Name :			Index No :	Class :	Calculator Model :
Expected Grade :	Marks Awarded :	/ 80	Actual Grade :	Parent's Signature :	

NORTHLANDSECONDARY SCHOOL

Motivated Learners, Assets to Community
Nurturing Minds, Shaping Character, Strengthening Vigour

MID YEAR EXAMINATION 2014	
Subject : ADDITIONAL MATHEMATICS	Date: 14 May 2014
Subject Code: 4047	Duration : 2 hours
Level : Secondary 3 Express	Time : 0900 – 1100
Setter : Ms. Liem PL	Vetter : Mr. Chen WZ

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use

This paper consists of 4printed pages including the cover page

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where$n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

1. The line $x-y=3$ intersects the curve $x^2-3xy+y^2+19=0$ at points P and Q. Find the coordinates of P and of Q. [5]

2. Given that $4x^2-6x+9=A(x-1)(2x+1)+B(x-1)+C$ for all values of x, find the values of A, B and C. [3]

3. The function $f(x)=x^3-6x^2+ax+b$, where a and b are constants, is exactly divisible by $x-3$ and leaves a remainder of -55 when divided by $x+2$.

   (i) Find the value of a and of b. [4]

   (ii) Solve the equation $f(x)=0$. [3]

4. Express $\dfrac{x^2-3x+4}{x-x^3}$ in partial fractions. [5]

5. The roots of the quadratic equation $x^2+5x-2=0$ are α and β. Find

   (i) $\alpha^2+\beta^2$, [3]

   (ii) $\alpha^3+\beta^3$, [2]

   (iii) the quadratic equation whose roots are $\dfrac{\beta}{\alpha^2+1}$ and $\dfrac{\alpha}{\beta^2+1}$. [3]

6. (i) Calculate the range of values of k for which $3x^2+kx+7>4+2x$ for all values of x. [3]

   (ii) The straight line $y=2p+1$ intersects the curve $xy=x^2+p^2$ at two distinct points. Find the range of values of p. [3]

7. (a) Solve $|x-6|=5-9x$. [3]

   (b) (i) Given that $y=x^2-6x+5$, express x^2-6x+5 in the form $(x-a)^2-b$. [2]

   (ii) Sketch the graph of $y=|x^2-6x+5|$ for $0\le x\le 6$, indicating on your graph the points where the graph meets the coordinate axes. [4]

8. (i) Write down and simplify the first three terms of the expansion $(2-x)^6$ in ascending powers of x. [3]

   (ii) Given that the coefficients of x and x^2 in the expansion of $(1-ax)^2(2-x)^6$ are 32 and b respectively, calculate the value of a and of b. [5]

9. (a) Sketch the graph $y=8^{-x}$. [1]

(b) Solve the equation $\frac{2^{x-3}}{8^{-x}}=\frac{32}{\sqrt{4^x}}$. [4]

10. Given that $2\sqrt{3}x-3=3x+\sqrt{3}$, find x in the form $p+q\sqrt{3}$, where p and q are integers. [4]

11. Solve the equation

(i) $2\log_2 6-\log_2(x-1)=3+\log_2(4x-7)$, [4]

(ii) $\log_2 x+3\log_x 2=4$. [4]

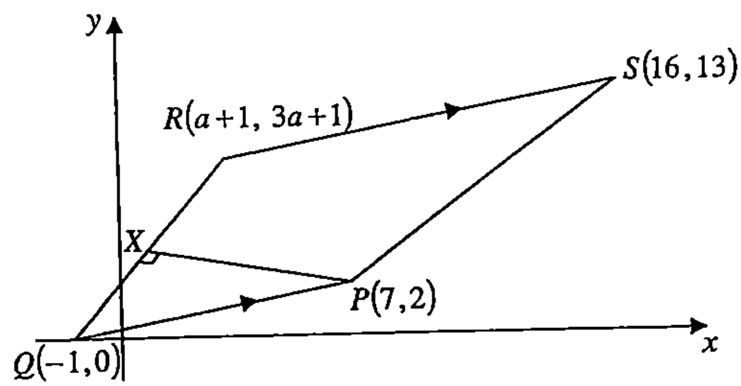
12. At the beginning of 2010, the number of a certain species of insects was estimated at 80000. The population decreased so that, after a period of t years, the population was $80000\,e^{-0.03t}$. Estimate

(i) the population at the beginning of 2020, [1]

(ii) the year in which the population would be expected to be halved. [3]

13. **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a quadrilateral $PQRS$ in which P is (7, 2), Q is (–1, 0), R is $(a+1, 3a+1)$ and S is (16, 13). RS is parallel to QP and X is the mid-point of QR.



