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NORTHBROOKS SECONDARY SCHOOL
End-of-Year Examination 2022
Secondary 3 Express

NORTHBROOKS

CANDIDATE NAME	

CLASS		REGISTER NUMBER		

ADDITIONAL MATHEMATICS — 4049/01

Paper 1 — **5 October 2022**

1 hour 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 60.

FOR EXAMINER'S USE											
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	
											60

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

This document consists of 13 printed pages.

Setter: Ng Puay Hoon

1 **Without using a calculator**, find the exact value of $\left(\sin\frac{\pi}{6} - \tan\frac{\pi}{3}\right)\sin\frac{\pi}{4}$ in the form $\frac{\sqrt{a} - b\sqrt{c}}{d}$, where a, b, c and d are integers. [3]

2 The path, y metres, of a diver above water can be modelled by the function $y = -3x^2 + 4.5x + 10$, where x is the horizontal distance of the diver from the end of the diving board in metres.

(a) Explain why the height of the diver above the water when he first left the diving board is 10 metres. [1]

(b) The diver said that he can reach a height of 12 metres above the water when executing his dive. Do you agree? Justify your answer. [3]

3 Solve the equation $(1+\tan x)\sec x = 6 \operatorname{cosec} x$ for $0° \le x \le 360°$. [4]

4 An equilateral triangle has a length of $\frac{6}{2+\sqrt{3}}$ cm for each side.

Without using a calculator, find the area of the triangle in the form $p+q\sqrt{3}$, where p and q are integers. [5]

5 The first three terms in the expansion of $\left(1-\dfrac{x}{5}\right)^6(5+bx)$ in ascending powers of x are $a-x+cx^2$. Find the values of each of the constants a, b and c. [5]

6 **(a)** Factorise $48x^3 + 162y^3$ completely. [3]

(b) Hence simplify $\dfrac{48x^3 + 162y^3}{-12x^2 + 18xy - 27y^2}$. [2]

7 Find the values of x and y which satisfy the equations

$$e^2e^x = e^{4y},$$
$$\log_4(x+2) = 1 + \log_2 y.$$ [5]

8 The equation of a curve is $y = 5\sin\left(\frac{x}{2}\right) - 1$.

(a) State the minimum and maximum values of y. [2]

(b) State the period of y. [1]

(c) Sketch the graph of $y = 5\sin\left(\frac{x}{2}\right) - 1$ for $-360° \leq x \leq 360°$. [2]

9 **(a)** Prove that $2^{x+3} - 2^{x}$ is exactly divisible by 7. [2]

(b) Use the substitution $u = 2^{y}$ to solve the equation $2^{2y+1} - 7(2^{y}) + 3 = 0$. [4]

10 **(a)** Prove the identity $\dfrac{1+\cos x}{\sin x}+\dfrac{\sin x}{1+\cos x}=2\operatorname{cosec} x$. [4]

(b) Hence solve the equation $\dfrac{1+\cos x}{\sin x}+\dfrac{\sin x}{1+\cos x}=3$ for $0 \le x \le 5$ radians. [3]

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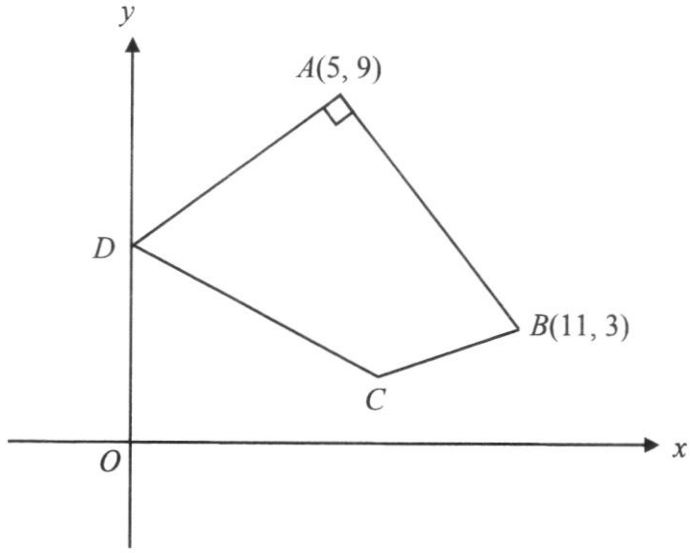

