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DUNEARN SECONDARY SCHOOL **Mid-Year Examination 2014** **Elementary Mathematics (4016)**

Secondary 3 Express

Friday

9th May 2014

0800 – 1030 hours

2 hrs 30 mins

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in
Write in dark blue or black ink.

You may use a pencil for any diagrams or graphs.

Do not use staples, paperclips, highlighters, glue or correction fluid.

Answer all **Section A** questions in the space provided in this paper.

Answer all **Section B** questions in the writing paper provided.

If working is needed for any question, it must be shown with the answer.

Omission of working will result in loss of marks.

You are expected to use a scientific calculator to evaluate explicit numerical expression.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.

Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 100.

PARENT'S SIGNATURE

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FOR EXAMINER'S USE

Section A	/40
Section B	/60
Total	/100

This question paper consists of 12 printed pages, including this cover page.

*Mathematical Formulae**Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Section A (40 marks)**Answer all questions in the answer space provided.**

- 1 Factorise $2p - q - 2p^2 + pq$ completely.

Answer _____ [2]

- 2 (a) Express 360 as a product of its prime factors.

Answer (a) _____ [1]

- (b) Find the smallest positive integer k such that $360k$ is a multiple of 196.

Answer (b) $k =$ _____ [1]

- 3 (a) Simplify $\left(\frac{2}{x}\right)^{-2} \times 4x^0$.

Answer (a) _____ [1]

- (b) Given that $\sqrt[3]{64^n} \times \frac{1}{16} = 4$, find n .

Answer (b) $n =$ _____ [2]

4

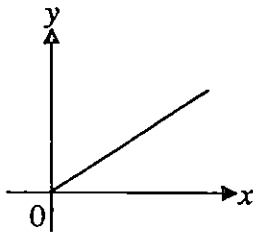


Figure 1

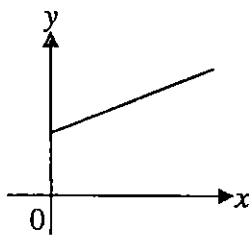


Figure 2

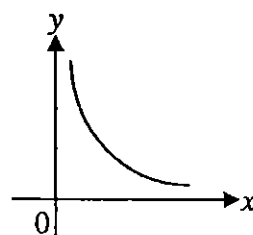


Figure 3

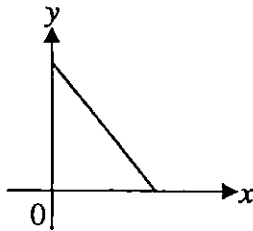


Figure 4

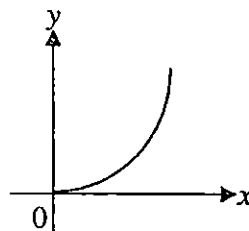


Figure 5

From the graphs above, select one which best illustrates each of the following statements.

- (a) The surface area y , of a sphere is proportional to the square of the radius, x .
- (b) The amount of pressure y , exerted is inversely proportional to the surface area of a cube, of sides x cm.
- (c) The total cost \$ y , of x number of apples, given that the cost of 1 apple is \$0.50.

Answer (a) Figure [1]

(b) Figure [1]

(c) Figure [1]

- 5 The line $2y = x - p$ passes through the point $A(-2, 4)$ and crosses the x -axis at B . Find

(a) the value of p ,

Answer (a) $p =$ [1]

(b) the coordinates of B ,

Answer (b) $B(\quad , \quad)$ [1]

(c) the length of AB .

