BEATTY SECONDARY SCHOOL
MID-YEAR EXAMINATION 2014

SUBJECT : Mathematics

LEVEL : Secondary 3 Express

PAPER : 4016 / 01

DURATION : 1 hour 15 min

SETTER : Mrs Samsol

DATE : 12 May 2014

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **50**.

For Examiner's Use
50

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

For Examiners use

1 **(a)** Given that $20120.3 = 2\times10^4 + 1\times10^n + 2\times10 + 3\times10^m$, state the values of n and m.

Answer (a) $n =$ ..... $m =$ ..... [1]

(b) Evaluate $(7.95\times10^6) \div (2.11\times10^{-3})$.
Give your answer in standard form correct to 3 significant figures.

Answer (b) ..... [1]

2 Simplify, giving your answers in positive indices

(a) $\left(\frac{3}{2x}\right)^{-3}$,

Answer (a).....[1]

(b) $\frac{(3x^{-2}y^3)^4}{\sqrt[3]{729x^{-9}y^6}} \times (4x^5)^0$.

Answer (b) ..... [2]

For Examiners use

3 **(a)** On the axes provided, the point (0,1) is shown.

Sketch the graphs of

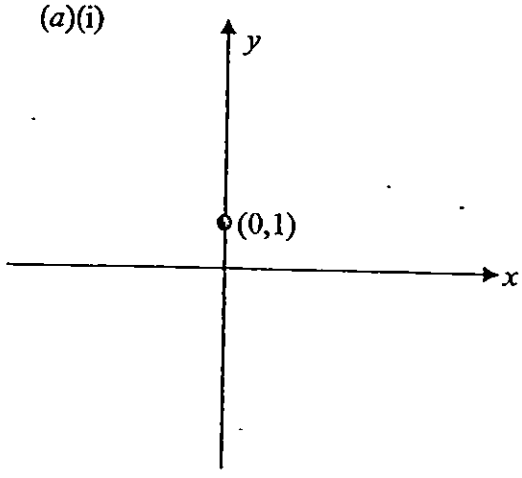
(i) $y = x^3$,

(ii) $y = 3^x$.

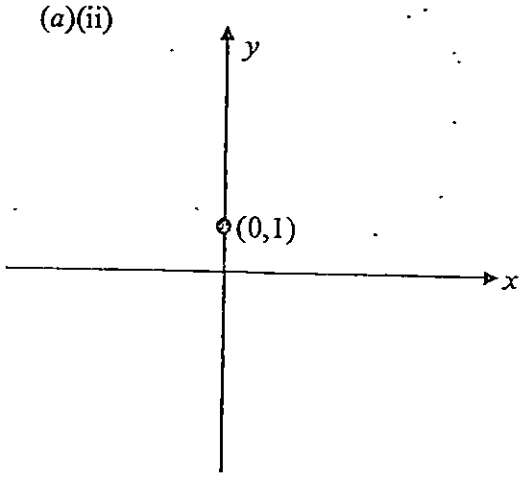
Answer

[2]

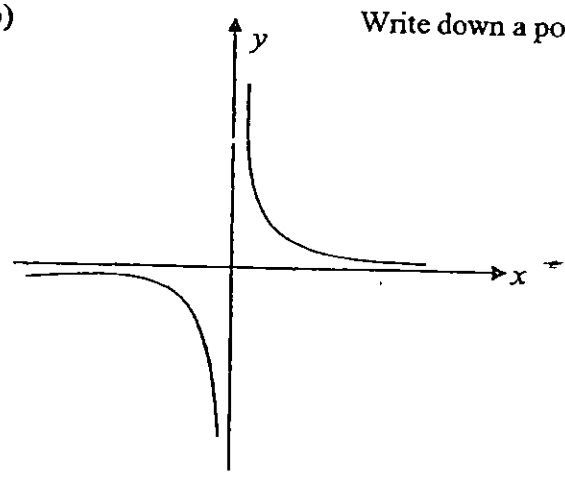
(a)(i)



(a)(ii)



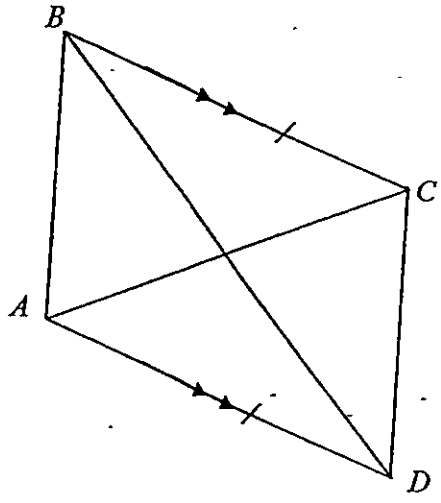
(b) Write down a possible equation for this graph.



Answer (b) ..... [1]

For Examiner's use

4 In the quadrilateral $ABCD$, $BC = AD$ and BC is parallel to AD.



(a) Show that triangles ABC and CDA are congruent.

Answer (*a*) In triangles ABC and CDA .....

.....

.....

.....

.....

.....

.....[2]

(b) Given that $\angle ADC = 72^\circ$ and $\angle ACB = 50^\circ$, find $\angle ACD$.

Answer (*b*) ..... [1]

For Examiner's use

5 **(a)** Solve the inequality $5(x-4) \leq 2x-3 < 9x+11$.

Answer (a)..... [3]

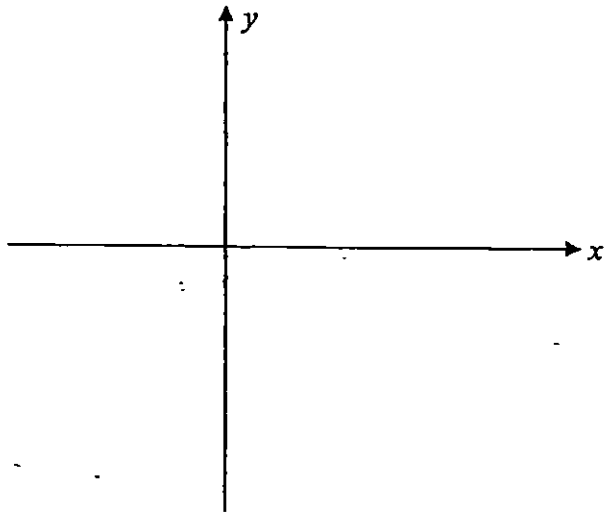
(b) Hence state the integer values of x.

Answer (b) $x =$ ..... [1]

For Examiner's use

6 **(a)** Sketch the graph of $y=-(x+1)(x-3)$.

Answer (a)



[2]

(b) Write down the equation of the line of symmetry of the graph.

Answer (b) .....[1]

(c) Find the coordinates of the maximum point.