(i) Find the value of a. [3]

(ii) Find the area of quadrilateral $PQRS$. [2]

(iii) Determine whether PX is perpendicular to QR. [3]

-- **End of Paper** --

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

wheren is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

1. The line $x-y=3$ intersects the curve $x^2-3xy+y^2+19=0$ at points P and Q. Find the coordinates of P and of Q. [5]

Substitute $x=3+y$ into $(3+y)^2-3(3+y)y+y^2+19=0$ [M1]

$9+6y+y^2-9y-3y^2+y^2+19=0$ [M1]

$-y^2-3y+28=0 \Rightarrow y^2+3y-28=0$

$(y-4)(y+7)=0$ [M1]

$x=-4, y=-7$ and $x=7, y=4$

$P(-4,-7)$ [A1] and $Q(7,4)$ [A1]

2. Given that $4x^2-6x+9=A(x-1)(2x+1)+B(x-1)+C$ for all values of x, find the values of A, B and C. [3]

Let $x=1,\ 4-6+9=C \Rightarrow \therefore C=7$ [B1]

Let $x=-\frac{1}{2},\ 1+3+9=-\frac{3}{2}B+7 \Rightarrow \therefore B=6\times(\frac{-2}{3})=-4$ [B1]

Let $x=0,\ 9=-A+4+7 \Rightarrow \therefore A=2$ [B1]

$4x^2-6x+9=2(x-1)(2x+1)-4(x-1)+7$

3. The function $f(x)=x^3-6x^2+ax+b$, where a and b are constants, is exactly divisible by $x-3$ and leaves a remainder of -55 when divided by $x+2$.

(i) Find the value of a and of b. [4]

$(3)^3-6(3)^2+3a+b=0 \Rightarrow 3a+b=27 \rightarrow i$ [M1]

$(-2)^3-6(-2)^2-2a+b=-55 \Rightarrow -2a+b=-23 \rightarrow ii$ [M1]

$ii-i \Rightarrow 5a=50, \therefore a=10$ [A1] and $b=-23+20=-3$ [A1]

(ii) Solve the equation $f(x)=0$. [3]

$x^3-6x^2+10x-3=(x-3)(x^2-3x+1)=0$

$$\begin{array}{r|rrrr} & 1 & -6 & 10 & -3 \\ 3 & 0 & 3 & -9 & 3 \\ \hline & 1 & -3 & 1 & 0 \end{array} \Rightarrow x^2-3x+1$$ [M1]

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}$$

$x=3$ [A1] or $x=2.62$ or $x=0.382$ [A1]

4. Express $\dfrac{x^2-3x+4}{x-x^3}$ in partial fractions. [5]

$$\frac{x^2-3x+4}{x(1-x^2)}=\frac{x^2-3x+4}{x(1-x)(1+x)}$$

$$=\frac{A}{x}+\frac{B}{1-x}+\frac{C}{1+x}$$ [M1]

$$x^2-3x+4=A(1-x)(1+x)+Bx(1+x)+Cx(1-x)$$

Let $x=1,\ 1-3+4=2B \Rightarrow \therefore B=\frac{2}{2}=1$ [B1]

Let $x=0,\ \therefore 4=A$ [B1]

Let $x=-1,\ 1+3+4=-2C \Rightarrow \therefore C=-4$ [B1]

$$\frac{x^2-3x+4}{x-x^3}=\frac{4}{x}+\frac{1}{(1-x)}-\frac{4}{(1+x)}$$ [A1]

