BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2014

SUBJECT : Mathematics | **LEVEL : Secondary 3 Express**

PAPER : 4016 / 01 | **DURATION : 1 hour 15 minutes**

SETTER : Chai Yoke Leng | **DATE : 7 October 2014**

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name. class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **50**.

For Examiner's Use
50

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For examiners use only

1 The bacteria population in Culture A is 3 560 000 000.

(a) Express 3 560 000 000 in standard form.

Answer (a) .... [1]

(b) The bacteria population in Culture B is 16.89 billion

Find the difference in population of these two cultures. Give your answer in standard form.

Answer (b) .... [2]

2 Simplify $\sqrt{\dfrac{p^2}{4q^4}} \div 16p^{-1}$, giving your answer in positive indices.

Answer .... [2]

For examiners use only

3 **(a)** Given that $a^m = 3$ and $a^n = 5$, find the value of

(i) a^{2m}

Answer (a)(i) .... [1]

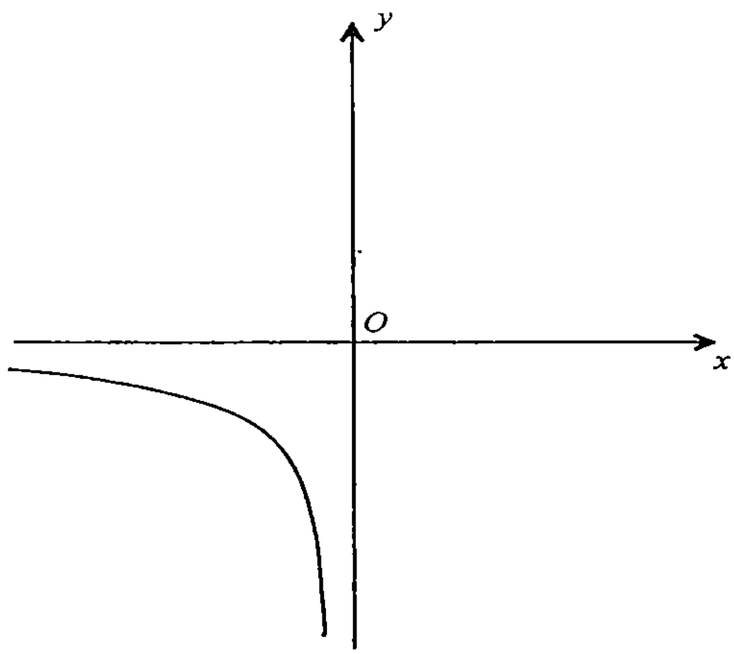
(ii) a^{m-n}

Answer (a)(ii) .... [1]

(b) Solve the equation $16^{2-x} = 8$

Answer (b) .... [2]

4 The diagram shows the sketch for the graph of $y = \dfrac{a}{x^2}$ for $x < 0$.



(a) Complete the sketch. [1]

(b) Write down a possible value of a.

Answer (b).................................... [1]

For examiners use only

5 Solve the following equations.

(a) $2(x+1)^2 = 3(x+1)$

Answer (a) .... [2]