Answer (c) (..... , ...) [1]

7 **(a)** Expressing your answer as a power of 5, find

(i) $5^7 \times 5^5 \div 5^3$

Answer (a)(i)..... [1]

(ii) $\sqrt[6]{25}$

Answer (a)(ii) ..... [1]

(b) Given that $3^a = 5$ and $3^b = 7$, find the value of 3^{2a-b}.

Answer (b) .....[2]

For Examiners use

8 Solve the following equations.

(a) $a^5 \times a \div \sqrt{a} = a^k$

Answer (a) $k =$ .....[1]

(b) $2^k \div 3 = \frac{1}{48}$

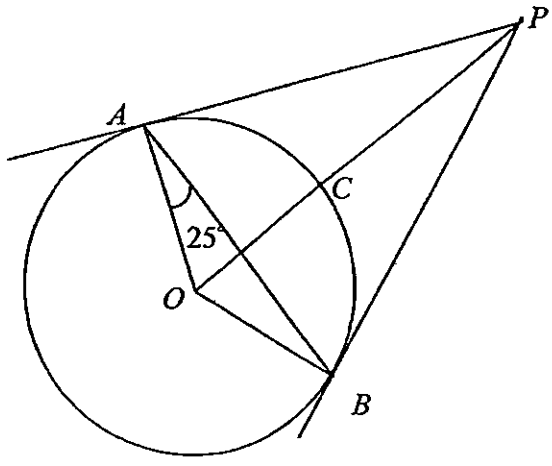
Answer (b) $k =$ ..... [1]

(c) $7^{3k+4} \times 7^{-5} = 1$

Answer (c) $k =$ .....[2]

For Examiner's use

9 The diagram shows a circle ABC, with centre O. AP and BP are tangents to the circle.
Angle $OAB = 25°$.



(i) Stating your reasons clearly, find

(a) angle AOB,

Answer (i)(*a*) $\angle AOB =$ ..... ° [1]

(b) angle PAB,

Answer (i)(*b*) $\angle PAB =$ ..... ° [1]

(c) angle APB.

Answer (i)(c) $\angle APB =$ ..... ° [1]

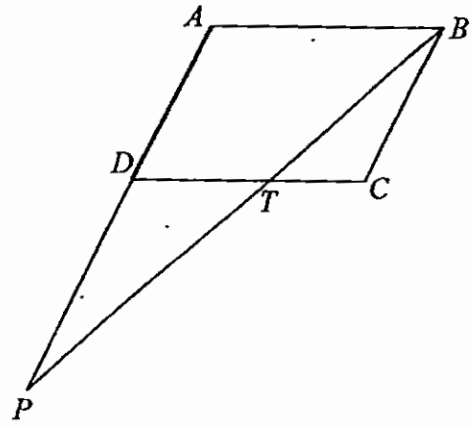
(ii) Given that the radius of the circle is 5 cm and $OP = 16$ cm., find the length of AP.

Answer (ii) ..... cm [2]

For Examiner's use

10 $ABCD$ is a parallelogram and T is a point on DC.

The line AD produced meets BT produced at P.



(a) Name a triangle similar to triangle ABP.

Answer (*a*) ..... [1]

(b) Given that $AD : DP = 2 : 3$, find

(i) $\dfrac{DT}{AB}$,

Answer (*b*)(i) ..... : ..... [1]

(ii) $\dfrac{\text{area of } \Delta PDT}{\text{area of } \Delta PAB}$.

Answer (*b*)(ii) .... : ..... [1]

(c) Given that the area of triangle PDT is 18 cm^2, find the area of the quadrilateral $ABTD$.

Answer (c) ..... cm^2 [2]

For Examiner's use

11 Solve the following equations.

(a) $(3x-1)(x+1)=20$

Answer (a) ..... [2]

(b) $\frac{x+1}{5x}=\frac{x-2}{4}$

Answer (b) .....[4]

For Examiner's use

12 **(a)** Express $x^2-6x+10$ in the form $(x-a)^2+b$.

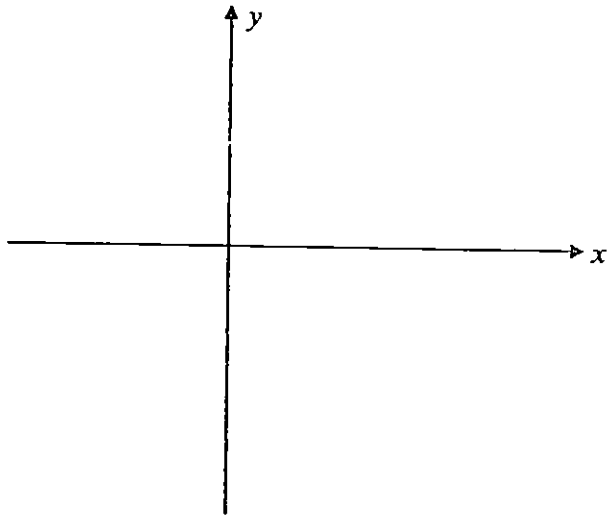
Answer (a)..... [2]

(b) Hence solve the equation $x^2-6x+10=7$, leaving your answers to 2 decimal places.

Answer (b) $x =$ ... . or ..... [3]

(c) Sketch the graph of $y=x^2-6x+10$.

Answer (c) [2]



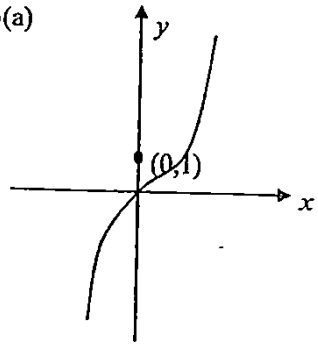
End of Paper

Answer Key

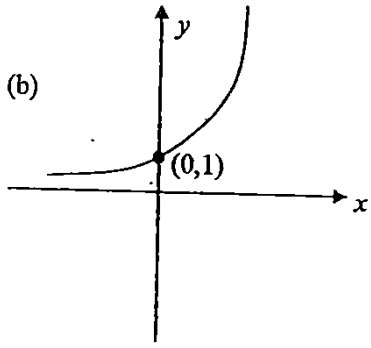
1(a) $n = 2$, $m = -1$ (b) 3.77×10^9

2(a) $\frac{8x^3}{27}$ (b) $\frac{9y^{10}}{x^5}$

3(a)



(b)



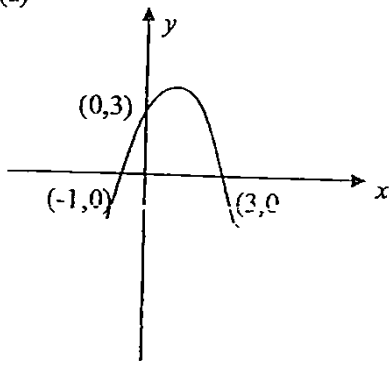
(c) $y = \frac{1}{x}$, $y = \frac{2}{x}$ .....

