聖嬰中學

# HOLY INNOCENTS' HIGH SCHOOL

## MID-YEAR EXAMINATION 2014
## SECONDARY 3 EXPRESS

## ADDITIONAL MATHEMATICS 4047

**Name** : ____________________

**Register No** : ____________________

**Class** : ____________________

**Date** : 12 May 2014

**Duration** : 2 h

**Additional Materials needed**
Writing Paper (10 sheets)

**Instructions to Candidates**

Write your register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
*Calculators should be used where appropriate.*
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Setter: Ms Koh Swee Kun

**This paper consists of 5 printed pages, inclusive of this cover page.**

*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

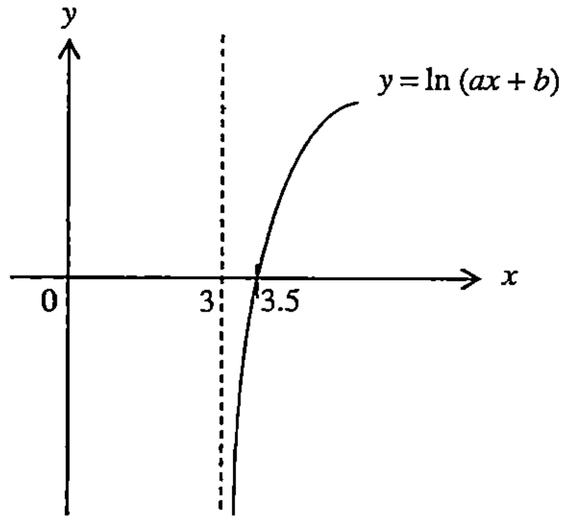
For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Answer all the questions.**

1 Find the set of values of $x$ for which $\dfrac{2x-5}{x-3} \leq 1$. [3]

2 The graph of $y = \ln(ax + b)$ is shown below. Find the value of $a$ and of $b$. [4]




3 The remainder when $x^3 + 3x^2 - 2x + c$ is divided by $(x+2)$ is twice the remainder when it is divided by $(x-3)$. Find the value of $c$. [4]

4 An object of temperature of $Y$ °C was taken out of the oven. It is then left to cool. Its temperature, $X$ °C, when it has left to cool for time $t$ minutes, is given by the equation

$$X = 27 + 63e^{-\frac{t}{8}}.$$

(i) Find the value of $Y$. [1]

(ii) Find the amount of time required for the object to cool to half of its initial temperature. Express your answer correct to the nearest minute. [2]

(iii) State the value that $X$ approaches as $t$ becomes very large. Explain the significance of this value of $X$. [2]

5 Find the coordinates of the points of intersection of the line $2x + 3y = 6$ and the curve $(2x+1)^2 + 6(y-2)^2 = 49$. [5]

6 (a) Show that $2^{n+4}+2^{n}-4(2^{n-1})$ is a multiple of 3 for all positive integer values of $n$. [3]

(b) Given that $3^{2x+3}\times5^{2x}=27^{x}\times125^{x+1}$, evaluate $15^{x}$. [3]

7 Given that the roots of $2x^2+6x+1=0$ are $\alpha$ and $\beta$, find a quadratic equation, with integer coefficients, whose roots are $(\alpha+2\beta)$ and $(2\alpha+\beta)$. [7]

8 (a) Find the range of values of $m$ for which the line $y=mx+6$ does not meet the curve $2x^2-xy-3=0$. [3]

(b) (i) Prove that the roots of $2qx^2+px=2q-p$ are real. [4]

(ii) **Hence** find the relationship between $p$ and $q$ for which the equation $2qx^2+px=2q-p$ has equal roots. [1]

9 Solve

(i) $\lg(1-2x)-2\lg x=1-\lg(2-5x)$. [4]

(ii) $\log_5 x+1=\log_{\sqrt{5}} x+2$, [3]

(iii) $4^{\log_4 x}=8$. [2]

10 (a) (i) Show that $\dfrac{1}{\sqrt{n}+\sqrt{n+1}}=\sqrt{n+1}-\sqrt{n}$ where $n>0$. [2]

(ii) **Hence** find the value of

$$\frac{2}{\sqrt{36}-\sqrt{37}}-\frac{2}{\sqrt{37}-\sqrt{38}}-\frac{2}{\sqrt{38}-\sqrt{39}}-\dots-\frac{2}{\sqrt{47}-\sqrt{48}}-\frac{2}{\sqrt{48}-\sqrt{49}}.$$ [2]

(b) Solve the equation $\sqrt{x}+\sqrt{x-3}=\sqrt{2x+1}$. [5]

11 (i) Given that $2x^2-5x-3$ is a factor of $2x^4-ax^3-2x^2+13x+b$, show that $a=7$ and $b=6$. [6]