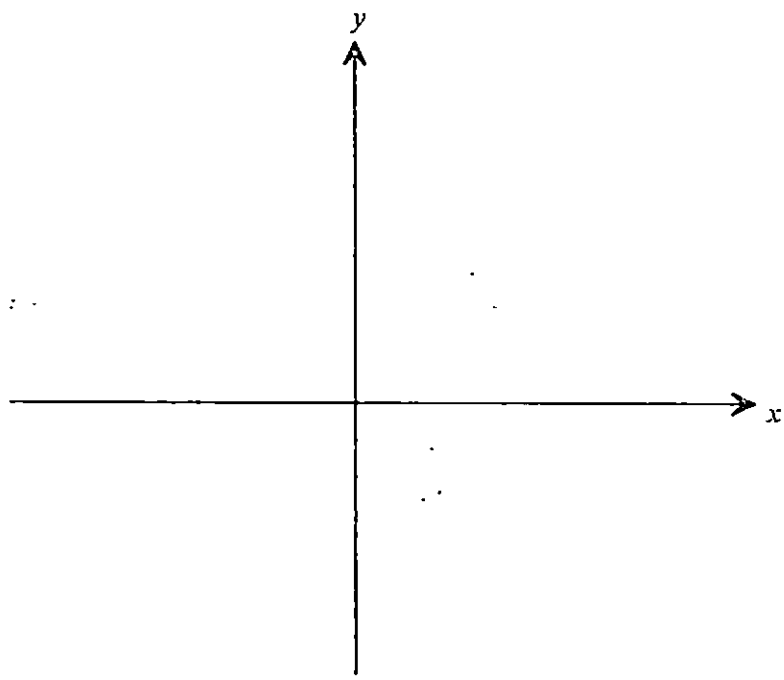
(b) $3x^2 = 5x + 1$

Answer (b) .... [2]

6 **(a)** Express x^2+4x-1 in the form $(x+p)^2+q$.

Answer (a) .... [2]

(b) Sketch the graph of $y=-(x-2)^2+3$, indicating clearly the y – intercept and the coordinates of the turning point. [2]



For examiners use only

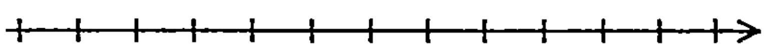
7 **(a)** Find the largest integer value of x for which $\frac{x}{3}-\frac{1}{4}\leq\frac{x+11}{12}$.

Answer (a) .... [2]

(b) **(i)** Solve the inequality $3x-1<5x+3\leq 2x+15$.

Answer (b)(i).. [2]

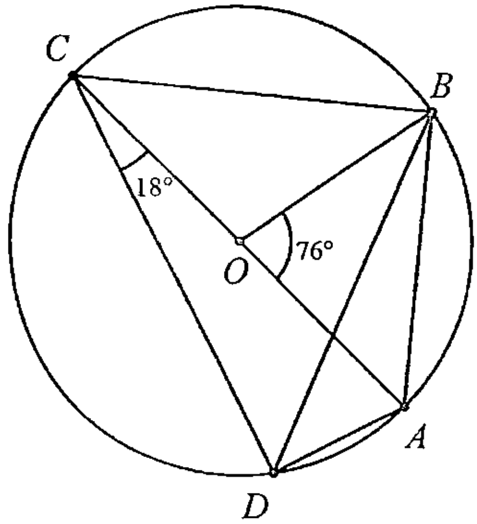
(ii) Illustrate the solution to **(b)(i)** on a number line. [1]



For examiners use only

8 In the diagram, AC is the diameter of the circle centre O. $\angle BOA = 76°$ and $\angle ACD = 18°$.

Find, stating reasons



(a) $\angle ABD$,

Answer (a) .... [1]

(b) $\angle DBC$,

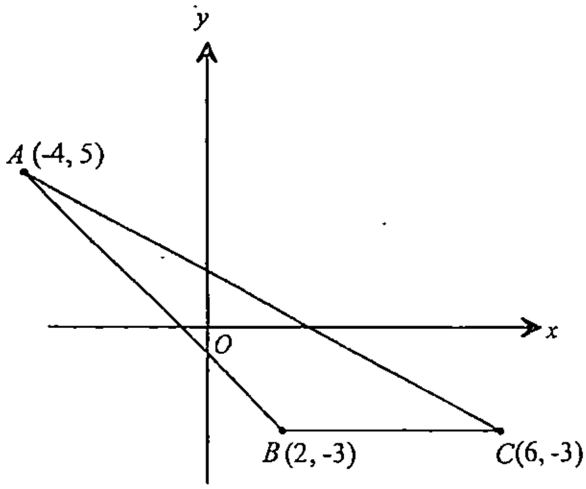
Answer (b) .... [1]

(c) $\angle BAD$.

Answer (c) .... [2]

For examiners use only

9 The diagram shows three points $A(-4,5)$, $B(2,-3)$ and $C(6,-3)$.



(a) Find the equation of AC.

Answer (a) .... [2]

(b) Find the length of AB.