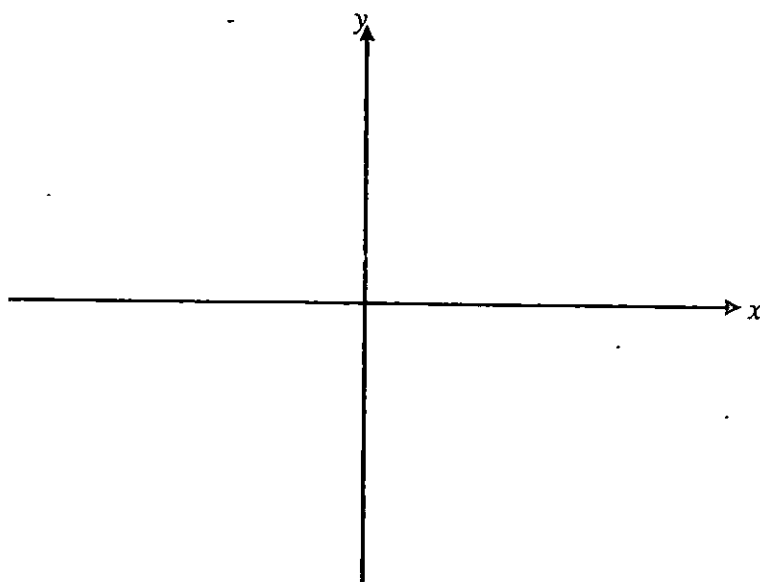
Answer (c) units [1]

- 6 (a) Express $x^2 - 4x + \frac{7}{4}$ in the form $(x - p)^2 + q$.

Answer (a) _____ [1]

- (b) Sketch the graph of $y = x^2 - 4x + \frac{7}{4}$ on the axes below, indicating clearly the x- and y-intercepts and turning point.

Answer



[2]

- 7 A school organised a Multiple-Choice Question (MCQ) quiz for all Secondary Three students. There were a total of 40 questions and the marks allocation were as follow:

Every correct question: 3 marks

Every incorrect question: Minus 2 marks

Questions not attempted: No marks awarded / deducted

A student was awarded the *Distinction* grade if he/she achieved a minimum of 100 marks.

Using **inequalities**, calculate the minimum number of questions Ali attempted correctly if he was awarded the *Distinction* grade.

Answer _____ questions [3]

- 8 (a) Solve the inequalities $2x + 3 \leq -x + 2 < 4$.

Answer (a) _____ [2]

(b) Hence, represent the solution in (a) on the number line below.

Answer



[1]

(c) State the largest rational number, x , which satisfies the inequalities in (a).

Answer (c) $x =$ _____ [1]

- 9 (a) 9 kilograms of sugar cost x cents. Find the cost, in **dollars**, of y kilogram of sugar, in terms of x and y .

Answer (a) \$ _____ [1]

- (b) Solve $\frac{x-4}{7} = \frac{5}{2x+3}$, giving your answers to **1 decimal place**.

Answer (b) $x =$ _____
or
 $x =$ _____ [4]

- 10 Consider the following sequence.

$$3 + 9 = 3 \times 2^2 = 12$$

$$3 + 9 + 15 = 3 \times 3^2 = 27$$

$$3 + 9 + 15 + 21 = 3 \times 4^2 = 48$$

- (a) Using the pattern above, find the sum of $3 + 9 + 15 + 21 + 27 + 33$.

Answer (a) _____ [2]

- (b) A given line of the sequence is as follows:

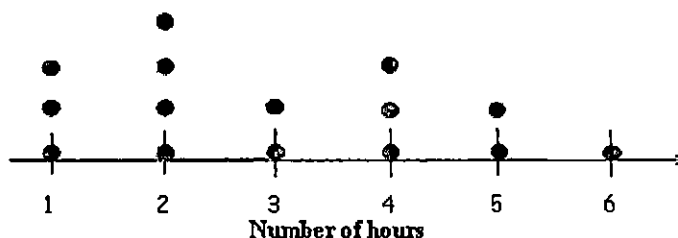
$$3 + 9 + 15 + 21 + 27 + \dots + x = 3 \times y^2 = 507$$

Find the value of x and of y .

Answer (b) $x =$ _____, $y =$ _____ [3]

- 11 The data below represents the number of hours spent on daily revision for 15 students in Class A.