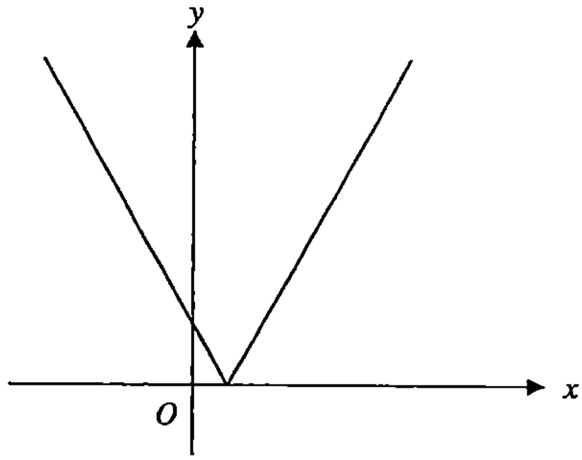
(ii) Hence solve the equation

$$2x^4-7x^3-2x^2+13x+6=0.$$ [3]

**12** (a) Solve the equation $|x+4|+3=2x$. [3]

(b) Sketch the curve $y=|-x^2+2x+3|$, showing clearly the coordinates of the maximum point and the points where the curve meets the axes. [3]

(c) The diagram below shows part of the graph of $y=|2x-1|$.




(i) Given that the line $y=mx+3$ intersects the graph $y=|2x-1|$ at only one point. Write down the possible values of $m$. [3]

(ii) Determine the number of points of intersection between the graph of $y=|2x-1|$ and the line $y=x+c$ where $c>0$. Explain your answer. [2]

***End of Paper***

**Holy Innocents' High School**
**End-of-Year Exam 2014**
**Secondary 3 Express Additional Mathematics**

1. $2 \le x < 3$

2. $a = 2, b = -6$

3. $c = -88$

4. (i) 90°C
   (ii) 10 minutes
   (iii) 27°C. The temperature of the environment in which the object is allowed to cool to is 27°C.

5. $\left(2\frac{2}{5}, \frac{2}{5}\right)$ and $(-3, 4)$.

6. (a)
$2^{n+4} + 2^n - 4(2^{n-1})$
$= 2^n \bullet 2^4 + 2^n - 2^{n+1}$ --- *M2 (Split* $2^{n+4} = 2^n \bullet 2^4$ *and simplify* $4(2^{n-1}) = 2^{n+1}$*)*
$= 2^n \bullet 2^4 + 2^n - 2^n \bullet 2$
$= 2^n(16+1-2)$
$= 2^n(15)$
$= 3(5)(2^n)$
***Showing 3 as a factor <u>or</u> for writing the following:Since 15 is a multiple of 3,*** $2^n(15)$ ***is also a multiple of 3***

   (b) $15^x = \frac{27}{125}$

7. $2x^2 + 18x + 37 = 0$

8. (a) $m > 5$
   (b) (i) Since $(p-4q)^2 \ge 0$, therefore, discriminant $\ge 0$.
   $\Rightarrow$ the roots are real.
   (ii) $p = 4q$

9. (i) $x = \frac{2}{9}$
   (ii) $x = \frac{1}{5}$ or 0.2
   (iii) $x = 8$

10. (a) (i)

$$= \frac{1}{\sqrt{n}+\sqrt{n+1}} \times \frac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}}$$ --- *M1 (Rationalise denominator)*

$$= \frac{\sqrt{n}-\sqrt{n+1}}{n-(n+1)}$$

$$= \frac{\sqrt{n}-\sqrt{n+1}}{-1}$$ --- *A1 (Expand denominator and arrive at denominator = –1)*

$= \sqrt{n+1}-\sqrt{n}$ (Shown)

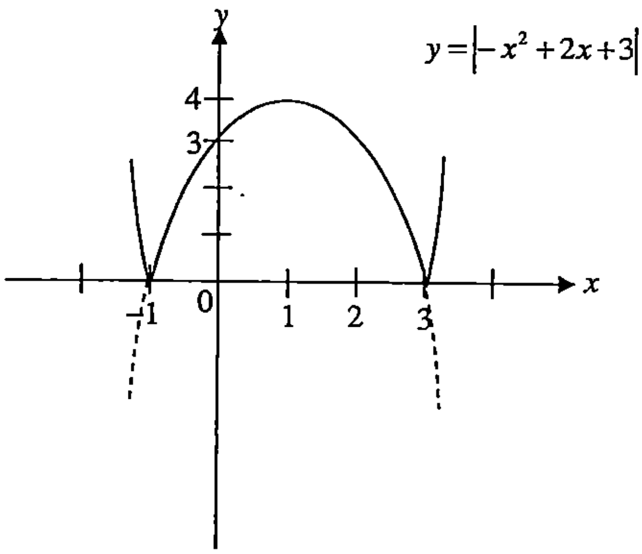
(ii) $4\sqrt{37}-26$ or $-1.67$

(b) $x=4$

11. (ii) $x=-\frac{1}{2}, 3, 2, -1$

12 (a) $x=7$

(b)




(c) (i)

Knowing that the vertical intercept (y-intercept) of the graph $y = mx + 3$ is 3 and is above the vertical-intercept of the graph $y=|2x-1|$ with vertical intercept = 1.