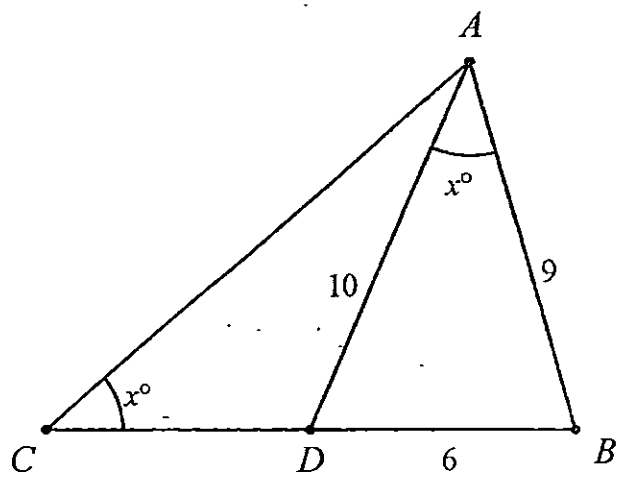
Answer (b) .... [1]

(c) Write down the exact value of $\cos\angle ABC$.

Answer (c) .... [1]

For examiners use only

10 In the diagram, $AB = 9$ cm, $AD = 10$ cm, $DB = 6$ cm and $\angle ACB = \angle BAD = x°$.



(a) Prove that ΔABC and ΔDBA are similar.

Answer (a)

....

....

.... [2]

(b) Find the length of AC.

Answer (b) .... [2]

11 Two similar containers have base areas of 36 cm^2 and 100 cm^2.

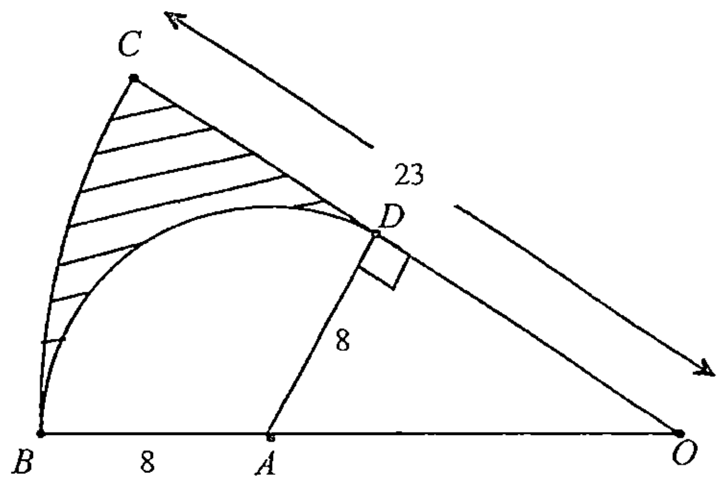
(a) Given that the height of the larger container is 50 cm, find the height of the smaller container.

Answer (a) .... cm [2]

(b) If the volume of the smaller container is 640 cm^3, find the volume of the larger container.

Answer (b) ....cm^3 [2]

12 The diagram shows a sector OBC of a circle centre O and radius 23 cm. BAD is a sector of the circle centre A and radius 8 cm and $\angle ADO = 90°$. Calculate



(a) $\angle AOD$,

Answer (a) .... [2]

(b) length of the arc BC,

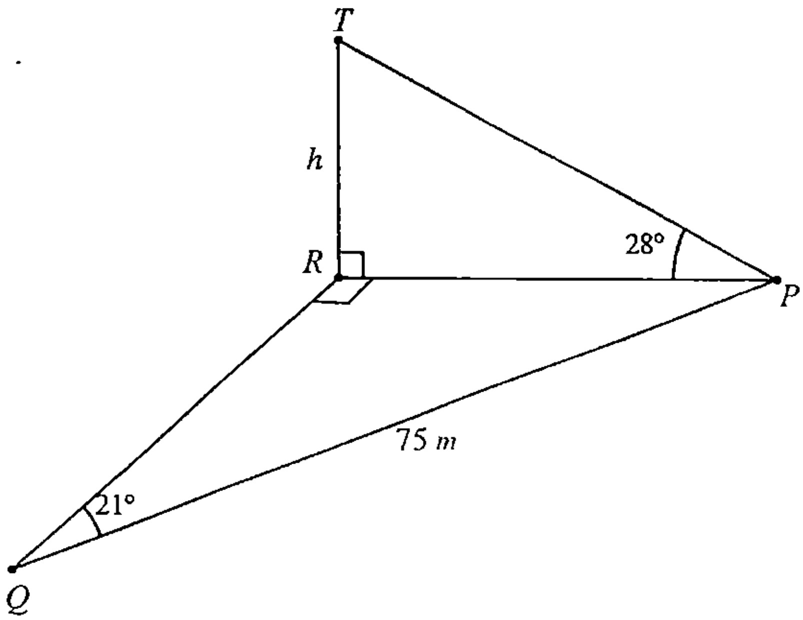
Answer (b) .... cm [2]

