Full Name	Class Index No	Class

Anglo-Chinese School
(Barker Road)

END-OF-YEAR EXAMINATION 2022
SECONDARY THREE EXPRESS

ADDITIONAL MATHEMATICS
4049

2 HOURS 15 MINUTES

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For Examiner's Use

This question paper consists of **19** printed pages and **1** blank page.

1 Given that $M = \log_2 x$ and $N = \log_4(2x)$, show that $2N - M = 1$. [3]

2 The volume, V m^3 of a rectangular box is $\left(26 + 11\sqrt{5}\right)$ and the cross-sectional area of the box is $\left(2 + \sqrt{5}\right)$ m^2. Without the use of a calculator, find the height of the box in the form $a + b\sqrt{5}$, where a and b are integers. [4]

3 Solve the equation $e^{2x+1} = 9e^{x+1} - 18e$. [4]

4 The equation of a circle is $x^2 + y^2 - 10x - 4y + 25 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [4]

(b) Given that the circle touches the x-axis, state the equation of the circle which is a reflection of the original circle in the x-axis. [1]

5 The function f is defined by $f(x) = 1 - 2\sin\left(\frac{x}{3}\right)$, for $0^\circ \le x \le 360^\circ$.

(a) State the maximum and minimum values of $f(x)$. [2]

(b) State the period of $f(x)$. [1]

(c) Sketch the graph of $y = f(x)$ for $0^\circ \le x \le 540^\circ$. [3]

6 A cup of Milo is heated until it reaches a temperature of $T^{\circ}\mathrm{C}$. It is then allowed to cool. Its temperature, $y^{\circ}\mathrm{C}$, when it has been cooled for t minutes, is given by the formula $y = 20 + 72\left(2^{-\frac{t}{8}}\right)$.

(a) Find

(i) the value of T, the initial temperature, [1]

(ii) the time taken, correct to the nearest minute, for the cup of Milo to reach $50^{\circ}\mathrm{C}$. [3]

(b) Is it possible for the cup of Milo to reach a temperature of $0^{\circ}\mathrm{C}$?
Explain your reasoning by showing your workings clearly. [2]

7 **(a)** The remainder when $mx^3 - 5x^2 + 7$ is divided by $2x - 1$ is twice the remainder when it is divided by $x - 2$. Find the value of the constant m. [4]

(b) Given that the root of the quadratic equation $x^2 = 2x - 1$ is a, show that $a^9 + 1 = \left(2a^2 - a + 1\right)\left(4a^4 - 4a^3 - a^2 + a + 1\right)$. [3]

8 During a basketball match, Lavin was fouled and took a free throw. The height of the ball, h m, when leaving his hand is given by $h=-\frac{1}{2}\left(t^2-5t+k\right)$, where t is the time in seconds after the ball left his hand and k is a constant.

Given further that the height of the ball was 4 m after 1 second,

(a) Show that the value of $k=-4$ and hence find the height of the ball when it first left Lavin's hand. [3]

(b) Deduce the maximum height of the ball and the time it takes to reach the maximum height. [4]

9 **(a)** Show that $x^2 - x + \frac{3}{4}$ is always positive for all real values of x. [3]

(b) Find the set of values of the constant c for which the line $y - 5x - c = 0$ meets the curve $y = 2x - \frac{3}{x}$. [4]

10 **(a)** Find the first three terms in the expansion of $(1+2x)^5$ in ascending powers of x. [2]

(b) Hence find the term independent of x in the expansion of $\left(1+\dfrac{1}{x}\right)^2(1+2x)^5$. [3]

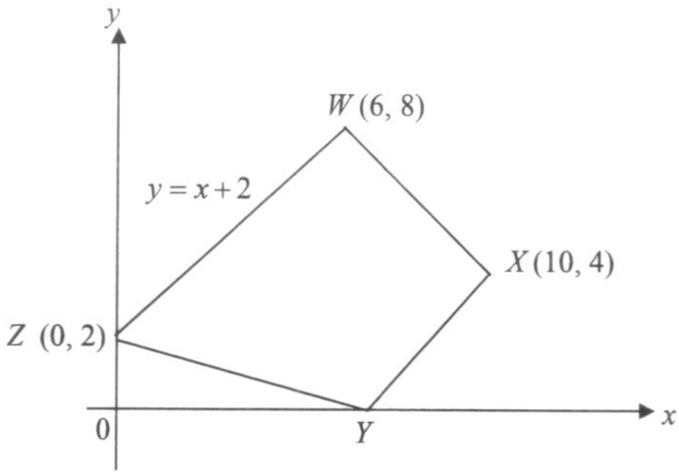
10 **(c)** By considering the general term in the binomial expansion of $\left(qx+\frac{1}{x}\right)^8$, where p is a constant, explain why there are no odd powers of x in this expansion. [3]

