Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

Mid-Year Examination 2014
Secondary Three

INTEGRATED MATHEMATICS 2 — **1 hour 30 minutes**

Thursday — **8 May 2014** — **08 00 – 09 30**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer all the questions.
2. Write your answers and working on the separate answer paper provided.
3. Write your name, register number and class on each separate sheet of paper that you use and fasten the separate sheets together with the string provided. Do not staple your answer sheets together.
4. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
5. Omission of essential working will result in loss of marks.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 60.
3. The use of an electronic calculator is expected, where appropriate.
4. You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1 Given that $x\sqrt{5} = (2x-1)\sqrt{10}$, express x as a fraction in the form $\frac{p+\sqrt{q}}{7}$, where p and q are integers. [4]

2 Find the range of values of q if the equation $x^2 - 4x + 6 = q$ has real distinct roots.
Hence state the value of q for which the equation $x^2 - 4x + 6 = q$ has equal roots. [4]

3 Given that $u = \log_4 x$, find, in terms of u,

(i) x, [1]

(ii) $\log_4 \frac{1}{\sqrt{x}}$, [1]

(iii) $\log_8 x$. [2]

4 It is given that the roots of $4x^2 - 16x + 9 = 0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, where $\alpha > 0, \beta < 0$ and $\alpha + \beta < 0$ [4]

5 Solve the equation $\frac{\lg(9x+2)}{2} - \lg x + \lg\sqrt{2} = \frac{1}{2}$. [5]

6 Sketch the graph of $y = \ln(x+3)$, showing clearly where it cuts the axes.
Find the equation of the straight line which must be drawn on the graph of $y = \ln(x+3)$ to obtain a solution of the equation $\sqrt{x+3} = \frac{1}{e^x}$. [6]
(It is not necessary to draw the line.)

7 (a) Solve $|x^2 - 4| - 5 = 0$. [3]

(b) Given that $y = |6 - 2x| - 3$, state the minimum value of y and the corresponding value of x. Hence sketch the graph of $y = |6 - 2x| - 3$, showing clearly where the graph cuts the coordinate axes. [4]

8 A curve has the equation $y = x(k - x) - 3$, where k is a constant.

(i) Find the values of k for which the line $y = k$ is a tangent to the curve. [4]

(ii) Find the smallest value of the integer k for which the graph of $y = x(k - x) - 3$ lies entirely below the x–axis. [4]

9 Solve the equation

(i) $8(4^x)^2 = \sqrt{\frac{1}{32^x}}$, [3]

(ii) $10^{2+x} - 10^x = 100^x - 100$. [5]

10 (a) Find the range of values of x for which $x(x - 3) > 2(x - 3)^2$. [3]

(b) Given that the range of values $-1 < x < 7$ satisfy the quadratic inequality $(x + h)^2 < k$ where h and k are constants, find the values of h and k. [5]

(c) Sketch the graph of $y = p(x + q)^2$ for $p < 0$ and $q > 0$ where p and q are constants. *(It is not necessary to label any intercept.)* [2]

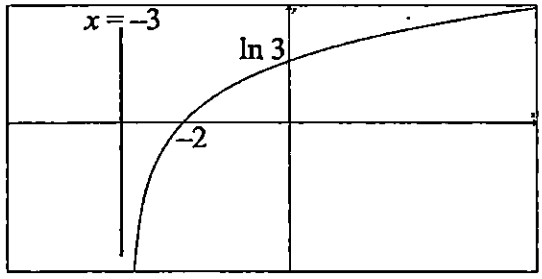
Bonus Question

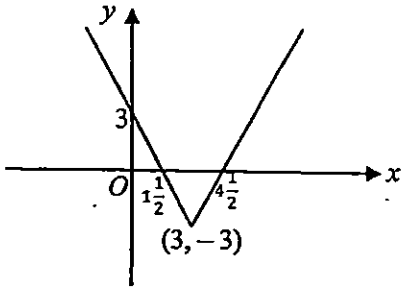
11 Solve $|x + 1| + |x - 2| + 2|x - 3| < 10$. [3]

