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聖嬰中學
HOLY INNOCENTS' HIGH SCHOOL

MID-YEAR EXAMINATION 2014
SECONDARY 3 EXPRESS

MATHEMATICS

4016 / 01

Paper 1

Name : _____

Date : 14 May 2014

Register No : _____

Duration : 1 hr 30 mins

Class : _____

Marks : / 60

Additional Materials needed: Nil

INSTRUCTIONS TO CANDIDATES

Write your register number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions in the answer space provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 60.

Setter: Mr Adrian Goh

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Geometry and Measurement

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

1. Solve $3 - x \leq 2x + 1 \leq 10 + x$.

Answer [2]

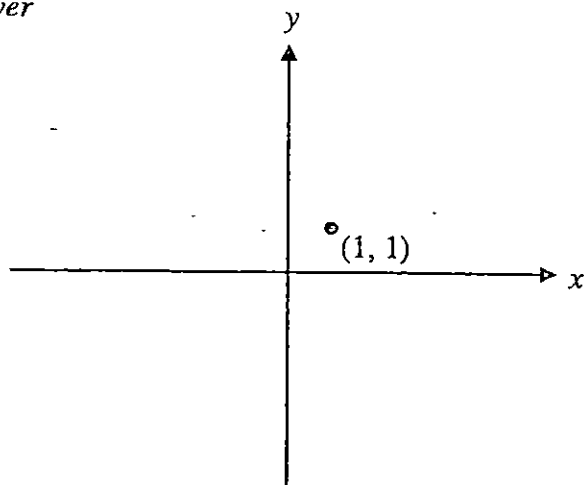
2. Express the sum of 213 picometres and 8.3 nanometres in metres, giving your answer in standard form.

Answer m [2]

3. The point (1, 1) is marked on the diagram below.

On the diagram, sketch the graph of $y = \frac{2}{x}$.

Answer



[2]

4. y is inversely proportional to x^2 . It is known that $y = 10$ for a particular value of x . Find the value of y when this value of x is doubled.

Answer [2]

5. Express as a single fraction in its simplest form

$$\frac{12}{a^2 - 4} + \frac{3}{a + 2}$$

Answer [2]

6. Calculate $\frac{3^5 \times 0.18 + \sqrt[3]{513}}{9 \times 0.2^2}$.

Write your answer correct to 4 significant figures.

Answer [2]

7. A room measuring 336 cm by 392 cm is to be completely covered by square tiles. Find the minimum number of square tiles needed to cover the room.

Answer [3]

8. Adam, Bernard and Charlie are running around the school field. They take 120 seconds, 105 seconds and 100 seconds respectively to complete one round. If they started running from the same starting point at 8 am, at what time would all three of them next meet at the same starting point?

Answer [3]

9. Town A and Town B are 300 km apart. Dominic leaves Town A at 9 am. In the first part of the journey, he travels at a speed of 70 km/h for 3 hours. In the second part of the journey, he travels at a speed of x km/h. Given that he arrived at Town B at 12.45 pm, find the value of x .

Answer $x =$ [3]

10. $\mathcal{E} = \{x : x \text{ is an integer such that } 2 \leq x \leq 20\}$

$A = \{x : x \text{ is a multiple of } 3\}$

$B = \{x : x \text{ is an integer such that } 6 < x \leq 17\}$

(a) List the elements contained in the set $A' \cap B$.

(b) If a number is selected at random, find the probability that it belongs to set A.

Answer (a) [2]

(b) [1]

11. Factorise completely

(a) $x^8 - y^4$,

(b) $ab - 2bc - ac + 2b^2$.

Answer (a) [2]

(b) [2]

12. (a) Solve $\frac{1}{3}p + 2 = 5p - 14$.
- (b) Given $10x - 9 = 9y$, find the value of $\frac{x}{y+1}$.

Answer (a) $p = \dots\dots\dots$ [2]

(b) $\frac{x}{y+1} = \dots\dots\dots$ [2]

13. A bag has 60 marbles. There are m yellow marbles, $(m + 5)$ red marbles and the rest are blue marbles. A marble is drawn at random from the bag. The probability of drawing a blue marble is $\frac{1}{4}$.
- (a) Find the probability that a green marble is drawn.
- (b) Find the number of red marbles in the bag.
- (c) Find the number of blue marbles that must be added to the bag so that the probability of drawing a blue marble becomes 0.4:

Answer (a) $\dots\dots\dots$ [1]

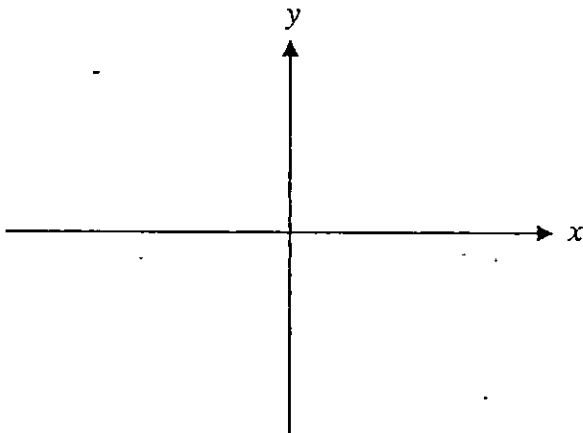
(b) $\dots\dots\dots$ [2]

(c) $\dots\dots\dots$ [2]

14. (a) Express $y = x^2 + 6x - 15$ in the form $y = (x + p)^2 + q$.
(b) Hence sketch the graph of $y = x^2 + 6x - 15$ on the axes provided.

Answer (a) $y = \dots\dots\dots$ [2]

Answer (b)



[2]

15. The table below shows the distribution of the number of hours spent by students on gaming.

Number of hours spent	0	1	2	3	4	5
Number of students	6	8	12	15	k	9

- (a) Calculate the value of k given that the mean is 2.7.
 (b) Write down the largest possible value of k given that the mode is 3.
 (c) Write down the smallest possible value of k given that the median is 3.