Showing the possible graphs of $y = mx + 3$ being parallel to any of the arms of the graph $y=|2x-1|$ by sketching or by explaining.

The possible values of m is 2 and –2.

(c) (ii)

Students consider gradient = 1 and sketching graph on the same axes to show that the graph $y = x + c$ is not parallel to the graph $y=|2x-1|$ and cuts at 2 points.

The number of points of intersection is 2.

聖嬰中學
# HOLY INNOCENTS' HIGH SCHOOL

**MID-YEAR EXAMINATION 2014**
**SECONDARY 3 EXPRESS**

# MARKING SCHEME (FINAL)

**ADDITIONAL MATHEMATICS** **4047**

**Name** : ______________________ **Date** : **12 May 2014**

**Register No** : ______________________ **Duration** : **2 h**

**Class** : ______________________

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*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Answer all the questions.**

1 Find the set of values of $x$ for which $\frac{2x-5}{x-3} \leq 1$. [3]

***Solution:***

$\frac{2x-5}{x-3} \leq 1$

$(x-3)^2 \times \frac{2x-5}{x-3} \leq (x-3)^2$

***--- M1 (Multiplying $(x-3)^2$ on both sides, since it is a perfect square, the inequality sign is not changed.)***

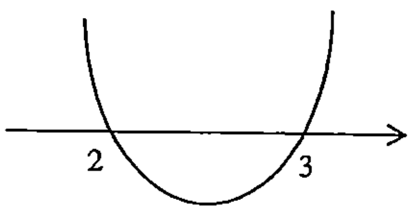
$(x-3)(2x-5) \leq (x-3)^2$

$(x-3)(2x-5)-(x-3)^2 \leq 0$

$(x-3)[2x-5-(x-3)] \leq 0$

***--- M1 (Factorisation. Award mark if student choose to expand, simplify then factorise)***

$(x-3)(x-2) \leq 0$

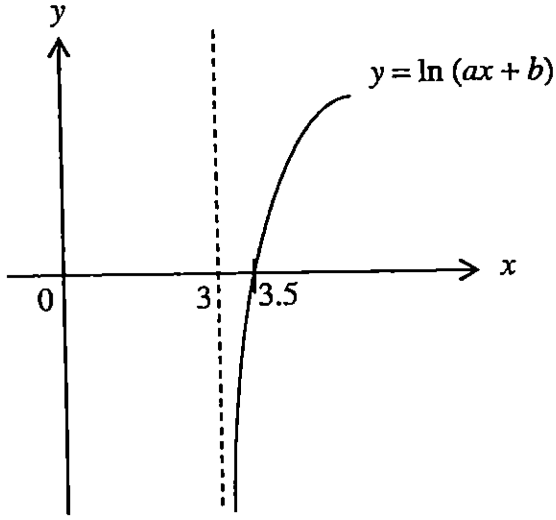



$2 \leq x \leq 3$

Since $x-3$ is the denominator, therefore $x \neq 3$.

Hence, the values of $x$ that satisfies the inequality is $2 \leq x < 3$. ***--- A1***

2 The graph of $y = \ln(ax + b)$ is shown below. Find the value of $a$ and of $b$. [4]




***Solution:***
For $x = 3.5$ and $y = 0$,
$\ln(3.5a + b) = 0$,
$3.5a + b = 1$ --- (1) --- ***M1***

The graph is undefined at $x = 3$, which means the graph is undefined when $3a + b = 0$ --- (2)
***--- M1 (Knowing that ln 0 is undefined and hence x = 3 is the asymptote)***

From (2), $b = -3a$ --- (3)

Sub (3) into (1):
$3.5a - 3a = 1$
$0.5a = 1$
$a = 2$ --- ***A1***

$b = -3(2) = -6$ --- ***A1***

3 The remainder when $x^3+3x^2-2x+c$ is divided by $(x+2)$ is twice the remainder when it is divided by $(x-3)$. Find the value of $c$. [4]

***Solution:***

$f(x)=x^3+3x^2-2\bar{x}+c$

$f(3)$

$=3^3+3(3)^2-2(3)+c$ --- ***M1 (Apply Remainder Theorem)***

$=48+c$

$f(-2)$

$=(-2)^3+3(-2)^2-2(-2)+c$ --- ***M1 (Apply Remainder Theorem)***

$=8+c$

$8+c=2(c+48)$ --- ***M1 (Forming equation)***

$8+c=2c+96$

$c=-88$ --- ***A1***

4 An object of temperature of $Y$ °C was taken out of the oven. It is then left to cool. Its temperature, $X$ °C, when it has left to cool for time $t$ minutes, is given by the equation

$$X = 27 + 63e^{-\frac{t}{8}}.$$

(i) Find the value of $Y$. [1]