11 **(a)** Explain why $x+1$ is a factor of $4x^3-3x+1$. [2]

(b) Factorise completely the cubic polynomial $4x^3-3x+1$. [2]

11 (c) Express $\dfrac{6x^2 - x + 2}{4x^3 - 3x + 1}$ in partial fractions. [5]

12 The diagram shows a quadrilateral $WXYZ$ in which W is (6, 8), X is (10, 4) and Z is (0, 2). The point Y lies on the perpendicular bisector of WZ and on the x-axis. The equation of line WZ is $y = x + 2$.



(a) Show that angle ZWX is 90°. [2]

(b) Find the equation of the perpendicular bisector of WZ. [3]

12 **(c)** State the coordinates of Y. [1]

(d) Find the area of the quadrilateral $WXYZ$. [2]

12 **(e)** Reuben extends the line WX until it reaches the point E. [3]

The extended line is such that $WX = \frac{2}{3}WE$.

Reuben claims that point E will lie on the x-axis.

Do you agree? Justify your answer with clear working.

13 **(a)** **Without using a calculator**, find the exact value of $\dfrac{\sin 60^\circ \times \tan 30^\circ}{2\cos^2 45^\circ}$. [3]

(b) Prove the identity $\dfrac{1+\sec\theta}{\tan\theta+\sin\theta} = \operatorname{cosec}\theta$. [3]

13 **(c)** Given that $\sin\theta = s$ and θ is acute, express, in terms of s, $\sec(90^\circ - \theta)$. [2]

(d) Solve the equation $2\cos^2 x + 3\sin x = 3$ for $0 \leq x \leq 2\pi$. [5]

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1		$2N = 2\log_4(2x)$			
		$= 2\left(\dfrac{\log_2 2x}{\log_2 4}\right)$ $= 2\left(\dfrac{\log_2 2 + \log_2 x}{2}\right)$			
		$2N = 1 + \log_2(x)$ $2N - M = 1$ (Shown)			
2		ht of box $= \dfrac{\left(26 + 11\sqrt{5}\right)}{2+\sqrt{5}}$ $= \dfrac{\left(26 + 11\sqrt{5}\right)}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$ $= \dfrac{52 - 26\sqrt{5} + 22\sqrt{5} - 11(5)}{4-5}$ $= 3 + 4\sqrt{5}$ $\therefore a = 3$ and $b = 4$			
3		$e^{2x+1} = 9e^{x+1} - 18e$ $\left(e^{2x}\right)(e) - (9e)e^x + 18e = 0$ $e^{2x} - 9e^x + 18 = 0$ $\left(e^x - 3\right)\left(e^x - 6\right) = 0$ $e^x = 3$ or $e^x = 6$ $x = 1.10$ or 1.79			
4	(a)	$x^2 + y^2 - 10x - 4y + 25 = 0.$			
		$x^2 - 10x + (-5)^2 + y^2 - 4y + (-2)^2 = -25 + 25 + 4$			
		$(x-5)^2 + (y-2)^2 = 2^2$			
		Centre $(5,2)$ and radius = 2 units			
	(b)	$(x-5)^2 + (y+2)^2 = 4$			

