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- 1 The straight line $x + y = 12$ meets the curve $y = \frac{10}{x} + x$ at two points A and B.
- (i) Calculate the coordinates of A and of B. [4]
- (ii) Hence, find the equation of the perpendicular bisector of AB. [4]
- 2 Solve the following simultaneous equations: [4]
- $$3^{x-1} = 9(3^y)$$
- $$2^x + 2^y = 9$$
- 3 (a) Find the value of c for which the line $y = 2x + c$ is a tangent to the curve
- $$y = 2x^2 - 6x + 5. \quad [3]$$
- (b) Show that the equation $(p+2)x^2 - 2(\sqrt{2p+1})x + 4 = 0$ cannot have real roots for all values of x as long as $p > 0$. [3]
- 4 (a) Solve the equation $9^{x+1} = 10(3^x) - 1$. [4]
- (b) Given that $k = \frac{1}{\sqrt{2}}$ and that $p = \frac{1+k}{1-k}$, prove that $p = 3 + 2\sqrt{2}$. [4]
- 5 (a) Express $\left(\frac{4\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}\right)$ in the form $a + b\sqrt{c}$ where a , b and c are constants. [3]
- (b) Given that $\left(\frac{\sqrt{40}}{3} - \frac{1}{\sqrt{10}}\right)\left(\frac{30}{\sqrt{5}}\right) = k\sqrt{2}$, find the integer value of k . [4]
- 6 It is given that $x^4 - x^3 - 7x^2 + x + 6 = (x^2 - 2)Q(x) + ax - 4$ for all values of x where $Q(x)$ is a polynomial in x and a is a constant. Find
- (i) $Q(x)$, [4]
- (ii) the value of constant a , [1]
- (iii) the remainder when $x^4 - x^3 - 7x^2 + x + 6$ is divided by $x + 3$. [2]

Start Question 7 on a fresh sheet of Answer Paper.

Hand in Questions 7 to Question 11 separately from Question 1 to Question 6.

- 7 (i) Find the range of values of m for which the line $y = mx - 4$ meets the curve

$$y = 2x^2 + 2x + m. \quad [5]$$

- (ii) Using the smallest positive integer value of m found in (i), find the coordinates of the point of intersection between the line and the curve in (i). [3]

- 8 (a) Solve the quadratic inequality $2x^2 - 8x + 14 \geq 3x^2 - 8x + 10$. [3]

- (b) Find the range of values of x for which $\frac{3(x+1)^2}{4} < x+2 \leq 2x(x-1)$. [5]

- 9 The roots of the equation $2x^2 - x + 6 = 0$ are α and β .

- (i) Find the value of $\alpha^2 + \beta^2$. [4]

- (ii) Factorise $\alpha^3 + \beta^3$ completely and show that $\alpha^3 + \beta^3 = -\frac{35}{8}$. [4]

- (iii) Hence, form a quadratic equation whose roots are α^3 and β^3 . [2]

- 10 If $5x^3 + Ax^2 + x + 3 = (x+1)(Bx^2 + Cx) + D$, find the values of A , B , C and D . [5]

- 11 (i) Given that $f(x) = x^3 + 2x^2 - 17x + 6$,

- (a) show that $x - 3$ is a factor, [1]

- (b) solve the equation $f(x) = 0$, giving your answers to 2 decimal places where appropriate. [3]

- (ii) Hence, or otherwise, express $\frac{11}{x^3 + 2x^2 - 17x + 6}$ in partial fractions. [5]

~End of Paper~

Have you checked your work?

Answer Key

- 1 (i) $A = (1, 11)$ $B = (5, 7)$ (ii) $y = x + 6$
- 2 $x = 3$, $y = 0$
- 3 (a) $c = -3$ (b) $D = -4(p+9) < 0$
- 4 (a) $x = -2$ or $x = 0$ (b) $3 + 2\sqrt{2}$
- 5 (a) $4\sqrt{10} + 12$ (b) $k = 17$
- 6 (i) $Q(x) = x^2 - x - 5$ (ii) $a = -1$ (iii) $f(-3) = 48$
- 7 (i) $m \leq -2$ or $m \geq 14$ (ii) $(3, 38)$
- 8 (a) $-2 < x \leq 2$ (b) $-1\frac{2}{3} < x \leq -\frac{1}{2}$
- 9 (i) $-5\frac{3}{4}$ (ii) $-\frac{35}{8}$ (proved)
- (iii) $8x^2 + 35x + 216 = 0$
- 10 (a) $A = 6$, $B = 5$, $C = 1$, $D = 3$
- 11 (i) (a) $f(3) = 0$ (b) $x = 3$ or $x = -5.37$ or $x = 0.37$
- (ii) $\frac{1}{2(x-3)} - \frac{x+8}{2(x^2+5x-2)}$

