Class	Register No	Name

Bukit Merah Secondary School
End-of-Year Examination 2015
Secondary 3 Express

E

MATHEMATICS

4048 / 01

Paper 1

5 Oct 2015

Candidates answer on the Question Paper.

2 hours

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

Calculator Model:

For Examiner's Use

This document consists of **15** printed pages.

102

Mathematical formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Using a calculator, evaluate $\dfrac{\left(-\frac{3}{5}\right)^2+\sqrt{47}}{0.165-2(4.65)}$, giving your answer correct to 3 significant figures.

Answer _____ [2]

---

2 Simplify the following, giving your answers in positive index notation.

(a) $\dfrac{\sqrt{x^3}\times\sqrt{y}}{x^0\times y^{-1}}$,

Answer (a) _____ [2]

(b) $\left(3a^{-2}\right)^4 \div a^4$.

Answer (b) _____ [2]

103

3 The hands of a clock show exactly 02 30 hours. If the minute hand of the clock rotates 150° clockwise, find

(i) the time the clock will now show,

Answer (i) _____ hours [1]

(ii) the angle through which the hour hand has rotated.

Answer (ii) _____ ° [1]

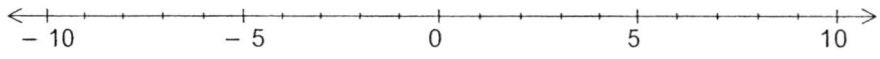
---

4 **(a)** Solve the inequalities $\frac{1}{2}(3-x) < 5 \leq 2(x-5)$.

Answer (a) _____ [2]

(b) Represent the solution on the number line below.

Answer (b) [1]



(c) Hence, state the smallest prime number that satisfies the inequalities

$\frac{1}{2}(3-x) < 5 \leq 2(x-5)$.

(c) _____ [1]

5 The population of fungi in an enclosed area weighs 2475 Megagrams.
Each fungus weighs 8.25 nanograms.

(i) Express 8.25 nanograms in milligrams, giving your answer in standard form.

Answer (i) _____ mg[1]

(ii) Find the number of fungi in the enclosed area, giving your answer in standard form.

Answer (ii) _____ [2]

---

6 t is inversely proportional to m^3.
It is known that $t = 12$ for a particular value of m.
Find the value of t when m is doubled.

Answer t = _____ [2]

10k

7 **(a)** Given that $s = \sqrt{\frac{2v - uv}{3a}}$, make v the subject of the formula.

Answer (a) _____ [2]

(b) Factorise completely $4x^2 - y^2 - 8x + 4y$.

Answer (b) _____ [2]

---

8 An object was taken out of a freezer and its temperature increases at a steady rate.

Its temperature at 10 00 hours was - 13°C. The temperature at 10 20 hours was 15°C.

(i) Write down the difference between the two temperatures.

Answer (i) _____ °C [1]

(ii) Find the temperature of the object at 10 15 hours.

Answer (ii) _____ °C [2]

---

9 A notebook costs x cents and a pen costs y cents.

(i) Judy bought 6 notebooks and 3 pens. The total cost was \$5.70.
Write down an equation in x and y and show that it reduces to $2x + y = 190$.

Answer (i) [1]

(ii) Jordon bought 4 notebooks and 5 pens. The total cost is \$4.70.
Write down another equation in x and y.

Answer (ii) _____ [1]

(iii) Showing your working, calculate the value of x and of y.

Answer (iii) $x =$ _____

$y =$ _____ [3]

10 The equation of line L_1 is $2x - 4y = 9$. Line L_2 passes through $(-2, 1)$. Given that the gradients of L_1 and L_2 are the same, find the equation of line L_2.

Answer _____ [3]

105

11 In the diagram, DCA is a straight line. $B\hat{A}D = 90°$, $AB = 7$ cm and $BC = 25$ cm.

(a) Find the length of CA.

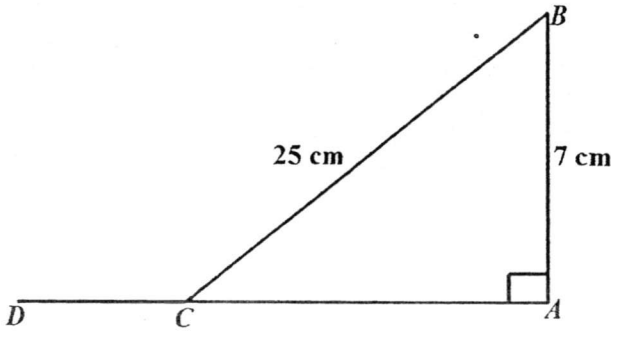
Answer (a) _____ cm [1]

(b) Express as a fraction in its lowest term, the value of

(i) $\tan B\hat{C}A$,

(ii) $\sin B\hat{C}D$,

(iii) $\cos B\hat{C}D$.



Answer (b)(i) _____ [1]

b(ii) _____ [1]

b(iii) _____ [1]

---

12 **(a)** Express $x^2 - 4x - 7$ in the form of $(x-h)^2 - k$.

Answer (a) _____ [1]

(b) Hence solve the equation $x^2 - 4x - 7 = 14$.

Answer (b) $x =$ _____ or _____ [2]

13 **(a)** Factorise $21x^2 - 5x - 6$.

Answer (a) _____ [1]

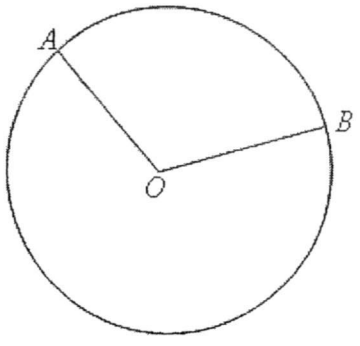
(b) Hence express $\frac{4(x+7)}{21x^2-5x-6} - \frac{4}{3x-2}$ as a fraction in its simplest form.

Answer (b) _____ [3]

---

14 In the diagram, the points A and B lie on a circle with centre O.
The area of the circle is $144\,\pi$ cm^2 and the length of the **minor** arc AB is $8\,\pi$ cm.
Calculate

(i) the radius of the circle,



Answer (i) _____ cm [1]

(ii) the area of the **major** sector OAB, leaving your answer in terms of π.

Answer (ii) _____ cm^2 [3]

106

15 **(a)** James bought a limited edition book at \$80. Several years later, he sold it at a profit of 150%. Calculate the selling price.

Answer (a) \$ _____ [2]

(b) Janice deposited \$5 000 in a bank account that paid a compound interest at the rate of 4% per annum. Calculate the interest she earned at the end of 3 years.