Class A



- (a) Calculate the mean number of hours for Class A.

Answer (a) _____ hours [2]

- (b) Calculate the interquartile range of the number of hours for Class A.

Answer (b) _____ hours [2]

- (c) Out of the 15 students in Class A, 5 are female students. The mean number of hours for the female students is 3.5 hours. Find the mean number of hours for the male students.

Answer (c) _____ hours [2]

- (d) During the Mid-Year Examination period, every student in Class A revised for an additional 2 hours daily. Explain how the **mean** of the number of hours spent on daily revision has been affected by this change.

Answer

[1]

Section B (60 marks)**Answer all questions on writing paper provided.**

12 (a) Simplify $p^2q \div 2pq^2 \times \left(\frac{-3p^4}{q^{-3}}\right)^2$ in positive indices. [3]

(b)(i) The Pacific Ocean covers approximately 165 250 000 km². Write this area in standard form. [1]

(ii) The ten largest oceans and seas in the world cover a total area of about 3.52×10^8 km². Calculate the percentage area of Pacific Ocean of the area of the ten largest oceans and seas. [2]

(iii) The area of the Earth's surface is given as 5.1×10^{10} km². Land covers about 30% of the Earth's surface. Calculate the difference between the land area and the area of the ten largest oceans and seas. [2]

13 (a) It is given that $v^2 = u^2 + 2as$. Express u in terms of v , a and s . [2]

(b)(i) Factorise completely $50x^2 - 2$. [2]

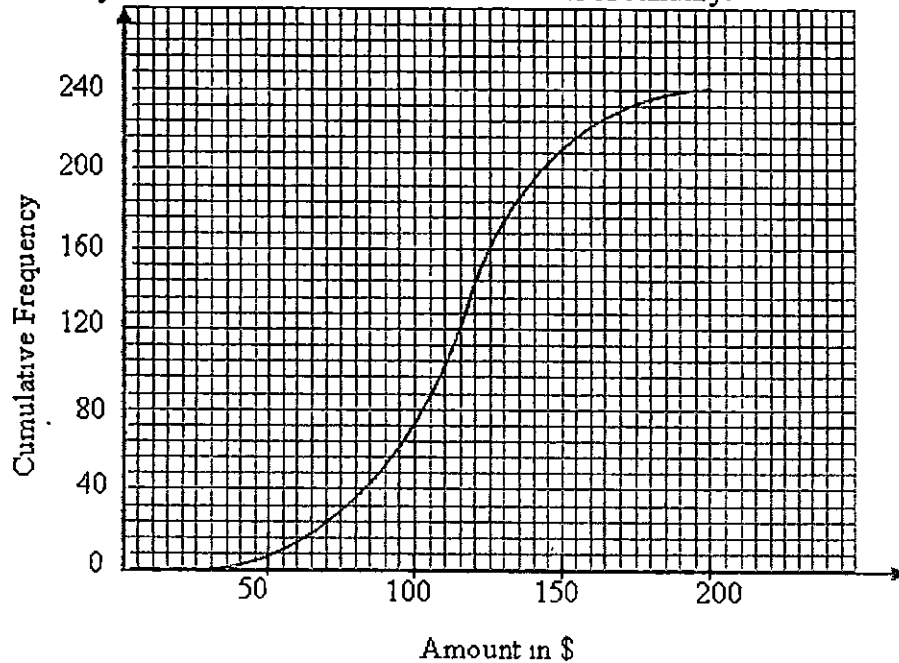
(ii) Simplify $\frac{50x^2 - 2}{10x^2 + 28x - 6}$. [2]

(c) Express as a single fraction $\frac{6}{2-x} - \frac{4}{1+3x}$. [2]

(d) Given that a and b are integers, $-4 \leq a \leq 4$ and $2 \leq b < 8$, state
(i) the smallest possible value of $a^2 - b^2$, [1]

(ii) the largest possible value of $b - a$. [1]