Sec 3 Express A. Maths MY 2016

1(i)

$$y = 12 - x \quad \text{--- (1)}$$

$$y = \frac{10}{x} + x \quad \text{--- (2)}$$

$$\therefore \frac{10}{x} + x = 12 - x \quad m1$$

$$10 + x^2 = 12x - x^2$$

$$2x^2 - 12x + 10 = 0$$

$$x^2 - 6x + 5 = 0 \quad m1$$

$$(x-5)(x-1) = 0$$

$$x = 1 \quad \text{or} \quad x = 5 \quad A1$$

$$\therefore y = 11 \quad \text{or} \quad y = 7 \quad A1$$

$$\therefore \underline{A \equiv (1, 11) \quad B \equiv (5, 7)}$$

(4)

(ii)

$$\begin{aligned} \text{Given } AB &= \frac{11-7}{1-5} \\ &= \frac{4}{-4} \end{aligned}$$

$$\text{Mid pt} \equiv \left(\frac{1+5}{2}, \frac{11+7}{2} \right)$$

$$= (3, 9)$$

$\therefore$ Eq of $\perp$ bisector is

$$\frac{y-9}{x-3} = -1$$

$$y-9 = -(x-3)$$

$$\underline{y = -x + 12} \quad (4)$$

(8)

2)

$$3^{x-1} = 9(3^y)$$

$$3^{x-1} = 3^2(3^y)$$

$$= 3^{2+y}$$

$$\therefore x-1 = 2+y$$

$$x = 3+y \quad \text{--- (1)}$$

m1

$$2^x + 2^y = 9$$

$$\therefore 2^{3+y} + 2^y = 9$$

m1

$$8(2^y) + 2^y = 9$$

$$9(2^y) = 9$$

$$2^y = 1$$

$$y = 0$$

1

$$\therefore x = 3$$

$$\underline{\text{Ans: } x = 3, y = 0}$$

1

(4)

3a)

$$y = 2x + c \quad \text{--- (1)}$$

$$y = 2x^2 - 6x + 5 \quad \text{--- (2)}$$

$$\text{Sub (1) } \rightarrow \text{ (2)} \quad 2x^2 - 6x + 5 = 2x + c$$

$$2x^2 - 8x + (5 - c) = 0 \quad |$$

Since only 1 root, $D = 0$

$$b^2 - 4ac = 0$$

$$64 - 4(2)(5 - c) = 0 \quad |$$

$$8 - (5 - c) = 0$$

$$3 + c = 0$$

$$\underline{\underline{c = -3}} \quad | \quad (3)$$

$$b) \quad (p+2)x^2 - 2(\sqrt{2p+1})x + 4 = 0$$

$$D = b^2 - 4ac$$

$$= 4(2p+1) - 4(p+2)(4) \quad |$$

$$= 4[2p+1 - 4p-8]$$

$$= 4[-2p-7]$$

$$= -4[p+7] \quad |$$

$$\underline{\underline{< 0}} \quad \text{if } p > 0 \quad |$$

$\therefore$ It cannot have real roots for all $p > 0$

$\forall x$ as long as $p > 0$

proved.

(6)

4 a)

$$9^{x+1} = 10(3^x) - 1$$

$$9(3^x)^2 = 10(3^x) - 1$$

$$\text{Let } y = 3^x$$

$$9y^2 - 10y + 1 = 0$$

$$(9y - 1)(y - 1) = 0$$

$$y = \frac{1}{9} \text{ or } y = 1$$

$$\therefore 3^x = 3^{-2} \text{ or } 3^x = 3^0$$

$$\therefore x = -2 \text{ or } x = 0$$

(4)

b)

$$p = \frac{1+k}{1-k}$$

$$= \frac{1 + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}+1}{\sqrt{2}} \div \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$= \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{2 + 2\sqrt{2} + 1}{1}$$

$$= 3 + 2\sqrt{2}$$

proved

(4)

(8)

5 a)

$$\frac{4\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{4\sqrt{2}(2\sqrt{5}+3\sqrt{2})}{(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})}$$

$$= \frac{8\sqrt{10} + 24}{20 - 18}$$

$$= \frac{8\sqrt{10} + 24}{2}$$

$$= \underline{\underline{4\sqrt{10} + 12}}$$

(3)

$$(b) \left(\frac{\sqrt{40}}{3} - \frac{1}{\sqrt{10}} \right) \frac{30}{\sqrt{5}} = k\sqrt{2}$$

$$\left(\frac{20-3}{3\sqrt{10}} \right) \left(\frac{30}{\sqrt{5}} \right) = k\sqrt{2}$$

$$\left(\frac{17}{3\sqrt{10}} \right) \left(\frac{30}{\sqrt{5}} \right) = k\sqrt{2}$$

$$\frac{170}{5\sqrt{2}} = k\sqrt{2}$$

$$\frac{170}{10} = k$$

$$\therefore \underline{\underline{k = 17}}$$

(7)