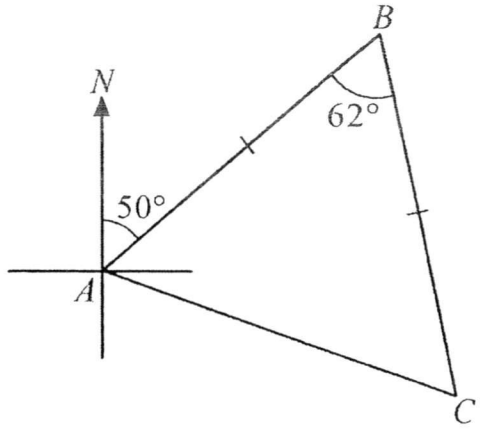
Answer (b) \$ _____ [2]

---

16 In the diagram, the points A, B and C form an isosceles triangle with $AB = CB$, $\angle ABC = 62°$ and the bearing of B from A is 050°.

Find

(i) the bearing of C from A,

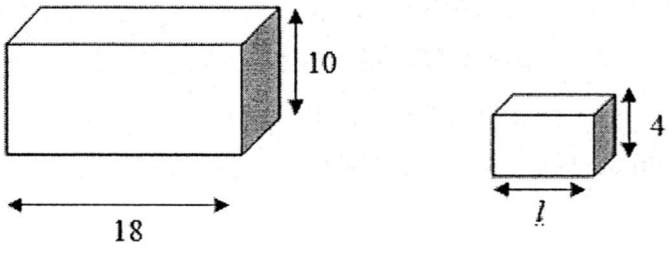


Answer (i) _____ ° [2]

(ii) the bearing of C from B.

Answer (ii) _____ ° [2]

17



The diagram above shows two types of similar wooden cuboid. The height of the smaller cuboid is 4 cm while the height of the bigger cuboid is 10 cm. The length of the bigger cuboid is 18 cm. Find

(i) the length l of the smaller cuboid,

Answer (i) _____cm [1]

(ii) the ratio of the surface area of the smaller cuboid to the bigger cuboid.

*Answer (ii)*_____ : _____ [1]

(iii) If the mass of the smaller cuboid is 16 g, find the mass of the bigger cuboid.

*Answer (iii)*_____ g [2]

107

18 The terms C_1, C_2, C_3 and C_4 of a sequence are given as follows:

$$C_1 = 0.5(1 \times 2) = 1$$
$$C_2 = 0.5(2 \times 3) = 3$$
$$C_3 = 0.5(3 \times 4) = 6$$
$$C_4 = 0.5(4 \times 5) = 10$$

(i) Write down the next 2 terms C_5 and C_6.

Answer (i) _____

_____ [1]

(ii) Find in terms of n, a formula for the n^{th} term.

Answer (ii) _____ [1]

(iii) Explain why 24 cannot be a number in the sequence.

Answer: (iii) [1]

_____

_____

_____

(iv) Show that $C_{n+1} - C_n$ is equal to (n + 1).

Answer: (iv) [2]

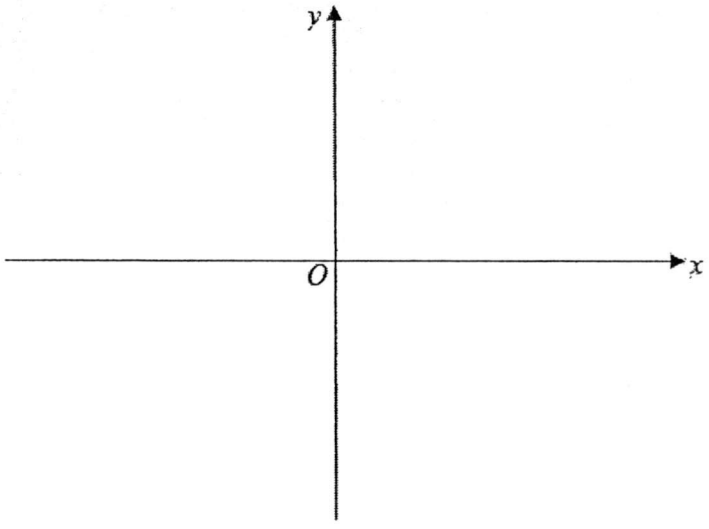
_____

_____

_____

19 **(a)** **(i)** Sketch the graph $y = (x+2)(x-3)$ in the space provided below, indicating clearly all the intercepts on the axes.

Answer: *(a)(i)* [2]



(ii) Write down the equation of line of symmetry.

Answer (a)(ii) _____ [1]

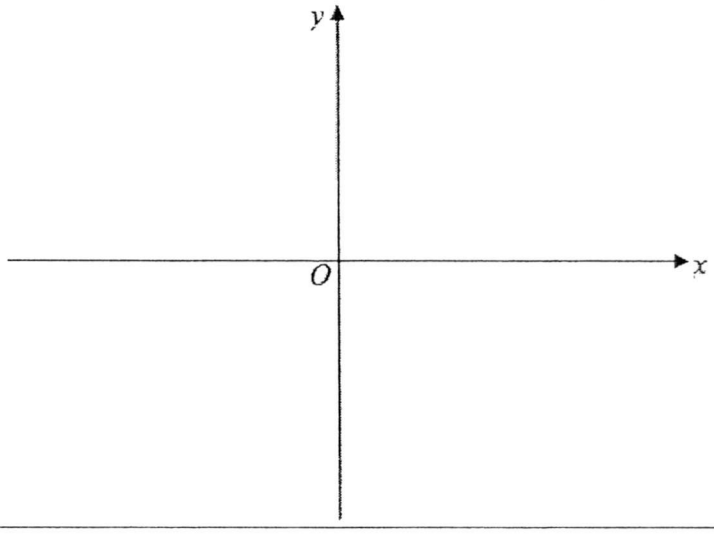
(b) The graph of $y = a^x - 1$ passes through the point (2, 3).

(i) Given that a is positive, find the value of a.

Answer (b)(i) a = _____ [1]

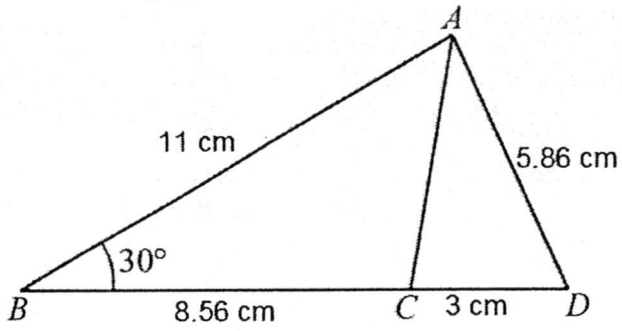
(ii) Sketch this graph, indicating the point (2, 3) clearly on your graph.