Answer (a) [2]

(b) [1]

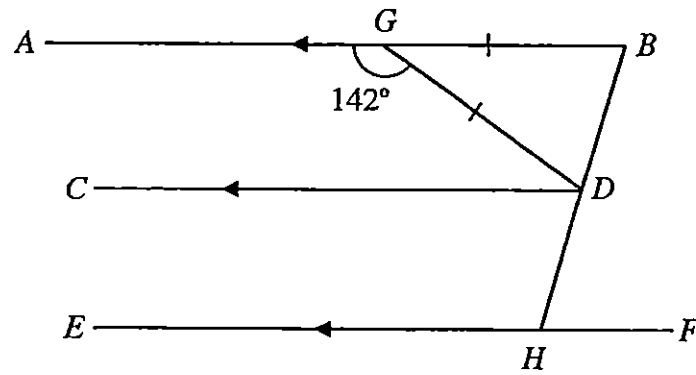
(c) [1]

16. (a) Factorise $4x^2 - 5x - 9$.
 (b) Hence solve $4y^4 - 5y^2 - 9 = 0$.

Answer (a) [1]

(b) $y =$ [3]

17. In the diagram, AB , CD and EF are parallel lines.
 $\angle AGD = 142^\circ$ and $GB = GD$.

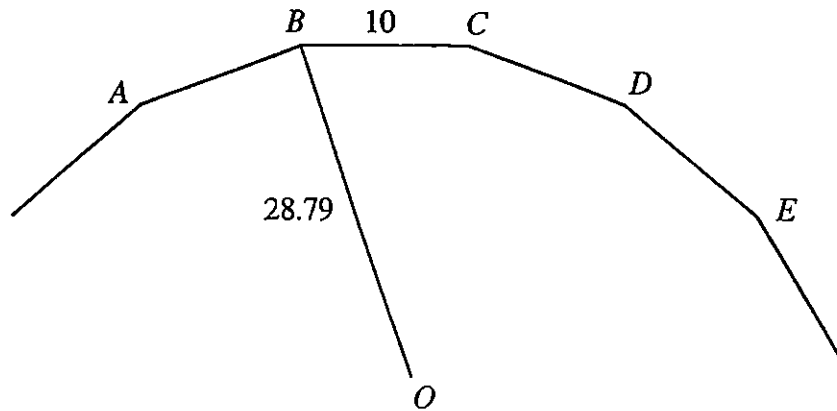


- (a) Find
 (i) $\angle GDH$,
 (ii) $\angle DHE$.
 (b) Given that $GH = BH$, find $\angle DGH$.

Answer

(a) (i) $\angle GDH = \dots\dots\dots^\circ$ [2]
 (ii) $\angle DHE = \dots\dots\dots^\circ$ [1]
 (b) $\angle DGH = \dots\dots\dots^\circ$ [2]

18. The diagram shows part of a regular polygon with n sides and centre O . Each interior angle of this polygon is 160° .



- Find the value of n .
- Find $\angle DCE$.
- Given that each side is 10 cm and BO is 28.79 cm, find the area of the whole polygon.

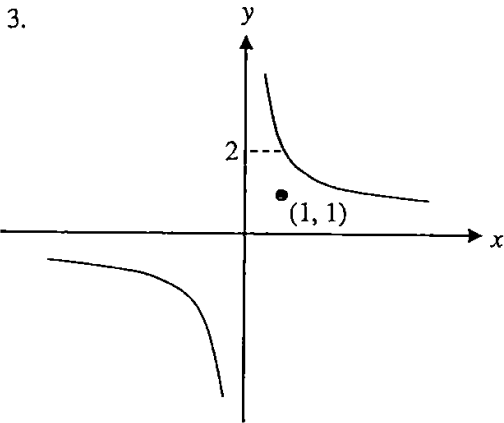
Answer (a) $n = \dots\dots\dots$ [1]
 (b) $\angle DCE = \dots\dots\dots^\circ$ [1]
 (c) $\dots\dots\dots \text{cm}^2$ [4]

End of Paper

**Answers for 3 Exp Math Paper 1
2014 MYE**

1. $\frac{2}{3} \leq x \leq 9$

2. 8.513×10^{-9}



4. $y = 2.5$

5. $\frac{3}{(a-2)}$

6. 143.7

7. 42

8. 9.10 am

9. 120 km/h

10. (a) 7, 8, 10, 11, 13, 14, 16, 17

(b) $\frac{6}{19}$

11. (a) $(x^2 - y)(x^2 + y)(x^4 + y^2)$

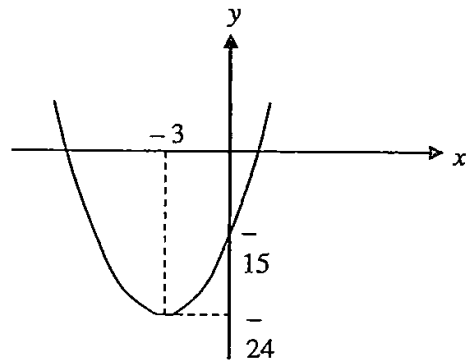
(b) $(a + 2b)(b - c)$

12. (a) $p = 3\frac{3}{7}$

(b) $\frac{9}{10}$

13. (a) 0
(b) 25
(c) 15

14. (a) $(x + 3)^2 - 24$
(b)



15. (a) 10
(b) 14
(c) 3

16. (a) $(4x - 9)(x + 1)$
(b) $\pm 1\frac{1}{2}$

17. (a) 109°
(b) 109°
(c) 33°

18. (a) 18
(b) 10°
(c) 2550.



