聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

Name of Student

Class | Index Number

100

END-OF-YEAR EXAMINATION 2015
SECONDARY 3 EXPRESS
ADDITIONAL MATHEMATICS

4047

Date: **6 Oct 2015**

Duration: **2 hr 30 min**

Additional Materials: **7 sheets of writing papers**
1 string

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction tape/fluid.

Answer **ALL** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown clearly and neatly.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is **100**.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Set by: Ms Lua Bee Hian

Vetted by: Mrs Rajammal Nathan and Mdm Hayati

*This document consists of **5** printed pages (including cover page).*

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Answer all questions.

1. (i) Sketch the graph of $y = 2 + |3x + 1|$. [3]

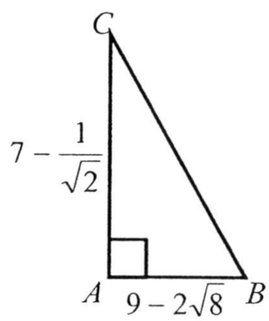
(ii) Find the set of values of the constant k for which the line $y = kx + 4$ intersects the graph $y = 2 + |3x + 1|$ at two distinct points. [2]

2. A curve has the equation $y = 5x^2 - 4x + k$, where k is a constant.

(i) In the case where $k = -9$, find the set of values of x for which $y > 0$. [3]

(ii) Find the value of k for which the line $y = 6x + 4$ is a tangent to the curve. [3]

3.



The diagram shows a right-angled triangle ABC in which $AB = \left(9 - 2\sqrt{8}\right)$ cm and $AC = \left(7 - \frac{1}{\sqrt{2}}\right)$ cm.

Express in the form of $a + b\sqrt{2}$, where a and b are rational numbers,

(i) the area of the triangle, [3]

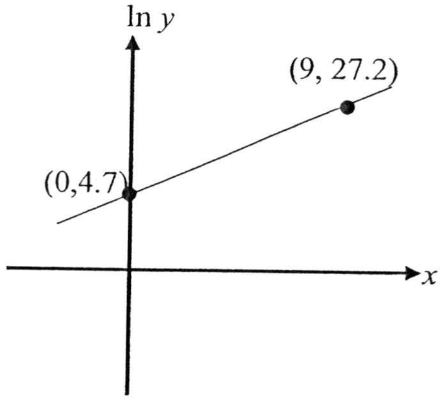
(ii) BC^2. [3]

4. Without using a calculator, find the value of

(i) 15^x, given that $75^{x-2} = 3^{2-x}$, [4]

(ii) p, given that $\left(\sqrt{5}\right)^9 + \left(\sqrt{5}\right)^7 + \left(\sqrt{5}\right)^5 + \left(\sqrt{5}\right)^3 - 155\sqrt{5} = 5^p$. [3]

5.



The variables x and y are connected by the equation $y = ab^x$, where a and b are constants. Experimental values of x and y were obtained. The diagram above shows the straight line graph, passing through the points (0, 4.7) and (9, 27.2), obtained by plotting ln y against x.

Estimate

(i) the value, to 2 significant figures, of a and of b, [6]

(ii) the value of y when $x = 2$. [2]

6. The mass, m grams, of a radioactive substance, present at time t years after first being observed, is given by the formula $m = 195(0.8)^t$.

(i) Find

(a) the initial mass of the substance, [1]

(b) the mass of the substance when $t = 6$, [1]

(c) the value of t when the mass of the substance is $\frac{1}{4}$ of its initial mass. [4]

Give your answers correct to three significant figures.

(ii) Explain why the mass of the substance can never be more than 195. [1]

(iii) Sketch the graph of m against t. [1]

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7. A circle, C_1, has equation $x^2+y^2+8x-12y+27=0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

A second circle, C_2 cuts C_1 on the y axis at the points F and G, where FG is the diameter of C_2.