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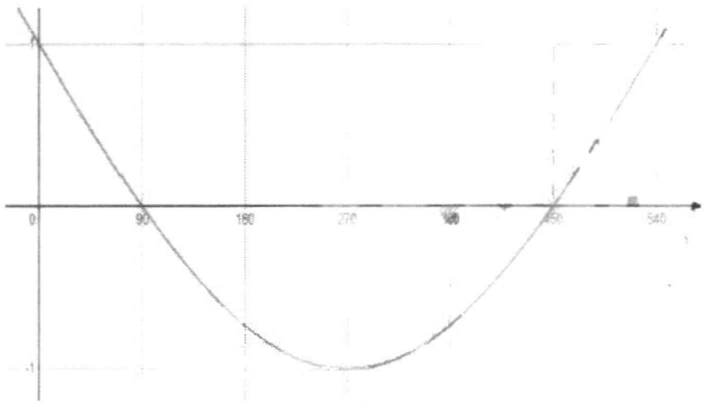
5	(a)	Max = 1 – 2 (-1) = 3 Min = 1 – 2 (1) = -1			
	(b)	$\text{Period} = \dfrac{360}{\left(\frac{1}{3}\right)}$ $= 1080^{\circ}$			
	(c)				
6	ai	$T = 20 + 72\left(2^{0}\right) = 92^{\circ}C$			
	aii	$50 = 20 + 72\left(2^{-\frac{t}{8}}\right)$ $2^{-\frac{t}{8}} = \frac{5}{12}$ $-\frac{t}{8}(\ln 2) = \ln\left(\frac{5}{12}\right)$ $t = 10.1$ secs			
	(b)	$2^{-\frac{t}{8}} > 0$ $72\left(2^{-\frac{t}{8}}\right) > 0$ $20 + 72\left(2^{-\frac{t}{8}}\right) > 20$ $\therefore y > 0$			

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		No it is not possible since the temp is always > 0 for all values of t.			
7	(a)	$f(x)=mx^3-5x^2+7$ $R_1=f(2)=8m-13$ $R_2=f\left(\frac{1}{2}\right)=\frac{m}{8}-\frac{5}{4}+7=\frac{m}{8}+\frac{23}{4}$ since $R_2=2R_1$ $\frac{m}{8}+\frac{23}{4}=16m-26$ $m+46=128m-208$ $m=2$			
	(b)	$x^2=2x-1$ Sub $x=a$ in equation.			
		$a^2=2a-1$ multiply by a throughout $a^3=2a^2-a$ $a^9+1=(2a^2-a+1)(4a^4-4a^3+a^2-2a^2+a+1)$ $a^9+1=(2a^2-a+1)(4a^4-4a^3-a^2+a-1)$			
8	(a)	$h=-\frac{1}{2}(t^2-5t+k)$ $4=-\frac{1}{2}(1^2-5+k)$ $4=-\frac{1}{2}(-4+k)$ $-8=-4+k$ $k=-4$			
		$h=-\frac{1}{2}(t^2-5t-4)$ $=-\frac{1}{2}(0-0-4)$ $=2$ m			

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	(b)	$h=-\frac{1}{2}\left(t^2-5t-4\right)$ $=-\frac{1}{2}\left[t^2-5t+\left(\frac{-5}{2}\right)^2-4-\left(\frac{-5}{2}\right)^2\right]$ $=-\frac{1}{2}\left[\left(t-\frac{5}{2}\right)^2-4-\left(\frac{25}{4}\right)\right]$ $=5\frac{1}{8}-\frac{1}{2}\left(t-\frac{5}{2}\right)^2$			
		Max height is $5\frac{1}{8}$ / 5.125 m and $t=2.5$ secs.			
9	(a)	$x^2-x+\frac{3}{4}$			
		Coefficient of $x^2>0$			
		$b^2-4ac=(-1)^2-4(-1)\left(\frac{3}{4}\right)$ $=-2<0$ D < 0			
		Hence, $x^2-x+\frac{3}{4}$ is always positive. (shown)			
	(b)	$5x+c=2x-\frac{3}{x}$ $3x^2+cx+3=0$			
		If line meets curve,			
		$b^2-4ac\geq 0$ $c^2-4(3)(3)\geq 0$ $(c+6)(c-6)\geq 0$ $\therefore c\leq -6$ or $c\geq 6$			
10	(a)	$(1+2x)^5=1+\binom{5}{1}(1)^4(2x)+\binom{5}{2}(1)^3(2x)^2+\ldots$ $=1+10x+40x^2+\ldots$			