4(a) In triangles ABC and CDA, BC = DA (given), $\angle BCA = \angle DAC$ (alternate angles) , AC is a common side . Hence triangle ABC and CDA are congruent (SAS Property)

(b) 58^0

5(a) $-2 < x \leq 5\frac{2}{3}$ (b) -1, 0 , 1, 2, 3, 4, 5

6(a)



(b) $x = 1$ (c) (1, 4)

7(a)(i) 5^9 (ii) $5^{\frac{1}{3}}$ (b) $\frac{25}{7}$

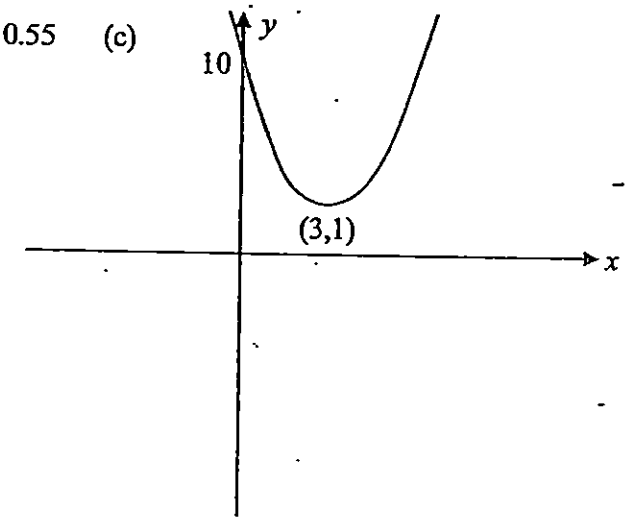
8(a) $k = 5.5$ (b) $k = -4$ (c) $k = \frac{1}{3}$

9(i) (a) 130^0 (b) 65^0 (c) 50^0 (ii) 15.2 cm

10(a) triangle DTP (b)(i) $\frac{3}{5}$ (ii) $\frac{9}{25}$ (c) 32 cm^2

11(a) $x=\frac{7}{3}=2\frac{1}{3}$ or $x=-3$ (b) $x=3.06$ or $x=-0.261$

12(a) $(x-3)^2+1$ (b) $x=5.45$ or $x=0.55$ (c)



BEATTY SECONDARY SCHOOL
MID-YEAR EXAMINATION 2014

SUBJECT : Mathematics | **LEVEL : Sec 3 Express**

PAPER : 4016 / 2 | **DURATION : 1 hour 30 minutes**

SETTER : Mdm See HY | **DATE : 19 May 2014**

CLASS :	NAME :	REG NO :

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Mathematical Formulae

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1 **(a)** Solve the simultaneous inequality $11-2x \le x-4 < \frac{6+3x}{5}$. [2]

(b) Represent your solution of part (a) on a number line. [1]

(c) Hence state the largest prime number that satisfies the inequality. [1]

2 **(a)** Express $\frac{1-3x}{9x^2-1} - \frac{3x+2}{3x+1}$ as a single fraction in its simplest form. [3]

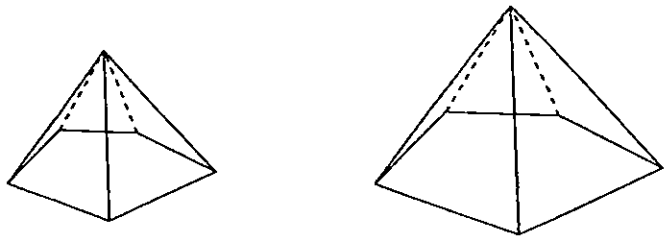
(b) Solve the equation $1728 = \frac{27}{16^{0.5x}}$. [3]

3 The diameter of a hydrogen atom is approximately 9800 picometres.

(a) Express 9800 picometres in kilometres, giving your answer in standard form. [1]

(b) The hydrogen atoms are placed side by side. Find the number of hydrogen atoms needed to form a line 3.5 cm long. Give your answer in standard form and correct to 3 significant figures. [2]

4

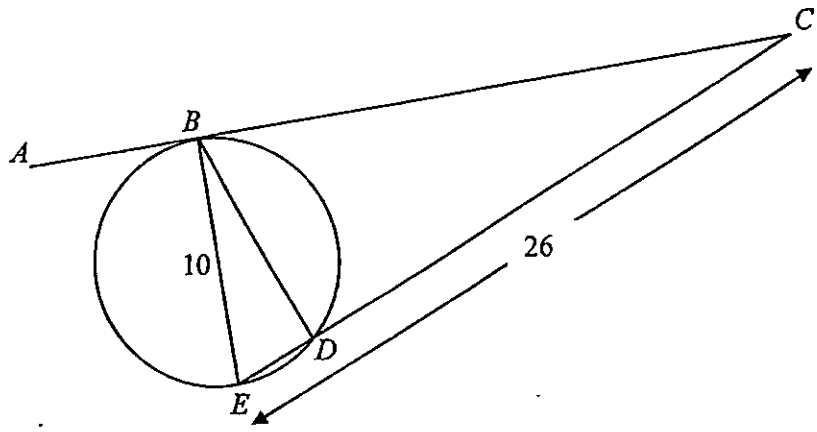


The surface areas of two geometrically similar pyramids made of the same material are 16 cm^2 and 49 cm^2 respectively.

(a) Find the ratio of the height of the smaller pyramid to the height of the bigger pyramid. [1]

(b) Both pyramids are filled with sand. Find the mass of the sand, in kg, in the bigger pyramid if the mass of sand in the smaller pyramid is 192 g. [2]

5



The diagram shows a circle, with diameter BE. The line AC is a tangent to the circle at the point B. The line CE cuts the circle at the point D.