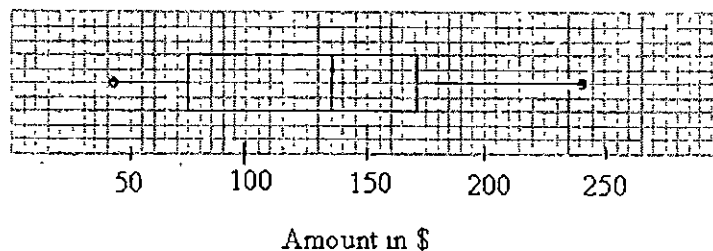
- 14 The graph below shows the cumulative frequency distribution of the amount of utility bills of 240 households for the month of January.



- (a) Use your graph to estimate
- (i) the median amount of utility bill, [1]
 - (ii) the interquartile range, [1]
 - (iii) the 20th percentile. [1]
- (b) Find the value of x if 10% of the families' utility bill is at least \$ x . [2]
- (c) Copy and complete the frequency table below. [2]

Amount (\$ x)	$0 < x \leq 50$	$50 < x \leq 100$	$100 < x \leq 150$	$150 < x \leq 200$
Frequency				

- (d) In February, a 'Use Less Water and Electricity' campaign was conducted for the same 240 households. The box and whisker plot below shows the amount of utility bills for March.



The organisers of the campaign felt that the campaign was an effective and successful one. Do you agree and why?

[2]

- 15 When turned on, Tap A alone can fill one tank in x minutes while Tap B alone can fill the same tank 4 minutes faster than Tap A.

(a) Find the fraction of the tank that can be filled in 1 minute for

(i) Tap A,

[1]

(ii) Tap B.

[1]

(b) When both taps are turned on at the same time, the time taken to fill the entire one tank is 50 minutes.

Write down an equation in x to represent this information, and show that it reduces to

$$x^2 - 104x + 200 = 0.$$

[3]

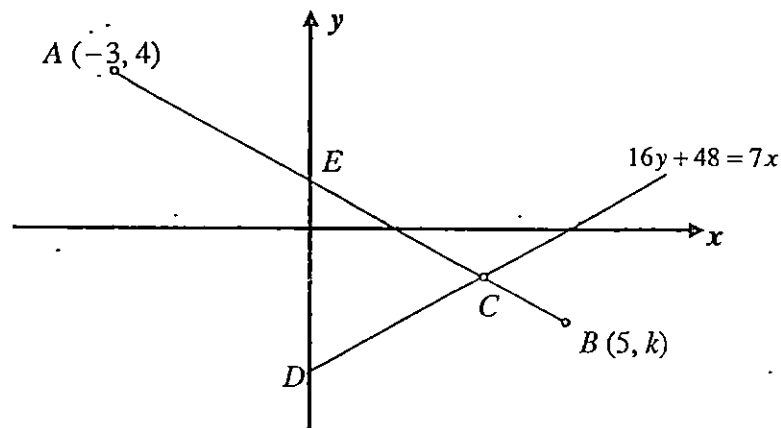
(c) Solve $x^2 - 104x + 200 = 0$, correct to 3 decimal places.

[3]

(d) Explain why one of the solutions in (c) is rejected. Hence, write down the time taken to fill up the entire tank by Tap A alone, to the nearest hours and minutes.

[2]

16



Points A and B have coordinates $(-3, 4)$ and $(5, k)$ respectively. Points E and D are on the y-axis.

(a) Given that the length of line AB is 10 units, show that the value of $k = -2$.

[3]

(b) Find the equation of line AB.

[2]

(c) Another line $16y + 48 = 7x$ intersects line AB at point C. Find the coordinates of point C.

[3]

(d) Calculate the area of triangle CED.

[2]

(e) State the equation of the line parallel to the y-axis and which passes through point C.