(c) area of the shaded region BCD.

Answer (c) .... cm^2 [2]

For examiners use only

13



In the diagram, P, Q and R are on level ground.

Alan is working out the height, h metres, of a building RT. He measures the angle TPR as 28°.

He was not able to measure the distance PR and so he walks 75 m from P to Q, where $\angle PQR = 21°$ and $\angle PRQ = 90°$. Calculate

(a) the distance PR,

Answer (a) .... m [2]

(b) the height of the building, RT.

Answer (b) .... m [2]

End of Paper

Answer Key:

1	(a)	3.56×10^9	(b)	1.333×10^{10}		
2	$\frac{p^2}{32q^2}$					
3	(a) (i)	9	(ii)	$\frac{3}{5}$	(b)	$1\frac{1}{4}$
4	(a)	sketch	(b)	any negative real number		
5	(a)	$-1, \frac{1}{2}$	(b)	$1.85, -0.180$		
6	(a)	$(x+2)^2 - 5$	(b)	sketch		
7	(a)	4	(b)	(i) $-2 < x \leq 4$	(ii)	number line
8	(a)	18°	(b)	72°	(c)	124°
9	(a)	$y = -\frac{4}{5}x + 1\frac{4}{5}$	(b)	10 units	(c)	$-\frac{3}{5}$

10 (a) $\angle ABC = \angle DBA$ (common angle), $\angle ACB = \angle DAB = x°$ (given), hence ΔABC is similar to ΔDBA (AA)

(b) 15 cm

11	(a)	30 cm	(b)	$2962\frac{26}{27}$ cm^3		
12	(a)	32.2°	(b)	12.9 cm	(c)	29.8 cm^2
13	(a)	26.9 m	(b)	14.3 m		

BEATTY SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2014

SUBJECT : Mathematics | **LEVEL : Sec 3 Express**

PAPER : 2 | **DURATION : 1 hour 30 minutes**

SETTER : Mr Bernard Lee | **DATE : 8 Oct 2014**

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
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Answer all questions.

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Calculators should be used where appropriate.
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three significant figures. Give answers in degrees to one decimal place.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This paper consists of 6 printed pages (including this cover page)

Mathematical Formulae

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$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

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Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

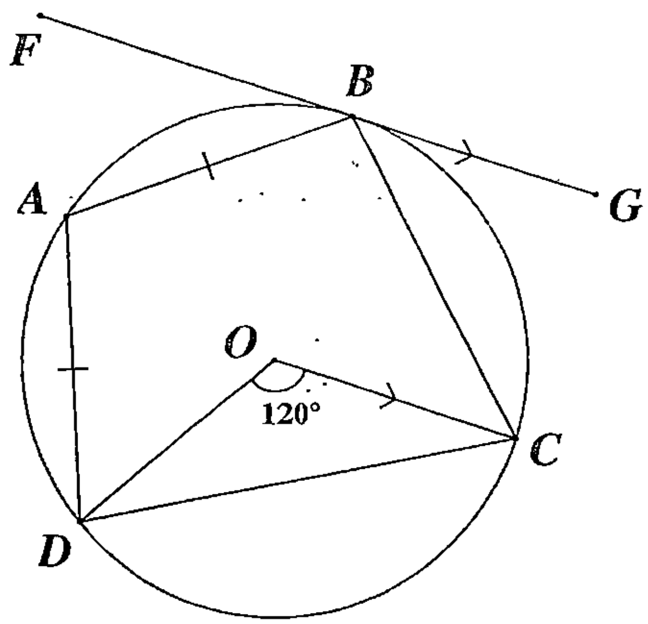
Answer **all** the questions.