(a) Show that triangle EBD is similar to triangle ECB. [3]

(b) Given that $BE = 10$ cm and $EC = 26$ cm, calculate the length of CD. [2]

6 **(a)** Samuel cycled 40 km from Town A to Town B at a constant speed of x km/h. Write down an expression, in terms of x, for the number of hours he took to cycle. [1]

(b) He then jogged 10 km from Town B to Town C at a constant speed of $(x-60)$ km/h. Write down an expression, in terms of x, for the number of hours he took to jog. [1]

(c) Given that Samuel took 15 minutes more for the second part of the journey, form an equation in x and show that it reduces to $x^2+60x-9600=0$. [2]

(d) Solve the equation $x^2+60x-9600=0$, giving your answers correct to 2 decimal places. [3]

(e) Hence calculate

(i) the time, in minutes, Samuel took to cycle from Town A to Town B, [2]

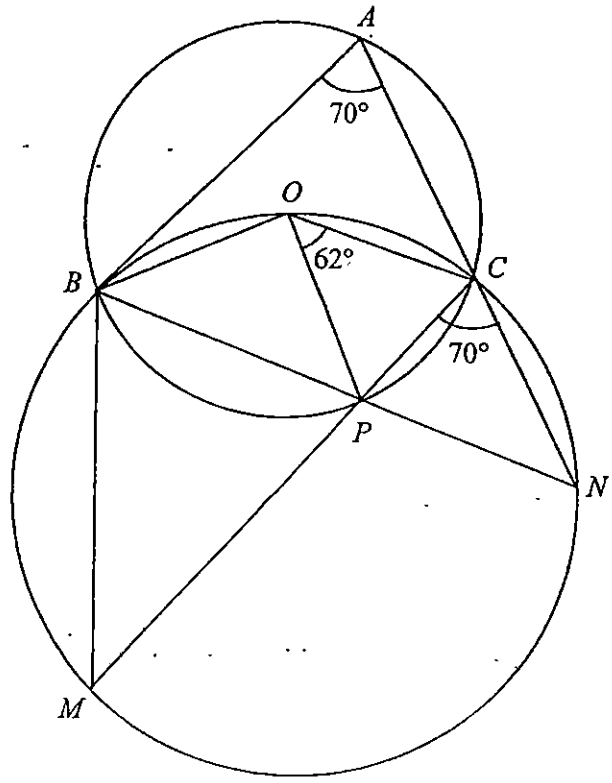
(ii) the average speed, in km/h, for the entire journey from Town A to Town C. [1]

7 In the diagram, O is the centre of circle $ABPC$.

$\angle BAC = \angle MCN = 70°$ and $\angle POC = 62°$.

The larger circle $BMNC$ passes through O.

ACN, BPN and CPM are straight lines.



Giving reasons, find the value of

(a) reflex $\angle BOC$, [2]

(b) $\angle ABP$, [1]

(c) $\angle MBN$, [1]

(d) $\angle BMP$, [1]

(e) $\angle ACO$. [2]

8. Answer the whole of this question on a sheet of graph paper.

The variables x and y are related by the equation $y = x + \frac{12}{x} - 6$.

Some corresponding values of x and y are given in the following table.

x	0.75	1	2	3	4	5	6	7	8
y	10.75	7	2	1	1	1.4	2	p	3.5

(a) Calculate the value of p, giving your answer correct to 2 decimal places. [1]

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = x + \frac{12}{x} - 6$ for $0.75 \leq x \leq 8$. [3]

(c) Estimate the minimum value of $y = x + \frac{12}{x} - 6$. [1]

(d) By drawing a tangent, calculate the gradient of the curve at the point $x = 6$. [2]

(e) Use your graph to solve the equation $x + \frac{12}{x} - 6 = 0$. [1]

(f) By drawing a suitable line on the graph, solve the equation

$$x + \frac{12}{x} - 14 = 0$$ [2]

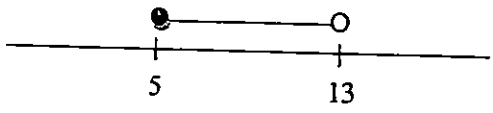
(g) On the same graph, draw the line $y = -x + 6$. Hence estimate the range of values of x for which $x + \frac{12}{x} - 6 < -x + 6$. [2]

End of Paper

Answers (3E MYE P2_2014)

1a) $5 \le x < 13$

b)



c) largest prime number = 11

2a) $\frac{-3(x+1)}{(3x+1)}$

b) $x = -3$

3a) 9.8×10^{-12} km

b) 3.57×10^{6}

4a) 4 : 7

b) 1.029 kg

5a) $\angle EDB = 90°$ ($\angle$ in semicircle)

$\angle EBC = 90°$ (rad $\perp$ tan)

$\therefore \angle EDB = \angle EBC = 90°$

$\angle BED = \angle CEB$ (common $\angle$)

$\therefore \Delta EBD$ is similar to ΔECB (AAA Similarity)

b) $22\frac{2}{13}$ cm

7a) 220°

b) 70°

c) 70°

d) 40°

e) 51°

6a) $\frac{40}{x}$

b) $\frac{10}{x-60}$

c) $$\frac{10}{x-60} - \frac{40}{x} = \frac{15}{60}\,\frac{3}{12}$$

$$10x(12) - 40(12)(x-60) = 3(x)(x-60)$$

$$120x - 480x + 28800 = 3x^2 - 180x$$

$$3x^2 + 180x - 28800 = 0$$

$$x^2 + 60x - 9600 = 0 \text{ (shown)}$$

d) -132.47 or 72.47

ei) 33.1 min

eii) 36.9 km/h

8a) 2.71

b) on graph

c) 0.9

d) accept 0.6 to 0.7

e) No solution

f) accept $0.8 \le x < 1$

g) $1.25\ (\pm 0.1) < x < 4.75\ (\pm 0.1)$

BEATTY SECONDARY SCHOOL
MID-YEAR EXAMINATION 2014

SOLUTIONS

SUBJECT : Mathematics

PAPER : 4016 / 01

SETTER : Mrs Samsol

LEVEL : Secondary 3 Express

DURATION : 1 hour 15 min

DATE : 12 May 2014

CLASS :	NAME :	REG NO :

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For Examiner's use

1 **(a)** Given that $20120.3 = 2\times10^4 + 1\times10^n + 2\times10 + 3\times10^m$, state the values of n and m.

Answer (a) n = ...2..... m =.....-1..... [1]

(b) Evaluate $(7.95\times10^6)\div(2.11\times10^{-3})$.
Give your answer in standard form correct to 3 significant figures.

Answer (b) .....3.77 x 10^9..... [1]

2 Simplify, giving your answers in positive indices

(a) $\left(\frac{3}{2x}\right)^{-3}$,

$= \left(\frac{2x}{3}\right)^3$

$= \frac{8x^3}{27}$

Answer (a)..... $\frac{8x^3}{27}$ .....[1]

(b) $\frac{(3x^{-2}y^3)^4}{\sqrt[3]{729x^{-9}y^6}}\times(4x^5)^0$.

$= \frac{81x^{-8}y^{12}}{9x^{-3}y^2}\times1$ M1 [M0 if more than 1 mistake]

$= \frac{9y^{10}}{x^5}$ A1

Answer (b) .... [2]

[Turn over

For Examiner's use

3 **(a)** On the axes provided, the point (0,1) is shown.