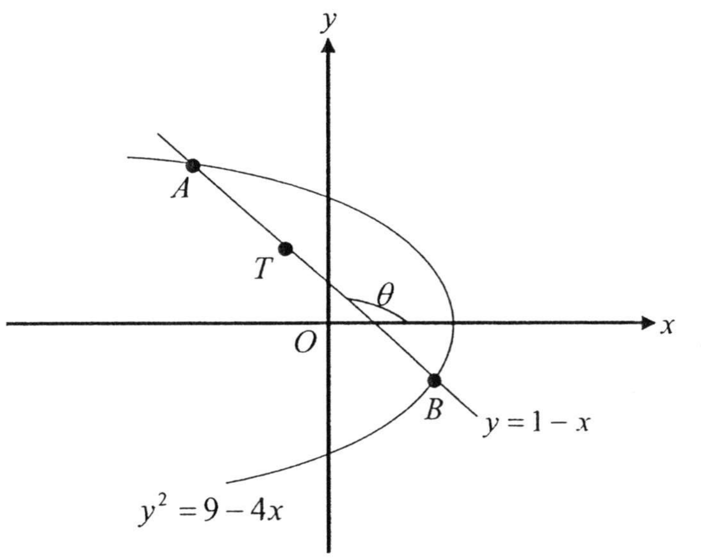
(ii) Find the equation of C_2 in the form of $(x-a)^2+(y-b)^2=r^2$. [4]

(iii) Explain why the point $(-3, 3)$ lies within only one of the circles C_1 and C_2. [2]

8. (a) Solve $2x^3+3x^2-11x-6=0$. [5]

(b) Express $\dfrac{2x+3}{(x+1)(x^2-5)}$ in partial fractions. [5]

9. The diagram shows part of the curve $y^2=9-4x$, which meets the line $y=1-x$ at the points A and B.



(i) Find

(a) the value of θ in degrees, [2]

(b) the coordinates of A and B, [4]

(c) the area of triangle ABC where point C is $(0, -3)$. [1]

(ii) A perpendicular line passes through the point T, where T lies on the line $y=1-x$ and the ratio of $AT : TB$ is $1 : 2$.
Find the equation of the perpendicular line. [3]

10. **(a)** Solve $2\log_7 p = 3 + \log_p 49$. [5]

(b) Given that $\log_3 x = a$ and $\log_9 y = b$, express in terms of a and b,

(i) $\log_x 9y$, [4]

(ii) $x^3 y$. [3]

11. **(a)** The roots of the quadratic equation $3x^2 - 5x + 1 = 0$ are α and β.

(i) Show that $\dfrac{\alpha^2 + \beta^2}{\alpha\beta} = 6\dfrac{1}{3}$. [4]

(ii) Find a quadratic equation whose roots are $1 - \dfrac{\alpha}{\beta}$ and $1 - \dfrac{\beta}{\alpha}$. [4]

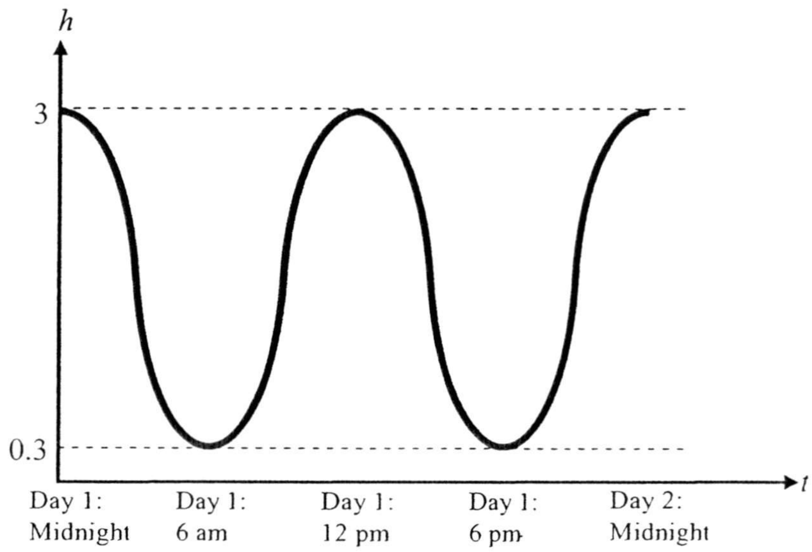
(b) **(i)** Without using a calculator, evaluate $\tan 45°(\sin 60° + \cos 30°)$. [2]