[1]

17 Answer the whole of this question on graph paper provided.

The variables x and y are connected by the equation $y = 5 + 6x^2 - x^3$ and some corresponding values of x and y are given in the following table.

x	-2	-1	0	1	2	3	4	5	6	7
y	37	12	5	p	21	32	37	30	5	-44

(a) Find the value of p . [1]

(b) Using a scale of 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 5 units on the vertical axis, draw the graph of $y = 5 + 6x^2 - x^3$ for $-2 \leq x \leq 7$. [3]

(c) Using your graph,

(i) find the value of y when $x = 2.5$, [1]

(ii) solve the equation $5 + 6x^2 - x^3 = 20$. [2]

(d) By drawing a tangent, find the gradient of the curve at $x = 4.5$. [2]

(e)(i) Draw the straight line $y = -3x + 10$ on the same axes. [1]

(ii) Show working to illustrate how the solutions to the

equation $-\frac{5}{x} + 6x - x^2 + 3 = 0$ are the points of intersection between the curve drawn in part (b) and the straight line drawn in part (e)(i). [1]

(iii) Hence, state the solutions of $-\frac{5}{x} + 6x - x^2 + 3 = 0$. [1]

Marking Scheme

Secondary 3 Express MYE 2014 (Section A)

Penalise from total: 1 mark for accuracy of ans + 1 mark for missing units in Section B.

Qns	Solutions	Marks / Remarks
1.	$2p - q - 2p^2 + pq$ $= 2p(1 - p) - q(1 - p)$ $= (2p - q)(1 - p)$	M1 (accept any valid factorisation in pairs) A1
2a.	$360 = 2^3 \times 3^2 \times 5$	B1
2b.	$k = 49$	B1
3a.	x^2	B1
3b.	$4^n \times \frac{1}{16} = 4$ Alternatively, change to base 2: $4^n \times 4^{-2} = 4^1 \quad 2^{2n} \times 2^{-4} = 2^2$ $n - 2 = 1$ $n = 3$	M1 (for reducing indices to base 4 or base 2) A1
4a.	5	B1
4b.	3	B1
4c.	1	B1
5a.	-10	B1
5b.	(-10,0)	B1
5c.	8.94 units	B1 (ans left in surd form not accepted) ECF: mark awarded if length found using incorrect 5b ans
6a.	$(x - 2)^2 - 2\frac{1}{4}$	B1
6b.		D1 (for general shape of graph: curve with 2 distinct roots) D1 (for identifying and indicating clearly intercepts and turning pt) No ecf allowed for wrong answer in 6a as students can sketch curve directly from original equation

Sub-Total: 16 marks

Marking Scheme

7.	Let the number of correct questions answered be x. $3x \geq 100$ $x \geq 33\frac{1}{3} \text{ while the rest of the questions remain non-attempted}$ Minimum no. of correct questions: 34 (the idea is to score as many correct questions and leave the rest blank)	M1 (for forming correct expression with correct inequality sign) M1 A1 (if students obtained correct final ans without using inequalities/use other methods, to award 1 mark only)
8a.	$2x+3 \leq -x+2 \text{ and } -x+2 < 4$ $x \leq -\frac{1}{3} \text{ and } x > -2$ $-2 < x \leq -\frac{1}{3}$	M1 (for obtaining two solutions from each side) A1
8b.		D1 (hollow and shaded circles respectively)
8c.	$\frac{1}{3}$	B1
9a.	$\$ \frac{xy}{900} \text{ or } \frac{1}{900}xy$	B1 (Not accepting: $\frac{0.01xy}{9}$ or algebraic expressions not simplified)
9b.	$\frac{x-4}{7} = \frac{5}{2x+3}$ $(x-4)(2x+3) = 35$ $2x^2 - 5x - 47 = 0$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-47)}}{4}$ $x = 6.3 \text{ or } -3.8$	M1 (for cross-multiplication) M1 (for obtaining this quadratic equation) M1 (for using quadratic formula) A1 (ans must be in 1dp, no ans mark if solving technique is not shown/if calculator is used to derive ans)
10a.	3×6^2 $= 108$	M1 A1 (no marks at all if students do not show working on the use of patterns)
10b.	$y^2 = 507 \div 3$ $y^2 = 169$ $y = 13$ $x = 75$	M1 A1 A1