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MATHEMATICS

4016 / 01

Paper 1

Name : _____

Date : 14 May 2014

Register No : _____

Duration : 1½ hours

Class : _____

Marks : / 60

Additional Materials needed:

INSTRUCTIONS TO CANDIDATES

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You may use a pencil for any diagrams or graphs.
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Answer all questions in the answer space provided.

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Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.
The total marks for this paper is 80.

Setter: Mr Adrian Goh

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Geometry and Measurement

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1. Solve $3 - x \leq 2x + 1 \leq 10 + x$.

$$\begin{array}{lll} 3 - x \leq 2x + 1 & \text{and} & 2x + 1 \leq 10 + x \\ \frac{2}{3} \leq x & \text{and} & x \leq 9 \quad \text{[M1]} \end{array}$$

$$\frac{2}{3} \leq x \leq 9 \quad \text{[A1]}$$

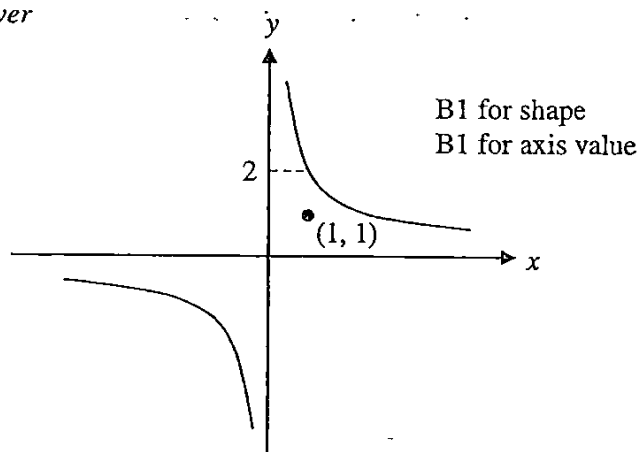
2. Express the sum of 213 picometres and 8.3 nanometres, in metres, giving your answer in standard form.

$$\begin{array}{ll} 213 \times 10^{-12} + 8.3 \times 10^{-9} & \text{[M1 for converting BOTH pico and nano correctly]} \\ = 8.513 \times 10^{-9} & \text{[A1]} \end{array}$$

3. The point (1, 1) is marked on the diagram below.

On the diagram, sketch the graph of $y = \frac{2}{x}$.

Answer



[2]

4. y is inversely proportional to x^2 . It is known that $y = 10$ for a particular value of x . Find the value of y when this value of x is doubled.

$$y = \frac{k}{x^2}$$

$$\text{Sub } y = 10$$

$$10 = \frac{k}{x^2} \quad [\text{M1}]$$

When x is doubled,

$$y = \frac{k}{(2x)^2}$$

$$y = \frac{k}{4x^2}$$

$$y = \left(\frac{1}{4}\right)10$$

$$y = 2.5 \quad [\text{A1}]$$

5. Express as a single fraction in its simplest form.

$$\frac{12}{a^2 - 4} + \frac{3}{a + 2}$$

$$\begin{aligned} & \frac{12}{(a+2)(a-2)} + \frac{3(a-2)}{(a+2)(a-2)} \\ &= \frac{12 + 3a - 6}{(a+2)(a-2)} \end{aligned}$$

[M1 for combining as a single fraction, even if they

$$\text{give } \frac{12(a+2) + 3(a^2 - 4)}{(a^2 - 4)(a+2)}]$$

$$= \frac{3}{(a-2)}$$

[A1]

6. Calculate $\frac{3^5 \times 0.18 + \sqrt[3]{513}}{9 \times 0.2^2}$.

Write your answer correct to 4 significant figures.

143.7 [B2, B1 if correct answer but not rounded off correctly]

7. A room measuring 336 cm by 392 cm is to be completely covered by square tiles. Find the minimum number of square tiles needed to cover the room.

HCF of 336 and 392 = 56 [M1, must show working]

Total number of tiles

$$= \frac{336}{56} \times \frac{392}{56} \quad [M1]$$

$$= 42 \quad [A1]$$

8. Adam, Bernard and Charlie are running around the school field. They take 120 seconds, 105 seconds and 100 seconds respectively to complete one round. If they started running from the same position at 8 am, at what time would they meet at the same position again.

LCM of 100, 105 and 120 = 4200 [M1]

4200 s = 70 minutes [M1]

They will meet again at 9.10 am [A1]

9. Town A and Town B are 300 km apart. Dominic leaves Town A at 9 am. In the first part of the journey, he travels at a speed of 70 km/h for 3 hours. In the second part of the journey, he travels at a speed of x km/h. Given that he arrived at Town B at 12.45 pm, find the value of x .

Distance remaining after travelling for 3 hours

$$= 300 - 210 \quad [M1]$$

$$= 90 \text{ km}$$

Time remaining

$$= \frac{3}{4} \text{ hrs}$$

Required speed

$$= 90 \div \frac{3}{4} \quad [M1]$$

$$= 120 \text{ km/h} \quad [A1]$$

10. $E = \{x : x \text{ is an integer such that } 2 \leq x \leq 20\}$

$$A = \{x : x \text{ is a multiple of } 3\}$$

$$B = \{x : x \text{ is an integer such that } 6 < x \leq 17\}$$

(a) List the elements contained in the set $A' \cap B$.

(b) If a number is selected at random, find the probability that it belongs to set A.

(a) M1 for listing elements in set A and B

A1 for answer 7, 8, 10, 11, 13, 14, 16, 17

(b) $\frac{6}{19}$ [B1]

11. Factorise completely

(a) $x^8 - y^4$

(b) $ab - 2bc - ac + 2b^2$

(a) $(x^4 - y^2)(x^4 + y^2)$ [M1]
 $= (x^2 - y)(x^2 + y)(x^2 + y^2)$ [A1]

(b) $ab - ac + 2b^2 - 2bc$
 $= a(b - c) + 2b(b - c)$ [M1 for factorizing by grouping]
 $= (a + 2b)(b - c)$ [A1]

12. (a) Solve $\frac{1}{3}p + 2 = 5p - 14$.