Answer: (b)(ii) [2]



108

20



In the diagram, $AB = 11$ cm, $BC = 8.56$ cm, $CD = 3$ cm, $AD = 5.86$ cm and $\angle ABC = 30°$.
Find

(i) the length of AC,

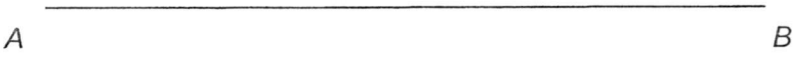
Answer (i) $AC =$ _____ cm [2]

(ii) $\angle ACD$.

Answer (ii) $\angle ACD =$ _____ ° [3]

21 ***(a)*** Triangle ABC is such that $\angle BAC = 46°$ and $BC = 8$ cm. Construct the two possible positions of C and label them C_1 and C_2. AB has already been drawn.

Answer (a), (b) [2]



(b) Join the points A, B and C to form the triangle where $\angle ACB$ is acute.

Construct

(i) the perpendicular bisector of BC, [1]

(ii) the bisector of angle ABC. [1]

(c) These two bisectors meet at P.

Complete the statement below.

Answer (c)

The point P is equidistant from the points _____ and _____ and equidistant from the lines _____ and _____ . [1]

End of Paper

ANSWER KEY FOR SEC 3 PAPER 1

1. ≈ -0.790

2(a) $(xy)^{\frac{3}{2}}$

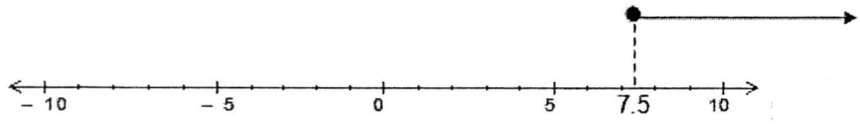
(b) $\dfrac{81}{a^{12}}$

3(i) 02 55 hours

(ii) 12.5°

4(a) $x \geq 7.5$

(b)



12(a) $(x-2)^2 - 11$

(b) $x = 7$ or -3

13(a) $(7x+3)(3x-2)$

(b) $\dfrac{-8}{(7x+3)}$

14(i) 12 cm

(ii) 96π cm^2

15(a) \$200

(b) \$624.32

4(c) 11

5(i) 8.25×10^{-6} milligrams

(ii) 3×10^{17}

6) 1.5

7(a) $v = \dfrac{3as^2}{2-u}$

(b) $(2x-y)(2x+y-4)$

8(i) 28° C

(ii) 8° C

9(i) $6x + 3y = 570$

$2x + y = 190$ (shown)

(ii) $4x + 5y = 470$

(iii) $x = 80,\ y = 30$

10. $y = \dfrac{1}{2}x + 2$

11(a) 24

11(b)(i) $\tan B\hat{C}A = \dfrac{7}{24}$

(ii) $\sin B\hat{C}D = \dfrac{7}{25}$

(iii) $\cos B\hat{C}D = -\dfrac{24}{25}$

16(i) 109°

(ii) 168°

17(i) 7.2 cm

(ii) 4: 25

(iii) 250 g

18(i) $C_5 = 0.5(5 \times 6) = 15$

$C_6 = 0.5(6 \times 7) = 21$

(ii) $C_n = 0.5(n)(n+1)$

(iii) $24 \times 2 = 48$

$6 \times 8 = 48$

Since 6 and 8 are not $m \times (m-1)$, 24 cannot be a number in the sequence

OR $0.5[m \text{ x } (m\text{-}1)] = 24$

$m(m\text{ -}1) = 48$

Since there are no integers to satisfy the equation, 24 cannot be a number in the sequence

(iv) $C_{n+1} = 0.5(n+1)(n+2)$

$C_{n+1} - C_n = 0.5(n+1)(n+2) - 0.5(n)(n+1)$

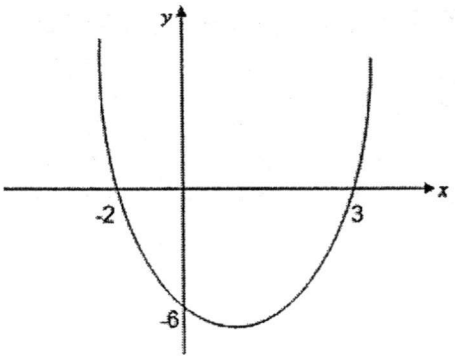
$C_{n+1} - C_n = 0.5(n+1)[(n+2) - (n)]$

$C_{n+1} - C_n = 0.5(n+1)[2]$

$C_{n+1} - C_n = (n+1)$ (Shown)

19.

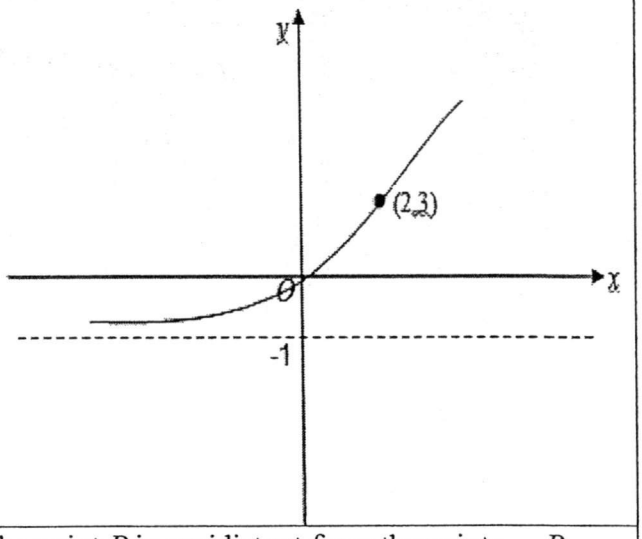
Ans:
(a)



aii) $x = 0.5$

19.
b(i) $a = 2$

Ans:
(b)



20(i) 5.58 cm (correct to 3 sig. fig.)
(ii) = 80.0^{o} (to 1 d.p.)

21(c) The point P is equidistant from the points __B__ and __C__ and equidistant from the lines __AB__ and __BC__ .

Class	Register No	Name

Bukit Merah Secondary School

End-of-Year Examination 2015

Secondary 3 Express

E

MATHEMATICS

4048 / 02

Paper 2

6 Oct 2015

Additional Materials: Writing papers (4 sheets)

Graph paper (1 sheet)

1 hour 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **60**.