5. The roots of the quadratic equation $x^2+5x-2=0$ are α and β. Find [3]

Sum of roots : $\alpha+\beta=-5$

Product of roots : $\alpha\beta=-2$ [M1]

(i) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(-5)^2-2(-2)$ [B1]

$=29$ [A1]

[2]

(ii) $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ [M1]

$=(-5)(29+2)$

$=-155$ [A1]

(iii) the quadratic equation whose roots are $\dfrac{\beta}{\alpha^2+1}$ and $\dfrac{\alpha}{\beta^2+1}$. [3]

Sum of roots : $\dfrac{\beta}{\alpha^2+1}+\dfrac{\alpha}{\beta^2+1}=\dfrac{\beta^3+\beta+\alpha^3+\alpha}{(\alpha\beta)^2+\alpha^2+\beta^2+1}$

$=\dfrac{-155-5}{4+29+1}=\dfrac{-160}{34}=-\dfrac{80}{17}$ [A1]

Product of roots : $\dfrac{\beta}{\alpha^2+1}\left(\dfrac{\alpha}{\beta^2+1}\right)=\dfrac{\alpha\beta}{(\alpha\beta)^2+\alpha^2+\beta^2+1}$

$=\dfrac{-2}{4+29+1}=-\dfrac{1}{17}$ [A1]

The quadratic equation : $x^2-(\text{Sum of roots})x+\text{Product of roots}=0$

$$x^2+\frac{80}{17}x-\frac{1}{17}=0$$

$$17x^2+80x-1=0$$ [A1]

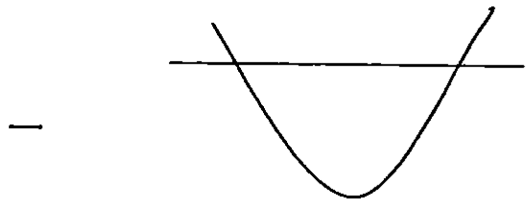
6.

(i) Calculate the range of values of k for which $3x^2 + kx + 7 > 4 + 2x$ for all values of x. [3]

$3x^2 + (k-2)x + 3 > 0$

$b^2 - 4ac < 0 \Rightarrow (k-2)^2 - 4(3)(3) < 0$ (No real roots) [M1]

$k^2 - 4k - 32 < 0 \Rightarrow (k-8)(k+4) < 0$ [M1]



$-4 < k < 8$ [A1]

(ii) The straight line $y = 2p + 1$ intersects the curve $xy = x^2 + p^2$ at two distinct points. Find the range of values of p. [3]

$x(2p+1) = x^2 + p^2 \Rightarrow x^2 - x(2p+1) + p^2 = 0$ [M1]

$b^2 - 4ac > 0 \Rightarrow [-(2p+1)]^2 - 4(1)(p^2) > 0$ [M1]

$4p^2 + 4p + 1 - 4p^2 > 0 \Rightarrow 4p + 1 > 0$

$p > -\frac{1}{4}$ [A1]

7. **(a)** Solve $|x-6| = 5 - 9x$. [3]

[M1]

$x - 6 = 9x - 5$ or $x - 6 = 5 - 9x$

$8x = -1 \Rightarrow x = -\frac{1}{8}$ [A1]

$x - 6 = 5 - 9x$

$10x = 11$

$\Rightarrow x = \frac{11}{10}$ (rejected) [A1]

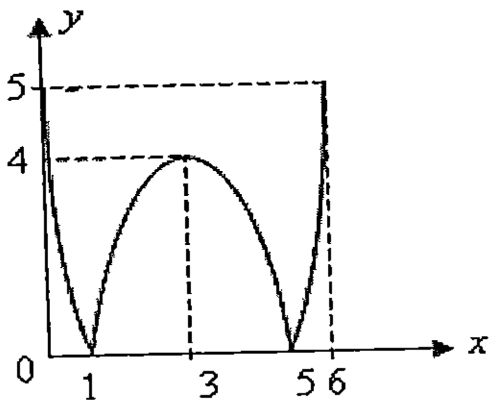
(b) (i)Given that $y = x^2 - 6x + 5$, express $x^2 - 6x + 5$ in the form $(x-a)^2 - b$. [2]