(ii) Given that $\sin A = \dfrac{3}{5}$ and that A is acute, find the value of

$3\tan A + \cos\left(\dfrac{\pi}{2} - A\right)$. [2]

12. **(a)** Solve $5\operatorname{cosec} x + 11 = 0$ for $0 \le x \le 2\pi$. [4]

(b)



The diagram shows the graph obtained on a particular day when the heights, h metres of the tides are recorded at a beach on the east of Singapore over time, t hours. The height is modelled by the equation $h = a + b\cos kt$, where a, b and k are constants.

(i) Show that the value of k is $\dfrac{\pi}{6}$ radians per hour. [1]

(ii) Find the value of a and of b. [2]

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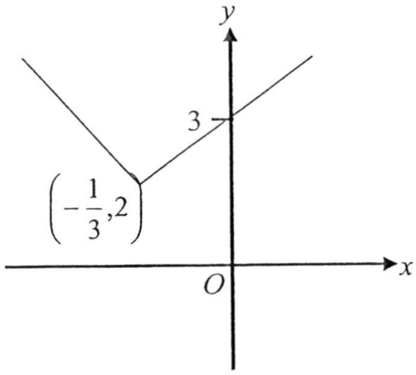
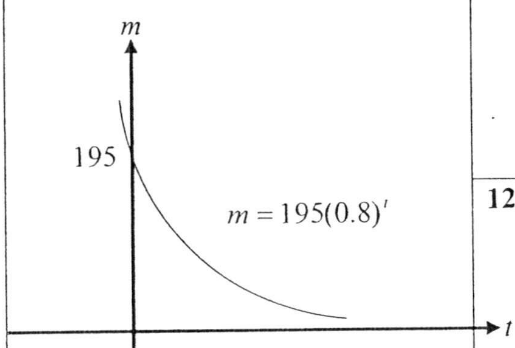
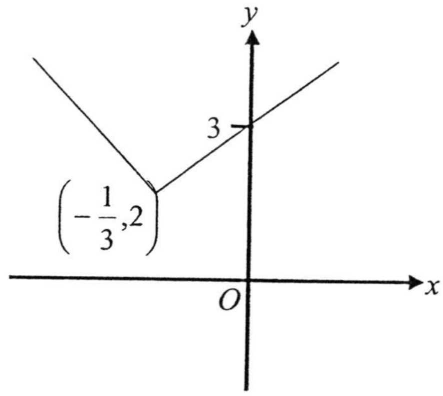
~ End of paper ~