(ii) Find the amount of time required for the object to cool to half of its initial temperature. Express your answer correct to the nearest minute. [2]

(iii) State the value that $X$ approaches as $t$ becomes very large. Explain the significance of this value of $X$. [2]

***Solution:***

*(i)*

When the object is first taken out from the oven right after being heated, $t = 0$.

Therefore, $Y = 27 + 63e^{0} = 90$ °C --- ***B1***

*(ii)*

$$27 + 63e^{-\frac{t}{8}} = \frac{90}{2}$$

$$27 + 63e^{-\frac{t}{8}} = 45$$

$$e^{-\frac{t}{8}} = \frac{45-27}{63}$$

$$-\frac{t}{8} = \ln\frac{18}{63}$$

--- ***M1 (For taking ln to solve for unknown in the power)***

$t = 10.022$

***Alternative Method:***

***Students solve*** $27 + 63e^{-\frac{t}{8}} = \frac{90}{2}$ ***and obtain t = 10.022***

10 minutes has passed when the object first reached half of its initial temperature. --- ***A1***

*(iii)*

As $t$ approaches $\infty$, $e^{-\frac{t}{8}}$ approaches 0.

Therefore $X = 27 + 63e^{-\frac{t}{8}}$ approaches 27. --- ***B1***

The temperature of the environment at which the object is allowed to cool to is 27°C. --- ***B1***

5 Find the coordinates of the points of intersection of the line $2x+3y=6$ and the curve $(2x+1)^2+6(y-2)^2=49$. [5]

***Solution***

$2x+3y=6$ --- (1)

$(2x+1)^2+6(y-2)^2=49$ --- (2)

From (1), $2x=6-3y$ --- (3)

Sub (3) into (2):

$(6-3y+1)^2+6(y-2)^2=49$ ***--- M1 (Substitution)***

$(7-3y)^2+6(y^2-4y+4)=49$

$49-42y+9y^2+6y^2-24y+24=49$

$15y^2-66y^2+24=0$

$3(5y^2-22y^2+8)=0$

$5y^2-22y^2+8=0$ ***--- M1 (Quadratic equation)***

$(5y-2)(y-4)=0$ ***--- M1 (Solving by factorisation or by quadratic formula)***

$y=\frac{2}{5}$ or $4$.

$y=\frac{2}{5}, x=\frac{6-3\left(\frac{2}{5}\right)}{2}=2\frac{2}{5}$ ***--- A1***

$y=4, x=\frac{6-3(4)}{2}=-3$ ***--- A1***

The two points are $\left(2\frac{2}{5}, \frac{2}{5}\right)$ and $(-3, 4)$.

6 (a) Show that $2^{n+4}+2^n-4\left(2^{n-1}\right)$ is a multiple of 3 for all positive integer values of $n$. [3]

(b) Given that $3^{2x+3}\times5^{2x}=27^x\times125^{x+1}$, evaluate $15^x$. [3]

***Solution:***

*(a)*

$2^{n+4}+2^n-4\left(2^{n-1}\right)$

$=2^n\bullet2^4+2^n-2^{n+1}$ ***--- M2 (Split** $2^{n+4}=2^n\bullet2^4$ **and simplify** $4\left(2^{n-1}\right)=2^{n+1}$**)***

$=2^n\bullet2^4+2^n-2^n\bullet2$

$=2^n(16+1-2)$

$=2^n(15)$

$=3(5)(2^n)$

***--- A1 (Showing 3 as a factor <u>or</u> for writing the following:Since 15 is a multiple of 3,*** $2^n(15)$ ***is also a multiple of 3)***

*(b)*

$3^{2x+3}\times5^{2x}=27^x\times125^{x+1}$

$3^{2x+3}\times5^{2x}=3^{3x}\times5^{3(x+1)}$ ***--- M1 (Change all to lowest base)***

$1=\dfrac{3^{3x}\times5^{3(x+1)}}{3^{2x+3}\times5^{2x}}$

$3^{3x-(2x+3)}\times5^{3(x+1)-2x}=1$

$3^{x-3}\times5^{x+3}=1$

$3^x\times3^{-3}\times5^x\times5^3=1$ ***--- M1 (Applying Laws of Indices to split the terms)***