The diagram shows a quadrilateral $ABCD$ with vertices $A(5, 9)$, $B(11, 3)$, C and D.
The point C lies on the line $2y = x - 3$. The point C is equidistant from A and D.
The point D lies on the y-axis and angle $DAB = 90°$.

Find

(a) the equation of the line AD, [3]

(b) the coordinates of D, [1]

(c) the equation of the perpendicular bisector of *AD*, [3]

(d) the coordinates of *C*, [2]

(e) the area of the quadrilateral *ABCD*. [2]

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NORTHBROOKS SECONDARY SCHOOL
End-of-Year Examination 2022
Secondary 3 Express

NORTHBROOKS

CANDIDATE NAME	

CLASS		REGISTER NUMBER		

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **6 October 2022**

1 hour 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

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You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 60.

FOR EXAMINER'S USE										
Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**	
										60

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Setter: Ng Puay Hoon

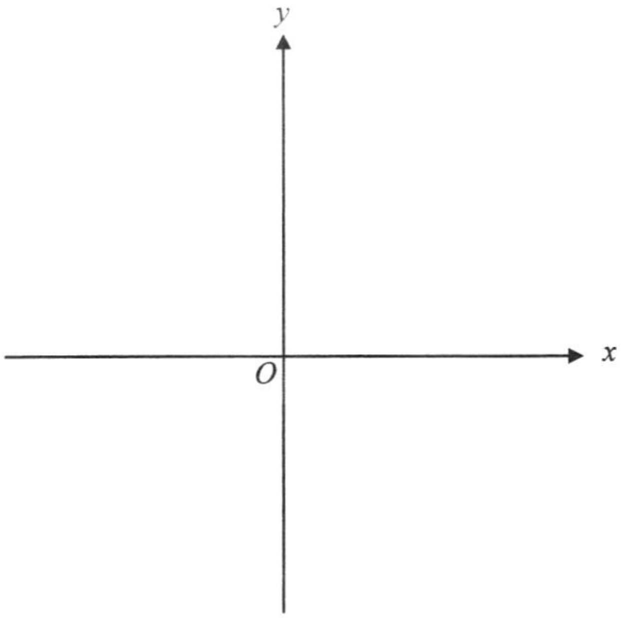
1 By considering the general term, determine the term independent of x in the expansion of $\left(2x^3 - \frac{1}{4x^5}\right)^8$. [3]

2 Given that $\operatorname{cosec} 70° = p$, express each of the following in terms of p.

(a) $\sin 250°$ [2]

(b) $\cot 290°$ [2]

3 **(a)** Sketch the graph of $y = \ln x$.
Indicate clearly the value where the graph crosses the x-axis. [1]



(b) In order to solve the equation $x^3 e^{2x-1} = 1$, a suitable straight line can be drawn on the same set of axes as the graph of $y = \ln x$. Find the equation of the straight line and the number of solution(s) to the equation $x^3 e^{2x-1} = 1$. [3]

4 Express $\dfrac{4x^2-4x-9}{2x^2-7x+5}$ in partial fractions. [5]

5 The equation of a curve is $y = ax^2 + (2a-4)x + 3a - 2$, where a is a non-zero constant.

(a) Find the range of values of a for which the curve lies completely above the x-axis. [3]

(b) In the case where $a = -2$, explain why there are no real values of x for which $y > 0$. [2]

6 The temperature, T °C, of a cup of coffee left to cool is given by the formula $T = 35 + 16e^{-kt}$, where k is a constant and t is the time in hours since the coffee was left to cool. 1 hour later, the temperature of the coffee is 45°C.