Sub-Total: 17 marks

Marking Scheme

11a.	Mean = $\frac{1(3) + 2(4) + 3(2) + 4(3) + 5(2) + 6}{15}$ = 3 hrs	M1 A1
11b.	IQR = 4 – 2 = 2 hours	M1 A1
11c.	$3.5 \times 5 = 17.5$ $\frac{45 - 17.5}{10} = 2.75$ hours	M1 A1
11d.	The mean no. of hours for Class A will increase by 2 hours.	B1 <i>(no mark if students showed working but no explanation made)</i>

Sub-Total: 7 marks

End of Section A

Total Marks: 40 marks

Marking Scheme

Secondary 3 Express MYE 2014 (Section B)

Qns	Solutions	Marks / Remarks
12a.	$p^2q \div 2pq^2 \times \left(\frac{-3p^4}{q^{-3}}\right)^2$ $= p^2q \div 2pq^2 \times \frac{9p^8}{q^{-6}}$ $= \frac{1}{2}pq^{-1} \times \frac{9p^8}{q^{-6}}$ $= \frac{9}{2}p^9q^5$	M1 (for expanding bracket) M1(for combining 1st 2 terms) A1(ans must be in positive indices)
12bi.	$1.6525 \times 10^8 \text{ km}^2$	B1 (accept if students accidentally round off to 3sf)
12bii.	$\frac{1.6525 \times 10^8}{3.52 \times 10^8} \times 100\%$ $= 46.9\% (3sf)$	M1 A1
12biii.	Land Area = $30\% \times 5.1 \times 10^{10}$ $= 1.53 \times 10^{10}$ Difference = $1.53 \times 10^{10} - 3.52 \times 10^8$ $= 1.49 \times 10^{10} \text{ km}^2 (3sf)$	M1(for finding amt of land area) A1(accept original 5 sfans)
13a.	$u^2 = v^2 - 2as$ $u = \pm \sqrt{v^2 - 2as}$	M1 A1
13bi.	$2(25x^2 - 1)$ $= 2(5x - 1)(5x + 1)$	M1 A1
13bii.	$\frac{50x^2 - 2}{10x^2 + 28x - 6}$ $= \frac{2(5x - 1)(5x + 1)}{2(5x - 1)(x + 3)}$ $= \frac{5x + 1}{x + 3}$	M1(for full factorisation of denominator) A1
13c.	$\frac{6}{2-x} - \frac{4}{1+3x}$ $= \frac{6(1+3x) - 4(2-x)}{(2-x)(1+3x)}$ $= \frac{22x - 2}{(2-x)(1+3x)}$	M1(for combining into single fraction) A1
13di.	-49	B1
13dii.	11	B1

Sub-Total: 18 marks

Marking Scheme

14ai.	\$115	B1(ans can be read easily hence no range of acceptable ans)
14aii.	$IQR = 132.5 - 95 = \$37.50 \pm 2.5$	B1(accept 3s.f ans)
14aiii.	$\$87.50 \pm 2.5$	B1(accept 3s.f ans)
14b.	$90\% \times 240 = 216$ $\$155 \pm 2.5$	M1 A1 or B2
14c.	$0 < x \leq 50$ Frequency: 8 $50 < x \leq 100$ Frequency: 64 $100 < x \leq 150$ Frequency: 136 $150 < x \leq 200$ Frequency: 32	B1: for at least 2 correct values OR B2: for all correct integral values and must add up to 240. *if values differ due to readings ± 4 , total should still add up to 240.
14d.	I disagree because the median amount of utility bills has increased to \$135 for the month of March which means the households on average used more water and electricity.	B1(for disagreeing) B1(supporting reasons using median and provided interpretation)