$3^x\times5^x\times\dfrac{125}{27}=1$

$15^x=\dfrac{27}{125}$ ***--- A1***

7 Given that the roots of $2x^2+6x+1=0$ are $\alpha$ and $\beta$, find a quadratic equation, with integer coefficients, whose roots are $(\alpha+2\beta)$ and $(2\alpha+\beta)$. [7]

***Solution:***

Sum of roots, $\alpha+\beta=-3$ --- ***M1***

Product of roots, $\alpha\beta=\frac{1}{2}$ --- ***M1***

For the equation whose roots are $(\alpha+2\beta)$ and $(2\alpha+\beta)$,

Sum of roots, $(\alpha+2\beta)$ + $(2\alpha+\beta)$ = $3(\alpha+\beta)$ $=-9$ --- ***M1***

Product of roots,

$(\alpha+2\beta)(2\alpha+\beta)$

$=2\alpha^2+5\alpha\beta+2\beta^2$

$=2(\alpha^2+\beta^2)+5\alpha\beta$

$=2[(\alpha+\beta)^2-2\alpha\beta]+5\alpha\beta$ --- ***M1 (Applying Special Algebraic Rules)***

$=2(\alpha+\beta)^2+\alpha\beta$

$=2(-3)^2+\frac{1}{2}$

$=\frac{37}{2}$ --- ***M1***

Therefore $x^2+9x+\frac{37}{2}=0$ --- ***M1(Forming the equation using sum and product of roots)***

$2x^2+18x+37=0$ --- ***A1 (Equation with integer coefficients)***

8 (a) Find the range of values of $m$ for which the line $y = mx + 6$ does not meet the curve $2x^2 - xy - 3 = 0$. [3]

(b) (i) Prove that the roots of $2qx^2 + px = 2q - p$ are real. [4]

(ii) **Hence** find the relationship between $p$ and $q$ for which the equation $2qx^2 + px = 2q - p$ has equal roots. [1]

***Solution:***

*(a)*

$y - mx = 6$

$y = mx + 6$

Sub $y = mx + 6$,

$2x^2 - x(mx + 6) - 3 = 0$

$(2 - m)x^2 - 6x - 3 = 0$ ***--- M1 (Quadratic Equation with correct terms)***

Since the line does not meet the curve, discriminant < 0.

$(-6)^2 - 4(2 - m)(-3) < 0$ ***--- M1 (Discriminant < 0)***

$36 + 24 - 12m < 0$

$12m > 60$

$m > 5$ ***--- A1 (No marks if coefficients are wrong to obtain answer)***

***(b)(i)***

$2qx^2 + px = 2q - p$

$2qx^2 + px + p - 2q = 0$

If the roots are real, then discriminant ≥ 0.

Discriminant

$= p^2 - 4(2q)(p - 2q)$ ***--- M1 (writing the discriminant with correct coefficients)***

$= p^2 - 8pq + 16q^2$ ***--- M1 (expansion done correctly and terms are simplified)***

$= (p - 4q)^2$ ***--- M1 (Factorisation to obtain a perfect square)***

Since $(p - 4q)^2 \geq 0$, therefore, discriminant ≥ 0.

⇒ the roots are real. ***--- A1 (Conclude that perfect square ≥ 0, therefore discriminant ≥ 0 which implies that the roots are real)***

***(b)(ii)***

If roots are equal, then discriminant = 0.

$(p - 4q)^2 = 0$

$p = 4q$ ***--- B1 (accept*** $q = \frac{p}{4}$ ***and*** $p - 4q = 0$***)***

**9** Solve

(i) $\lg(1-2x)-2\lg x=1-\lg(2-5x)$. [4]

(ii) $\log_5 x+1=\log_{\sqrt{5}} x+2$, [3]

(iii) $4^{\log_4 x}=8$. [2]

***Solution:***

*(i)*

$\lg(1-2x)-2\lg x=1-\lg(2-5x)$

$\lg(1-2x)-\lg x^2=\lg 10-\lg(2-5x)$ ***--- M1 (Changing 1 = lg 10)***

$\lg\frac{1-2x}{x^2}=\lg\frac{10}{2-5x}$ ***--- M1 (Applying Laws of Logarithms to combine the terms)***

$\frac{1-2x}{x^2}=\frac{10}{2-5x}$

$(1-2x)(2-5x)=10x^2$

$2-5x-4x+10x^2=10x^2$

$9x=2$ ***--- M1***

$x=\frac{2}{9}$ ***--- A1***

*(ii)*

$\log_5 x+1=\log_{\sqrt{5}} x+2$

$\log_5 x+1=\frac{\log_5 x}{\log_5 \sqrt{5}}+2$ ***--- M1 (Change to same base)***

$\log_5 x+1=\frac{\log_5 x}{\frac{1}{2}}+2$ ***--- M1 (Knowing that*** $\log_5\sqrt{5}=\log_5 5^{\frac{1}{2}}=\frac{1}{2}$***)***