(b) Given $10x - 9 = 9y$, find the value of $\frac{x}{y+1}$.

(a) $\frac{14p}{3} = 16$ [M1 for grouping p on one side]

$p = 3\frac{3}{7}$ [A1]

(b) $10x = 9(y + 1)$ [M1]

$\frac{x}{y+1} = \frac{9}{10}$ [A1]

OR

$x = \frac{9y+9}{10}$ (Student could also make y the subject)

Sub into $\frac{x}{y+1}$ [M1, for "subbing" x or y depending on which is the subject]

$= \frac{9y+9}{10} \times \frac{1}{y+1}$

$= \frac{9}{10}$ [A1]

13. A bag has 60 marbles. There are m yellow marbles, $(m + 5)$ red marbles and the rest are blue marbles. A marble is drawn at random from the bag. The probability of drawing a blue marble is $\frac{1}{4}$.

- (a) Find the probability that a green marble is drawn.
 (b) Find the number of red marbles in the bag.
 (c) Find the number of blue marbles that must be added to the bag so that the probability of drawing a blue marble becomes 0.4.

(a) 0 [B1]

(b) Number of blue marbles = $\frac{1}{4} \times 60$
 $= 15$

No. of red and yellow marbles = $2m + 5 = 45$ [M1]

$m = 20$

Number of red marbles = 25 [A1]

OR

$\frac{m + 5 + m}{60} = \frac{3}{4}$ [M1]

$m = 20$

Number of red marbles = 25 [A1]

- (c) Let the number of blue marbles to be added be b .

$\frac{15 + b}{60 + b} = 0.4$ [M1]

$15 + b = 24 + 0.4b$

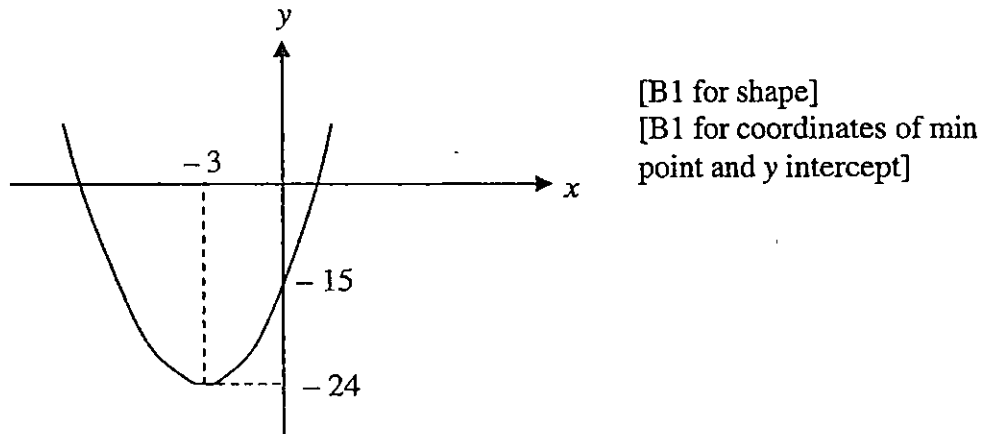
$0.6b = 9$

$b = 15$ [A1]

14. (a) Express $y = x^2 + 6x - 15$ in the form $y = (x + p)^2 + q$.
 (b) Hence sketch the graph of $y = x^2 + 6x - 15$ on the axes provided.

(a) $y = x^2 + 6x + 3^2 - 15 - 3^2$
 $y = (x + 3)^2 - 24$ [B2 for value of p and q]
 or $y = (x + 3)^2 + (-24)$

Answer (b)



[2]

15. The table below shows the distribution of the number of hours spent by students on gaming.

Hours	0	1	2	3	4	5
Number of students	6	8	12	15	k	9

- (a) Calculate the value of k given that the mean is 2.7.
 (b) Write down the largest possible value of k given that the mode is 3.
 (c) Write down the smallest possible value of k given that the median is 3.

(a) $\frac{8 + 24 + 45 + 4k + 45}{6 + 8 + 12 + 15 + k + 9} = 2.7$ [M1]
 $122 + 4k = (50 + k)2.7$
 $1.3k = 13$
 $k = 10$ [A1]

- (b) 14 [B1]
 (c) 3 [B1]

16. (a) Factorise $4x^2 - 5x - 9$.
 (b) Hence solve $4y^4 - 5y^2 - 9 = 0$

(a) $(4x - 9)(x + 1)$ [B1]

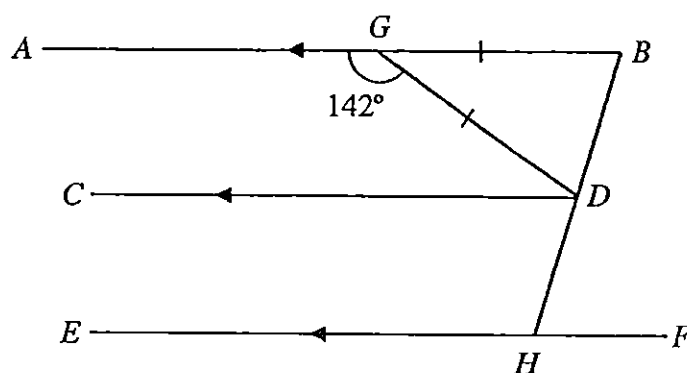
(b) $(4y^2 - 9)(y^2 + 1) = 0$ [M1]

$y^2 = \frac{9}{4}$ or $(2y - 3)(2y + 3) = 0$ [M1]

$y = \pm 1\frac{1}{2}$ [A1]