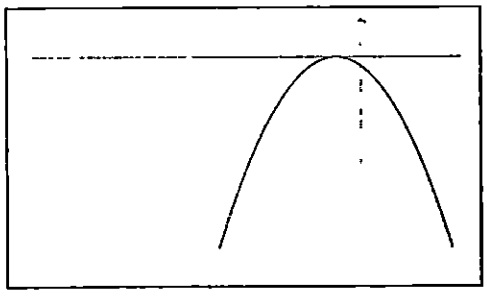
~ End of Paper ~

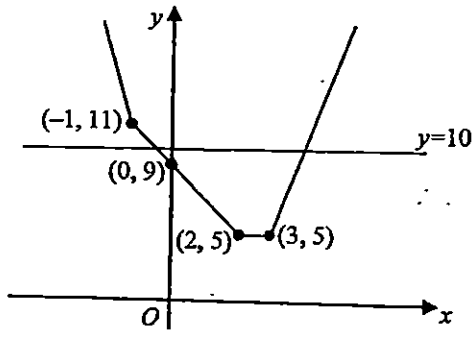
2014 S3 Mid-Year IM2 Marking Scheme

No.		
1	$x\sqrt{5} = (2x-1)\sqrt{10}$ $x\sqrt{5} = 2\sqrt{10}x - \sqrt{10}$ $(\sqrt{5} - 2\sqrt{10})x = -\sqrt{10}$ $x = \dfrac{-\sqrt{10}}{(\sqrt{5} - 2\sqrt{10})}$ $= \dfrac{-\sqrt{10}}{(\sqrt{5} - 2\sqrt{10})} \times \dfrac{(\sqrt{5} + 2\sqrt{10})}{(\sqrt{5} + 2\sqrt{10})}$ $= \dfrac{-\sqrt{50} - 20}{5 - 40}$ $= \dfrac{20 + \sqrt{50}}{35}$ $= \dfrac{20 + 5\sqrt{2}}{35}$ $= \dfrac{4 + \sqrt{2}}{7}, p = 4, q = 2$	
2	$x^2 - 4x + 6 - q = 0$ $b^2 - 4ac > 0$ $(-4)^2 - 4(6 - q) > 0$ $16 - 24 + 4q > 0$ $4q > 8$ $q > 2$ For equal roots, $q = 2$.	
3 (i) (ii) (iii)	$x = 4^u$ $\log_4 \dfrac{1}{\sqrt{x}} = \log_4 x^{-\frac{1}{2}} = -\dfrac{1}{2}\log_4 x = -\dfrac{1}{2}u$ $\log_8 x = \dfrac{\log_4 x}{\log_4 8}$ $= \dfrac{\log_4 x}{\log_4 4^{\frac{3}{2}}} = \dfrac{\log_4 x}{\frac{3}{2}} = \dfrac{2}{3}u$	
4	$4x^2 - 16x + 9 = 0$ $\alpha^2 + \beta^2 = 4$ $\alpha^2\beta^2 = \dfrac{9}{4}$ $\Rightarrow \alpha\beta = -\dfrac{3}{2}$ $(\alpha\beta < 0)$ $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ $(\alpha + \beta)^2 = 4 + 2\left(-\dfrac{3}{2}\right)$ $(\alpha + \beta)^2 = 1$ $\alpha + \beta = -1$ $x^2 + x - \dfrac{3}{2} = 0$ $\Rightarrow 2x^2 + 2x - 3 = 0$	