$\log_5 x+1=2\log_5 x+2$

$\log_5 x+1=2\log_5 x+2$

$\log_5 x=-1$

$x=5^{-1}=\frac{1}{5}$ ***--- A1 (Accept 0.2 as answer as well. Do not accept*** $5^{-1}$***)***

*(iii)*

$\log_4 x=\log_4 8$ ***--- M1 (Changing to logarithmic form)***

$x=8$ ***--- A1***

***Alternative Method:***

$4^{\log_4 x}=8$

$(2^2)^{\log_4 x}=2^3$

$2\log_4 x=3$ ***--- M1 (Comparing powers after changing both sides to the same base)***

$\log_4 x^2=3$

$x^2 = 4^3$
$x^2 = 64$
$x = 8$ or $-8$ (rejected) *--- A1 (Must reject $-8$)*

**10** **(a)** **(i)** Show that $\dfrac{1}{\sqrt{n}+\sqrt{n+1}}=\sqrt{n+1}-\sqrt{n}$ where $n>0$. [2]

**(ii)** **Hence** find the value of

$$\frac{2}{\sqrt{36}-\sqrt{37}}-\frac{2}{\sqrt{37}-\sqrt{38}}-\frac{2}{\sqrt{38}-\sqrt{39}}-....-\frac{2}{\sqrt{47}-\sqrt{48}}-\frac{2}{\sqrt{48}-\sqrt{49}}.$$ [2]

**(b)** Solve the equation $\sqrt{x}+\sqrt{x-3}=\sqrt{2x+1}$. [5]

***Solution:***

***(a)(i)***

$$\frac{1}{\sqrt{n}+\sqrt{n+1}}$$

$$=\frac{1}{\sqrt{n}+\sqrt{n+1}}\times\frac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}}$$ *--- M1 (Rationalise denominator)*

$$=\frac{\sqrt{n}-\sqrt{n+1}}{n-(n+1)}$$

$$=\frac{\sqrt{n}-\sqrt{n+1}}{-1}$$ *--- A1 (Expand denominator and arrive at denominator = –1)*

$$=\sqrt{n+1}-\sqrt{n}$$ (Shown)

***(a)(ii)***

$$\frac{2}{\sqrt{36}-\sqrt{37}}-\frac{2}{\sqrt{37}-\sqrt{38}}-\frac{2}{\sqrt{38}-\sqrt{39}}-....-\frac{2}{\sqrt{47}-\sqrt{48}}-\frac{2}{\sqrt{48}-\sqrt{49}}$$

$$\frac{2}{\sqrt{1}+\sqrt{2}}+\frac{2}{\sqrt{2}+\sqrt{3}}+\frac{2}{\sqrt{3}+\sqrt{4}}+....+\frac{2}{\sqrt{98}+\sqrt{99}}+\frac{2}{\sqrt{99}+\sqrt{100}}$$

$$=2\left(\frac{1}{\sqrt{36}-\sqrt{37}}-\frac{1}{\sqrt{37}-\sqrt{38}}-\frac{1}{\sqrt{38}-\sqrt{39}}-....-\frac{1}{\sqrt{47}-\sqrt{48}}-\frac{1}{\sqrt{48}-\sqrt{49}}\right)$$

$$=2\left(\sqrt{37}-\sqrt{36}-(\sqrt{38}-\sqrt{37})-(\sqrt{39}-\sqrt{38})-....-(\sqrt{47}-\sqrt{48})-(\sqrt{49}-\sqrt{48})\right)$$

$$=2\left(2\sqrt{37}-\sqrt{36}-\sqrt{49}\right)$$ *--- M1*

$$=2\left(2\sqrt{37}-6-7\right)$$

$$=4\sqrt{37}-26 \text{ or } -1.67$$ *--- A1*

***(b)***

$$\sqrt{x}+\sqrt{x-3}=\sqrt{2x+1}$$

$$\left(\sqrt{x}+\sqrt{x-3}\right)^2=\left(\sqrt{2x+1}\right)^2$$ *--- M1 (Square both sides)*

$$x+2\sqrt{x}\sqrt{x-3}+(x-3)=2x+1$$

$$2\sqrt{x}\sqrt{x-3}=4$$ *--- M1 (Simplifying after correct expansion)*

$$\sqrt{x}\sqrt{x-3}=2$$

$$\sqrt{x(x-3)}=2$$

$$x(x-3)=4$$

$$x^2-3x-4=0$$ *--- M1 (Obtaining quadratic equation)*

$$(x-4)(x+1)=0$$ *--- M1 (Solving quadratic equation)*

$x=4$ or $-1\ (rej)$ *--- A1 (must reject x = –1)*

11 (i) Given that $2x^2-5x-3$ is a factor of $2x^4-ax^3-2x^2+13x+b$, show that $a=7$ and $b=6$. [6]

(ii) Hence solve the equation
$$2x^4-7x^3-2x^2+13x+6=0.$$ [3]

***Solution:***
*(i)*
Let $\mathrm{f}(x)=2x^4-ax^3-2x^2+13x+b$

$2x^2-5x-3=(2x+1)(x-3)$ *--- M1 (Factorisation to find two linear factors of f(x))*
Since $2x^2-5x-3$ is a factor, then $(2x+1)$ and $(x-3)$ are factors of f($x$).