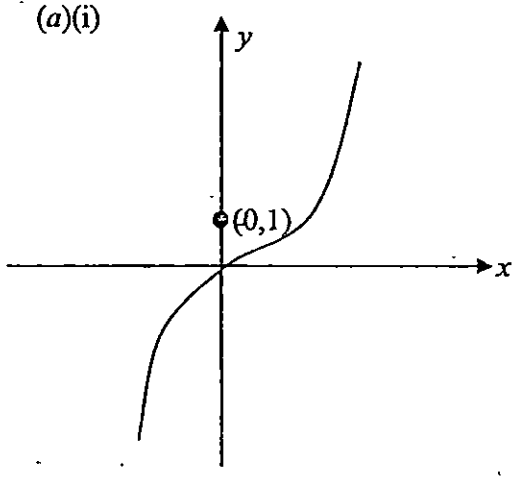
Sketch the graphs of

(i) $y = x^3$,

(ii) $y = 3^x$.

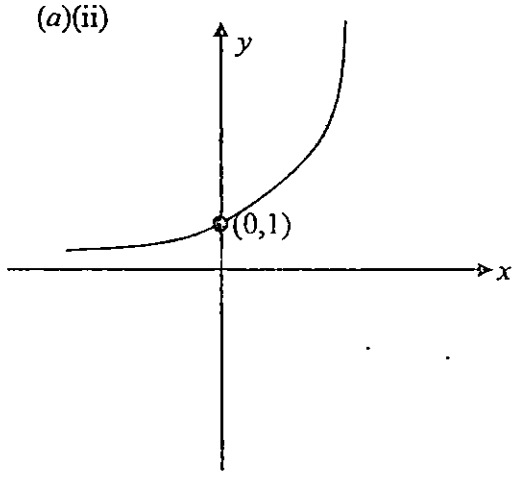
Answer [2]

(a)(i)



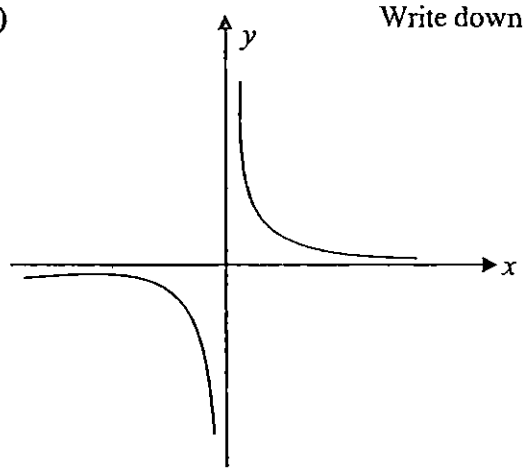
B1

(a)(ii)



B1

(b) Write down a possible equation for this graph.



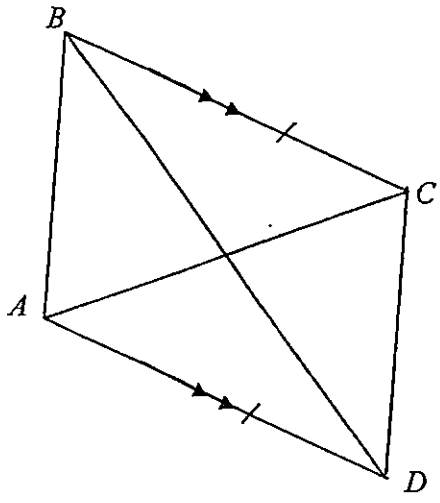
$y = \frac{a}{x^n}$ where a > 0 and n is an odd integer B1

eg : $y = \frac{1}{x}$, $y = \frac{2}{x}$, $y = \frac{1}{x^3}$

Answer (b) ..... [1]

For Examiners use

4 In the quadrilateral $ABCD$, $BC = AD$ and BC is parallel to AD.



(a) Show that triangles ABC and CDA are congruent.

Answer (*a*) In triangles ABC and CDA .....

BC = DA (given), $\angle BCA = \angle DAC$ (alternate angles) , AC is a common side..... B1

.... .hence triangle ABC and CDA are congruent (SAS Property)...B1 [awarded only if above conditions are correct]

....

....

....[2]

(b) Given that $\angle ADC = 72^\circ$ and $\angle ACB = 50^\circ$, find $\angle ACD$.

$\angle ACD = 180^0 - 50^\circ - 72^\circ = 58^\circ$ B1

Answer (*b*) .. .. [1]

For Examiners use

5 **(a)** Solve the inequality $5(x-4) \leq 2x-3 < 9x+11$.

$5x-20 \leq 2x-3$

$3x \leq 17$

$x \leq 5\frac{2}{3}$ M1

$2x-3 < 9x+11$

$2x-9x < 11+3$

$-7x < 14$

$x > -2$ M1

Ans: $-2 < x \leq 5\frac{2}{3}$ A1

Answer (a)..... [3]

(b) Hence state the integer values of x.

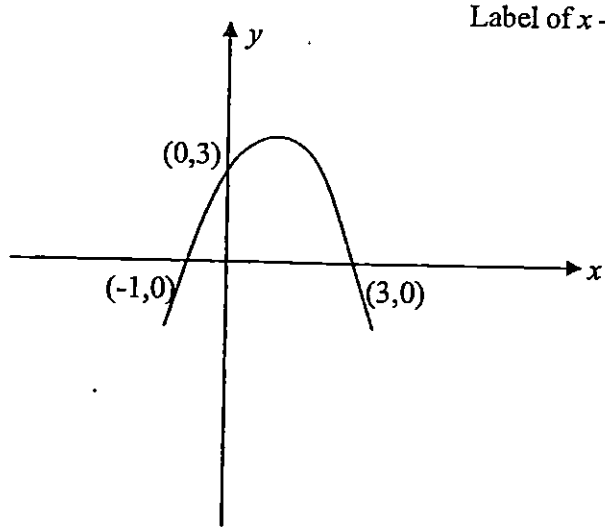
Answer (b) x = ...-1, 0, 1, 2, 3, 4, 5.. ..... [1]

For Examiners use

6 **(a)** Sketch the graph of $y=-(x+1)(x-3)$.

Answer (a) Shape of curve and y-intercept M1

Label of x-intercepts A1



[2]

(b) Write down the equation of the line of symmetry of the graph.

Answer (b) ..... $x=1$ .....[1]

(c) Find the coordinates of the maximum point.

Answer (c) (...1..... , ...4.....) [1]

7 **(a)** Expressing your answer as a power of 5, find

(i) $5^7 \times 5^5 \div 5^3$

Answer (a)(i)..... 5^9 ..... [1]

(ii) $\sqrt[6]{25}$

Answer (a)(ii) ..... $5^{\frac{1}{3}}$ ..... [1]

(b) Given that $3^a = 5$ and $3^b = 7$, find the value of 3^{2a-b}.