1 **(a)** Simplify $\dfrac{x^2+3x-4}{4x-4}$. [2]

(b) Find the largest prime number x which satisfies the inequality $-\dfrac{1}{4}x > -5$ [2]

(c) Simplify $\left(\dfrac{5}{2x^2}\right)^{-3} \div 5x$. [2]

2



A, B, C and D lie on a circle, centre O. FBG is tangent to the circle at B, and is parallel to OC. $AB = AD$ and $C\hat{O}D = 120°$.

(a) Show clearly that $B\hat{C}O = 45°$. [2]

(b) Stating your reasons clearly, find

(i) $O\hat{C}D$, [1]

(ii) $D\hat{A}B$, [1]

(iii) $D\hat{B}C$, [1]

(iv) $A\hat{B}C$. [2]

3 The cost of a \$500 birthday present is to be shared equally by x people.

(a) Write down the amount of money that each person has to pay in terms of x. [1]

4 more people decide to share the cost of the present as well.

(b) Write down the new amount of money that each person has to pay in terms of x. [1]

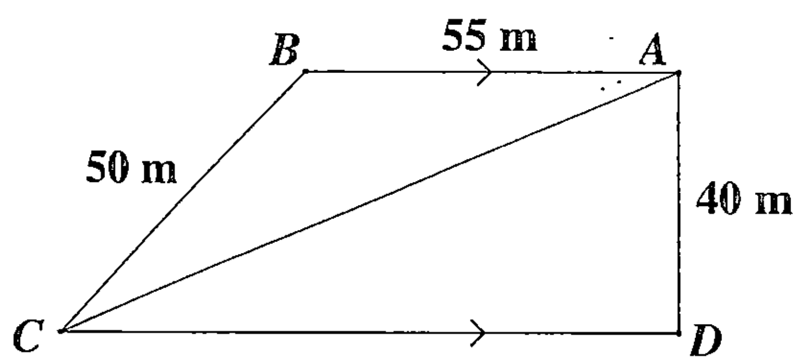
After the addition of 4 people, each person pays \$6.25 less.

(c) Form an equation in x and shows that it reduces to $x^2 + 4x - 320 = 0$. [3]

(d) Solve the equation $x^2 + 4x - 320 = 0$. [3]

(e) Hence, find the new amount of money that each person has to pay. [1]

4 Points A, B, C and D are on level ground, where $AB = 55$ m, $BC = 50$ m and $AD = 40$ m. B is due west of A and D is due south of A. BA is parallel to CD.



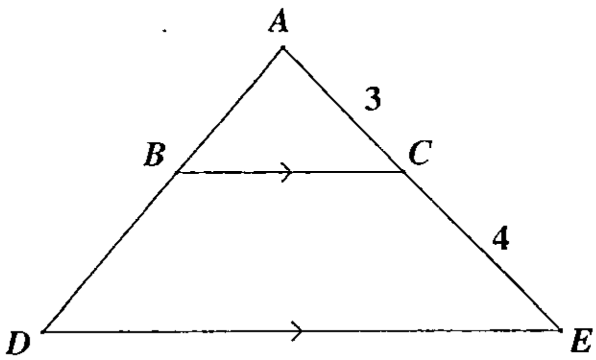
(a) Calculate the bearing of D from B. [2]

(b) Show that $CD = 85$ metres. [1]

(c) Calculate $A\hat{B}C$. [3]

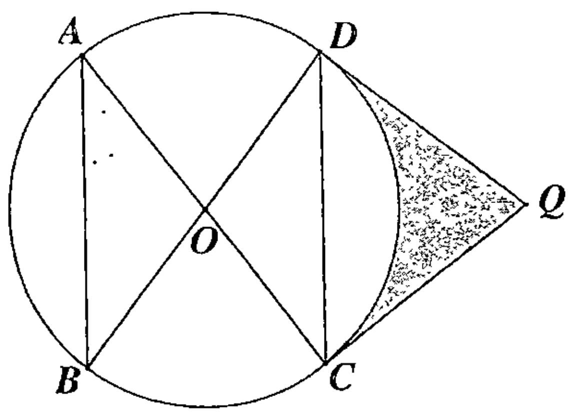
(d) Calculate the area of the trapezium $ABCD$. [2]

5(a) In the diagram below, B and C are points on AD and AE such that BC is parallel to DE. $AC = 3$ cm and $CE = 4$ cm.