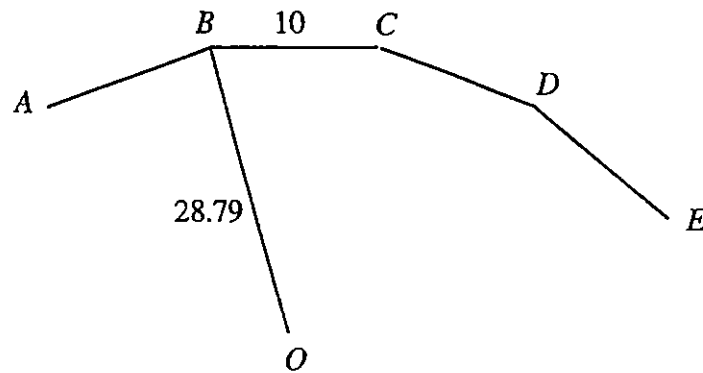
(If students attempt to solve $y^2 = -1$, and write the answer on the answer line, A1 should not be awarded)

17. In the diagram, AB , CD and EF are parallel lines.
 $\angle AGD = 142^\circ$ and $GB = GD$.



- (a) Find
 (i) $\angle GDH$,
 (ii) $\angle DHE$.
 (b) Given that $GH = BH$, find $\angle DGH$.
- (a) $\angle GDB = 142^\circ / 2$ (Isos. Triangle) [M1]
 $= 71^\circ$
 $\angle GDH = 180^\circ - 71^\circ$
 $= 109^\circ$ [A1]
- (b) $\angle DHE = 180^\circ - \angle GBH$
 $= 180^\circ - 71^\circ$
 $= 109^\circ$ [B1]
- (c) $\angle HGB = \angle HBG$ (Isos triangle)
 $\angle HGB = 71^\circ$ [M1]
 $\angle BGD = 180^\circ - 142^\circ$ (Angle on a st line)
 $= 38^\circ$
 $\angle DGH = 71^\circ - 38^\circ$
 $= 33^\circ$ [A1]

18. The diagram shows part of a regular polygon with n sides and centre O . Each interior angle of this polygon is 160° .



- (a) Find the value of n .
 (b) Find $\angle DCE$.
 (c) Given that each side is 10 cm and BO is 28.79 cm, find the area of the whole polygon.

- (a) Exterior angle $= 20^\circ$
 Number of sides $= 360 / 20$
 $= 18$ [B1]

OR

$$(n-2) \times 180^\circ = 160^\circ n$$

$$n = 18$$
 [B1]

- (b) $\angle DCE = 20^\circ / 2$ (Isos triangle)
 $= 10^\circ$ [B1]

- (c) Let the height of the triangle BOC be h .
 $h^2 + 5^2 = 28.79^2$ [M1 for applying Pythagoras]
 $h^2 = 803.8641$
 $h = 28.352497$ [M1]
 Area of 1 triangle $= \frac{1}{2} \times 28.352497 \times 10$
 $= 141.762486$ [M1]
 Area of polygon $= 141.762486 \times 18$
 $= 2551.72$
 $= 2550$ (3 s.f) [A1]

End of Paper

Answer all the questions

1. The employees of a company are offered two schemes for an increase in salary.

Scheme A: 7% increase of the present weekly salary

Scheme B: 5% increase of the present weekly salary and an extra \$8 per week

- (a) Adi earns \$350 per week.

Calculate his new weekly salary under **Scheme A**.

[1]

- (b) Benny finds that his new weekly salary would be the same under either scheme.

Calculate his present weekly salary.

[2]

If there are n employees in the company and all of them select **Scheme B**, the total weekly salary of the company increases from \$100 000 to \$107 240.

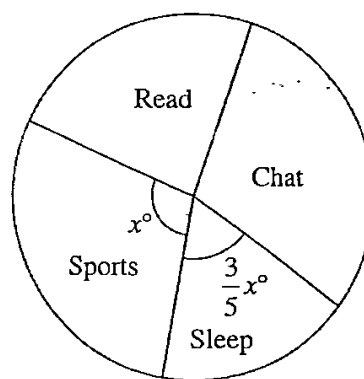
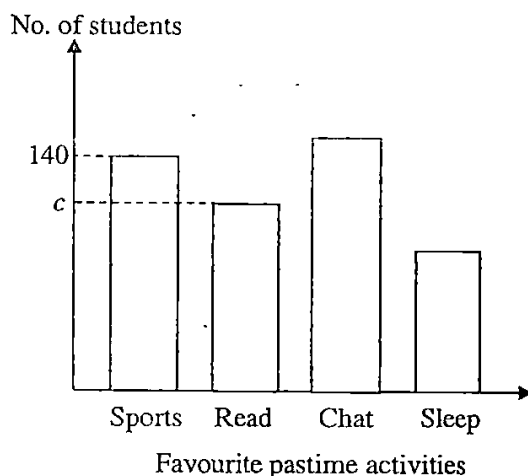
- (c) (i) Write an expression, in terms of n , for the present weekly salary of each employee, assuming that all the employees receive the same weekly salary.

[1]

- (ii) Calculate the number of employees, n , in the company.

[2]

2. A survey was conducted with some students to find out their favourite pastime activities. The result of the survey was presented using a bar chart and a pie chart as shown below.