Mathematical formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

1. **(a)** Solve the following equations

(i) $2^{-x} \times 4^{x} = 8$, [2]

(ii) $6 - y = \frac{9}{y}$. [3]

(b) x is an odd number and the sum of the next two consecutive even numbers is less than 90. By forming an inequality in terms of x, find the largest possible value of x. [3]

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2. A group of y tourists is on a sightseeing tour.

(a) The total cost of the tour, \$90 000, is shared equally among the tourists.

Write down an expression, in terms of y, for the cost per tourist. [1]

(b) 5 tourists decided to drop out of the tour.

Write down an expression, in terms of y, for the cost per tourist when there is a shortfall of 5 tourists. [1]

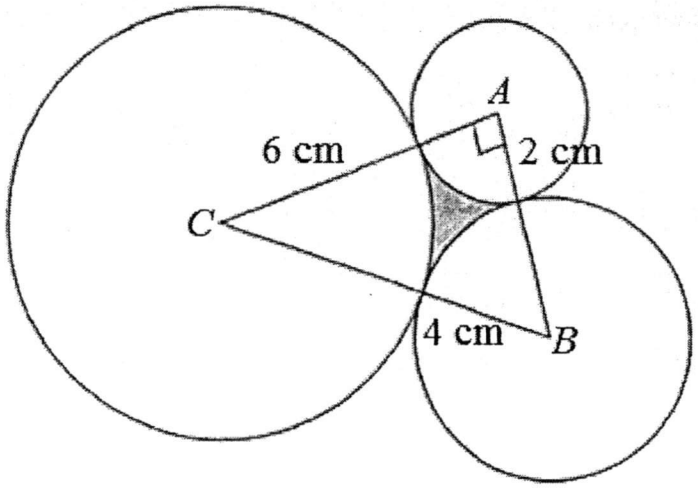
(c) Due to the shortfall, each tourist has to pay \$250 more.

Form an equation in y and show that it reduces to $y^2 - 5y - 1800 = 0$. [3]

(d) Solve the equation $y^2 - 5y - 1800 = 0$. [2]

(e) Find the amount each tourist must pay if less than 5 tourists join the tour than originally planned. [1]

3.



In the diagram, three circles with centres A, B and C touch one another. The radii of the circles are 2 cm, 4 cm and 6 cm respectively. A, B and C join to form a right-angled triangle with $\angle BAC = 90^o$.

(a) Show that $\angle ACB = 0.644$ radians. [1]

Find

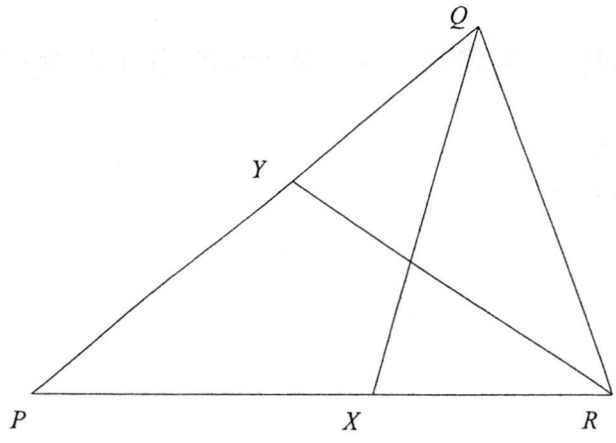
(b) $\angle ABC$ in radians, [1]

(c) the perimeter of the shaded region, [3]

(d) the area of triangle ABC. [1]

(e) Calculate the area of the shaded region. [3]

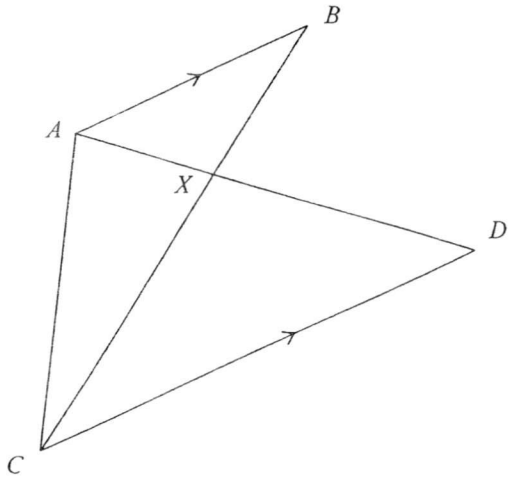
4. (a)



In the diagram, $PQ = PR$, QX bisects $\angle PQR$ and RY bisects $\angle QRP$.

Prove that triangles PQX and PRY are congruent. [3]

(b)



In the diagram, AB is parallel to CD. AD and BC meet at X.

(i) Prove that triangles ABX and DCX are similar. [2]

(ii) Given that $\frac{BX}{BC} = \frac{3}{8}$ and $AB = 15\,\text{cm}$, find CD. [1]

(iii) Given the area of triangle ABX is $81\,\text{cm}^2$. Find

(a) the area of triangle DCX, [1]

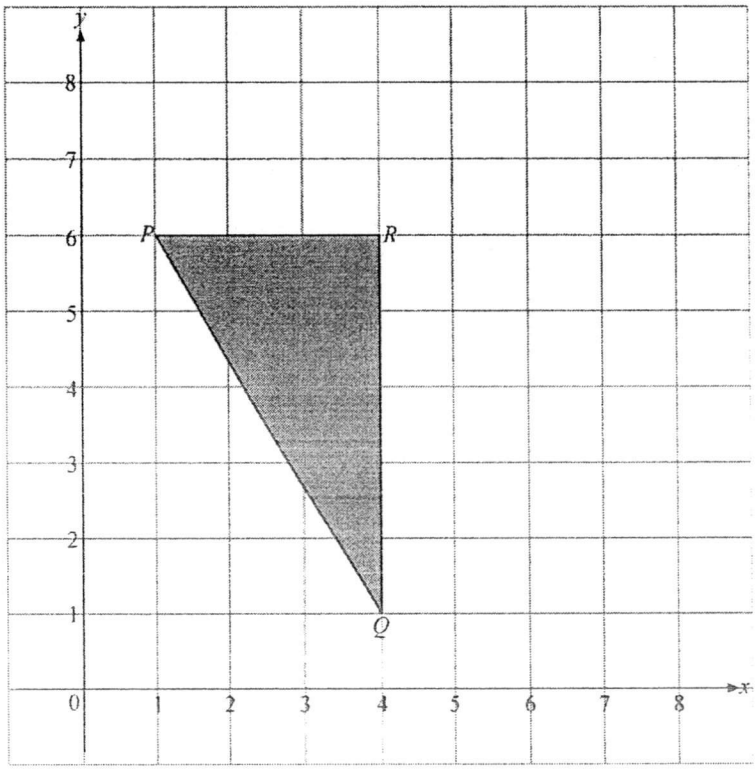
(b) the area of triangle ACX. [1]

