YISHUN SECONDARY SCHOOL

We Seek, We Strive, We Soar

MID-YEAR EXAMINATION

Name : _____ Reg. No : _____ Class: _____

Calculator Model:_____

Sec 3 Express **Date: 16 May 2014**

ADDITIONAL MATHEMATICS (4047/1)

PAPER 1 **Duration: 2 hours**

MAX MARKS: 80

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of 5 printed pages including the cover page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}bc\sin A.$$

Answer all the questions.

1. An object is heated until it reaches a temperature of θ °C. It is then allowed to cool. Its temperature, θ °C, when it has been cooling for t minutes, is given by $\theta = 28 + 50(3^{-0.3t})$.

Find

(i) the initial value of θ, [1]

(ii) the value of θ when $t = 20$, [1]

(iii) the value of θ as t becomes very large. [1]

2. Express the following as powers of 6

(i) $2^{2r}9^r$ [1]

(ii) $\left(\frac{1}{\sqrt{216}}\right)^r$ [1]

Hence, or otherwise, solve the equation $\frac{2^{2r}9^r}{6} \times \left(\frac{1}{\sqrt{216}}\right)^r = 36^{\frac{3}{4r}}$. [3]

3. The straight line $2y - x = 4$ intersects the curve $2x^2 + 4y^2 = 38 + xy$ at the points A and B.
Find the coordinates of A and B. [5]

4. Given that the roots of the equation $2x^2 - 7x - 5 = 0$ are α and β,

(i) find the value of $\alpha^2 + \beta^2$,

(ii) form an equation, with integer coefficients, whose roots are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$. [8]

5. A rectangular block of wood has a square base.
The length of each side of the base is $\left(\sqrt{3} - \sqrt{2}\right)$ cmand the volume of the block is $\left(4\sqrt{2} - 3\sqrt{3}\right)$ cm^3.

(i) Find, **without using a calculator,** the

(a) base area of the rectanglular block, leaving your answer in surd form, [1]

(b) height of the block in the form $\left(a\sqrt{2} + b\sqrt{3}\right)$cm, where a and b are integers, [4]

(ii) Evaluate the total surface area of the block. [3]

6. (a) Find the range of values of x for which $(1+x)(6-x) \leq -8$. [3]

(b) Find the range of values of m for which $2x^2 > m(1-4x)$ for all real values of x. [3]

(c) Show that $my = x(x-2)+1$ meets the curve $y = x^2 - 3x + 2$ at two distinct points for all non-zero values of m. [5]

7. (a) Simplify and evaluate $\dfrac{5\times 2^n - 8\times 2^{n-2}}{2^n - 2^{n+3}}$ [3]

(b) Express $\dfrac{15x^2+5x+2}{(2-x)(1+2x^2)}$ in partial fractions. [5]

8. (a) The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 2 and the roots of the equation $f(x) = 0$ are $-1, -3, \dfrac{3}{2}$.

(i) Express $f(x)$ as a product of linear factors $ax + b$, where a and b are all integers. [2]

(ii) Hence, solve the equation $f(x-1) = 0$. [3]

(b) The expression $px^3 + x^2 + qx + 2$ has a factor $2x-1$ but leaves a remainder of 6 when divided by $x+1$.

(i) Find the value of p and of q. [3]

(ii) With the value of p and q found in (i), solve $px^3 + x^2 + qx + 2 = 0$. [4]