Find the ratio $\dfrac{\text{Area of } \Delta ABC}{\text{Area of quadrilateral } BCED}$. [2]

(b)



AOC and BOD are diameters of a circle with O as the centre. DQ and CQ are tangents to the circle.

(i) Show that $\Delta ABO \equiv \Delta DCO$. [3]

(ii) It is given that $D\hat{O}C = 1.92$ radians and the radius of the circle is 8 cm.

(a) Show that the area of quadrilateral $DOCQ$ is 91.4 cm^2. [2]

(b) Calculate the shaded area. [2]

6 Answer the whole of this question on the graph paper provided.

The variables t and v are connected by the equation $v = \frac{800}{t^2} + 32 - \frac{320}{t}$. Some corresponding values of t and v are given in the table below.

t	2.5	3	4	5	10	15	20	25	30
v	32	p	2	0	8	14.2	18	20.5	22.2

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 5 units on each axis, draw a horizontal t-axis for $2.5 \le t \le 40$ and a vertical v-axis for $0 \le v \le 25$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Write down the coordinates of the turning point of the curve. [1]

(d) By drawing a tangent, find the gradient of the curve at the point (20, 18). [2]

(e) On the same axes, draw the graph of $v = 32 - t$ for $0 \le t \le 40$. [2]

(f) (i) Write down the t-coordinates of the points where the two graphs intersect. [1]

(ii) These values of t are solutions to the equation $t^3 + At^2 + Bt + C = 0$.