5. The diagram shows triangle PQR where P is (1,6), R is (4, 6) and Q is (4, 1).

(a) Find the length of PQ. [2]

(b) Find the acute angle PQR. [2]

(c) Find the perpendicular distance of PQ from R. [2]



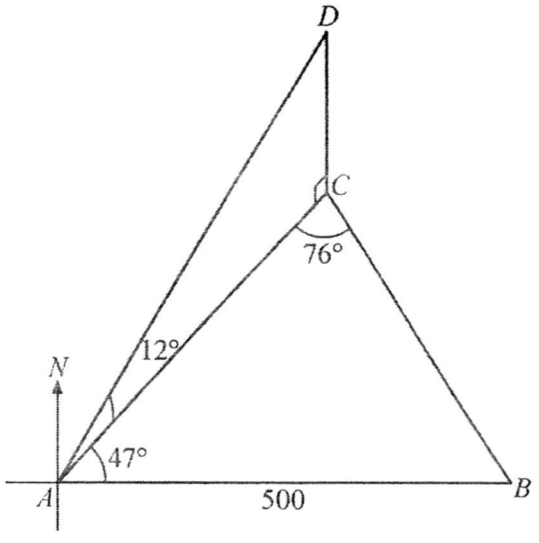
6. In the diagram, points A, B and C are on level ground and CD is a building at the point C. Given that $\angle ACB = 76°$, $\angle BAC = 47°$, $AB = 500$ m and the angle of elevation of the top of the building from A is 12°. Find

(a) the bearing of C from A, given that B is due east of A, [1]

(b) the distance AC. [3]

(c) the height of the building, correct to the nearest metre, [2]

(d) A cat was crawling along AB. Find the greatest angle of depression if Cathy was watching the cat from the top of the building CD. [4]



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7. **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = 2 + \frac{9}{2}x - \frac{1}{6}x^3$.

The table below shows some values of x and the corresponding values of y correct to 1 decimal place, where necessary.

x	-4	-3	-2	-1	0	1	2	3	4
y	-5.3	-7	-5.6	p	2	6.3	9.7	11	9.3

(a) Calculate the value of p, correct to 1 decimal place. [1]

(b) Using a scale of 2cm to 1 unit, draw a horizontal x-axis for $-4 \le x \le 4$.

Using a scale of 1cm to 1 unit, draw the vertical y-axis for $-8 \le y \le 12$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve when $x = -2$. [2]

(d) Use your graph to find the solutions to the equation $\frac{1}{6}x^3 - \frac{9}{2}x + 8 = 0$, in the range $-4 \le x \le 4$. [2]

(e) (i) On the same axes, draw the graph $y = -3x + 6$ for $-2 \le x \le 4$. [1]

(ii) Write down the x-coordinate of the point where this line intersects the curve. [1]

(iii) Write down the equation, without simplifying, which is satisfied by the value of x found in part **(e)(ii)**. [1]

----------End of Paper----------

ANSWER KEY FOR SEC 3EXP PAPER 2

1. (a) $2^x = 2^3 \Rightarrow x = 3$

(b) $y = 3$

(c) $x < 43$

x is 41

2(a) $\$\dfrac{90000}{y}$

2(b) $\$\dfrac{90000}{y-5}$

2(c) $\dfrac{90000}{y-5} - \dfrac{90000}{y} = 250$

$y^2 - 5y - 1800 = 0$

2(d) $y = 45$ or $y = -40$

2(e) \$2 250

---

3 (a) $\sin \angle ACB = \dfrac{2+4}{4+6} = \dfrac{6}{10}$

$\angle ACB = 0.64350... \approx 0.644$ rad (3 sf)

(b) $= 0.92729... \approx 0.927$ rad (3 sf)

(c) Perimeter of the shaded region
$= 10.711 \approx 10.7$ cm (3 sf)

(d) Area of triangle $ABC = 24$ cm^2

(e) Area of the shaded region
$= 1.8570$
$= 1.86$ cm^2 (3 sf)

4(a) $\angle PQR = \angle PRQ$ (Base $\angle$s of isos Δ)

$\dfrac{1}{2}\angle PQR = \dfrac{1}{2}\angle PRQ$

$\therefore \angle PQX = \angle PRY$

$PQ = PR$ (Given)

$\angle QPX = \angle RPY$ (*Common angle*)

$\therefore \Delta PQX$ and ΔPRY are congruent. **(ASA)**

P/S: Deduct 1 mark if conclusion is not stated

4 (b)(i) $\angle ABX = \angle DCX$ (alternate angles, // lines)

$\angle AXB = \angle DXC$ (vert. opp. angles)

$\therefore \Delta ABX$ and ΔDCX are similar (AA)

P/S: Deduct 1 mark if conclusion is not stated.

(ii) $CD = 25$ cm

(iii)(a) $\text{area of } \Delta DCX = \frac{25}{9} \times 81$

$= 225 \text{cm}^2$

(b) $\frac{\text{area of } \Delta ACX}{81} = \frac{5}{3}$

$\text{area of } \Delta ACX = \frac{5}{3} \times 81$

$= 135 \text{ cm}^2$

5(a) 5.83 units

(b) $\angle PQR = 31.0°$ (correct to 1 d.p.)

(c) $d = 2.57$ units

6

(a) Bearing of C from $A = 043°$

(b) $\angle ABC = 57°$

$AC = 432$ m (correct to 3 sig. fig.)

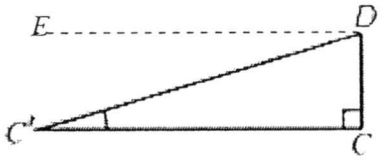
(c) $CD = 432.17 \times \tan 12°$

$= 92$ m (correct to the nearest metre)

(d) Let d be the shortest distance from C to AB.

$d = 432.17 \times \sin 47°$

$= 316$ m (correct to 3 sig. fig.)



Greatest $\angle$ of depression $EDC' = \angle DC'C$ (alt $\angle$s, // lines)

$\tan \angle DC'C = \frac{91.860}{316.069}$

Greatest $\angle$ of depression $EDC' = \angle DC'C = 16.2°$

---

7. (a) $p = -2.3$

(b) Points [**2 m**], A smooth curve [**1 m**], Deduct 1 m for wrong scale or axes are not labelled

(c) Draw a **long** tangent at $x = -2$.

