Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

Mid-Year Examination 2014
Secondary Three

INTEGRATED MATHEMATICS 1 — **1 hour 30 minutes**

Monday — **12 May 2014** — **0845 – 1015**

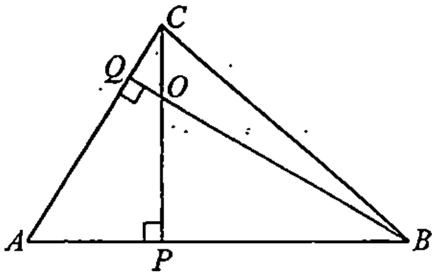
READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer <u>all</u> the questions.
2. Write your answers and working on separate answer paper provided.
3. Write your name, register number and class on each separate sheet of paper that you use and fasten the sheets together with the string provided. Do not staple your answer sheets together.
4. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 60.
3. Electronic calculators may be used, where appropriate.
4. You are reminded of the need for clear presentation in your answers.

1 

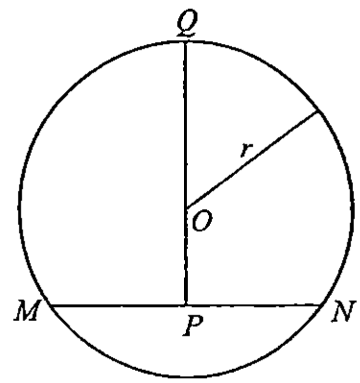
In the figure shown, $\angle APC = \angle AQB = 90°$.

$OP = 20$ cm, $OB = 30$ cm and $OC = 6$ cm.

(i) Name any two triangles that are geometrically similar to ΔQOC. [2]

(ii) Calculate the length of OQ in cm. [2]

2 The diagram shows the circular cross-section of a tunnel. The circle has centre O and radius r metres. MN represents the horizontal road surface. P is the mid-point of MN and Q is a point on the circumference such that QOP is a straight line. $MN = PQ = 8$ metres.



(i) Express OP in terms of r. [1]

(ii) Explain why $\angle OPM = 90°$. [1]

(iii) Form an equation in r and solve it to find the value of r. [2]

3(a) Factorise completely $3ac - 2bd + 6bc - ad$. [2]

(b) Express the following as a single fraction in its simplest form:

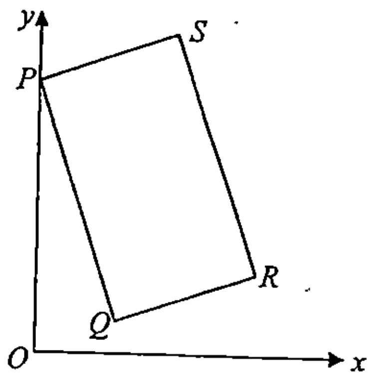
$$\frac{4}{2x+3y} + \frac{11y}{6x^2+7xy-3y^2} + \frac{3}{y-3x}.$$ [4]

4(a) Given that $x - 3 = \sqrt{p - 6x + 7}$, express p in terms of x in the simplest form. [3]

(b) Solve the equation $2 - \frac{2}{x+1} = \frac{1}{x^2-1}$, giving your answers to 2 decimal places. [4]

5 The equation of a circle is $x^2 + y^2 + 4x - 8y = 29$. Determine the radius and the coordinates of the centre. A tangent to the circle passes through the point $(5, 28)$. Find the distance between this point and the point of contact of the tangent with the circle. [6]

6 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, $PQRS$ is a rectangle. P is the point (0, 9), Q is the point (2, 1) and R is (6, 2).

(i) Find the coordinates of S. [2]

(ii) Calculate the area of the rectangle. [2]

(iii) A point T lies on RQ produced such that $RQ:RT = 1:3$. Find the coordinates of T. [2]

7 **Solutions to this question by accurate drawing will not be accepted.**

The coordinates of points A, B and C are $(-6, -1)$, $(0, 2)$ and $(8, k)$ respectively. L is a line with equation $3x + 2y - 5 = 0$. Find

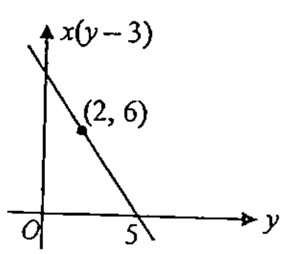
(i) the value of k if A, B and C lie on the same straight line, [2]

(ii) the equation of the line through A and parallel to the x-axis, [1]

(iii) the equation of the line through B and parallel to the line L, [2]

(iv) the equation of the perpendicular bisector of AB, in the general form $ax + by + c = 0$, where a, b and c are integers. [4]

8(a)



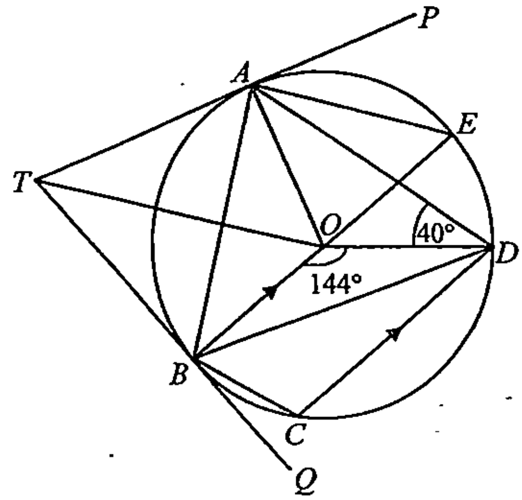
The figure shows parts of a straight line graph obtained by plotting values of $x(y-3)$ against those of y. The line passes through (2, 6) and (5, 0).

