M1 A1 [2]