9. (a) By using a suitable substitution or otherwise, solve the equation

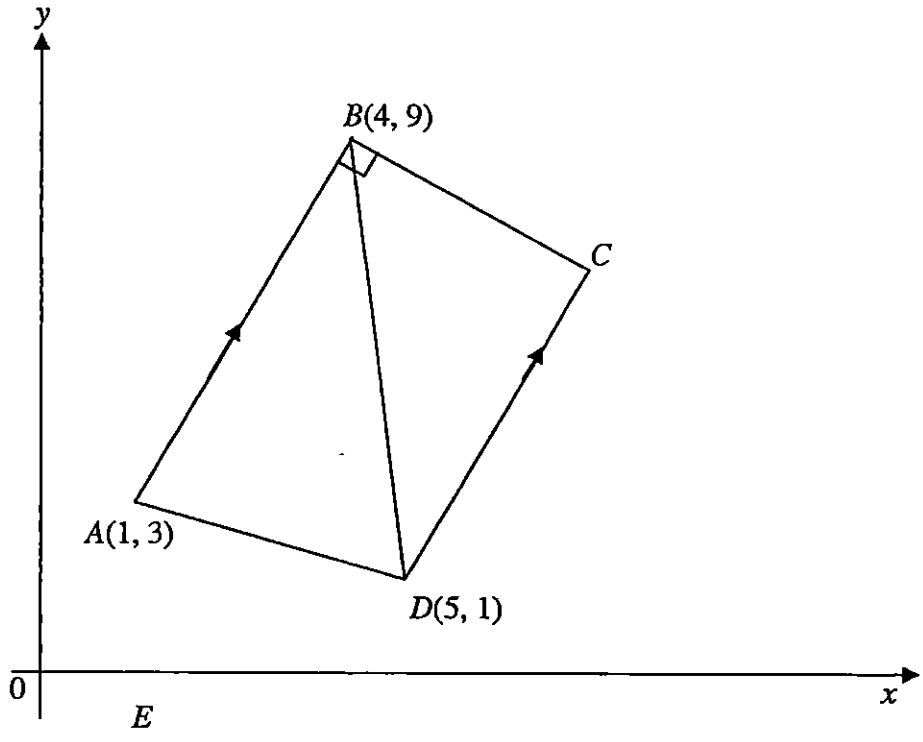
$$2^{2y+1} + 32 = 16(2^y).$$ [3]

(b) Solve the simultaneous equations. [5]

$$(4^x)^y = 32$$

$$5^{3x}\left(\frac{1}{25}\right)^y = 25$$

10. Solutions to this question by accurate drawing will not be accepted.



The diagram shows a trapezium $ABCD$ in which AB is parallel to $DC$and BC is perpendicular to AB.
The coordinates of A, B, and $D$are (1,3), (4,9)and(5,1) respectively.

(a) Find the equations of the line BC and DC.
Hence find the coordinates of C. [6]

(b) Find the area of $ABCD$. [2]

(c) The point $T$is such that $ADBT$ forms a parallelogram.
Find the coordinates of T. [2]

(d) E is a point on the$x$-axis such that $AE = ED$.
Find the coordinates of E. [2]

- End of Paper-

Answer-

1a) 78 b) 28.1 (c) 28

2a) (i) 6^{2r} (ii) $6^{-\frac{3}{2}r}$

(b) r = -1 or 3

3) (-4.4,-0.2) or (2,3)

4) a) 17.25 (exact) (b) $5x^2 + 69x + 20 = 0$

5)a) (i) $5 - 2\sqrt{6}$ (ii) $2\sqrt{2} + \sqrt{3}$

5 b) 6 cm^2

6a) $x \leq -2 or x \geq 7$

b) -0.5< m< 0

c) show that b^2-4ac = 0, since m non values values therefore discriminant is >0 and hence there is 2 distinct roots.

7a) -3/7

b) $\frac{8}{2-x} + \frac{x-3}{1\_2x^2}$

8a) (i) (x+1)(x+3)(2x-3)

(ii) x= 0,-2, 2.5

8b) (i) p = 2, q =-5

(ii) x=0.5, 1 , -2

9a) y = 2

b) x = -1, 5/3 and y = -2.5, 1.5

10a) Eqn of BC: y =- 0.5x + 11

Eqn of DC: y = 2x -9

C=(8,7)

10b) 30 units ^{2}

c) T=(0, 11)

d) E= (2,0)