Questions			Answers
1	(i)		
	(ii)		$-3 < k < 3$
2	(i)		$x > 1\frac{4}{5}$ or $x < -1$
	(ii)		$k = 9$
3	(i)		$\frac{67}{2} - \frac{65}{4}\sqrt{2}$ cm^2
	(ii)		$BC^2 = \frac{325}{2} - 79\sqrt{2}$
4	(i)		$15^x = 225$
	(ii)		$p = 4.5$
5	(i)		$a \approx 110$ and $b \approx 12$
	(ii)		$y \approx 16300(3s.f.)$
6	(i)	(a)	$m = 195$ gram
		(b)	$m \approx 51.1$ gram
		(c)	$t \approx 6.21$ years
	(ii)		Since the mass decreases with time, thus, the mass cannot never be more than 195 grams. *Possible acceptable answer:* $t \to \infty$, $0.8^t \to 0$, $195(0.8^t) \to 0$ Therefore the mass can never be more than 195
	(iii)		
7	(i)		centre of circle $= (-4, 6)$ Radius = 5 units
7	(ii)		Equation of C_2 is $x^2 + (y-6)^2 = 3^2$
	(iii)		Circle 2: when $x = -3$ and $y = 3$, $(-3)^2 + (3-6)^2 = 18$, which is more than 3^2 but less than 5^2. Therefore the point $(-3, 3)$ lies in C_1 only. ***Alternate method*** Circle 1: radius = 5 units Circle 2: radius = 3 units Distance from centre of Circle 1 to point $= \sqrt{(-4+3)^2 + (6-3)^2}$ $= \sqrt{10}$ units Since $3 < \sqrt{10} < 5$, it lies within circle 1.
8	(a)		$x = 2$ or $x = -\frac{1}{2}$ or $x = -3$
	(b)		$\frac{2x+3}{(x+1)(x^2-5)} = -\frac{1}{4(x+1)} + \frac{x+7}{4(x^2-5)}$
9	(i)	(a)	$\theta = 135°$
		(b)	Coordinates of A is $(-4, 5)$ and coordinates of B is $(2, -1)$.
		(c)	12 units^2
	(ii)		$y = x + 5$
10	(i)		$p \approx 0.378$ or $p = 49$
	(ii)	(a)	$\log_x 9y = \frac{2+2b}{a}$
		(b)	3^{3a+2b}
11	(a)	(i)	$\frac{\alpha^2 + \beta^2}{\alpha\beta} = 6\frac{1}{3}$
		(ii)	$x^2 + \frac{13}{3}x - \frac{13}{3} = 0$
	(b)	(i)	$\sqrt{3}$
		(ii)	$2\frac{17}{20}$
12	(a)		$x \approx 3.61$ or $x \approx 5.81$
	(b)	(i)	Period = 12 hours ➔ $\cos(\frac{\pi}{6}t)$ Therefore $k = \frac{\pi}{6}$ (shown)
		(ii)	$a = 1.65$, $b = 1.35$

Questions			Marking Scheme	Marks allocation	Remarks
1	(i)			B1: General shape B1: Labelled y-intercept or coordinates of y-axis B1: Labelled 'turning' pt or coordinates	General shape must be in 2nd quad. Line passing through y-intercept must be –ve gradient.
	(ii)		$-3<k<3$	B2	Show $-3<k$ or $k<3$ B1 each
2	(i)		$5x^2-4x-9>0$ $(5x-9)(x+1)>0$ $5x-9>0$ or $x+1<0$ $x>1\frac{4}{5}$ $\quad x<-1$	M1: factorization or $x=\dfrac{4\pm\sqrt{196}}{10}$ A2: 1 mark each	
	(ii)		$y=5x^2-4x+k$ -------- (1) $y=6x+4$ -------- (2) (1) = (2), $\quad 5x^2-4x+k=6x+4$ $5x^2-10x+k-4=0$ Since line is tangent to curve → $D=0$ $(-10)^2-4(5)(k-4)=0$ $100-20k+80=0$ $20k=180$ $k=9$	M1: obtain an equation in terms of x using simultaneous equation method M1: formulate equation with discriminant = 0 A1	e.c.f. is given
3	(i)		Area of right-angled triangle $=\frac{1}{2}\left(9-2\sqrt{8}\right)\left(7-\frac{1}{\sqrt{2}}\right)$ $=\frac{1}{2}\left(63-\frac{9}{\sqrt{2}}-14\sqrt{8}+\frac{2\sqrt{8}}{\sqrt{2}}\right)$ $=\frac{1}{2}\left(63-\frac{9}{\sqrt{2}}-28\sqrt{2}+4\right)$	M1: expansion 85	