(a) Calculate the value of k. [2]

(b) Find the least number of hours needed for the coffee to cool to 38°C. Give your answer to the nearest hour. [3]

(c) Explain why the temperature of the coffee can never reach 35°C. [2]

7 **(a)** Factorise $2x^3 - 9x^2 + 3x + 4$ completely. [3]

(b) Hence solve $2+3x^2=9x-4x^3$. [4]

8 The equation of a circle is $x^2 + y^2 - 4x - 6y = 12$.

(a) Find the coordinates of the centre of the circle and the radius of the circle. [3]

The line $y + 2x = 5$ intersects the circle at A and B.

(b) Show that AB is not the diameter of the circle. [5]

9 **(a)** Solve the equation $\lg(x+4)-\lg(x-3)=3\lg 2$. [4]

(b) Given that $\log_{27} x^6 = \log_{81} y^2$, express y in terms of x. [4]

10 The table below shows experimental values of two variables x and y.

x	2.0	3.0	4.0	5.0	6.0
y	1.6	3.0	5.2	7.0	9.6

It is known that x and y are related by the equation $y = ax + bx^2$, where a and b are constants. It is believed that an error was made in recording one of the values of y.

(a) On the grid on page 13, plot $\frac{y}{x}$ against x and draw a straight line graph. [3]

Use your graph to

(b) determine the inaccurate reading of y and estimate its correct reading, [2]

(c) estimate the values of a and b, [2]

(d) estimate the value of x when $2y = 3x$. [2]

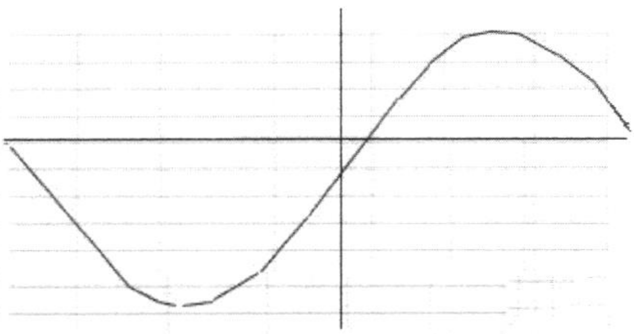
NORTHBROOKS SECONDARY SCHOOL
MATHEMATICS DEPARTMENT
3 EXPRESS
ADDITIONAL MATHEMATICS

END-OF-YEAR EXAMINATION PAPER 1
MARKING SCHEME 2022

Qn	Working	Marks	Remarks
1	$\sin\frac{\pi}{4}\left(\sin\frac{\pi}{6}-\tan\frac{\pi}{3}\right)$		
	$=\frac{\sqrt{2}}{2}\left(\frac{1}{2}-\sqrt{3}\right)$	B(2, 1, 0)	3 correct: 2 marks 2 correct: 1 mark
	$=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{2}$		
	$=\frac{\sqrt{2}-2\sqrt{6}}{4}$	A1	
2(a)	When $x=0$,		
	Height $=-3(0)^2+4.5(0)+10$		
	$=10$ metres	B1	
(b)	$y=-3x^2+4.5x+10$		
	$=-3\left(x^2-1.5x-\frac{10}{3}\right)$	M1	factorise
	$=-3\left[(x-0.75)^2-\frac{187}{48}\right]$		
	$=-3(x-0.75)^2+11.6875$	M1	completed square
	Since the **maximum height** is only about **11.7 metres**,		
	I do not agree with him.	A1	
	Method 2:		
	$-3x^2+4.5x+10=12$	M1	
	$-3x^2+4.5x-2=0$		
	Discriminant $=(4.5)^2-4(-3)(-2)$	M1	
	$=-3.75$		
	Since **discriminant is negative**, there is **no solution** for		
	$-3x^2+4.5x+10=12$, hence I do not agree with him.	A1	