$\frac{(3^a)^2}{3^b}$ M1

$= \frac{(5)^2}{7} = \frac{25}{7}$ A1

Answer (b) ....[2]

For Examiners use

8 Solve the following equations.

(a) $a^5 \times a \div \sqrt{a} = a^k$

Answer (a) $k =$ .....5.5.....[1]

(b) $2^k \div 3 = \frac{1}{48}$

$2^k = \frac{3}{48} = \frac{1}{16}$

$k = -4$

Answer (b) $k =$ .....- 4..... [1]

(c) $7^{3k+4} \times 7^{-5} = 1$

$7^{3k+4-5} = 7^0$

$3k + 4 - 5 = 0$ M1

$3k = 1$

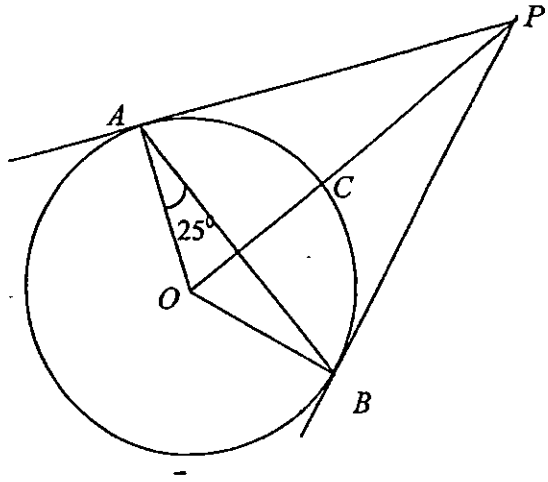
$k = \frac{1}{3}$ A1

Answer (c) $k = \frac{1}{3}$ .....[2]

For Examiners use

9 The diagram shows a circle ABC, with centre O. AP and BP are tangents to the circle.

Angle $OAB = 25^0$.



(i) Stating your reasons clearly, find

(a) angle AOB,

$= 180^0 - 2(25^0) = 130^0$ (isosceles triangle) B1

Answer (i)(*a*) $\angle AOB =$ .....^{0} [1]

(b) angle PAB,

$= 90^0 - 25^0 = 65^0$ (tangent perpendicular to radius) B1

Answer (i)(*b*) $\angle PAB =$ .....^{0} [1]

(c) angle APB.

$= 360^0 - 180^0 - 130^0 = 50^0$ (sum of angles in quad) B1

or angle $APB = 180^\circ - 2(65^\circ) = 50^\circ$ (tangents from ext point)

Answer (i)(c) $\angle APB =$ .....^{0} [1]

(ii) Given that the radius of the circle is 5 cm and $OP = 16$ cm, find the length of AP.

$AT = \sqrt{16^2 - 5^2}$ M1

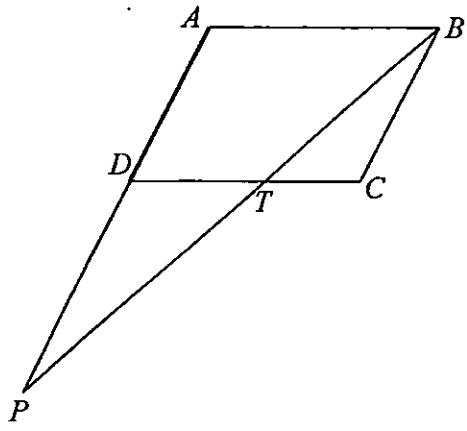
$AT = 15.2$ cm (3sf) A1

Answer (ii) ..... cm [2]

For Examiners use

10 $ABCD$ is a parallelogram and T is a point on DC.

The line AD produced meets BT produced at P.



(a) Name a triangle similar to triangle ABP.

Answer (*a*) triangle DTP or CTB..... [1]

(b) Given that $AD : DP = 2:3$, find

(i) $\dfrac{DT}{AB}$,

Answer (*b*)(i) ... $\dfrac{3}{5}$ or ...3: 5..... [1]

(ii) $\dfrac{\text{Area of } \Delta PDT}{\text{Area of } \Delta PAB} = \left(\dfrac{3}{5}\right)^2$

Answer (*b*)(ii) $\dfrac{9}{25}$... or ...9 : 25... ... [1]

(c) Given that area of triangle PDT is 18 cm^2, find the area of the quadrilateral $ABTD$

Area of $\Delta PAB = \dfrac{25}{9} \times 18$ M1

$= 50$

Area of $ABTD = 50\text{-}18 = 32$ cm^2 A1

Answer (c) .... cm^2 [2]

For Examiners use

11 Solve the following equations.

(a) $(3x-1)(x+1)=20$

$3x^2+2x-1-20=0$

$3x^2+2x-21=0$

$(3x-7)(x+3)=0$ M1

$x=\frac{7}{3}=2\frac{1}{3}$ or $x=-3$ A1

Answer (a) ..... [2]

(b) $\frac{x+1}{5x}=\frac{x-2}{4}$

$5x^2-10x=4x+4$ M1

$5x^2-14x-4=0$

$x=\frac{14\pm\sqrt{(-14)^2-4(5)(-4)}}{2(5)}$ M1

$x=3.06$ or $x=-0.261$ (3 sf) A2

Answer (b) .....[4]

For Examiners use

12 (a) Express $x^2-6x+10$ in the form $(x-a)^2+b$.

$= (x-3)^2-3^2+10$ M1

$= (x-3)^2+1$ A1

Answer (a)..... [2]

(b) Hence solve the equation $x^2-6x+10=7$, leaving your answers to 2 decimal places.

$(x-3)^2+1=7$ M1

$(x-3)^2=6$

$x=\pm\sqrt{6}+3$

$x=5.45$ or $x=0.55$ (2 dp) A2

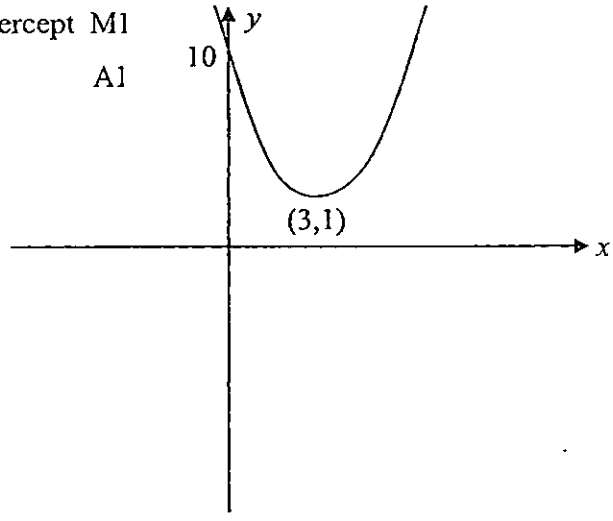
Answer (b) x = ... or ..... [3]

(c) Sketch the graph of $y=x^2-6x+10$.