Therefore, $\mathrm{f}\left(-\frac{1}{2}\right)=0$

$\Rightarrow 2\left(-\frac{1}{2}\right)^4-a\left(-\frac{1}{2}\right)^3-2\left(-\frac{1}{2}\right)^2+13\left(-\frac{1}{2}\right)+b=0$ *--- M1 (Form equation)*

$\frac{1}{8}+\frac{a}{8}-\frac{1}{2}-\frac{13}{2}+b=0$
$1+a-56+8b=0$
$a=55-8b$ --- (1)

$\mathrm{f}(3)=0$
$\Rightarrow 2(3)^4-a(3)^3-2(3)^2+13(3)+b=0$ *--- M1 (Form equation)*
$162-27a-18+39+b=0$
$b=27a-183$ --- (2)

Sub (1) into (2):
$b=27(55-8b)-183$
*--- M1 (Solving simultaneous equations by substitution or elimination)*

$b=1485-216b-183$
$217b=1302$
$b=6$ *--- A1*
$a=55-8(6)=7$ *--- A1*

*(ii)*
$2x^4-7x^3-2x^2+13x+6\equiv(2x^2-5x-3)(px^2+qx+r)$

By comparing coefficients of...
$x^4$: $p=1$

constant term: $6=-3r$
$r=-2$

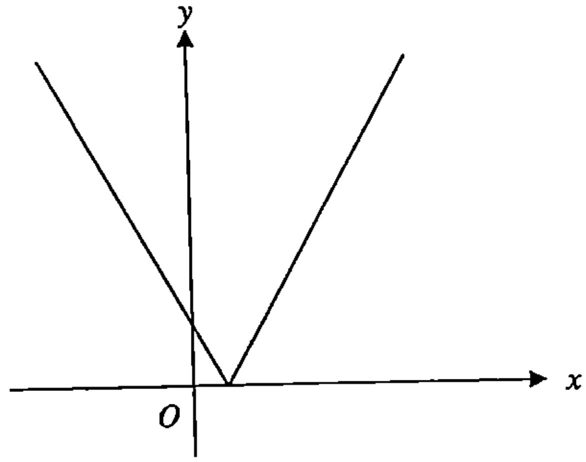
$x^2$: $-2x^2 = -4x^2 - 3x^2 - 5qx^2$
$-2 = -4 - 3 - 5q$
$q = -1$

Therefore,
$2x^4 - 7x^3 - 2x^2 + 13x + 6 = 0$
$(2x^2 - 5x - 3)(x^2 - x - 2) = 0$ --- ***M1 (Finding the other quadratic factor)***
$(2x+1)(x-3)(x-2)(x+1) = 0$ --- ***M1 (Factorisation)***
$x = -\frac{1}{2}, 3, 2, -1$ --- ***A1 (For all correct answers)***

12 (a) Solve the equation $|x+4|+3=2x$. [3]

(b) Sketch the curve $y=|-x^2+2x+3|$, showing clearly the coordinates of the maximum point and the points where the curve meets the axes. [3]

(c) The diagram below shows part of the graph of $y=|2x-1|$.




(i) Given that the line $y=mx+3$ intersects the graph $y=|2x-1|$ at only one point. Write down the possible values of $m$. [3]

(ii) Determine the number of points of intersections between the graph of $y=|2x-1|$ and the line $y=x+c$ where $c>0$. Explain your answer. [2]

***Solution:***

***(a)***

$|x+4|+3=2x$

$|x+4|=2x-3$

$x+4=2x-3$ or $x+4=-(2x-3)$ --- ***M1 (Splitting)***

$x=7$ or $-\frac{1}{3}$ (rej) --- ***A2 (A1 each; accepting $x = 7$, rejecting $x = -\frac{1}{3}$, no A1 if never reject $x = -\frac{1}{3}$)***

*(b)*

$y = -x^2 + 2x + 3$

$y = -(x+1)(x-3)$

Therefore, $x$-intercepts are $-1$ and 3.