Gradient = 2.5 [2 to 2.75]

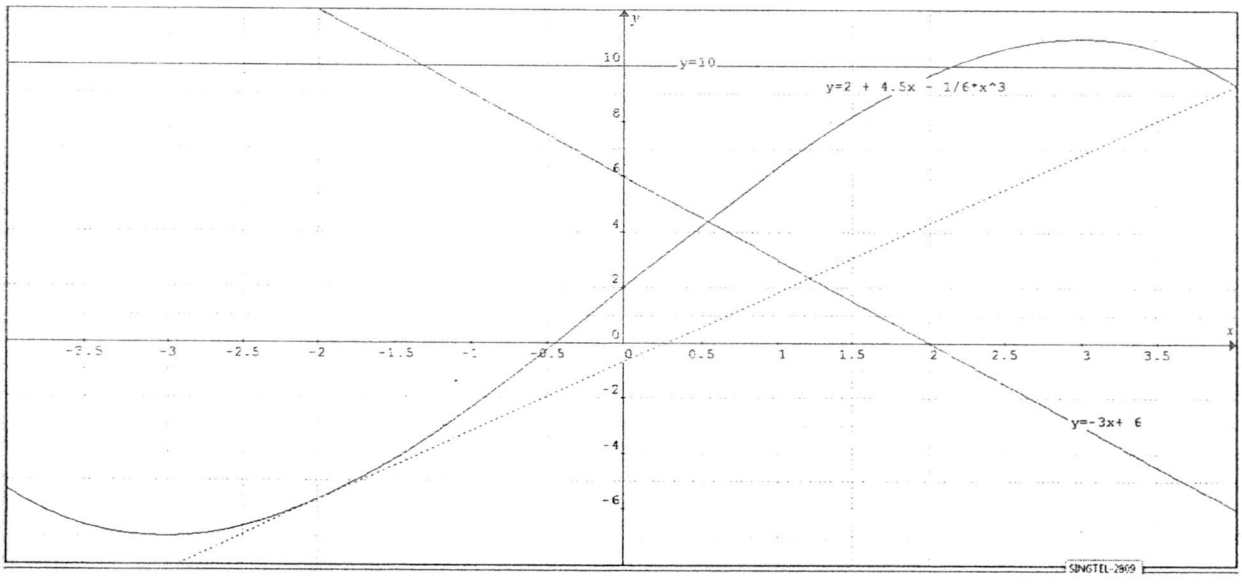
(d) $\frac{1}{6}x^3 - \frac{9}{2}x + 8 = 0 \Rightarrow y = 10$

$x = 2.14$ [2.1 to 2.2] , $x = 3.78$ [3.75 to 3.85]

(e)(i) Draw the graph $y = -3x + 6$ for $-2 \le x \le 4$

(ii) $x = 0.537$ [0.5 to 0.6]

(iii) $2 + \frac{9}{2}x - \frac{1}{6}x^3 = -3x + 6$



----------End of Paper----------

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Answer <u>ALL</u> questions.

1. Evaluate $\dfrac{3.28^2}{\sqrt{36.36}+5}-(9.327-\pi)^2$, giving your answer correct to 5 significant figures.

Answer ..... [2]

---

2. Given that $-2 < x \leq 5$ and $-6 < y < 2$, find

   (a) the least integer value of xy,

Answer (a) ..... [1]

   (b) the value of y when $y^2 = 25$.

Answer (b) ..... [1]

---

3. Factorise $2y^2 - 18 + (ay)^2 - 9a^2$ completely.

Answer ..... [2]

---

4. (a) Solve the inequalities $\frac{x-10}{5} \leq x+1 < \frac{1}{3}x+2$.

Answer (a) ..... [2]

(b) Write down all the integers which satisfy $\frac{x-10}{5} \leq x+1 < \frac{1}{3}x+2$.

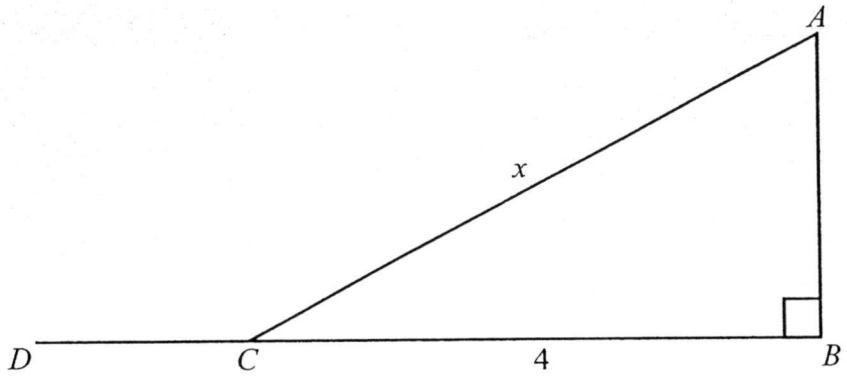
(b) ..... [1]

---

5. Simplify $\frac{2x+3}{x^2+x-6} - \frac{3}{2-x}$.

Answer ..... [3]

6. In the triangle ABC, angle $ABC = 90°$, $AC = x$ cm, $BC = 4$ cm and BC is produced to D.



(a) Show that $AB = \sqrt{x^2 - 16}$.

Answer (a) .....

.....

..... [1]

(b) Find in terms of x

(i) $\sin A\hat{C}B$,

(ii) $\cos A\hat{C}D$.

Answer (b) (i) ..... [1]

(b) (ii) ..... [1]

7. Leonardo owns 540 cubes each of side 1cm.
He decides to build a large cube using these existing cubes and additional cubes that he would have to purchase.
Each additional cube costs \$0.25.
What is the least amount of money he has to spend to complete the large cube?

Answer \$..... [3]

---

8. On a drawing, the length of a cruise ship is 300 mm.
It's actual length is 360 metres.
(a) Express the scale of the drawing in the form 1 : n.

Answer (a) 1 : ..... [1]

(b) The ship has a swimming pool in the shape of a square on its deck.
The area of the pool is 250 m^2.
Find the length of one side of the pool on the drawing, giving your answer in mm.

Answer (b) ..... mm [2]

---

9. The area of a rectangle, A cm^2, is given by $A = x^2 - 4x + 7$.

(a) Express $x^2 - 4x + 7$ in the form $(x - a)^2 + b$.