15ai.	$\frac{1}{x}$	B1
15aii.	$\frac{1}{x-4}$	B1
15b.	$\frac{1}{x-4} + \frac{1}{x-50} = 1$ or $50 \times \left(\frac{1}{x} + \frac{1}{x-4} \right) = 1$ $\frac{x-4+x-1}{x(x-4)} = 50$ $x^2 - 104x + 200 = 0$	M1 ECF: method mark awarded if equation is formed using incorrect expressions found in 15a. M1(for combining into single denominator) A1(shown)
15c.	$x^2 - 104x + 200 = 0$ $x = \frac{104 \pm \sqrt{(-104)^2 - 4(1)(200)}}{2}$ $x = 102.03998$ or $x = 1.960016$ $x = 102.040$ or $x = 1.960$ (3 dp)	M1 A1 A1(only 1 mark awarded if any correct ans not rounded off to 3dp / if students reject 1 ans) Award 2 marks out of 3 if students derive answers from the calculator
15d.	Reject $x = 1.960$ because the timing for Tap B will be a negative value / timing for individual taps cannot be faster than timing when both taps combined Time taken by Tap A = 1 hour 42 minutes	B1 B1

Sub-Total: 19 marks

Marking Scheme

16a.	$\sqrt{(-3-5)^2 + (4-k)^2} = 10$ $64 + (4-k)^2 = 100$ $4 - k = \pm 6$ $k = -2$ or $k = 10(rej)$	M1 M1 A1(shown)
16b.	Gradient = $\frac{4+2}{-3-5}$ $= -\frac{3}{4}$ $4 = -\frac{3}{4}(-3) + c$ $c = 1\frac{3}{4}$ Equation: $y = -\frac{3}{4}x + 1\frac{3}{4}$ or $4y = -3x + 7$	M1 A1(accept equation in integral coefficients)
16c.	$16\left(-\frac{3}{4}x + 1\frac{3}{4}\right) + 48 = 7x$ $x = 4, y = -1\frac{1}{4}$ $C\left(4, -1\frac{1}{4}\right)$	M1(solve simultaneous) <u>ECF: method mark awarded if simultaneous working performed using incorrect equation in 16b.</u> M1(for solving x and y values) A1(no A1 for missing brackets)
16d.	$D(0, -3)$ Area = $\frac{1}{2} \times 4 \times \frac{3}{4} \times 4$ $= 9.5 \text{ units}^2$	M1 <u>ECF: method mark awarded if working for area of triangle done using incorrect coordinates of C in 16c.</u> A1
16e.	$x = 4$	B1