Find the values of A, of B and of C. [1]

~~~ *End of Paper* ~~~
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Qn	Answer		Qn	Answer
1(a)	$\frac{x+4}{4}$		4(a)	126.0°
1(b)	19		4(b)	*Show*
1(c)	$\frac{8x^5}{625}$		4(c)	126.9°
			4(d)	2800 m^3
2(a)	*Show*			
2(b)(i)	30°		5(a)	$\frac{9}{40}$
2(b)(ii)	105°		5(b)(i)	*Proof*
2(b)(iii)	60°		5(b)(ii)	29.96 cm^2 or 30.0 cm^2
2(b)(iv)	37.5°			
			6(a)	14.2
3(a)	$\frac{500}{x}$		6(b)	*Graph*
3(b)	$\frac{500}{x+4}$		6(c)	(5, 0)
3(c)	*Show*		6(d)	0.6 ± 0.1
3(d)	$x = -20$ or $x = 16$		6(e)	*Graph*
3(e)	$25		6(f)(i)	$t = 2.5$ and 16.5 ($\pm$ 0.5)
			6(f)(ii)	$A = 0, B = -320, C = 800$

3E EOY 2014 P2 Soln

Qn	Solution	Marks
1(a)	$\dfrac{x^2+3x-4}{4x-4}$	
	$=\dfrac{(x-1)(x+4)}{4(x-1)}$	M1
	$=\dfrac{(x+4)}{4}$	A1
1(b)	$-\dfrac{1}{4}x>-5$	
	$x<20$	M1
	Largest prime number $x=19$	A1
1(c)	$\left(\dfrac{5}{2x^2}\right)^{-3}\div 5x$	
	$=\dfrac{8x^6}{125}\div 5x$	M1
	$=\dfrac{8x^5}{625}$	A1
2(a)	$\angle GBO=90°$ (tan $\perp$ rad)	
	$\angle BOC=180°-90°=90°$ (int $\angle$s)	M1
	$\angle BCO=\dfrac{180°-90°}{2}=45°$ (isos Δ) (shown)	A1
	**Note that question says to show CLEARLY*	
2(b)(i)	$\angle OCD=\dfrac{180°-120°}{2}=30°$ (isos Δ)	B1
2(b)(ii)	$\angle DAB=180°-(30°+45°)=105°$ ($\angle$s in opp segments)	B1
2(b)(iii)	$\angle DBC=\dfrac{120°}{2}=60°$ ($\angle$ at ctr $=2\angle$ at circumference)	B1
2(b)(iv)	$\angle ABD=\dfrac{180°-105°}{2}=37.5°$ (isos Δ)	M1
	$\angle ABC=37.5°+60°=97.5°$	A1
3(a)	$\dfrac{500}{x}$	B1
3(b)	$\dfrac{500}{x+4}$	B1

3(c)	$\frac{500}{x}-\frac{500}{x+4}=6.25$	M1 o.e.
	$500(x+4)-500x=6.25x(x+4)$	M1
	$6.25x^2+25x-2000=0$	
	$x^2+4x-320=0$ (shown)	A1
3(d)	$x^2+4x-320=0$	
	$(x+20)(x-16)=0$	M1
	$x=-20$ or $x=16$	A1 + A1
3(e)	$\frac{500}{16+4}=\$25$	B1
4(a)	$\tan\angle ABD=\frac{40}{55}$	
	$\angle ABD=\tan^{-1}\frac{40}{55}=36.027°$	M1
	Bearing of D from $B=90°+36.027°=126.0°$ (to 1dp)	A1
4(b)	Let E be the foot of the perpendicular of B on CD.	
	$BE=40$ m	
	$CE=\sqrt{50^2-40^2}=30$ m	
	$CD=30+55=85$ m (shown)	B1
4(c)	$AC=\sqrt{40^2+85^2}=\sqrt{8825}$	M1
	$\cos\angle ABC=\frac{50^2+55^2-8825}{2(50)(55)}=-0.6$	M1
	$\angle ABC=\cos^{-1}(-0.6)$	
	$=126.870°$	
	$=126.9°$ (to 1dp)	A1
4(d)	Area of $ABCD=\frac{1}{2}(50)(55)\sin 126.87°+\frac{1}{2}(40)(85)$	M1
	$=2800\text{ m}^3$ (to 3sf)	A1
	Alternatively, Area of $ABCD=0.5\ (40)\ (55+85)$ M1	
	$=2800\text{ m}^3$ A1	
5(a)	$\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta ADE}=\left(\frac{3}{7}\right)^2=\frac{9}{49}$	M1
	$\frac{\text{Area of }\Delta ABC}{\text{Area of quadrilateral }BCED}=\frac{9}{40}$	A1

5(b)(i)	$DO = AO$ (radii of circle)*	B1
	$CO = BO$ (radii of circle)*	B1
	$\angle AOB = \angle DOC$ (vert opp $\angle$s)	B1
	$\therefore \Delta ABO \equiv \Delta DCO$ (SAS)	
	**markers need to take note of the naming of the sides.*	
	Students may also use $\angle$s at same segment. 0 marks for AA proof	
5(b)(ii)	(a) $\tan\left(\frac{1.92}{2}\right) = \frac{DQ}{8}$	
	$DQ = 8\tan 0.96 = 11.425$ cm	M1
	Area of $DOCQ = 2\left(\frac{1}{2}\times 11.425\times 8\right) = 91.4$ cm^2 (to 3sf) (shown)	A1
	(b) Area of minor sector $DOC = \frac{1}{2}\times 8^2\times 1.92 = 61.44$ cm^2	M1
	Shaded area $= 91.4 - 61.44 = 29.96$ cm^2	A1
	Accept also 30.0 cm^2 (to 3 sig fig)	
6(a)	$p = 14.2$ (to 1dp)	B1
6(b)	Graph	Curve – G1 Points – G1 Axes – G1
6(c)	(5, 0)	B1 √
6(d)	Tangent at (20, 18)	B1
	Gradient = 0.6 (Accept ±0.1)	B1
6(e)	Graph of $v = 32 - t$	G2 – correct start point, correct gradient
6(f)(i)	$t = 2.5$ and 16.5 (Accept ± 0.5)	B1
6(f)(ii)	$\frac{800}{t^2} + 32 - \frac{320}{t} = 32 - t$	
	$t^3 - 320t + 800 = 0$	
	$A = 0, B = -320, C = 800$	A1