- (a) If the total sum of angle of the sectors for Sports and Sleep is 168° , calculate the angle of the sector for Sleep. [2]
- (b) Calculate the total number of students involved in the survey. [2]
- (c) If the angles of the sectors for Read and Chat are in the ratio of 7 : 9, find the value of c in the bar chart. [3]

3. (a) (i) Solve the simultaneous equations [3]

$$4x = y + 9,$$

$$2x - 3y = -23.$$

- (ii) Explain why the following pair of simultaneous equations

$$x - 2y = -5,$$

$$4x - 8y = 12$$

has no solution. [2]

- (b) (i) Without the use of calculator, evaluate $9^{\frac{1}{2}} + 9^{-1} - 9^0$. [2]

- (ii) Solve $\sqrt{125^{x-8}} = \frac{1}{5^{x+2}}$. [3]

4. (a) Express $\frac{4d+2}{d^2+d-6} - \frac{2}{d-2}$ as a single fraction in its simplest form. [3]

- (b) (i) Make E the subject in the formula $f = \sqrt[3]{\frac{4E}{2E-1}}$ [3]

- (ii) Hence find the value of E when $f = -1$. [1]

- (c) (i) Simplify $2gh - (g+h)^2$. [2]

- (ii) Hence evaluate $180 - 33^2$. [2]

5. Figure I shows a container made by joining a cylindrical tube of height h cm to a hemisphere of radius r cm.

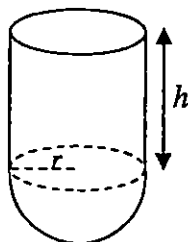


Figure I

- (a) Given that the volume of the hemisphere is $\frac{1}{4}$ of the volume of the container and has a volume of $144\pi \text{ cm}^3$,
- show that $r = 6$ cm, [2]
 - calculate the value of h . [2]
- (b) Water is poured into the container at a rate of $8\pi \text{ cm}^3$ per minute. It takes 54 minutes for the water level to reach a height k cm above the base of the container, as shown in Figure II.

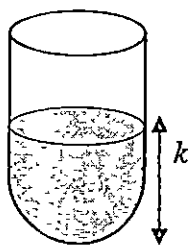


Figure II

Find

- the volume of water in the container, leaving your answer in terms of π , [1]
 - the value of k . [2]
- (c) Water in the container is then poured into a right conical container placed upright as shown in Figure III. The water level reaches a height of 16 cm in the conical container.

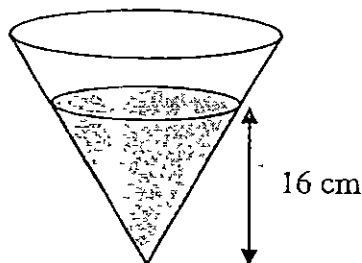


Figure III

Calculate

- the radius of the water surface, [2]
- the area of the cone which is in contact with water. [3]

6. Answer the whole of this question on a fresh piece of graph paper.

A stone is thrown upwards from the top of a vertical cliff.

The height of the stone above the top of the cliff, y metres, during its motion is represented by the equation $y = -10x^2 + 42x$, where x is the time, in seconds, after the stone has been thrown.

The table shows the corresponding values of y and x during the motion.

x (s)	0	1	1.5	2	2.5	3	4	4.5
y (m)	0	32	r	44	42.5	36	s	-13.5

- (a) Explain the significance of the value $y = -13.5$ and $x = 4.5$. [1]
- (b) Calculate the value of r and of s . [1]
- (c) Using a scale of 2 cm to represent 1 second on the horizontal axis and 2 cm to represent 10 metres on the vertical axis, draw the graph of $y = -10x^2 + 42x$ for $0 \leq x \leq 4.5$. [3]
- (d) From your graph, find
- the greatest height from the top of the cliff reached by the stone and the corresponding time, [2]
 - the time at which the stone is at the same height as the cliff for the second time. [1]
- (e) (i) By drawing a tangent, find the gradient of your graph at $x = 2.5$. [2]
- (ii) Explain the significance of this gradient. [1]
-

7. A fruit seller bought several durians from the wholesaler for \$100.
The cost price was \$ w per kilogram.

(a) Write down an expression, in terms of w , for the mass of the durians he bought from the wholesaler. [1]

Back at his store, the fruit seller found out that 1 kilogram of the durians were inedible so he threw them away.

He then divided the rest into two types, Y and Z , such that there was 10 kilogram of type Z durians.

He sold all type Z durians at \$5 per kilogram more than the cost price and all type Y durians at \$10 per kilogram more than the cost price.

(b) Write down an expression in its simplest form, in terms of w , for the sales amount he received in dollars from selling all the

(i) type Z durians, [1]

(ii) type Y durians. [2]

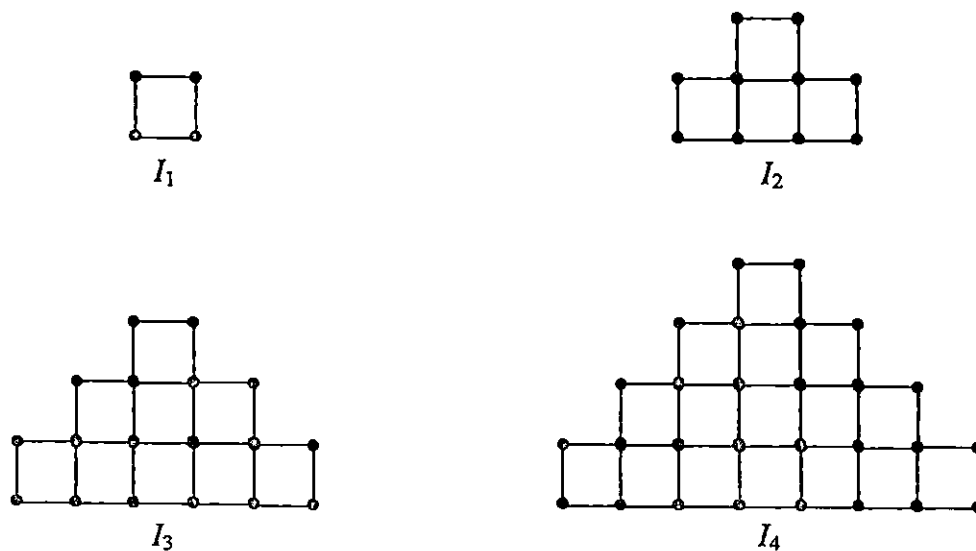
(c) The fruit seller made a profit of \$200 from the sales of all the durians.