(4)

6)

$$x^4 - x^3 - 7x^2 + x + 6 = (x^2 - 2) Q(x) + ax - 4$$

$$\begin{array}{r} x^2 - 2 \overline{) x^4 - x^3 - 7x^2 + x + 6} \\ \underline{x^4 - x^3 - 5x^2 + x} \\ -2x^2 \end{array}$$

$$\begin{array}{r} -2x^2 \\ \underline{-2x^2 + 4x} \\ 4x \end{array}$$

M1

$$\begin{array}{r} 4x \\ \underline{4x - 8} \\ 12 \end{array}$$

M1

$$\begin{array}{r} 12 \\ \underline{12x - 24} \\ 12x - 24 \end{array}$$

$$\begin{array}{r} 12x - 24 \\ \underline{12x - 24} \\ 0 \end{array}$$

M1

$$\begin{array}{r} 0 \\ \underline{0} \\ 0 \end{array}$$

$$\begin{array}{r} 0 \\ \underline{0} \\ 0 \end{array}$$

$$\therefore x^4 - x^3 - 7x^2 + x + 6 = (x^2 - 2)(x^2 - x - 5) - x - 4$$

$$(i) \quad Q(x) = \underline{\underline{x^2 - x - 5}} \quad A1$$

$$(ii) \quad a = \underline{\underline{-1}} \quad A1$$

(5)

$$(iii) \quad \text{Let } f(x) = x^4 - x^3 - 7x^2 + x + 6$$

$$f(-3) = 81 + 27 - 63 - 3 + 6 \quad M1$$

$$\underline{\underline{48}}$$

A1

(2)

(7)

7 (1)

$$y = mx - 4 \quad \text{--- (1)}$$

$$y = 2x^2 + 2x + m \quad \text{--- (2)}$$

sub (1) $\rightarrow$ (2) $2x^2 + 2x + m = mx - 4$

$$2x^2 + (2-m)x + m + 4 = 0$$

m1

meeting the curve $\Rightarrow D \geq 0$

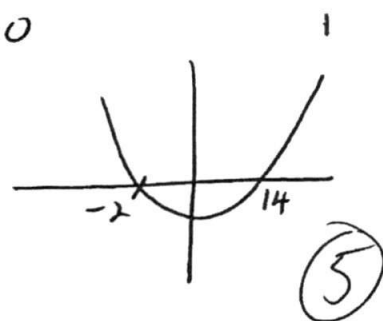
$$\therefore (2-m)^2 - 4(2)(m+4) \geq 0$$

$$4 - 4m + m^2 - 8m - 32 \geq 0$$

$$m^2 - 12m - 28 \geq 0$$

$$(m - 14)(m + 2) \geq 0$$

$$\therefore \underline{m \leq -2 \text{ or } m \geq 14}$$



(11) let $m = 14$

$$y = 14x - 4 \quad \text{--- (1)}$$

$$y = 2x^2 + 2x + 14 \quad \text{--- (2)}$$

$$2x^2 + 2x + 14 = 14x - 4$$

$$2x^2 - 12x + 18 = 0$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3$$

$$\therefore y = 14(3) - 4$$

$$= 38$$

$\therefore$ The pt is (3, 38)

(8)

(3)

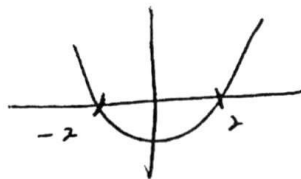
8 a)

$$2x^2 - 8x + 14 \geq 3x^2 - 8x + 10$$

$$x^2 - 4 \leq 0$$

$$(x-2)(x+2) \leq 0$$

$$\therefore -2 \leq x \leq 2$$



(3)

b)

$$\frac{3(x+1)^2}{4} < x+2 \leq 2x(x-1)$$

$$\frac{3}{4}(x+1)^2 < x+2 \quad \text{and} \quad x+2 \leq 2x(x-1)$$

$$3(x^2 + 2x + 1) < 4x + 8 \quad \text{and} \quad x+2 \leq 2x^2 - 2x$$

$$3x^2 + 6x + 3 < 4x + 8$$

$$2x^2 - 3x - 2 \geq 0$$

$$3x^2 + 2x - 5 < 0$$

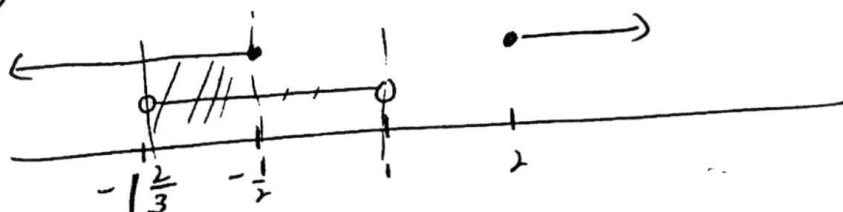
$$(2x+1)(x-2) \geq 0$$

$$(3x+5)(x-1) < 0$$



$$x \leq -\frac{1}{2} \quad \text{or} \quad x \geq 2$$

$$\therefore -\frac{5}{3} < x < 1$$



Final answer

$$\therefore -\frac{5}{3} < x < 1$$

8

5

9)