Answer (c) [2]

Shape of graph and y-intercept M1

Label for minimum point A1



End of Paper

MARKING SCHEME

BEATTY SECONDARY SCHOOL
MID-YEAR EXAMINATION 2014

SUBJECT : Mathematics

LEVEL : Sec 3 Express

PAPER : 4016 / 2

DURATION : 1 hour 30 minutes

SETTER : Mdm See HY

DATE : May 2014

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

MARKING SCHEME

1 **(a)** Solve the simultaneous inequality $11-2x \le x-4 < \frac{6+3x}{5}$. [2]

$$11-2x \le x-4 < \frac{6+3x}{5}$$

$11-2x \le x-4$ and $x-4 < \frac{6+3x}{5}$

$-3x \le -15$ $5x-20 < 6+3x$

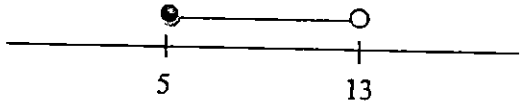
$x \ge 5$ $2x < 26$

$x < 13$

Either one [M1]

$\therefore 5 \le x < 13$ **[A1]**

(b) Represent your solution of part (a) on a number line. [1]



[B1 — FT]

(c) Hence state the largest prime number that satisfies the inequality. [1]

largest prime number = 11 **[B1]**

---

2 **(a)** Express $\frac{1-3x}{9x^2-1} - \frac{3x+2}{3x+1}$ as a single fraction in its simplest form. [3]

$$\frac{1-3x}{9x^2-1} - \frac{3x+2}{3x+1} = \frac{1-3x-(3x+2)(3x-1)}{(3x+1)(3x-1)}$$ **[M1]**

$$= \frac{1-3x-(9x^2+3x-2)}{(3x+1)(3x-1)}$$

$$= \frac{-9x^2-6x+3}{(3x+1)(3x-1)}$$ **[M1]**

$$= \frac{-3(3x^2+2x-1)}{(3x+1)(3x-1)}$$

$$= \frac{-3(x+1)(3x-1)}{(3x+1)(3x-1)}$$

$$= \frac{-3(x+1)}{(3x+1)}$$ **[A1]**

2 **(b)** Solve the equation $1728 = \dfrac{27}{16^{0.5x}}$. [3]

$$1728 = \frac{27}{16^{0.5x}}$$

$$64 = 16^{-0.5x} \quad \textbf{[M1]}$$

$$4^3 = \left(4^2\right)^{-0.5x} \quad \textbf{[M1]}$$

$$4^3 = 4^{-x}$$

$$x = -3 \quad \textbf{[A1]}$$

---

3 The diameter of a hydrogen atom is approximately 9800 picometres.

(a) Express 9800 picometres in kilometres, giving your answer in standard form. [1]

$$9800 \text{ picometres} = 9800 \times 10^{-12} \text{ m}$$

$$= 9.8 \times 10^{-9} \times 10^{-3} \text{ km}$$

$$= 9.8 \times 10^{-12} \text{ km} \quad \textbf{[B1]}$$

(b) The hydrogen atoms are placed side by side. Find the number of hydrogen atoms needed to form a line 3.5 cm long Give your answer in standard form and correct to 3 significant figures. [2]

$$9800 \text{ picometres} = 9800 \times 10^{-12} \text{ m}$$

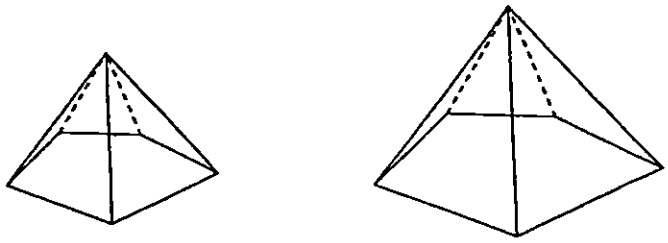
$$= 9.8 \times 10^{-9} \times 10^{2} \text{ cm}$$

$$\text{number of hydrogen atoms} = 3.5 \div \left(9.8 \times 10^{-9} \times 10^{2}\right) \quad \textbf{[M1]}$$

$$= 3571428\,57 \ldots\ldots$$

$$\approx 3.57 \times 10^{6} \text{ (3 s.f.)} \quad \textbf{[A1]}$$

4



The surface areas of two geometrically similar pyramids made of the same material are 16 cm^2 and 49 cm^2 respectively.

(a) Find the ratio of the height of the smaller pyramid to the height of the bigger pyramid. [1]

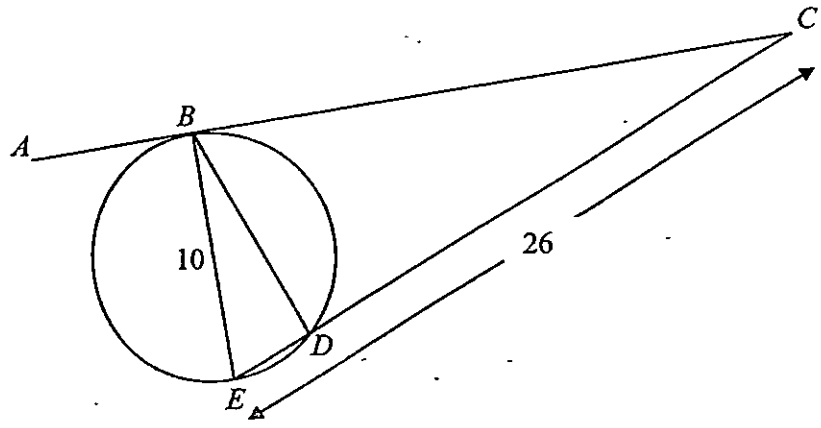
$$\text{Ratio} = \sqrt{16} : \sqrt{49}$$
$$= 4 : 7$$ **[B1]**

(b) Both pyramids are filled with sand. Find the mass of the sand, in kg, in the bigger pyramid if the mass of sand in the smaller pyramid is 192 g. [2]

$$\frac{mass}{192} = \left(\frac{7}{4}\right)^3$$ **[M1]**

$$mass = 1029 \text{ g}$$
$$= 1.029 \text{ kg}$$ **[A1]**

5



The diagram shows a circle, with diameter BE. The line AC is a tangent to the circle at the point B. The line CE cuts the circle at the point D.