(i) Form an equation in w , and show that it reduces to $w^2 + 260w - 1000 = 0$. [3]

(ii) Solve this equation, giving your answers correct to 2 decimal places. [3]

(d) Hence find the total mass of type Y durians, leaving your answer correct to the nearest kilogram. [2]

8. (a) The diagram shows a sequence of shapes I_1 , I_2 , I_3 and I_4 . Each shape consists of a number of squares. A dot is placed at each point where there is a corner of one or more squares.

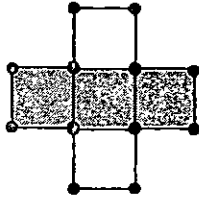
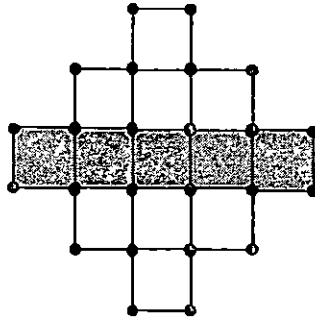
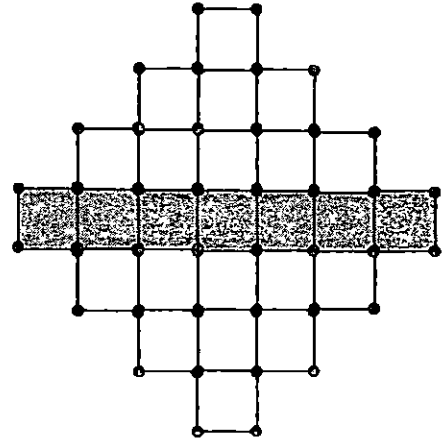


The letter n represents the number of rows of squares in each shape. The number of squares, S , and the number of dots, D , in the first four shapes are recorded in the table below.

Shape		I_1	I_2	I_3	I_4
Number of rows	n	1	2	3	4
Number of squares	S	1	4	m	16
Number of dots	D	4	10	p	28
	$D - n^2$	3	6	9	12

- (i) Find the value of m and of p . [2]
 (ii) Write a formula of S in terms of n . [1]
 (iii) Write a formula of D in terms of n . [2]

- (b) Another sequence of shapes $J_1, J_2, J_3, \dots$ is formed using the shapes $I_1, I_2, I_3, \dots$ and a row of shaded squares.

 J_1  J_2  J_3

For example, J_2 is formed by joining two I_2 shapes and a row of five shaded squares.

- (i) Write down the number of shaded squares in J_4 . [1]
- (ii) Write down an expression, in terms of n , for the total number of squares in J_n . Explain how you arrive at your answer. [3]
- (iii) Using the formula of D in (a), find the number of dots in J_{10} . [2]

~ End of Paper 2 ~

Answer

Qn	Answer
1a	\$374.50
1b	$x = \$400$
1ci	$\frac{100000}{n}$
1cii	$n = 280$
2a	63°
2b	480
2c	112°
3ai	$x = 5, y = 11$
3aii	Comparing with the other equation $4x - 8y = 12$, both lines have the same gradient, they are parallel and will not meet. Hence, the simultaneous equations have no solution.
3bi	$2\frac{1}{9}$
3bii	$x = 4$
4a	$\frac{2}{d+3}$
4bi	$E = \frac{f^3}{2f^3 - 4}$
4bii	$\frac{1}{6}$
4ci	$-g^2 - h^2$ or $-(g^2 + h^2)$
4cii	-909
5aii	$h = 12$
5bi	$432\pi \text{ cm}^2$
5bii	$k = 14$
5ci	$r = 9$
5cii	519 cm^2
6a	At 4.5 sec after being thrown, the stone is at a height of 13.5 m below the top of the cliff.
6b	$r = 40.5, s = 8$
6di	44 to 45 m at 2 to 2.2 sec
6dii	4.15 to 4.3 sec
6ei	Gradient -7 to -9
6eii	The stone is falling at a speed of (gradient value) m/s towards the ground.

Qn	Answer
7a	$\frac{100}{w} \text{ kg}$
7bi	$10w + 50$
7bii	$\frac{1000}{w} - 11w - 10$ o.e.
7cii	3.79 or -263.79
7d	15 kg
8ai	$m = 9, p = 18$
8aii	$S = n^2$
8aiii	$D = n^2 + 3n$
8bi	9
8bii	$2n + 1 + 2n^2$
8biii	264

MID YEAR EXAM 2014

SECONDARY 3 EXPRESS

Mathematics Paper 2

Qn	Solution and Answer	Marks allocation
1a	$1.07 \times \$350 = \374.50	B1
1b	Let Benny's present weekly salary be \$x. $0.07x = 0.05x + 8$ $\therefore x = \$400$	M1: form equation on increase under both schemes A1
1ci	$\frac{100000}{n}$	B1
1cii	$0.05\left(\frac{100000}{n}\right) + 8 = \frac{7240}{n}$ $\therefore n = 280$	M1: form equation on increase per employee under Scheme B A1
2a	$1\frac{3}{5}x = 168^\circ$ $\Rightarrow x = 105^\circ$ $\therefore \text{angle of sector for Sleep} = \frac{3}{5} \times 105^\circ = 63^\circ$	M1: value of x A1
2b	$\frac{360^\circ}{105^\circ} \times 140 = 480$	√M1: value of x from (a), A1
2c	Read : Chat : Total of both 7 : 9 : 16 $\frac{7}{16} \times 192^\circ$: : 192° 84° : : 192° $\therefore c = \frac{84^\circ}{360^\circ} \times 480 = 112$ or $\frac{140}{105} \times 84 = 112^\circ$	√M1: angle of sector for Read M1, A1
3ai	Subst. $2x = 3y - 23$ into $4x = y + 9$ $2(3y - 23) = y + 9$ $\therefore y = 11, x = 5$	M1: method of solving A1: each answer
3aai	$4(x - 2y = -5) \Rightarrow 4x - 8y = -20$ Or $2y = x + 5 \Rightarrow y = \frac{1}{2}x + \frac{5}{2}$ and $8y = 4x + 12 \Rightarrow y = \frac{1}{2}x + \frac{3}{2}$ Comparing with the other equation $4x - 8y = 12$, both lines have the same gradient, they are parallel and will not meet. Hence, the simultaneous equations have no solution.	M1: expressing one equation with the same gradient as the other A1: keywords – parallel o.e. and will not meet
3bi	$= 3 + \frac{1}{9} - 1$ $= 2\frac{1}{9}$	M1: simplifying with laws of indices A1