5	$\frac{\lg(9x+2)}{2} - \lg x + \lg\sqrt{2} = \frac{1}{2}$ $\lg\frac{(9x+2)^{\frac{1}{2}}(\sqrt{2})}{x} = \lg\sqrt{10}$ $\frac{(9x+2)^{\frac{1}{2}}(\sqrt{2})}{x} = \sqrt{10}$ $\frac{(9x+2)(2)}{x^2} = 10$ $(9x+2)(2) = 10x^2$ $9x+2 = 5x^2$ $5x^2 - 9x - 2 = 0$ $(5x+1)(x-2) = 0$ $x = -\frac{1}{5}(\text{na}), x = 2$ or alternatively, $\lg(9x+2) - 2\lg x + 2\lg\sqrt{2} = 1$ $\lg(9x+2) - \lg x^2 + \lg 2 = \lg 10$ $\lg\frac{(9x+2)(2)}{x^2} = \lg 10$ $\frac{(9x+2)(2)}{x^2} = 10$	
6	$y = \ln(x+3)$  $\sqrt{x+3} = \frac{1}{e^x}$ $\ln\sqrt{x+3} = \ln\frac{1}{e^x}$ $\frac{1}{2}\ln(x+3) = \ln e^{-x}$ $\ln(x+3) = -2x$ st line: $y = -2x$	

7 (a)	$\lvert x^2-4\rvert = 5$ $x^2-4 = \pm 5$ $x^2 = \pm 5+4$ $x^2 = 9$ or -1(na) $x = \pm 3$	
(b)	Min value of $y=-3$ when $x=3$  	
8. (i)	$x(k-x)-3 = k$ $x^2-kx+k+3 = 0$ $(-k)^2-4(k+3) = 0$ $k^2-4k-12 = 0$ $(k-6)(k+2) = 0$ $k = 6$ or -2	
(ii)	$x^2-kx+3 = 0$ or $-x^2+kx-3 = 0$ $(-k)^2-4(3) < 0$ or $k^2-4(-1)(-3) < 0$ $k^2 < 12$ $-\sqrt{12} < k < \sqrt{12}$ $-3.46 < k < 3.46$ Smallest integer $k=-3$	
9 (i)	$8(4^x)^2 = \sqrt{\frac{1}{32^x}}$ $2^3 \times 2^{4x} = 2^{-\frac{5x}{2}}$ $3+4x = -\frac{5x}{2}$ $6+8x = -5x$ $13x = -6$ $x = -\frac{6}{13}$	
(ii)	$10^{2+x}-10^x = 100^x-100$ Let $y = 10^x$ $100y-y = y^2-100$ $y^2-99y-100 = 0$ $(y+1)(y-100) = 0$ $y = -1$ or 100 $10^x = -1$(na) $10^x=100 \Rightarrow x=2$	

10 (a)	$x(x-3) > 2(x-3)^2$ $x^2 - 3x > 2(x^2 - 6x + 9)$ $x^2 - 3x > 2x^2 - 12x + 18$ $x^2 - 9x + 18 < 0$ $(x-3)(x-6) < 0$ $3 < x < 6$	
(b)	$(x+1)(x-7) < 0$ $x^2 - 6x - 7 < 0$ $(x-3)^2 - 9 - 7 < 0$ $(x-3)^2 < 16$ $h = -3, k = 16$	
(c)	or alternatively, $(x+1)(x-7) < 0$ $x^2 - 6x - 7 < 0$ $(x+h)^2 < k$ $x^2 + 2xh + h^2 - k < 0$ By comparing, $2h = -6$ $h^2 - k = -7$ $h = -3, k = 16$ or alternatively, using substitution method, $(x+h)^2 = k$ Let $x = -1$, $(-1+h)^2 = k$ Let $x = 7$, $(7+h)^2 = k$ $(-1+h)^2 = (7+h)^2$ $1 + 2h = 49 + 14h$ $12h = -48$ $h = -3, k = 16$ 	

Bonus 11	$\|x+1\|+\|x-2\|+2\|x-3\|<10$ Solve by graphical method For $x<-1, y=-4x+7$ For $-1\le x<2, y=-2x+9$ For $2\le x<3, y=5$ For $x\ge 3\ y=4x-7$  $-2x+9=10\Rightarrow x=-\frac{1}{2}$ $4x-7=10\Rightarrow x=\frac{17}{4}=4\frac{1}{4}$ $-\frac{1}{2}<x<4\frac{1}{4}$	