$y = (x-3)^2 - (-3)^2 + 5$ [M1]

$= (x-3)^2 - 9 + 5$

$= (x-3)^2 - 4$ [A1]

(ii)Sketch the graph of $y=|x^2-6x+5|$ for $0 \le x \le 6$, indicating on your graph thepoints where the graph meets the coordinate axes. [4]



[B1]

y-intercept: (0, 5)

x-intercepts:

$y = x^2 - 6x + 5$

$= (x-1)(x-5)$ [B1]

[B1]

x-intercepts: (1, 0) and (5, 0) [B1]

max turning point (3,4)

curve and end points of (0,5) and (6,5)

8. (i) Write down and simplify the first three terms of the expansion $(2-x)^6$ in ascending powers of x. [3]

$$(2-x)^6 = 2^6 - \binom{6}{1}2^5 x + \binom{6}{2}2^4 x^2 + ...$$

$$= 64 - 192x + 240x^2 + ...$$

[B1], [B1], [B1]

(ii) Given that the coefficients of x and x^2 in the expansion of $(1-ax)^2(2-x)^6$ are 32 and b respectively, calculate the value of a and of b. [5]

$$(1-ax)^2(2-x)^6 = (1-2ax+a^2x^2)(64-192x+240x^2+....)$$ [M1]

$$= 64 - 192x + 240x^2 - 128ax + 384ax^2 + 64a^2x^2 + ...$$

$$= 64 - (192+128a)x + (240+384a+64a^2)x^2 + ...$$

Equating the coefficient of x, $-(192+128a) = 32 \Rightarrow 128a = -32-192$ [M1]

$$a = -\frac{224}{128} = -\frac{7}{4}$$ [A1]

Equating the coefficient of x^2, $b = 240 + 384a + 64a^2$ [M1]

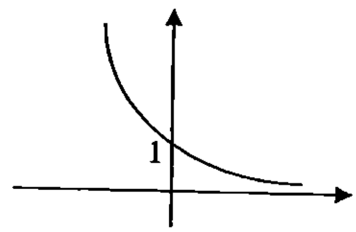
$$= 240 + 384\left(-\frac{7}{4}\right) + 64\left(-\frac{7}{4}\right)^2$$

$$= -236$$ [A1]

9. (a) Sketch the graph $y=8^{-x}$. [1]

y

x



[B1]

(b) Solve the equation $\frac{2^{x-3}}{8^{-x}}=\frac{32}{\sqrt{4^x}}$. [4]

$\frac{2^{x-3}}{2^{-3x}}=\frac{2^5}{2^x}$ [M1]

$2^{x-3-(-3x)}=2^{5-x}$ [M1]

Equating the indices, $x-3+3x=5-x$ [M1]

$5x=8 \Rightarrow x=\frac{8}{5}$ [A1]

10. Given that $2\sqrt{3}x-3=3x+\sqrt{3}$, find x in the form $p+q\sqrt{3}$, where p and q are integers. [4]

$2\sqrt{3}x-3x=3+\sqrt{3} \Rightarrow (2\sqrt{3}-3)x=3+\sqrt{3}$ [M1]

$x=\frac{3+\sqrt{3}}{2\sqrt{3}-3}\times\frac{2\sqrt{3}+3}{2\sqrt{3}+3}$ [M1]

$=\frac{6\sqrt{3}+3\sqrt{3}+6+3(3)}{4(3)-3(3)}$ [M1]

$=\frac{9\sqrt{3}+15}{3}$ [A1]

$=5+3\sqrt{3}$

11. Solve the equation [4]

(i) $2\log_2 6-\log_2(x-1)=3+\log_2(4x-7)$,

$2\log_2 6-\log_2(x-1)-\log_2(4x-7)=3$

$\log_2\frac{6^2}{(x-1)(4x-7)}=3 \Rightarrow \log_2\frac{36}{4x^2-11x+7}=3$ [M1]

$\frac{36}{4x^2-11x+7}=2^3 \Rightarrow 36=8(4x^2-11x+7)$ [M1]