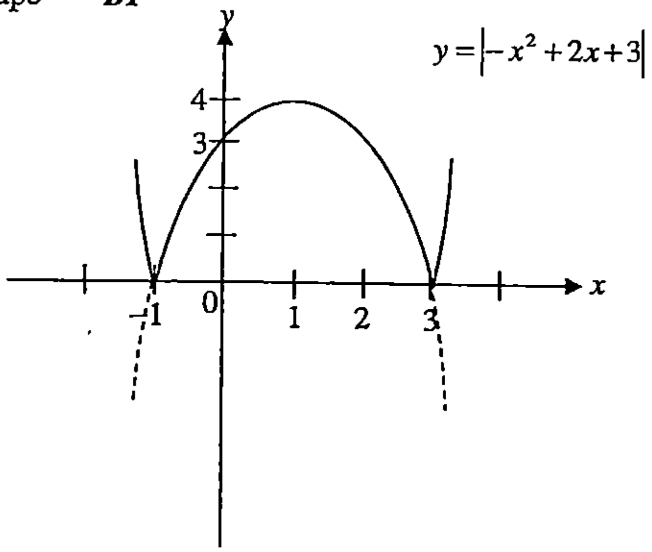
$x = 0$, $y$-intercept = 3

Maximum point at mid-point of $x$-intercepts, therefore (1, 4).

$x$ & $y$-intercepts --- ***B1***

Max point at (1, 4) --- ***B1***

Shape --- ***B1***




*(c)(i)*

Knowing that the vertical intercept ($y$-intercept) of the graph $y = mx + 3$ is 3 and is above the vertical-intercept of the graph $y = |2x-1|$ with vertical intercept = 1. --- ***M1***

Showing the possible graphs of y = mx + 3 being parallel to any of the arms of the graph $y = |2x-1|$ by sketching or by explaining. --- ***M1***

The possible values of m is 2 and –2. --- ***A1 (Both answers must be correct)***

*(c)(ii)*

Students consider gradient = 1 and sketching graph on the same axes to show that the graph $y = x + c$ is not parallel to the graph $y = |2x-1|$ and cuts at 2 points. --- ***M1***

The number of points of intersection is 2.

***--- A1 (Since the gradients are different, the graph $y = x + c$ will definitely cut through 2 points as $y = |2x-1|$ is symmetrical)***

**Holy Innocents' High School**
**Secondary 3 Express**
**End-of-Year Exam 2012**
**Additional Mathematics**

Answers

1. $x=-\frac{2}{7}$ , $y=-3\frac{3}{14}$

   or $x=2$, $y=-1\frac{1}{2}$

2. $2+\frac{2}{x}-\frac{1}{x^2}+\frac{4}{3x+1}$

3. $x=70.5°$, $289.5°$

4. $\frac{900\sqrt{5}+500}{19}$

5. (i) $f(x) = 2x^3+9x^2-6x-40$

   (ii) $-40\frac{1}{2}$

6. (b) $x=\frac{\pi}{2},\frac{3\pi}{2}$

7. (a) (i) $-3$ (ii) $-4\frac{1}{2}$

   (b) $x^2-16x+\frac{13}{4}=0$ or

   $4x^2-64x+13=0$

8. (a) $a<-2$ or $a>2$

   (b) $x=2.46$ or $-4.46$ (rej)

9. (a) (i) Centre = $(-2, 6)$ , Radius = 5

   (ii) Distance between (0, 2) and centre of circle $=\sqrt{2^2+4^2}=\sqrt{20}<5$.
   Thus (0, 2) is inside the circle.

   (b) (i) $a=-3\frac{1}{2}$, $b=2$ , $c=7$

   (ii) $x=2\frac{1}{14}$ or $3\frac{13}{14}$

10. (i) Amplitude = 3

    (ii) Period = 720°

    (iii) Min pt = (540, –2)
    Max pt = (180, 4)

    (iv) (398.9 , 0) , (681.1 , 0)

11. (a) $y=x-3$

    (b) $(0,-3)$

    (c) $D=(-3,0)$

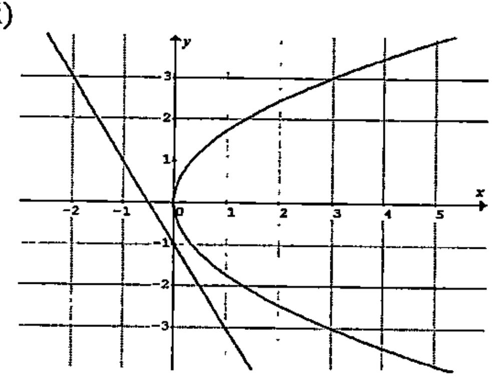
    (d) $B=(3,-6)$

    (e) 24 units$^2$

12. (a) $-3<k<-2$ *or* $k>6$

    (b) (i) The graphs do not intersect.

    (ii)




13. (a) (ii) When $x=5$, correct $\frac{y}{x}$ value = 28
    Corrected $y$-value = $28\times5=140$ .

    (iii) $a=1$, $b=3$

    (b) $y=\frac{10^7}{x^2}$