Qn	Solution and Answer	Marks allocation
3bii	$5^{\frac{3x-24}{2}} = 5^{-x-2}$ $\Rightarrow \frac{3x-24}{2} = -x-2$ $\therefore x = 4$	M1: expressing both sides in same base √M1: equation in x by comparing exponents A1
4a	$\frac{4d+2-2d-6}{(d-2)(d+3)}$ $= \frac{2d-4}{(d-2)(d+3)}$ $= \frac{2(d-2)}{(d-2)(d+3)} = \frac{2}{d+3}$	M1: factorise $d^2 + d - 6$ into $(d+2)(d-3)$ M1: combining as 1 fraction with same denominator A1: simplest form
4bi	$f^3 = \frac{4E}{2E-1}$ $2Ef^3 - f^3 = 4E$ $2Ef^3 - 4E = f^3$ $E(2f^3 - 4) = f^3$ $\therefore E = \frac{f^3}{2f^3 - 4}$	M1: getting rid of $\sqrt[3]{}$ M1: grouping terms with E A1
4bii	$\therefore E = \frac{(-1)^3}{2(-1)^3 - 4} = \frac{1}{6}$	B1: working using previous part expected
4ci	$= 2gh - (g^2 + 2gh + h^2)$ $= -g^2 - h^2 \text{ or } -(g^2 + h^2)$	M1: expansion A1
4cii	$2(3)(30) - (3+30)^2$ $= -(3^2 + 30^2) = -909$	M1: expressing in same form A1: application of results from previous part
5ai	$\frac{2}{3}\pi r^3 = 144\pi \Rightarrow r^3 = 216$ $\therefore r = 6$	M1: vol. of hemisphere A1
5aai	$\pi(6)^2 h = 3(144\pi)$ $\therefore h = 12$	M1: vol. of cylinder A1
5bi	$54 \times 8\pi = 432\pi \text{ cm}^3$	B1
5bii	$144\pi + \pi(6)^2(k-6) = 432\pi$ $\therefore k = 14$	M1: total vol. of water A1
5ci	$\frac{1}{3}\pi(\text{radius of water level})^2 16 = 432\pi$ $\therefore \text{radius of water level} = 9 \text{ cm}$	√M1: vol. of water in cone = to 5bi A1
5cii	Slant height of cone by water = $\sqrt{9^2 + 16^2} = \sqrt{337}$ $\therefore$ area of cone in contact with water = curved surface area of smaller cone $= \pi \times 9 \times \sqrt{337} \approx 519 \text{ cm}^2$	√M1: using radius from 5ci √M1: using slant height A1: not awarded if include πr^2 in working)

Qn	Solution and Answer	Marks allocation
6a	At 4.5 sec after being thrown, the stone is at a height of 13.5 m below the top of the cliff.	B1: to use given values
6b	$r = 40.5, s = 8$	B1: both values
6c	Refer to attached graph paper	P2: all points plotted correctly [P1: at least 5 points correct] C1: smooth curve -1 mark if axes not labelled
6di	Greatest height = 44 m at $t = 2$	B1, B1
6dii	4.25sec [Calculated value = 4.2 sec]	B1: accept 3.5 – 3.6
6ei	Appropriate tangent at $x = 2.5$ Gradient at $x = 2.5$ is -7.33 [Calculated value = -8]	B1: even if gradient is out of range B1: accept -7 to -9
6eii	The stone is falling at a speed of (gradient value) m/s towards the ground.	B1: to use value of gradient from 6ei
7a	$\frac{100}{w}$ kg	B1
7bi	$10(w + 5) = 10w + 50$	B1: expanded out
7bii	$\left(\frac{100}{w} - 11\right)(w + 10) = \frac{1000}{w} - 11w - 10$	M1, A1: expanded out
7ci	$10w + 50 + \frac{1000}{w} - 11w - 10 - 100 = 200$... $w^2 + 260w - 1000 = 0$	$\sqrt{M1}$: forming equation using S.P – C.P = Profit and 7bi, ii $\sqrt{M1}$: simplification step(s) A1: required equation
7cii	$w = \frac{-260 \pm \sqrt{260^2 - 4(1)(-1000)}}{2(1)}$ $= 3.79$ or -263.79	M1: method of solving A1 each: not to reject any
7d	Since cost cannot be negative, $w = 3.7908$ $\therefore$ Total mass of Type Y durians $= \frac{100}{3.7908} - 11 \approx 15$ kg (nearest kg)	$\sqrt{M1}$: using higher degree of w , A1
8ai	$m = 9, p = 18$	B1 each
8aaii	$S = n^2$	B1
8aiiii	$D - n^2 = 3n$ $\therefore D = n^2 + 3n$	M1: formula for last row A1 s.o.i
8bi	No. of shaded squares in $J_4 = 9$	B1
8bii	No. of shaded squares in $J_n = 2n + 1$ $\therefore$ Total no. of squares in $J_n = 2n + 1 + 2S$ $= 2n + 1 + 2n^2$	M1 M1 A1
8biii	No. of dots in $J_n = 2D + 4 = 2(n^2 + 3n) + 4$ $\therefore$ No. of dots in $J_{10} = 2(10^2 + 3 \times 10) + 4 = 264$	M1 A1