$8x^2-22x+14-9=0 \Rightarrow 8x^2-22x+5=0$

$(4x-1)(2x-5)=0$

$x=\frac{1}{4}$ (rejected), [A1] or $x=\frac{5}{2}$ [A1]

[4]

(ii) $\log_2 x + 3\log_x 2 = 4.$ [M1]

$$\log_2 x + 3\left(\frac{\log_2 2}{\log_2 x}\right) = 4$$ [M1]

Let $a = \log_2 x$, $a + \frac{3}{a} = 4 \Rightarrow a^2 - 4a + 3 = 0$

$(a-1)(a-3) = 0 \Rightarrow a = 1, a = 3$

$\log_2 x = 1$, $x = 2$ [A1] or $\log_2 x = 3$, $x = 2^3 = 8$ [A1]

12. At the beginning of 2010, the number of a certain species of insects was estimated at 80000. The population decreased so that, after a period of t years, the population was $80000e^{-0.03t}$. Estimate [1]

(i) the population at the beginning of 2020,

$80000e^{-0.03(10)} \approx 59266$ [B1]

(ii) the year in which the population would be expected to be halved. [3]

$e^{-0.03t} = 40000 \div 80000 = \frac{1}{2}$ [M1]

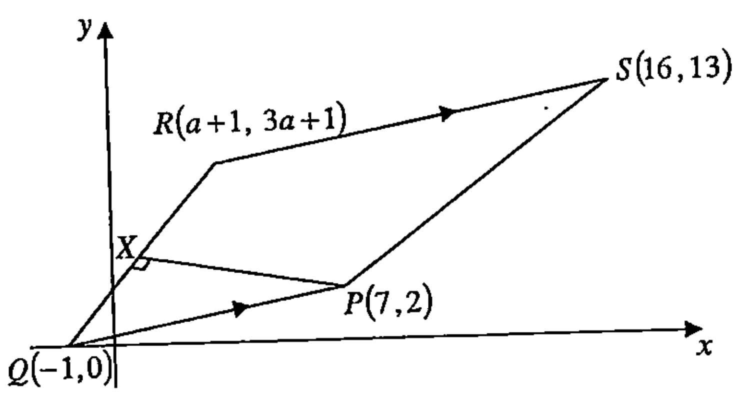
$$-0.03t = \ln 0.5 \Rightarrow t = \frac{\ln 0.5}{-0.03}$$ [M1]

$\therefore t = 23.1$ [A1]

The year is $2010 + 23 \approx 2033$

13. **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a quadrilateral $PQRS$ in which P is $(7, 2)$, Q is $(-1, 0)$, R is $(a+1, 3a+1)$ and S is $(16, 13)$. RS is parallel to QP and X is the mid-point of QR.



(i) Find the value of a. [3]

Gradient of PQ [M1] = Gradient of RS [M1] (// lines)

$$\frac{3a+1-13}{0-2} = \frac{a+1-16}{-1-7} \Rightarrow \frac{3a-12}{-2} = \frac{a-15}{-8}$$

$4(3a-12) = a - 15 \Rightarrow 12a - 48 = a - 15$ [A1]

$11a = 33 \Rightarrow a = 3$

(ii) Find the area of quadrilateral $PQRS$. [2]

Area of quadrilateral $PQRS = \frac{1}{2}\begin{vmatrix} -1 & 7 & 16 & 4 & -1 \\ 0 & 2 & 13 & 10 & 0 \end{vmatrix}$

$= \frac{1}{2}\left|(-2+91+160)-(32+52-10)\right| = \frac{1}{2}\left|175\right|$ [M1]

$= 87\frac{1}{2}$ unit^2. [A1]

(iii) Determine whether PX is perpendicular to QR. [3]

Mid-point, $X\left(\frac{4-1}{2},\ \frac{10+0}{2}\right) = \left(\frac{3}{2},\ 5\right)$ [M1]

Gradient of $PX = \frac{5-2}{1.5-7} = -\frac{6}{11}$

[A1]

Gradient of $QR = \frac{10-0}{4+1} = 2$

Since the product of the gradients of PX and QR is not -1, [A1]
$\therefore PX$ is not perpendicular to QR.

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