YUAN CHING SECONDARY SCHOOL
2015 MID–YEAR EXAMINATION
SECONDARY THREE EXPRESS

ADDITIONAL MATHEMATICS
4038 / 01

NAME:____________________________( ) TIME: 2 hours 30 minutes

CLASS: Sec ______

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staple, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of the marks for this paper is **100**.

100

This document consists of **5** printed pages.

[Turn Over

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 The straight line $3x+2y=1$ intersects the curve $3x^2+2y^2=11$ at points P and Q. Find the midpoint of PQ. [5]

2 **(a)** Given that $P(x)=3x^3-5x^2+4x-9$ and $Q(x)=2x^4+4x^3-6x-5$ share a common factor $x-a$, find the remainder when $P(x)+Q(x)$ is divided by $x-a$. [2]

(b) **(i)** Factorise x^3-y^3. [1]

(ii) Hence, write down 11^3-8^3 as the product of 2 integers. [1]

3 **(i)** Given that $f(x)=2x^3+ax^2+bx-9$, find the value of a and of b if $f(x)$ is exactly divisible by $x-3$ and leaves a remainder of 8 when divided by $x+1$. [5]

(ii) Hence, factorise $f(x)$ completely. [2]

4 Express $\dfrac{-3x^2+7x+6}{x^3+2x^2}$ in partial fractions. [5]

5 Given that the equation $x^2+(2-k)x+k=0$ has non-zero roots which differ by 2, find

(i) the value of each root, [5]

(ii) the value of k. [1]

6 The equation $x(x-2)=4$ has roots α and β where $\alpha>\beta$.

(i) Find $\alpha^2+\beta^2$. [3]

(ii) Find $\alpha-\beta$, leaving your answer in the form $a\sqrt{b}$ where a and b are integers. [2]

(iii) Form an equation whose roots are α^2 and β^2. [2]

7 **(a)** Find the range of values of m for $mx^2+1>2mx-3m$ for all real values of x. [4]

(b) Find the value(s) of p for which the line $y=px-5$ is a tangent to the curve $x^2=2y+1$. [3]

(c) Show that $x^2+(1-n)x-n=0$ has real roots for all real values of n. [2]

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8 (i) Express $f(x) = -2x^2 + 8x + 7$ in the form $a(x+h)^2 + k$, where a, h and k are constants. [2]

(ii) Hence, state the maximum or minimum value of $f(x)$ and the corresponding value of x. [2]

9 (i) Find the first 3 terms in the expansion, in ascending powers of x, of $(2-3x)^7$. [2]

(ii) Hence, estimate the value of $(1.997)^7$. [2]

(iii) Using your answer in part **(i)**, find the values of the constants h and k given that $(6+2x-3x^2)(2-3x)^7 = 768 + hx + kx^2 + \cdots$ [2]

10 (a) Find the term independent of x in the binomial expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$. [2]

(b) Find the coefficient of $\frac{1}{x^3}$ in the expansion of $\left(2x^2 - \frac{1}{2x}\right)^9$. [2]

11 A rectangle has length $\left(5-\sqrt{12}\right)$ m and breadth $\left(4+\frac{6}{\sqrt{3}}\right)$ m. Express in the form $\left(a+b\sqrt{3}\right)$, where a and b are integers,

(i) the area of the rectangle, [3]

(ii) the value of D^2, where D m is the length of the diagonal of the rectangle. [4]

12 Solve

(a) $9^x + 2(3^x) = 3^{x+2} - 12$, [4]

(b) $e^x = 5 + 6e^{-x}$. [3]

13 (a) Given that $2\lg xy = 2 + \lg(1+x) - \lg y$, express y in terms of x. [4]

(b) Given that $\ln 2 = a$ and $\ln 5 = b$, express $\ln\sqrt[3]{10e}$ in terms of a and b. [3]

(c) Prove that $\log_a b + \log_{\frac{1}{a}} b = 0$. [3]

14 After a newly invented pesticide has been used on a crop farm for t days, the number of caterpillars, N, on the farm is given by $N = 2500\left(3 + e^{-0.163t}\right)$. Find, correct to the nearest whole number,

(i) the initial number of caterpillars on the farm before the pesticide is used, [1]