3	$(1+\tan x)\sec x = 6\ \text{cosec}\ x$		
	$\frac{1}{\cos x}(1+\tan x) = \frac{6}{\sin x}$	M1	
	$\tan x(1+\tan x) = 6$		
	$\tan^2 x + \tan x - 6 = 0$	M1 s.o.i.	
	$\tan x = 2$ or $\tan x = -3$		
	$x = 63.435°, 243.435°$ or $x = 108.435°, 288.435°$		
	$x = 63.4°, 243.4°$ or $x = 108.4°, 288.4°$	A1, A1	
4	$\sin 60° = \text{height} \div \frac{6}{2+\sqrt{3}}$	M1 o.e.	Form trigo ratio
	$\frac{\sqrt{3}}{2} = \frac{\text{height}(2+\sqrt{3})}{6}$		
	$\text{height} = \frac{3\sqrt{3}}{2+\sqrt{3}}$	M1	Attempt to find height
	$\text{Area} = \frac{1}{2} \times \frac{6}{2+\sqrt{3}} \times \frac{3\sqrt{3}}{2+\sqrt{3}}$	M1	
	$= \frac{9\sqrt{3}}{7+4\sqrt{3}}$		
	$= \frac{9\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$	M1	Rationalise
	$= \frac{63\sqrt{3}-108}{49-48}$		
	$= 63\sqrt{3} - 108$	A1	
	<u>Method 2:</u>		
	$\frac{1}{2} \times \left(\frac{6}{2+\sqrt{3}}\right)^2 \sin 60°$	M1	Area of triangle
	$= \frac{1}{2} \times \frac{36}{4+4\sqrt{3}+3} \times \frac{\sqrt{3}}{2}$	M1, M1	
	$= \frac{9\sqrt{3}}{7+4\sqrt{3}}$		
	$= \frac{9\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$	M1	Rationalise
	$= 63\sqrt{3} - 108$	A1	

5	$\left(1-\frac{x}{5}\right)^6$		
	$=1+\binom{6}{1}\left(-\frac{x}{5}\right)^1+\binom{6}{2}\left(-\frac{x}{5}\right)^2+\binom{6}{3}\left(-\frac{x}{5}\right)^3+\ldots$	M1	Apply binomial theorem to expand at least 1st 3 terms.
	$=1-\frac{6}{5}x+\frac{3}{5}x^2-\frac{4}{25}x^3+\ldots$		
	$\left(1-\frac{x}{5}\right)^6(5+bx)$		
	$=\left(1-\frac{6}{5}x+\frac{3}{5}x^2-\frac{4}{25}x^3+\ldots\right)(5+bx)$		
	$=1(5+bx)-\frac{6}{5}x(5+bx)+\frac{3}{5}x^2(5+bx)+\ldots$	M1	Show expansion at least till x^2 terms
	$=5+bx-6x-\frac{6b}{5}x^2+3x^2+\ldots$		
	$5+bx-6x-\frac{6b}{5}x^2+3x^2=a-x+cx^2$		
	By comparison constant terms, $a=5$	A1	
	By comparison x terms, $b-6=-1$		
	$b=5$	A1	
	By comparison x^2 terms, $-\frac{6b}{5}+3=c$		
	$c=-3$	A1	
6(a)	$48x^3+162y^3$		
	$=6\left(8x^3+27y^3\right)$	M1	
	$=6\left[(2x)^3+(3y)^3\right]$		
	$=6\left(2x+3y\right)\left[(2x)^2-(2x)(3y)+(3y)^2\right]$	M1	
	$=6\left(2x+3y\right)\left(4x^2-6xy+9y^2\right)$	A1	
(b)	$\frac{48x^3+162y^3}{-12x^2+18xy-27y^2}$		
	$=\frac{6(2x+3y)(4x^2-6xy+9y^2)}{-3(4x^2-6xy+9y^2)}$	M1 ECF	
	$=-2(2x+3y)$	A1	

