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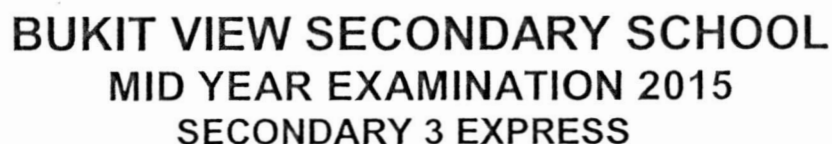
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2 hours

READ THESE INSTRUCTIONS FIRST

Do not use staples, paper clips, highlighters, glue or correction fluid.

The total number of marks for this paper is 80.

Answer **all** the questions

1. Show that the equation $2x^2 + x(3x + h) = -3k$ has two real and distinct roots for all negative values of k . [3]

2. Simplify $\sqrt{32} - \sqrt{8} + \sqrt{\frac{1}{8}}$, giving your answer in the form $k\sqrt{2}$. [3]

3. The equation $(k + 2)x^2 + 4k = (4k + 2)x$ has no real roots.
Find the range of values of k . [4]

4. The line $y = kx - 3$ is a tangent to the curve $y = 2x^2 + 7$.
Find the possible values of k . [4]

5. Find the value of $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$. [4]

6. (i) Simplify $2\log_4(x + 2y) - \log_4 x - \log_4 y$ into a single logarithmic function. [2]
 (ii) Given that $x^2 + 4y^2 = 12xy$, hence, show that

$$2\log_4(x + 2y) - \log_4 x - \log_4 y = 2. \quad [2]$$

7. (i) Show that $\frac{1}{\sqrt{p+1} + \sqrt{p}} = \sqrt{p+1} - \sqrt{p}$. [3]
 (ii) Hence, find the value of

$$\frac{1}{\sqrt{16} + \sqrt{15}} + \frac{1}{\sqrt{15} + \sqrt{14}} + \frac{1}{\sqrt{14} + \sqrt{13}} + \dots + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{1}}. \quad [2]$$

8. (i) Solve the simultaneous equations.

$$x = 1 + 2y$$

$$x = 6xy - 2y - 3$$

[5]

- (ii) Explain the geometrical meaning of the solution.

[1]

9. By using the substitution $u = 2^x$ or otherwise,

- (a) prove that $2^{x+3} - 2^x$, is exactly divisible by 7.

[2]

- (b) find the values of x such that $2^{2x+1} - 7(2^x) + 3 = 0$.

[4]

10. Find the range of values of x which satisfy the inequality $x^2 \leq x^2 - 2x < 4x^2 - 1$.

Represent the solution on a number line.

[6]

11. Express $y = 3x^2 - 6x + 11$ in the form $y = a(x + h)^2 + k$, where a , h and k are constants.

- (i) State the minimum value of y and the corresponding value of x .

[4]

- (ii) Sketch the graph $y = 3x^2 - 6x + 11$ showing clearly the coordinates of the minimum point of the curve and y -intercept.

[3]

12. A cuboid, as shown below, of volume $(18 + 11\sqrt{2})\text{cm}^3$ stands on its square base.

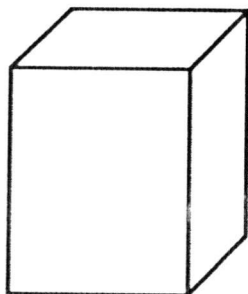
Given that the area of the base is $(3 + 2\sqrt{2})\text{cm}^2$, without using a calculator,

- (i) show that the length of its base is $(1 + \sqrt{2})\text{cm}$,

[4]

- (ii) find the height of the cuboid in the form $(p + q\sqrt{2})\text{cm}$, where p and q are integers.

[3]



13. (a) On the same axes, sketch the graphs $y = 3(2^{-0.2x})$ and $y = 3(2^{0.2x})$. [2]

(b) The amount, y milligrams, of a radioactive substance present after t years, is given by $y = Ae^{-0.2t}$, where A is a constant. At the beginning of 2000, there was an estimated amount of 500 mg. Find

(i) the value of A , [1]

(ii) the amount present in the year 2005, [2]

(iii) the year in which the amount of substance would be expected to be halved. [3]

(iv) the amount that has decayed after one decade. [3]

14. Solve each of the following equations.

(a) $e^{3x+2} = 43.7$ [2]

(b) $\frac{2}{\log_7 2} = \log_2(7 - x)$ [4]

(c) $4^x = 32^{2x+3}$ [4]