			$=\frac{1}{2}\left(67-\frac{9\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}-28\sqrt{2}\right)$ $=\frac{1}{2}\left(67-\frac{9}{2}\sqrt{2}-28\sqrt{2}\right)$ $=\frac{1}{2}\left(67-\frac{9}{2}\sqrt{2}-28\sqrt{2}\right)$ $=\frac{1}{2}\left(67-\frac{65}{2}\sqrt{2}\right)$ $=\frac{67}{2}-\frac{65}{4}\sqrt{2}$ cm^2	M1: rationalizing denominator A1	
			Alternate method Area of right-angled triangle $=\frac{1}{2}\left(9-2\sqrt{8}\right)\left(7-\frac{1}{\sqrt{2}}\right)$ $=\frac{1}{2}\left(9-2\sqrt{8}\right)\left(7-\frac{\sqrt{2}}{2}\right)$ $=\frac{1}{2}\left(63-\frac{9}{2}\sqrt{2}-28\sqrt{2}+4\right)$ $=\frac{1}{2}\left(67-\frac{65\sqrt{2}}{2}\right)$ $=\frac{67}{2}-\frac{65}{4}\sqrt{2}$ cm^2	M1: rationalize denominator M1: expansion A1	
	(ii)		BC^2 $=\left(9-2\sqrt{8}\right)^2+\left(7-\frac{1}{\sqrt{2}}\right)^2$ $=81-36\sqrt{8}+32+49-\frac{14}{\sqrt{2}}+\frac{1}{2}$ $=81-72\sqrt{2}+32+49-\frac{14\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}+\frac{1}{2}$ $=81-72\sqrt{2}+32+49-7\sqrt{2}+\frac{1}{2}$ $=\frac{325}{2}-79\sqrt{2}$ ***Alternate method*** BC^2 $=\left(9-4\sqrt{2}\right)^2+\left(7-\frac{\sqrt{2}}{2}\right)^2$ $=81-72\sqrt{2}+32+49-7\sqrt{2}+\frac{1}{2}$	M1: expansion M1: rationalizing denominator A1 M1: rationalise denominator M1: expansion	

			$=\dfrac{325}{2}-79\sqrt{2}$	A1	
4	(i)		$75^{x-2}=3^{2-x}$ $(3\times5^2)^{x-2}=3^{2-x}$ $3^{x-2-2+x}\times5^{2x-4}=1$ $3^{2x-4}\times5^{2x-4}=1$ $15^{-4}\times15^{2x}=1$ $15^{2x}=15^4$ $15^x=15^2$ $15^x=225$ or -225 (rejected since $15^x>0$)	M1: express 75 in its prime factors M1: simplify base 3 and 5 M1(same base) A1: negative must be rejected	
	(ii)		$\left(5^{\frac{1}{2}}\right)^9+\left(5^{\frac{1}{2}}\right)^7+\left(5^{\frac{1}{2}}\right)^5+\left(5^{\frac{1}{2}}\right)^3-155\left(5^{\frac{1}{2}}\right)=5^p$ $625\left(5^{\frac{1}{2}}\right)+125\left(5^{\frac{1}{2}}\right)+25\left(5^{\frac{1}{2}}\right)+5\left(5^{\frac{1}{2}}\right)-155\left(5^{\frac{1}{2}}\right)=5^p$ $625\left(5^{\frac{1}{2}}\right)=5^p$ $5^4\times5^{\frac{1}{2}}=5^p$ $5^{\frac{9}{2}}=5^p$ Comparing indices, $p=4.5$	M1: simplify the power M1: change to same base A1	
5	(i)		$y=ab^x$ $\ln y=\ln ab^x$ $\ln y=\ln a+x\ln b$ $\ln a=4.7$ $a=109.9$ $a\approx110$ $\dfrac{27.2-4.7}{9-0}=\ln b$ $b=12.182$ $b\approx12$ *Alternative Method* $\ln y=2.5x+4.7$ $y=e^{2.5x+4.7}$ $y=e^{4.7}\times e^{2.5x}$ By comparing with $y=ab^x$,	M2: power law and quotient law M1 A1 M1 A1	