(a) Show that triangle EBD is similar to triangle ECB. [3]

$\angle EDB = 90°$ ($\angle$ in semicircle)
$\angle EBC = 90°$ (rad $\perp$ tan) } [M1]

$\therefore \angle EDB = \angle EBC = 90°$ [M1]

$\angle BED = \angle CEB$ (common $\angle$) [M1]

$\therefore \Delta EBD$ is similar to ΔECB (AAA Similarity)

(b) Given that BE = 10 cm and EC = 26 cm, calculate the length of CD. [2]

$$\frac{BE}{CE} = \frac{ED}{EB}$$

$$\frac{10}{26} = \frac{ED}{10}$$ [M1]

$$ED = 3\frac{11}{13} \text{ cm}$$

$$CD = 26 - 3\frac{11}{13}$$

$$= 22\frac{2}{13} \text{ cm}$$ [A1]

6 (a) Samuel cycled 40 km from Town A to Town B at a constant speed of x km/h. Write down an expression, in terms of x, for the number of hours he took to cycle. [1]

number of hours he took $= \frac{40}{x}$ **[B1]**

(b) He then jogged 10 km from Town B to Town C at a constant speed of $(x-60)$ km/h. Write down an expression, in terms of x, for the number of hours he took to jog. [1]

number of hours he took $= \frac{10}{x-60}$ **[B1]**

(c) Given that Samuel took 15 minutes more for the second part of the journey, form an equation in x and show that it reduces to $x^2+60x-9600=0$. [2]

$$\frac{10}{x-60}-\frac{40}{x}=\frac{15}{60}\left(\frac{1}{4}\right)$$ **[M1]**

$$\left.\begin{aligned} 10x(4)-40(4)(x-60) &= x(x-60) \\ 40x-160x+9600 &= x^2-60x \\ x^2+60x-9600 &= 0 \end{aligned}\right\}$$ **[M1]**

(d) Solve the equation $x^2+60x-9600=0$, giving your answers correct to 2 decimal places. [3]

$$x^2+60x-9600=0$$

$$x=\frac{-60\pm\sqrt{60^2-4(1)(-9600)}}{2(1)}$$ **[M1]**

$$=\frac{-60\pm\sqrt{42000}}{2}$$

$=-132.4695$ ..... or $=72.4695$ .....

≈ -132.47 ≈ 72.47 **[A2]**

(e) Hence calculate

(i) the time, in minutes, Samuel took to cycle from Town A to Town B, [2]

time taken $= \frac{40}{72.4695}$ h **[M1]**

$\approx$ 33.1 min (3 s.f) **[A1]**

(ii) the average speed, in km/h, for the entire journey from Town A to Town C. [1]

average speed $= 50 \div \left(\frac{40}{72.4695}+\frac{10}{72.4695-60}\right)$

$=36.9299$.....

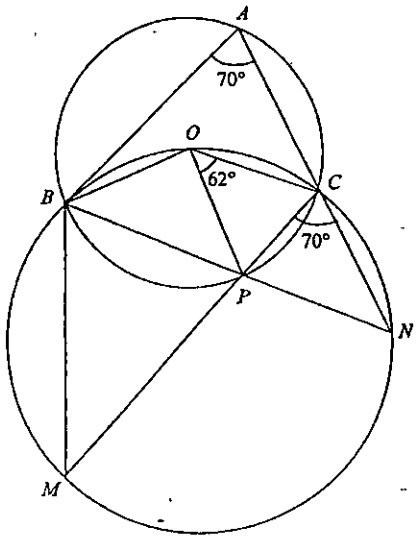
$\approx$ 36.9 km/h (3 s.f) **[A1]**

7 In the diagram, O is the centre of circle $ABPC$.

$\angle BAC = \angle MCN = 70°$ and $\angle POC = 62°$.

The larger circle $BMNC$ passes through O.

ACN, BPN and CPM are straight lines.



Giving reasons, find the value of

(a) reflex $\angle BOC = 360 - (70 \times 2)$ ($\angle$s at a pt., $\angle$ at centre $= 2\angle$ at circ) **[M1]** [2]

$= 220°$ **[A1]**

(b) $\angle ABP = 70°$ (ext. $\angle$ of cyclic quad) **[B1]** [1]

(c) $\angle MBN = 70°$ ($\angle$ in same segment) **[B1]** [1]

(d) $\angle BMP = 180 - (70 \times 2)$ ($\angle$ in opp. segments) [1]

$= 40°$ **[B1]**

(e) $\angle OCP = \dfrac{180 - 62}{2}$ ($\angle$ sum of isos. ΔOPC) [2]

$= 59°$ **[M1]**

$\angle ACO = 180 - 70 - 59$ (adj. $\angle$s on str. line)

$= 51°$ **[A1]**

8. Answer the whole of this question on a sheet of graph paper.

The variables x and y are related by the equation $y = x + \frac{12}{x} - 6$.

Some corresponding values of x and y are given in the following table.

x	0.75	1	2	3	4	5	6	7	8
y	10.75	7	2	1	1	1.4	2	p	3.5

(a) Calculate the value of p, giving your answer correct to 2 decimal places. [1]

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = x + \frac{12}{x} - 6$ for $0.75 \leq x \leq 8$. [3]

(c) Estimate the minimum value of $y = x + \frac{12}{x} - 6$. [1]

(d) By drawing a tangent, calculate the gradient of the curve at the point $x = 6$. [2]

(e) Use your graph to solve the equation $x + \frac{12}{x} - 6 = 0$. [1]

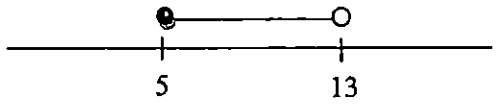
(f) By drawing a suitable line on the graph, solve the equation

$$x + \frac{12}{x} - 14 = 0$$ [2]

(g) On the same graph, draw the line $y = -x + 6$. Hence estimate the range of values of x for which $x + \frac{12}{x} - 6 < -x + 6$. [2]

Answers (3E MYE P2_2014)

1a) $5 \le x < 13$

b) 

c) largest prime number = 11

2a) $\dfrac{-3(x+1)}{(3x+1)}$

b) $x = -3$

3a) 9.8×10^{-12} km

b) 3.57×10^{6}

4a) 4 : 7

b) 1.029 kg

5a) $\angle EDB = 90°$ ($\angle$ in semicircle)

$\angle EBC = 90°$ (rad $\perp$ tan)

$\therefore \angle EDB = \angle EBC = 90°$

$\angle BED = \angle CEB$ (common $\angle$)

$\therefore \Delta EBD$ is similar to ΔECB (AAA Similarity)

b) $22\dfrac{2}{13}$ cm

7a) 220°

b) 70°

c) 70°

d) 40°

e) 51°

6a) $\dfrac{40}{x}$

b) $\dfrac{10}{x-60}$

c) $$\frac{10}{x-60} - \frac{40}{x} = \frac{15}{60}\,\frac{1}{4}$$

$$10x(4) - 40(4)(x-60) = x(x-60)$$

$$40x - 160x + 9600 = x^2 - 60x$$

$$x^2 + 60x - 9600 = 0 \text{ (shown)}$$

d) -132.47 or 72.47

ei) 33.1 min

eii) 36.9 km/h

8a) 2.71

b) on graph

c) 0.9

d) accept 0.6 to 0.7

e) No solution

f) accept $0.8 \le x < 1$

g) 1.25 ($\pm$0.1) $< x <$ 4.75 ($\pm$0.1)

End of Paper