THE END

A Maths M1E 2015

i) $2x^2 + x(3x+h) = -3k$

[3m] $2x^2 + 3x^2 + xh + 3k = 0$

$5x^2 + (h)x + 3k = 0$ [m1]

$\therefore b^2 - 4ac = h^2 - 4(5)(3k)$ [m1]
 $= h^2 - 60k$

For All negative values of k , $-60k > 0$

$\therefore h^2 - 60k > 0$ # shown. [A1]

[3m] 2) $\sqrt{32} - \sqrt{8} + \sqrt{\frac{1}{8}}$

$= \sqrt{16 \times 2} - \sqrt{4 \times 2} + \sqrt{\frac{1}{4 \times 2}}$

$= 4\sqrt{2} - 2\sqrt{2} + \frac{1}{2\sqrt{2}}$ [m1] $= 2\sqrt{2} + \frac{1}{2\sqrt{2}}$

$= 2\sqrt{2} + \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ [m1]

$= 2\sqrt{2} + \frac{\sqrt{2}}{4}$

$= \frac{8\sqrt{2} + \sqrt{2}}{4}$

$= \frac{9\sqrt{2}}{4}$ or $\frac{9}{4}\sqrt{2}$ # [A1]

[4m] 3) $(k+2)x^2 + 4k = (4k+2)x$

$(k+2)x^2 - (4k+2)x + 4k = 0$ [m1]

$\therefore b^2 - 4ac < 0$

$[-(4k+2)]^2 - 4(k+2)(4k) < 0$ [m1]

$16k^2 + 16k + 4 - 16k(k+2) < 0$

$16k^2 + 16k + 4 - 16k^2 - 32k < 0$

$-16k + 4 < 0$ [m1]

$-16k < -4$

$k > \frac{1}{4}$ # [A1]

[4m] 4) $kx - 3 = 2x^2 + 7$

$2x^2 - kx + 7 + 3 = 0$

$2x^2 - kx + 10 = 0$ [m1]

$\therefore b^2 - 4ac = 0$

$(-k)^2 - 4(2)(10) = 0$ [m1]

$k^2 = 80$

$k = \pm\sqrt{80}$ or ± 8.94 # [A2]

[4m] 5) $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

$= \frac{\log_a a}{\log_a abc} + \frac{\log_b b}{\log_b abc} + \frac{\log_c c}{\log_c abc}$ [m1]

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$ [m1]

$= \log_{abc} abc$ [m1]

$= 1$ # [A1] OR $\log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc} abc = 1$

[4m] 6) (i) $2\log_4(x+2y) - \log_4 x - \log_4 y$ [3m]

$= \log_4(x+2y)^2 - \log_4 x - \log_4 y$ [m1]

$= \log_4 \left(\frac{x^2 + 4xy + 4y^2}{xy} \right)$ # [A1]

(ii) $2\log_4(x+2y) - \log_4 x - \log_4 y$ [3m]

$= \log_4 \left(\frac{12xy + 4xy}{xy} \right)$

$= \log_4 \left(\frac{16xy}{xy} \right)$ [m1]

$= \log_4 16$

$= \log_4 4^2$ [m1]

$= 2\log_4 4 = 2$ # shown.

[5m] 7) (i) $\frac{1}{\sqrt{P+1} + \sqrt{P}} \times \frac{\sqrt{P+1} - \sqrt{P}}{\sqrt{P+1} - \sqrt{P}}$ [m1]

$= \frac{\sqrt{P+1} - \sqrt{P}}{(\sqrt{P+1})^2 - (\sqrt{P})^2}$ [m1]

$= \frac{\sqrt{P+1} - \sqrt{P}}{P+1 - P}$ [m1]

$= \sqrt{P+1} - \sqrt{P}$ # shown.

(ii) $\frac{1}{\sqrt{16} + \sqrt{15}} + \frac{1}{\sqrt{15} + \sqrt{14}} + \frac{1}{\sqrt{14} + \sqrt{13}} + \dots + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{2} + 1}$

$= \sqrt{16} - \sqrt{15} + \sqrt{15} - \sqrt{14} + \sqrt{14} - \sqrt{13} + \dots + \sqrt{3} - \sqrt{2} + \sqrt{2} - 1$ [m1]

$= \sqrt{16} - 1$

$= 4 - 1$

$= 3$ # [A1]

8) (1) $x = 1 + 2y$ — (1)

(2m) $x = 6xy - 2y - 3$ — (2)