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	(b)	$\left(1+\frac{1}{x}\right)^2(1+2x)^5$			
		$=\left(1+\frac{2}{x}+\frac{1}{x^2}\right)(1+10x+40x^2+\ldots)$			
		Term independent of x			
		$=1+\frac{2}{x}(10x)+\left(\frac{1}{x^2}\right)(40x^2)$ $=1+20+40$ $=61$			
	(c)	$\left(qx+\frac{1}{x}\right)^8$			
		$T_{r+1}=\binom{8}{r}(qx)^{8-r}\left(\frac{1}{x}\right)^r$ $=\binom{8}{r}(q)^{8-r}(x)^{8-2r}$			
		Since 8 and $2r$ are even numbers for all positive integers of r, the $8-2r$ must be an even number. Hence, there are no odd powers of x in the expansion.			
11	(a)	Let $f(x)=4x^3-3x+1$.			
		$f(-1)=-4+3+1=0$			
		Since the remainder is zero, $(x+1)$ is a factor.			
		(alternative: By Factor Theorem…)			
	(b)	$f(x)=(x+1)(4x^2-4x+1)$ $=(x+1)(2x-1)^2$			
11	(c)	let $\frac{3x^2+x-1}{x^2(2x^2+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{(2x^2+1)}$ $3x^2+x-1=Ax(2x^2+1)+B(2x^2+1)+(Cx+D)x^2$ let x = 0: $B=-1$ By			

		$\frac{6x^2-x+2}{(x+1)(2x-1)^2}=\frac{1}{(x+1)}+\frac{1}{(2x-1)}+\frac{2}{(2x-1)^2}$			
12	(a)	Grad of $WX=\frac{8-4}{6-10}=-1$			
		Grad of $WZ=1$			
		(Grad of WX) (Grad of WZ) = -1 X 1 = -1			
		Therefore, angle $ZWX=90°$ (Shown)			
	(b)	the Mid-Pt of $WZ=\left(\frac{0+6}{2},\frac{2+8}{2}\right)=(3,5)$			
		Grad of $WZ=1$			
		the equation of the perpendicular bisector of WZ:			
		$y-5=-1(x-3)$ $y=8-x$			
	(c)	Y = $(8,0)$			
	(d)	area of the quadrilateral $WXYZ$			
		$=\frac{1}{2}\begin{vmatrix}6 & 0 & 8 & 10 & 6\\ 8 & 2 & 0 & 4 & 8\end{vmatrix}$ $=\frac{1}{2}\lvert(12+0+32+80)-(24+0+16+0)\rvert$ $==42$ units^2			
12	(e)	Grad of WZ:			
		$y-8=-1(x-6)$ $y=-x+14$ sub $y=0$, $x=14$ Assuming $E(14,0)$, $\frac{\text{Hor. Dist from W to X}}{\text{Hor. Dist from W to E}}=\frac{10-6}{14-6}=\frac{1}{2}\neq\frac{2}{3}$			
		Hence, point E does not lie on the x-axis.			
		Alternative Solution:			

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		Let $E(x, y)$:			
		$\frac{4-8}{y-8}=\frac{2}{3}$ $2y-16=-12$ $2y=4$ $y=2$			
		Hence, point E does not lie on the x-axis.			
13	(a)	$\frac{\sin 60^\circ \times \tan 30^\circ}{2\cos^2 45^\circ}$			
		$\frac{\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}}}{2\left(\frac{\sqrt{2}}{2}\right)^2}$ $=\frac{\frac{1}{2}}{1}$ $=\frac{1}{2}$			
	(b)	LHS $=\frac{1+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\sin\theta}$ $=\frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}}$ $=\frac{\cos\theta+1}{\sin\theta(1+\cos\theta)}$ $=\frac{1}{\sin\theta}$ $=\text{cosec}\,\theta$ = RHS			

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	(c)	$\sec(90-\theta)=\frac{1}{\cos(90-\theta)}$ $=\frac{1}{\sin\theta}$ $=\frac{1}{s}$			
	(d)	$2\cos^2 x+3\sin x=3$ $2(1-\sin^2 x)+3\sin x=3$ $0=3-2+2\sin^2 x-3\sin x$ $2\sin^2 x-3\sin x+1=0$ let $\sin x=u$ $2u^2-3u+1=0$ $(2u-1)(u-1)=0$ $u=\frac{1}{2}$ or 1 $\sin x=\frac{1}{2}$ or 1 basic angle $=\frac{\pi}{6}$ $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{\pi}{2}$			