Express y in terms of x. [4]

(b) The variables x and y are related by the equation $a(x+y-b) = bx^2$, where a and b are constants. When the graph of $x+y$ against x^2 is drawn, a straight line is obtained. This line passes through (9, 8) and has a gradient of $\frac{2}{3}$.

Determine the value of a and of b. [5]

9 In the diagram, O is the centre of the circle. TP and TQ are tangents to the circle at A and B respectively. BOE is a straight line which is parallel to CD. $\angle BOD = 144°$ and $\angle ADO = 40°$.



(a) Show that BD bisects $\angle ODC$. [2]

(b) Briefly explain why $\angle AEB = \angle AD$B. [1]

(c) Find (i) $\angle BAD$,
(ii) $\angle CBQ$,
(iii) $\angle ABC$. [3]

(d) Explain clearly why ΔOAT is congruent to ΔOBT. [3]

10 **Bonus Question**

Simplify $\dfrac{2ab(a^2 + c^2 - b^2 + 2ac)(b + c - a)}{(c^2 - a^2 - b^2 + 2ab)(b + c + a)}$. [3]

~ ~ ~ END OF PAPER ~ ~ ~

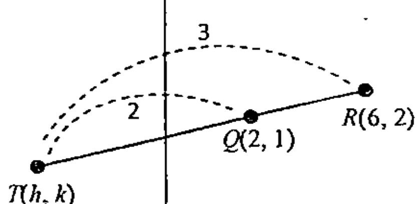
1	ΔQOC is similar to ΔPOB, ΔPAC, ΔQAB (any two). As ΔQOC is similar to ΔPOB, $\frac{QO}{PO} = \frac{OC}{OB}$ OR $\frac{QO}{20} = \frac{6}{30}$. $QO = \frac{6}{30} \times 20$ cm $= 4$ cm.	

2(i)	$OP = 8 - r$.	
2(ii)	A straight line drawn from the centre of a circle to bisect a chord, is perpendicular to the chord.	
2(iii)	In ΔOPM, by the Pythagoras' Theorem, $r^2 = 4^2 + (8-r)^2$ $= 16 + 64 - 16r + r^2$. $16r = 80 \Rightarrow r = 5$.	

3(a)	$3ac - 2bd + 6bc - ad$ $= a(3c-d) + 2b(3c-d)$ OR $3c(a+2b) - d(a+2b)$ $= (3c-d)(a+2b)$ OR $(a+2b)(3c-d)$.	
3(b)	$\frac{4}{2x+3y} + \frac{11y}{6x^2+7xy-3y^2} + \frac{3}{y-3x}$ $= \frac{4}{2x+3y} + \frac{11y}{(2x+3y)(3x-y)} - \frac{3}{3x-y}$ $= \frac{4(3x-y)+11y-3(2x+3y)}{(2x+3y)(3x-y)}$ $= \frac{12x-4y+11y-6x-9y}{(2x+3y)(3x-y)}$ $= \frac{6x-2y}{(2x+3y)(3x-y)}$ $= \frac{2}{2x+3y}$.	

4(a)	$x - 3 = \sqrt{p - 6x + 7}$. $(x-3)^2 = p - 6x + 7$. $x^2 - 6x + 9 = p - 6x + 7$. $p = x^2 + 2$.	Qn: ... p in terms of x ..., $\therefore$ $x^2 + 2 = p$ is NOT acceptable
4(b)	$2 - \frac{2}{x+1} = \frac{1}{x^2-1}$. $2(x^2-1) - 2(x-1) = 1$. $2x^2 - 2x - 1 = 0$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$ $= \frac{2 \pm \sqrt{4+8}}{4}$. $x = 1.37$ or -0.37.	

5	$x^2+y^2+4x-8y=29$. $x^2+4x+4+y^2-8y+16=29+4+16$. $(x+2)^2+(y-4)^2=49$. $\Rightarrow$ centre $=(-2, 4)$ and radius $=7$ units. Distance from $(5, 28)$ to $(-2, 4)$ $=\sqrt{(-2-5)^2+(4-28)^2}$ units $=25$ units. Let the length of the tangent be l. $l^2+7^2=25^2$. Length of tangent $=24$ units.	