Sub-Total: 11 marks

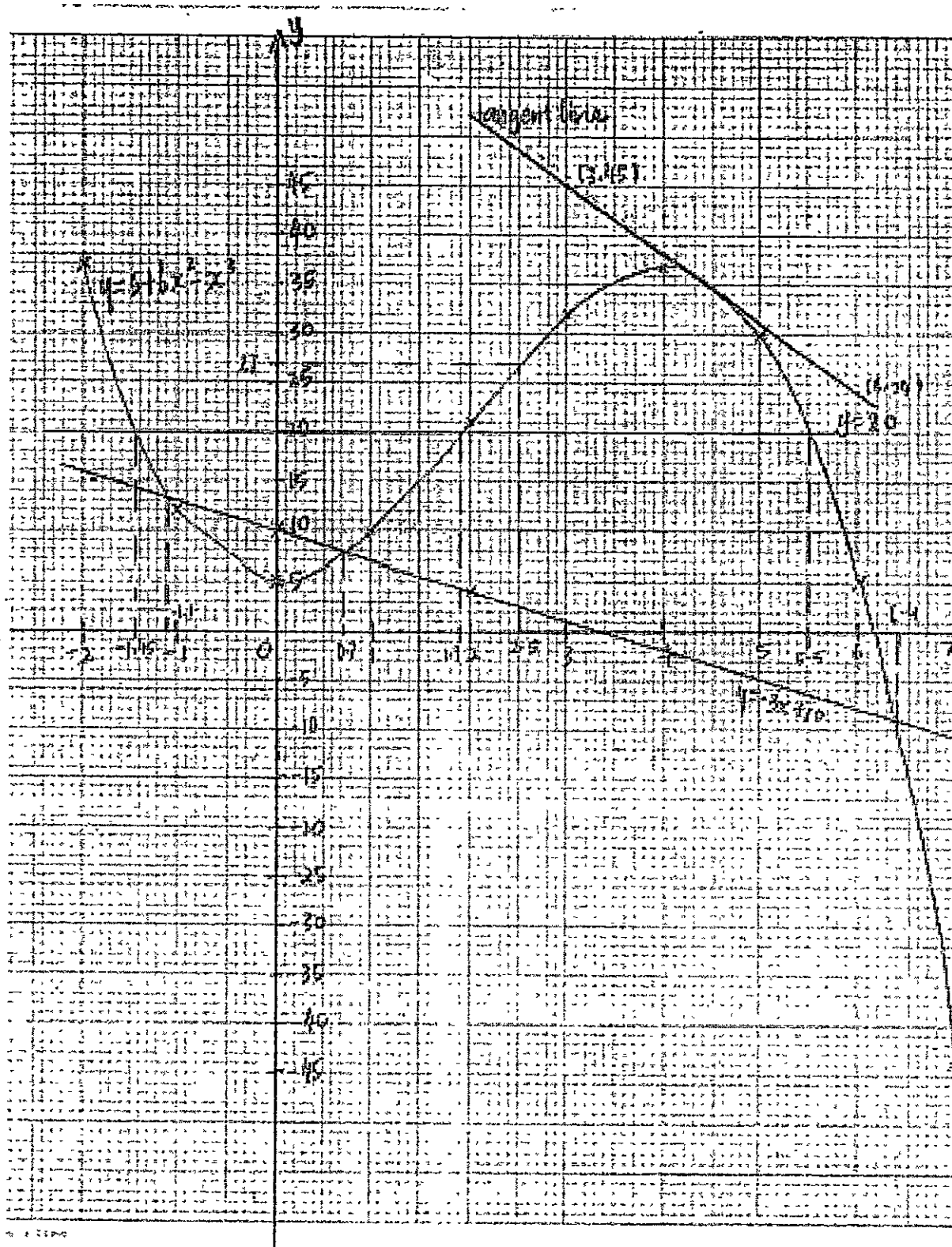
Marking Scheme

17a.	$p = 10$	B1
17b.	See graph below (for actual illustration on graph paper refer to the last page)	D1: correct scale and labelling of axes D1: correct location of points <i>E.C.F.</i> (for point plotted incorrectly using p found in (a)) D1: Smooth Curve
17ci.	$2f+1$	B1
17cii.	Draw line / dotted line $y = 20$ $x = -1.4, 1.9, 5.5 \pm 0.1$	D1 B1 (all three values)
17d.	Draw appropriate tangent line and using the gradient formula using 2 points Gradient = $-6/5 \pm 2.5$	M1 A1
17ei.		D1 (for drawing of straight line)
17eii.	$\frac{5}{x^2} + 6x = x^2 + 3 = 0$ $5 + 6x^2 = x^3 + 3x = 0$ $6x^2 - x^3 = -3x + 5$ $5 + 6x^2 - x^3 = -3x + 10$	B1 (multiply x throughout and adding of constant to derive final form where equation of curve = equation of straight line)
17eiii.	$x = -1.1, 0.7, 6.4 \pm 0.1$	B1 (for all three values)

Sub-Total: 12 marks

Marking Scheme

Scanned image of Q17 graph (all parts included)



End of Section B

Total Marks: 60 marks