7	$e^2e^x = e^{4y}$, $\log_4(x+2) = 1+\log_2 y$		
	$2+x=4y$ ---(1)	M1	
	$\dfrac{\log_2(x+2)}{2} = 1+\log_2 y$	M1	Change of log base
	$\log_2(x+2) = 2\log_2 2 + 2\log_2 y$		
	$x+2=4y^2$ ---(2)	M1	Obtain non-linear eqn
	By substitution,		
	$(4y-2)+2=4y^2$	Students likely to cancel y from both sides.	
	$y^2-y=0$		
	$y(y-1)=0$		
	$y=0$ or $y=1$		
	When $y=0$, $x=-2$ (rejected as $\log_2 0$ is undefined)	A1	Mark given to rejection of answer.
	When $y=1$, $x=2$	A1	
8(a)	Minimum value = -6	B1	
	Maximum value = 4	B1	
(b)	Period = 720°	B1	
(c)		B1 – sine curve passing through (0°, -1), (180°, 4) and (360°, -1) B1 – sine curve passing through (-180°, -6) and (-360°, -1)	

9(a)	$2^{x+3}-2^x$		
	$=2^x(2^3-1)$	M1	
	$=7(2^x)$	A1	
	Hence, $2^{x+3}-2^x$ is exactly divisible by 7.		
(b)	$2^{2y+1}-7(2^y)+3=0$		
	$2u^2-7u+3=0$	M1	
	$u=3$ or $u=\dfrac{1}{2}$	M1	
	$2^y=3$ or $2^y=\dfrac{1}{2}$		
	$y=\dfrac{\lg 3}{\lg 2}$ or $y=-1$		
	$y=1.58$ or $y=-1$ (to 3 s.f.)	A1, A1	
10(a)	LHS $=\dfrac{1+\cos x}{\sin x}+\dfrac{\sin x}{1+\cos x}$		
	$=\dfrac{1+2\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}$	M1	Make denominator same
	$=\dfrac{2+2\cos x}{\sin x(1+\cos x)}$	M1	Trigo identities
	$=\dfrac{2}{\sin x}$	M1	Cancellation of common terms
	$=2\operatorname{cosec} x$ (shown)	A1	
(b)	$2\operatorname{cosec} x=3$		
	$\sin x=\dfrac{2}{3}$	M1	
	$x=0.72973,\ \pi-0.72973$		
	$=0.72973,\ 2.4119$		
	$=0.730,\ 2.41$ (to 3 s.f.)	A1, A1	

11(a)	grad of $AB = \dfrac{9-3}{5-11}$	M1	
	$= -1$		
	grad of $AD = 1$	M1	
	Equation of AD:		
	$y-9=1(x-5)$		
	$y=x+4$	A1	
(b)	$y=4$		
	$D(0,4)$	B1	ECF from (a)
(c)	Midpoint of $AD = \left(\dfrac{0+5}{2}, \dfrac{4+9}{2}\right)$	M1	ECF from (b)
	$= \left(\dfrac{5}{2}, \dfrac{13}{2}\right)$		
	Equation of perp. bisector of AD:		
	$y-\dfrac{13}{2}=-1\left(x-\dfrac{5}{2}\right)$	M1	
	$y=-x+9$	A1	
(d)	Solving **(c)** answer and $2y = x - 3$ simultaneously,	M1	ECF from (c)
	$x = 7, y = 2$		
	$C(7, 2)$	A1	
(e)	Area $= \dfrac{1}{2}\begin{vmatrix} 5 & 0 & 7 & 11 & 5 \\ 9 & 4 & 2 & 3 & 9 \end{vmatrix}$	M1	ECF from (b), (d)
	$= \dfrac{1}{2}(140-65)$		
	$= 37.5$ units^2	A1	