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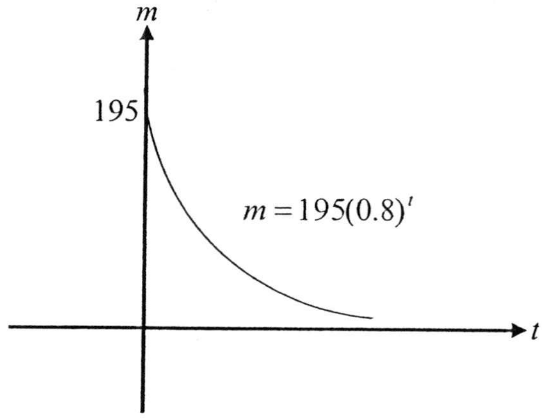
			$a = e^{4.7}$ $a \approx 110$ $b = e^{2.5}$ $b \approx 12$		
	(ii)		when $x = 2,\ y = 109.9(12.182)^2$ $y \approx 16300(3s.f.)$ *Alternate method* Gradient = 2.5 $\ln y = 2.5x + 4.7$ $\ln y = 2.5(2) + 4.7$ $y = e^{9.7} \approx 16300$	M1: subst. truncated value A1	e.c.f is given provided exact or truncated value is used.
6	**(i)**	**(a)**	when $t = 0,\quad m = 195(0.8)^0$ $m = 195(1)$ $m = 195$ gram	B1	
		(b)	when $t = 6,\quad m = 195(0.8)^6$ $m \approx 51.1(3s.f.)$ gram	B1	
		(c)	$\frac{1}{4}(195) = 195(0.8)^t$ $\frac{1}{4} = (0.8)^t$ $\ln\left(\frac{1}{4}\right) = \ln(0.8)^t$ $\ln\left(\frac{1}{4}\right) = t\ln(0.8)$ $t = \ln\left(\frac{1}{4}\right) \div \ln(0.8)$ ≈ 6.21 years	M1: formulate equation. o.e. M1: using natural log o.e. M1: apply power law A1	
	(ii)		Since the mass decreases with time, thus, the mass cannot never be more than 195 grams. *Possible acceptable answer:* $t \to \infty,\ 0.8^t \to 0,\ 195(0.8^t) \to 0$ Therefore the mass can never be more than 195	B1: o.e.	Explain with respect to the context

	(iii)			B1: general shape and interception with vertical axis shown clearly and correct	
7	(i)		centre of circle = $\left(\frac{8}{-2},\frac{-12}{-2}\right)$ $=(-4,6)$ Radius = $\sqrt{(-4)^2+6^2-27}=5$ units ***Alternate method:*** $x^2+y^2+8x-12y+27=0$ $x^2+8x+4^2-4^2+y^2-12y+6^2-6^2+27=0$ $(x+4)^2-4^2+(y-6)^2-6^2+27=0$ $(x+4)^2+(y-6)^2=25$ centre of circle = $(-4,6)$ Radius = 5 units	B2: 1 mark for x and y coordinate each B1	
	(ii)		when $x=0$, $y^2-12y+27=0$ $(y-9)(y-3)=0$ $y-9=0$ or $y-3=0$ $y=9$ $y=3$ Centre of C_2 = $\left(0,\frac{9+3}{2}\right)$ $=(0,6)$ Radius = 3 units Equation of C_2 is $x^2+(y-6)^2=3^2$	M1 M1 M1 A1	*Accept:* $(x-0)^2+(y-6)$
	(iii)		Circle 2: when $x=-3$ and $y=3$, $(-3)^2+(3-6)^2=18$, which is more than 3^2 but less than 5^2. Therefore the point $(-3, 3)$ lies in C_1 only.	M1: value of 18 for comparison A1: explain the comparison of r^2 value for both circles and conclude	