Sub (1) into (2):

$$1 + 2y = 6y(1 + 2y) - 2y - 3 \quad [m1]$$

$$1 + 2y = 6y + 12y^2 - 2y - 3$$

$$12y^2 + 6y - 2y - 2y - 3 - 1 = 0$$

$$12y^2 + 2y - 4 = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} [m1]$$

$$2(6y^2 + y - 2) = 0$$

$$(3y + 2)(2y - 1) = 0 \quad [m1]$$

$$y = -\frac{2}{3} \quad \text{or} \quad y = \frac{1}{2}$$

$$\therefore x = 1 + 2\left(-\frac{2}{3}\right) \quad x = 1 + 2\left(\frac{1}{2}\right)$$

$$= -\frac{1}{3} \quad = 2$$

$$\therefore x = -\frac{1}{3}, y = -\frac{2}{3} \quad \# \quad [A1]$$

$$x = 2, y = \frac{1}{2} \quad \# \quad [A1]$$

(ii) The line intersects the curve at the points $(2, \frac{1}{2})$ and $(-\frac{1}{3}, -\frac{2}{3})$

or

The line intersects the curve at 2 points. [B1]

9) (a) $2^{x+3} - 2^x = 2^x \times 2^3 - 2^x \quad [m1]$

$$= 8(2^x) - 2^x$$

$$= 7(2^x) \quad [A1]$$

$\therefore 2^{x+3} - 2^x$ is exactly divisible by 7.

(b) $2^{2x+1} - 7(2^x) + 3 = 0$

$2^{2x} \times 2 - 7(2^x) + 3 = 0 \quad [m1]$

$$2u^2 - 7u + 3 = 0$$

$$(2u - 1)(u - 3) = 0 \quad [m1]$$

$$u = \frac{1}{2} \quad \text{or} \quad u = 3$$

$$2^x = 2^{-1} \quad 2^x = 3$$

$$\therefore x = -1 \quad \# \quad [A1] \quad x = \log_2 3$$

$$= 1.58 \quad \# \quad [A1]$$

10) $x^2 \leq x^2 - 2x < 4x^2 - 1$

$$x^2 \leq x^2 - 2x$$

$$x^2 - x^2 + 2x \leq 0$$

$$2x \leq 0$$

$$x \leq 0 \quad [A1]$$

$$x^2 - 2x < 4x^2 - 1$$

$$x^2 - 4x^2 - 2x + 1 < 0$$

$$-3x^2 - 2x + 1 < 0$$

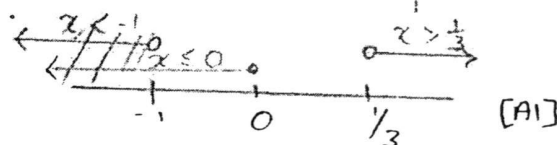
$$3x^2 + 2x - 1 > 0 \quad [m1]$$

$$(3x - 1)(x + 1) > 0 \quad [m1]$$



$$x < -1 \quad \text{or} \quad x > \frac{1}{3}$$

[A1]



$$\therefore x < -1 \quad \# \quad [A1]$$

11) $y = 3x^2 - 6x + 11$

$$[2m] = 3(x^2 - 2x + \frac{11}{3}) \quad [m1]$$

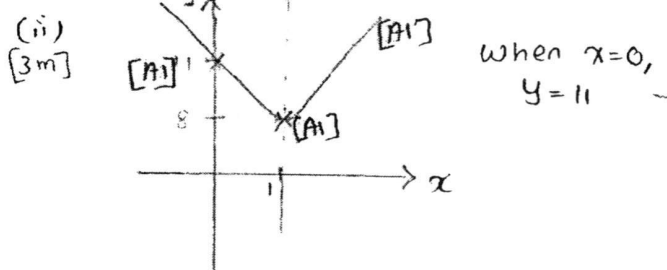
$$= 3[(x - 1)^2 - 1 + \frac{11}{3}]$$

$$= 3[(x - 1)^2 + 2\frac{2}{3}]$$

$$= 3(x - 1)^2 + 8 \quad \# \quad [A1]$$

(i) minimum value of $y = 8 \quad \# \quad [A1]$

[2m] corresponding value of $x = 1 \quad \# \quad [A1]$



when $x = 0$,
 $y = 11$

12) (i) let $\sqrt{3 + 2\sqrt{2}} = a + b\sqrt{2}$

$$[4m] \quad 3 + 2\sqrt{2} = (a + b\sqrt{2})^2 \quad [m1]$$

$$= a^2 + 2ab\sqrt{2} + 2b^2$$

$$= a^2 + 2b^2 + 2ab\sqrt{2}$$

$$\therefore 2ab = 2 \Rightarrow ab = 1 \quad \left. \begin{array}{l} a = \frac{1}{b} \text{ — (1)} \\ a^2 + 2b^2 = 3 \text{ — (2)} \end{array} \right\} [m1]$$