$$2x^2 - x + 6 = 0$$

$$\alpha + \beta = \frac{1}{2} \quad B1$$

$$\alpha\beta = 3 \quad B1$$

$$(i) \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \quad M1$$

$$= \frac{1}{4} - 6$$

$$= -5\frac{3}{4} \quad A1$$

(4)

$$(ii) \quad \alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) \quad B1$$

$$= \left(\frac{1}{2}\right) \left(-5\frac{3}{4} - 3\right) \quad 1$$

$$= \left(\frac{1}{2}\right) \left(-8\frac{3}{4}\right) \quad 1$$

$$= -4\frac{3}{8}$$

$$= -\frac{35}{8} \quad \text{proved} \quad 1 \quad (4)$$

$$(ii) \quad \alpha^3\beta^3 = (\alpha\beta)^3$$

$$= 27 \quad 1$$

$\therefore$ the eqn is

$$x^2 - \left(-\frac{35}{8}\right)x + 27 = 0$$

$$\therefore x^2 + \frac{35}{8}x + 27 = 0 \quad 1 \quad (2)$$

(10)

$$\underline{\underline{8x^2 + 35x + 216 = 0}}$$

$$10) \quad 5x^3 + Ax^2 + x + 3 = (x+1)(Bx^2 + cx) + D$$

$$\text{When } x=0 \quad \underline{\underline{3 = D}} \quad B1$$

$$\text{Equating coeff of } x^3 : \underline{\underline{5 = B}} \quad B1$$

$$\text{When } x=-1 : -5 + A - 1 + 3 = D$$

$$A - 3 = 3$$

$$\underline{\underline{A = 6}} \quad B1$$

$$\text{When } x=1 : 5 + A + 1 + 3 = 2(B+c) + D$$

$$5 + 6 + 1 + 3 = 2(5+c) + 3 \quad M1$$

$$15 = 10 + 2c + 3$$

$$= 13 + 2c$$

$$2 = 2c$$

$$\underline{\underline{c = 1}} \quad A1$$

$$\text{Ans: } \underline{\underline{A=6, B=5, c=1, D=3}}$$

(5)

$$11) \quad f(x) = x^3 + 2x^2 - 17x + 6$$

$$a) \quad f(3) = 27 + 18 - 51 + 6 \\ = 0$$

$\therefore x-3$ is a factor.

$$b) \quad \begin{array}{r|rrrr} 1 & 2 & -17 & 6 \\ + & 0 & 3 & 15 & -6 \\ \hline & 1 & 5 & -2 & 0 \end{array}$$

$$\therefore f(x) = (x-3)(x^2 + 5x - 2)$$

$$\text{For } (x-3)(x^2 + 5x - 2) = 0$$

$$x = 3 \text{ or } x = \frac{-5 \pm \sqrt{25 - 4(-2)}}{2}$$

$$x = 3 \text{ or } x = -5.37 \text{ or } 0.37 \text{ (2 d.p.)}$$

$$(11) \quad \frac{11}{x^3 + 2x^2 - 17x + 6} = \frac{A}{x-3} + \frac{Bx+C}{x^2 + 5x - 2}$$

$$\therefore 11 = A(x^2 + 5x - 2) + (Bx + C)(x-3)$$

$$\text{When } x=3, \quad 11 = 22A \Rightarrow A = \frac{1}{2}$$

$$\text{Equating coeff of } x^2: \quad 0 = A + B \Rightarrow B = -\frac{1}{2}$$

$$\text{When } x=0$$

$$11 = -2A - 3C$$

$$\Rightarrow C = -4$$

$$\therefore \frac{11}{x^3 + 2x^2 - 17x + 6} = \frac{\frac{1}{2}}{2(x-3)} + \frac{-\frac{1}{2}x - 4}{x^2 + 5x - 2}$$

$$= \frac{1}{2(x-3)} - \frac{\frac{1}{2}x + 4}{x^2 + 5x - 2}$$

$$= \frac{1}{2(x-3)} - \frac{x+8}{2(x^2 + 5x - 2)}$$