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			Alternate method Circle 1: radius = 5 units Circle 2: radius = 3 units Distance from centre of Circle 1 to point $=\sqrt{(-4+3)^2+(6-3)^2}$ $=\sqrt{10}$ units Since $3<\sqrt{10}<5$, it lies within circle 1.	 M1 A1	
8	**(a)**		let f(x) = $2x^3+3x^2-11x-6$ $f(2) = 2(2)^3+3(2)^2-11(2)-6=0$ $x-2$ is a factor of f(x). $2x^3+3x^2-11x-6=(x-2)(2x^2+bx+3)$ Comparing coeff. of x, $-11=-2b+3$ $b=7$ $2x^3+3x^2-11x-6=(x-2)(2x^2+7x+3)$ $=(x-2)(2x+1)(x+3)$ $2x^3+3x^2-11x-6=0$ $(x-2)(2x+1)(x+3)=0$ $x-2=0$ or $2x+1=0$ or $x+3=0$ $x=2$ $\quad x=-\frac{1}{2}$ $\quad x=-3$	 M1: finding 1st factor M1: obtain quadratic form M1: factorise completely A2, minus 1 mark for 1 mistake	
	(b)		$\frac{2x+3}{(x+1)(x^2-5)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-5}$ $2x+3=A(x^2-5)+(Bx+C)(x+1)$ Subst. $x=-1$, $2(-1)+3=-4A$ $1=-4A$ $A=-\frac{1}{4}$ Subst. $x=0$, $3=-\frac{1}{4}(-5)+(C)(-0+1)$ $3=-\frac{1}{4}(-5)+(C)(1)$ $C=1\frac{3}{4}$	M1 M1 M1	For values of A, B and C, accept improper fraction or decimals

			Compare coeff. of x^2, $0 = -\frac{1}{4} + B$ $B = \frac{1}{4}$ therefore, $\frac{2x+3}{(x+1)(x^2-5)} = -\frac{1}{4(x+1)} + \frac{x+7}{4(x^2-5)}$	M1 A1	
9	**(i)**	**(a)**	$\tan\theta = -1$ θ lies in 2^{nd} quadrant Basic angle, $\alpha = \tan^{-1}1$ $= 45°$ Therefore, $\theta = 180° - 45°$ $= 135°$	M1 A1	
		(b)	$y^2 = 9 - 4x$ ----- (1) $y = 1 - x$ ----- (2) Subst. (2) into (1), $(1-x)^2 = 9 - 4x$ $1 - 2x + x^2 = 9 - 4x$ $x^2 + 2x - 8 = 0$ $(x+4)(x-2) = 0$ $x + 4 = 0$ or $x - 2 = 0$ $x = -4$ $x = 2$ Subst. $x = -4$ into (2), $y = 5$ Subst. $x = 2$ into (2), $y = -1$ Coordinates of *A* is $(-4, 5)$ and coordinates of *B* is $(2, -1)$.	M1: substitution M1: quadratic equation A2: must express in terms of coordinates, if not, minus 1 mark	
		(c)	area of triangle $ABC = \frac{1}{2}\begin{vmatrix} 2 & -4 & 0 & 2 \\ -1 & 5 & -3 & -1 \end{vmatrix}$ $= \frac{1}{2}(10 + 12 - 4 + 6)$ $= 12$ units^2	B1	
	(ii)		Since $AT : TB$ is $1 : 2$, coordinates of *T* is $(-2, 3)$ Gradient of perpendicular line $= 1$ $1 = \frac{y-3}{x+2}$ $y - 3 = x + 2$ $y = x + 5$	M1 M1 A1	

10	(i)		$2\log_7 p = 3 + 2\log_p 7$ $2\log_7 p = 3 + \dfrac{2}{\log_7 p}$ Let $u = \log_7 p$, $2u = 3 + \dfrac{2}{u}$ $2u^2 = 3u + 2$ $2u^2 - 3u - 2 = 0$ $(2u+1)(u-2) = 0$ $2u + 1 = 0$ or $u - 2 = 0$ $u = -\dfrac{1}{2}$ $\quad u = 2$ $\log_7 p = -\dfrac{1}{2}$ $\quad \log_7 p = 2$ $p = 7^{-0.5}$ $\quad p = 7^2$ $p \approx 0.378$ $\quad p = 49$	M1: change of base M1: subst. method M1: solving for quadratic A2: 1 mark each	
	(ii)	**(a)**	$\log_x 9y = \log_x 9 + \log_x y$ $= 2\log_x 3 + \dfrac{\log_3 y}{\log_3 x}$ $= 2\log_x 3 + \dfrac{\log_3 y}{\log_3 x}$ $= \dfrac{2}{a} + \dfrac{1}{a}(\log_3 y)$ $= \dfrac{2}{a} + \dfrac{1}{a}\left(\dfrac{\log_9 y}{\log_9 3}\right)$ $= \dfrac{2}{a} + \dfrac{2b}{a}$ $= \dfrac{2+2b}{a}$	M1: product law M2: power law and change of base s.o.i. A1	
			Alternate method: $\log_x 9y = \log_x 9 + \log_x y$ $= \dfrac{\log_3 9}{\log_3 x} + \dfrac{\log_9 y}{\log_9 x}$ $= \dfrac{2}{a} + \dfrac{b\log_3 9}{\log_3 x}$ $= \dfrac{2}{a} + \dfrac{2b}{a}$ $= \dfrac{2+2b}{a}$	M1: product law M2 A1	