$$\sqrt{3 + 2\sqrt{2}}$$

Sub (1) into (2):

$$\begin{aligned} \left(\frac{1}{b}\right)^2 + 2b^2 &= 3 \\ \frac{1}{b^2} + 2b^2 &= 3 \\ 1 + 2b^4 &= 3b^2 \\ 2b^4 - 3b^2 + 1 &= 0 \end{aligned}$$

$$(2b^2 - 1)(b^2 - 1) = 0 \quad [m1]$$

$$b^2 = \frac{1}{2} \quad \text{or} \quad b^2 = 1$$

$$b = \pm \frac{1}{\sqrt{2}} \quad \text{or} \quad b = \pm 1$$

$$\therefore a = \pm \sqrt{2} \quad a = \pm 1$$

Reject negative values.

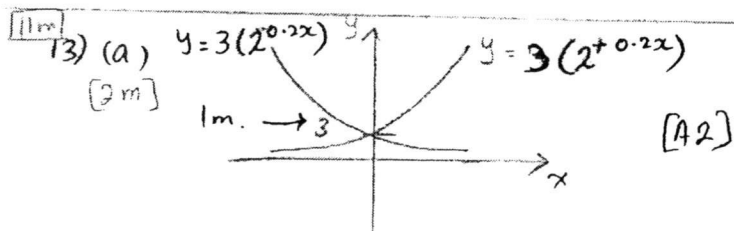
$$\therefore \left. \begin{aligned} b &= \frac{1}{\sqrt{2}}, a = \sqrt{2} \\ b &= 1, a = 1 \end{aligned} \right\} [m1]$$

$$\therefore \text{length} = 1 + \sqrt{2} \# \text{ shown.}$$

(ii) $\frac{18 + 11\sqrt{2}}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}} \quad [m2]$

$$= \frac{54 - 36\sqrt{2} + 33\sqrt{2} - 44}{9 - 8}$$

$$= 10 - 3\sqrt{2} \# [A1]$$



(b) (i) $500 = Ae^{-0.2(0)}$
 $\therefore A = 500 \# [B1]$

(ii) When $t = 5$, $y = 500e^{-0.2(5)}$
 $= 183.9$
 $\approx 184 \# [A1]$

(iii) $\frac{500}{2} = 500e^{-0.2t} \quad [m1]$

$$\frac{1}{2} = e^{-0.2t}$$

$$-0.2t = \ln\left(\frac{1}{2}\right) \quad [m1]$$

$$\therefore \text{The year is } 2000 + 3 = 2003 \# [A1]$$

(iv) When $t = 10$, $y = 500e^{-0.2(10)}$
 $= 67.6676 \quad [m1]$

$\therefore$ Amount decayed

$$= 500 - 67.6676 \quad [m1]$$

$$\approx 432 \# [A1]$$

(10m)

14) (a) $e^{3x+2} = 43.7$

[2m] $3x+2 = \ln(43.7) \quad [m1]$

$$x = 0.592 \# [A1]$$

(b) $\frac{2}{\log_7 2} = \log_2(7-x)$
[4m]

$$2 \log_2 7 = \log_2(7-x) \quad [m1]$$

$$\log_2 7^2 = \log_2(7-x) \quad [m1]$$

$$\therefore 49 = 7-x \quad [m1]$$

$$x = 7 - 49$$

$$= -42 \# [A1]$$

(c) $4^x = 32^{2x+3}$
[4m]

$$2^{2x} = 2^{5(2x+3)} \quad [m1]$$

$$\therefore 2x = 10x + 15 \quad [m1]$$

$$-8x = 15$$

$$x = -\frac{15}{8} \# [A1]$$

$$\text{or } -1.875 \#$$

(b) $\frac{2}{\log_7 2} = \log_2(7-x)$

$$2 = \log_2(7-x) \times \log_7 2$$

$$= \frac{\log_2(7-x)}{\log_2 7} \quad [m1]$$

$$2 = \log_7 7-x \quad [m1]$$

$$7^2 = 7-x$$

$$49 = 7-x \quad [m1]$$

$$x = -42 \quad [A1]$$