6(i)	Let $S=(x_S, y_S)$. $x_S+2=0+6,\ y_S+1=9+2$. **OR** $\frac{x_S+2}{2}=\frac{0+6}{2},\ \frac{y_S+1}{2}=\frac{9+2}{2}$. $\therefore S=(4, 10)$.	
6(ii)	Area $=\sqrt{(0-2)^2+(9-1)^2}\times\sqrt{(2-6)^2+(1-2)^2}$ sq units $=\sqrt{68}\times\sqrt{17}$ sq units $=34$ sq units . **OR** Area $=\frac{1}{2}\begin{vmatrix}0 & 2 & 6 & 4 & 0\\ 9 & 1 & 2 & 10 & 9\end{vmatrix}$ sq units $=\frac{1}{2}\lvert 0+4+60+36-18-6-8\rvert$ sq units $=34$ sq units.	
6(iii)	Let T be (h, k). By similar triangles, $\frac{h-2}{2-6}=\frac{2}{1}\ ,\ \frac{k-1}{1-2}=\frac{2}{1}$. **OR** Adapt info to $RQ:QT=1:2$. By the ratio theorem (internal division), $\left(\frac{1(h)+2(6)}{1+2}, \frac{1(k)+2(2)}{1+2}\right)=(2, 1)$. **OR** Adapt info to $RT:TQ=3:2$. By the ratio theorem (external division), $T=\left(\frac{3(2)-2(6)}{3-2}\ ,\ \frac{3(1)-2(2)}{3-2}\right)$.  $T=(-6, -1)$	

7(i)	$\frac{k-2}{8-0}=\frac{2+1}{0+6}$ **OR** $k-2=\frac{2+1}{0+6}(8-0)$ **OR** $\frac{k+1}{8+6}=\frac{2+1}{0+6}$ **OR** $k+1=\frac{2+1}{0+6}(8+6)$ $k=6$	
7(ii)	$y=-1$ **OR** $y+1=0$	
7(iii)	Gradient $=-\frac{3}{2}$ $\therefore$ Equation: $y=-\frac{3}{2}x+2$ **OR** Let equation be $3x+2y+a=0$ Use $(0,2)$, $a=-4$. $\therefore$ Equation: $3x+2y-4=0$	
7(iv)	Mid-point of $AB=\left(-3,\ \frac{1}{2}\right)$ Gradient of $\perp$ bisector $=-2$ $y-\frac{1}{2}=-2(x+3)$ $y=-2x-5\frac{1}{2}$ $\times 2$, rearrange, $4x+2y+11=0$.	

8(a)	Gradient $=\frac{6-0}{2-5}=-2$ Using $(5,0)$, $0=-2(5)+c$, **OR** using $(2,6)$, $6=-2(2)+c$, giving $c=10$. Linear equation: $x(y-3)=-2y+10$ $xy-3x+2y=10$ $y(x+2)=3x+10$ giving $y=\frac{3x+10}{x+2}$.	
8(b)	Using $Y=mX+c$, we have $8=\frac{2}{3}(9)+c$. $c=2$. Thus $Y=\frac{2}{3}X+2$. From $a(x+y-b)=bx^2$, $x+y=\frac{b}{a}x^2+b$. $\Rightarrow$ $b=2$, and $\frac{2}{a}=\frac{2}{3}$, giving $a=3$.	

9(a)	$\angle OBD = \angle ODB$ (base $\angle$s of isos. Δ) $\angle OBD = \angle BDC$ (alt. $\angle$s, $CD // BOE$). $\therefore \angle ODB = \angle BDC$ $\therefore BD$ bisects $\angle ODC$.	
9(b)	$\angle$s in the same segment	
9(c)	(i) $\angle BAD = 72°$	
	(ii) $\angle CBQ = 18°$	
	(iii) $\angle ABC = 104°$	
9(d)	In ΔOAT and ΔOBT, $\angle OAT = \angle OBT = 90°$ (tangent $\perp$ radius) $OT = OT$ (common line) $OA = OB$ (radii of the same circle) $\therefore \Delta OAT \equiv \Delta OBT$ (RHS)	

10 Bonus Qn	$\dfrac{2ab(a^2+c^2-b^2+2ac)(b+c-a)}{(c^2-a^2-b^2+2ab)(b+c+a)}$ $=\dfrac{2ab[a^2+2ac+c^2-b^2](b+c-a)}{[c^2-(a^2-2ab+b^2)](a+b+c)}$ $=\dfrac{2ab\,[(a+c)^2-b^2](b+c-a)}{[c^2-(a-b)^2](a+b+c)}$ $=\dfrac{2ab\,(a+c+b)(a+c-b)(b+c-a)}{(c+a-b)(c-a+b)(a+b+c)}$ $=\dfrac{2ab\,(a+b+c)(a-b+c)(-a+b+c)}{(a-b+c)(-a+b+c)(a+b+c)}$ $=2ab$.	