		(b)	$x = 3^a$ and $y = 9^b$ $x^3 y = 3^{3a} \times 9^b$ $= 3^{3a} \times 3^{2b}$ $= 3^{3a+2b}$	M1: change to index form M1: change to same base A1	
11	(a)	(i)	$\alpha + \beta = -\frac{-5}{3} = \frac{5}{3}$ $\alpha\beta = \frac{1}{3}$ $\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ $= \frac{\left(\frac{5}{3}\right)^2 - 2\left(\frac{1}{3}\right)}{\frac{1}{3}}$ $= 6\frac{1}{3}$	M1: sum of roots M1: product of roots M1 A1	
		(ii)	$1 - \frac{\alpha}{\beta} + 1 - \frac{\beta}{\alpha} = 2 - \frac{\alpha^2}{\alpha\beta} - \frac{\beta^2}{\alpha\beta}$ $= 2 - \frac{\alpha^2 + \beta^2}{\alpha\beta}$ $= 2 - 6\frac{1}{3}$ $= -4\frac{1}{3}$ $\left(1 - \frac{\alpha}{\beta}\right)\left(1 - \frac{\beta}{\alpha}\right) = 1 - \frac{\beta}{\alpha} - \frac{\alpha}{\beta} + 1$ $= 2 - \frac{\alpha^2 + \beta^2}{\alpha\beta}$ $= 2 - 6\frac{1}{3}$ $= -4\frac{1}{3}$ Quadratic equation is $x^2 + \frac{13}{3}x - \frac{13}{3} = 0$	M1: simplify M1 M1 A1	accept improper fraction

	(b)	**(i)**	$\tan 45°(\sin 60° + \cos 30°)$ $= 1\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)$ $= \sqrt{3}$	M1: express in exact form A1	
		(ii)	$3\tan A + \cos\left(\frac{\pi}{2} - A\right)$ $= 3\left(\frac{3}{4}\right) + \frac{3}{5}$ $= 2\frac{17}{20}$	M1 A1	Calculator is allowed for this part, hence working involving use of calculator is acceptable.
12	**(a)**		$5\operatorname{cosec} x + 11 = 0$ $\operatorname{cosec} x = -\frac{11}{5}$ $\sin x = -\frac{5}{11}$ x lies in 3rd and 4th quadrant basic angle, $\alpha = \sin^{-1}\left(\frac{5}{11}\right)$ $x = \pi + \sin^{-1}\left(\frac{5}{11}\right)$ or $x = 2\pi - \sin^{-1}\left(\frac{5}{11}\right)$ $x \approx 3.61$ $x \approx 5.81$	M1: simplify M1: identify basic angle A2	
	(b)	**(i)**	Period = 12 hours → $\cos(\frac{\pi}{6}t)$ Therefore $k = \frac{\pi}{6}$ (shown)	B1: need to show period = 12 hr or some form of working.	***Working:*** $k = \frac{2\pi}{12}$ $k = \frac{\pi}{6}$
		(ii)	Graph is shifted by 0.3 units upward Therefore $a = 1.65$ Amplitude of oscillation = $\frac{3 - 0.3}{2} = 1.35$ Therefore $b = 1.35$	B1 B1	