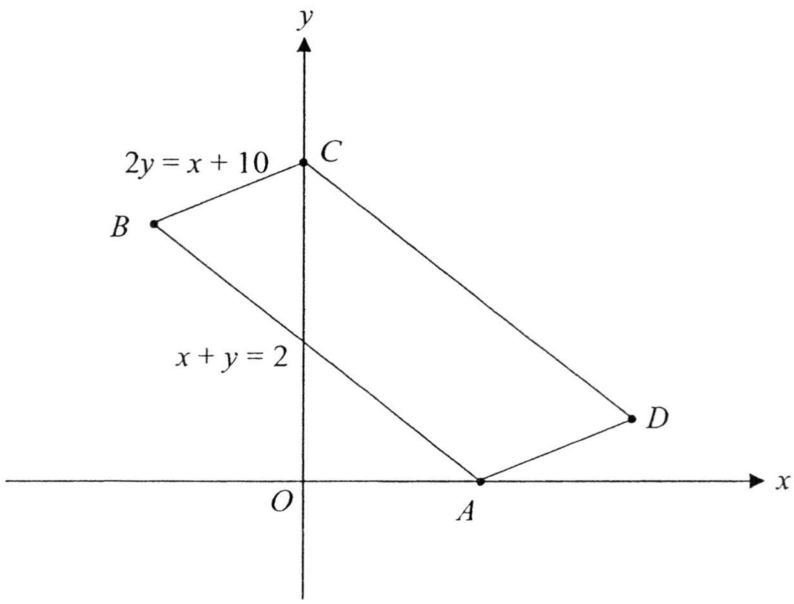
(ii) the number of caterpillars after 1 week, [2]

(iii) the time taken for the number of caterpillars to reduce to less than 8500, [3]

(iv) the number of caterpillars that died off in the long run. [1]

15 **Solution to this question by accurate drawing will not be accepted.**

The diagram shows a parallegram $ABCD$ with A and C on the x-axis and y-axis respectively. The equation of AB is $x + y = 2$ and the equation of BC is $2y = x + 10$.



Find

(i) the coordinates of A, B, C and D, [5]

(ii) the equation of line AD, [2]

(iii) the area of the parallegram $ABCD$, [2]

(iv) the equation of the perpendicular bisector of AC. [3]

-----END OF PAPER-----

Efforts today, Rewards tomorrow

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Answer Key

Qn	Answers	Qn	Answers
1	$\left(\frac{1}{5}, \frac{1}{5}\right)$		
2(a)	0		
2(b)(i)	$(x-y)(x^2+xy+y^2)$	2(b)(ii)	$(3)(273)$
3(i)	$a=1$, $b=-18$	3(ii)	$f(x)=(x-3)(x+3)(2x+1)$
4	$\frac{-3x^2+7x+6}{x^3+2x^2}\ \frac{-3x^2+7x+6}{x^2(x+2)}=\frac{2}{x}+\frac{3}{x^2}-\frac{5}{x+2}$		
5(i)	$\alpha=2$, $\alpha=-2$ (rejected) $\alpha+2=4$, $\alpha+2=0$ (rejected)	5(ii)	$k=8$
6(i)	12	6(ii)	$\pm 2\sqrt{5}$
6(iii)	$x^2-12x+16=0$		
7(a)	$m<-0.5$ (N.A.) or $m>0$	7(b)	$p=\pm 3$
8(i)	$f(x)=-2x^2+8x+7=-2(x-2)^2+15$	8(ii)	Maximum of $y=15$ corresponding value of $x=2$
9(i)	$(2-3x)^7=128-1344x+6048x^2+\ldots$	9(ii)	126.662048
9(iii)	$h=-7808$, $k=33216$		
10(a)	7920	10(b)	$-\frac{9}{8x^3}$
11(i)	$8+2\sqrt{3}$	11(ii)	$64-4\sqrt{3}$
12(a)	1, 1.26 (3s.f)	12(b)	ln 6/1.79 (3s.f)
13(a)	$y=\sqrt[3]{\frac{100(1+x)}{x^2}}$	13(b)	$\frac{a+b+1}{3}$
14(i)	10 000	14(ii)	8 299
14(iii)	6 days	14(iv)	2 500
15(i)	$A=(2,0)$, $B=(-2,4)$, $C=(0,5)$, $D=(4,1)$	15(ii)	$2y=x-2$
15(iii)	12	15(iv)	$10y=4x+21$