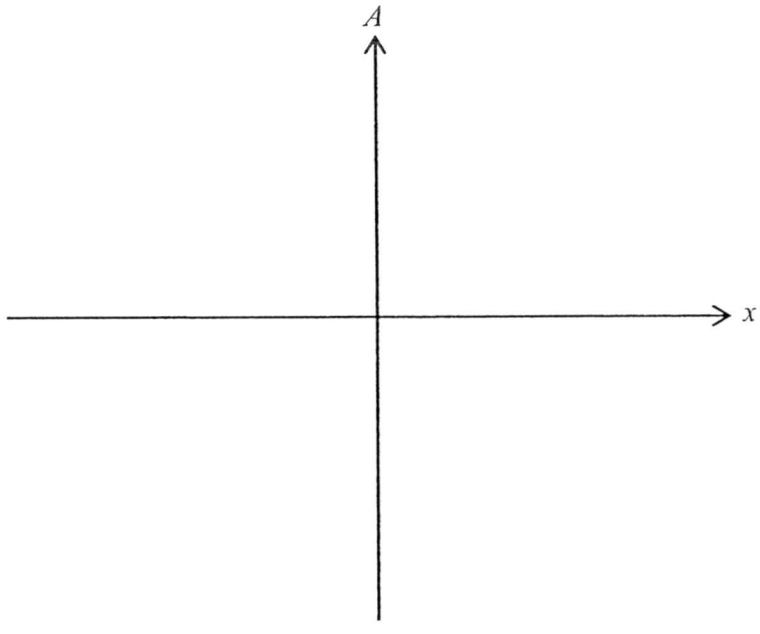
Answer (a) ..... [2]

(b) State the minimum area of the rectangle.

Answer (b) ..... cm^2 [1]

(c) Sketch the graph of $A = x^2 - 4x + 7$ on the axes below, clearly stating the turning point and the vertical intercept.

Answer (c)



[1]

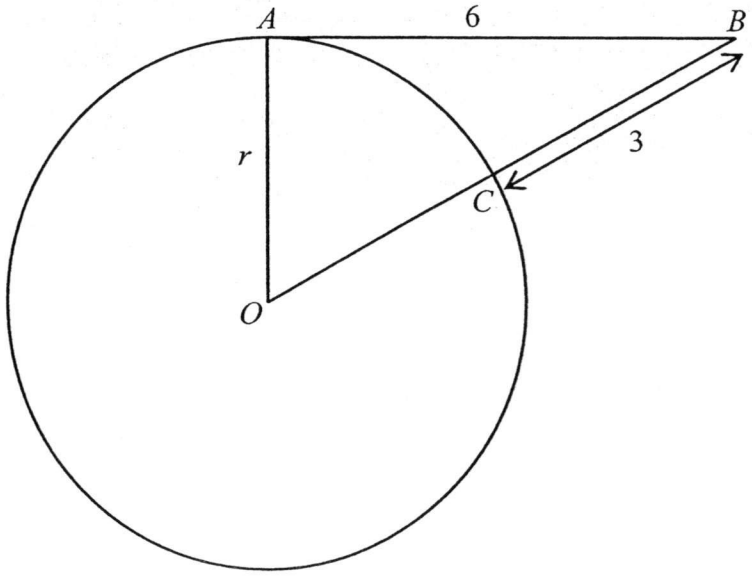
10. (a) Simplify $\left(\frac{2}{3x}\right)^{-2} \div \frac{2}{x^3}$.

Answer (a) ..... [2]

(b) Given that $3^{0.2x} - 3^3 = 54$, find x.

Answer (b) x = ... [2]

11. The diagram shows a circle with centre O and radius r cm.
AB is a tangent to the circle such that $AB = 6$ cm.
C lies on the circumference of the circle and $BC = 3$ cm .



(a) Find the radius, r, of the circle.

Answer (a) $r =$ ..... cm [2]

(b) Find angle AOB.

Answer (b) $\angle AOB =$ ..... ° [2]

12. There are 45 tickets in a lucky draw.
The tickets come in the colour of gold, silver and bronze.
A ticket is picked at random.

The probability of drawing a gold ticket is $\frac{1}{9}$ and the probability of drawing a silver ticket is $\frac{1}{5}$.

(a) Find the number of bronze tickets in the lucky draw.

Answer (a) ..... [2]

(b) Before the draw begins, a sponsor adds an equal number of bronze and silver tickets and the probability of drawing a gold ticket becomes $\frac{1}{13}$.

Find the number of bronze tickets added.

Answer (b) ..... [2]

13. A shopkeeper sells fruits at the prices shown in the table below.

Apples	40 cents each
Durians	\$3 per kg
Kiwis	\$2.20 each

(a) Adrian buys $2\frac{3}{4}$ kg of durians and 2 apples.

Calculate how much he pays.

Answer (a) \$ ..... [1]

(b) The shopkeeper buys

100 apples for \$30,

40 kg of durians at \$3.20 per kg,

30 kiwis for \$70.

He sells all the fruits at the prices shown in the table.
Explain if he makes a profit or loss.
Express his profit/loss as a percentage of his cost price.

Answer (b) .....

.....

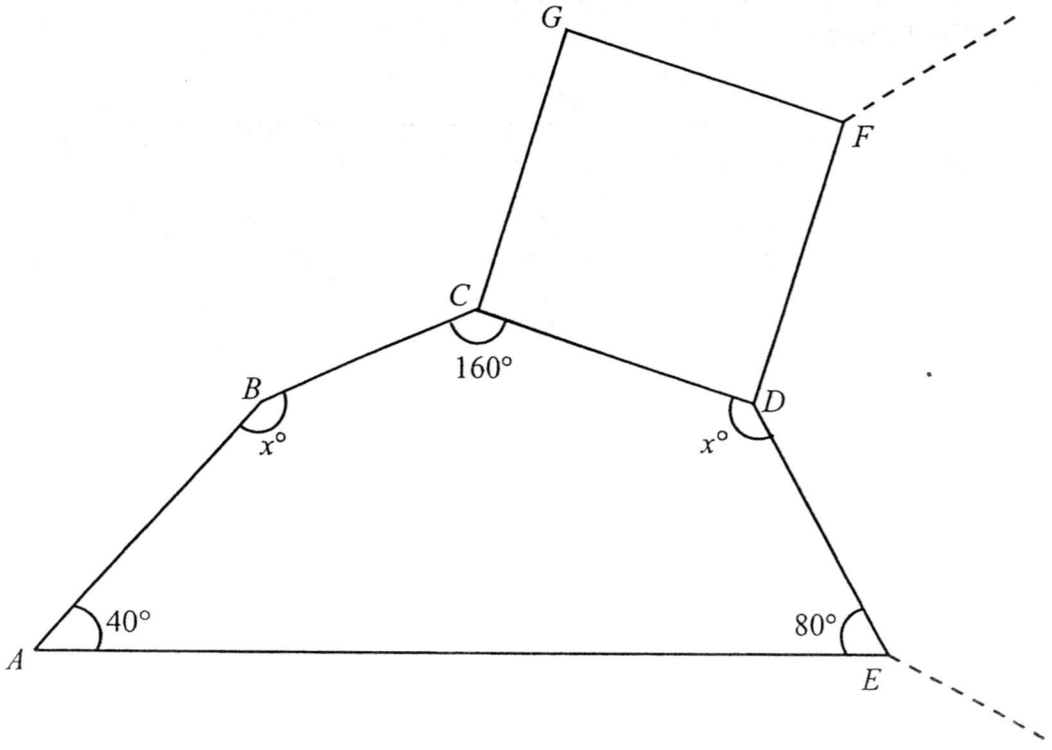
.....

.....

.....

..... [3]

14. In the diagram, *ABCDE* is a pentagon, *CDFG* is a square and *FDE* is part of a regular polygon with n sides.



Find

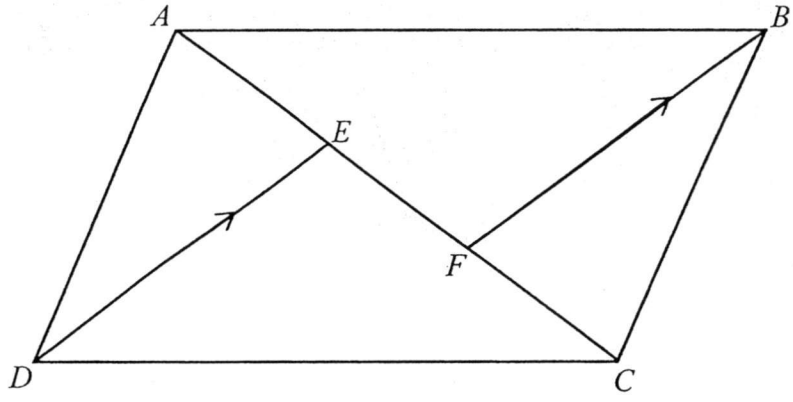
(a) the value of x.

Answer (a) x =..... [2]

(b) the value of n.

Answer (b) n =..... [2]

15. In the diagram, $ABCD$ is a quadrilateral.
E and F are points on AC such that $AF = CE$.
$DE = BF$ and $DE \parallel BF$.



(a) Prove that triangle ADE is congruent to triangle CBF.

Answer (a) .....

.....

.....

.....

..... [3]

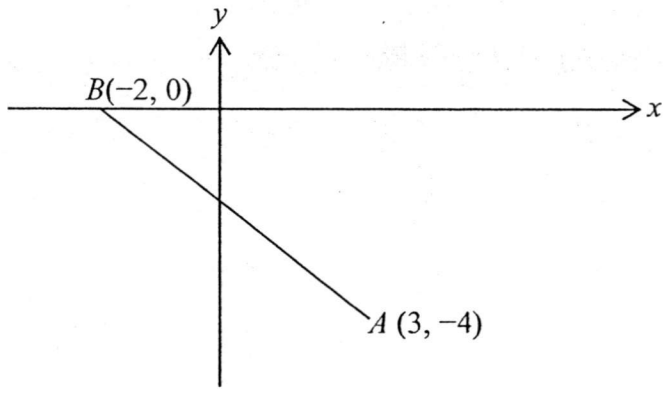
(b) Using your result from part (a), show that $ABCD$ is a parallelogram.

Answer (b) .....

.....

..... [2]

16. The diagram shows the points A (3,−4) and B(−2, 0).



Find

(a) the gradient of AB,

Answer (a) ..... [1]

(b) the length of AB,

Answer (b) ..... [1]

(c) the equation of the line AB,

Answer (c) ..... [1]

(d) The point C lies on the x-axis such that $AB = AC$.
Write down the coordinates of point C.

Answer (d) $C($ ..... , ... $)$ [1]

(e) Find the area of triangle ABC.

Answer (e) ..... [1]

(f) The line $x = 2$ intersects AB at the point D.
Find the coordinates of point D.

Answer (f) $D($ ..... , ... $)$ [1]

17. The scale drawing below shows the positions, on a map, of three houses A, B and C.

(a) Find the bearing of B from A.

Answer (a) .....° [1]

(b) On the diagram below construct the

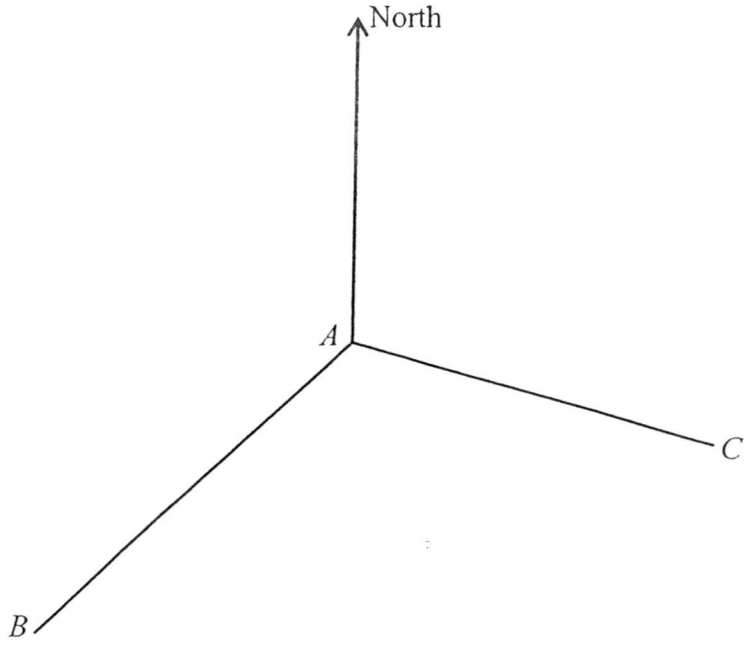
(i) perpendicular bisector of AB,

(ii) bisector of angle BAC.

(c) A house D is equidistant from the lines AB and AC and equidistant from the houses A and B.

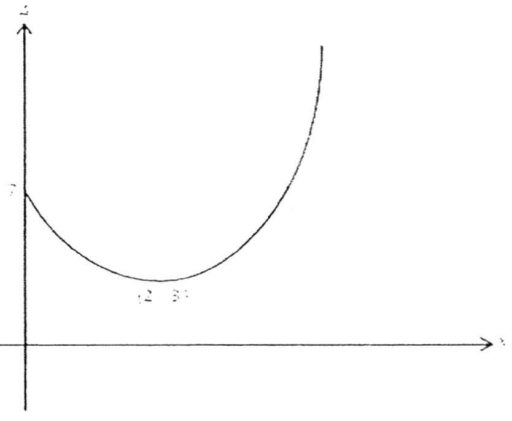
Mark the position of house D clearly on the diagram.

Answer (b)(i), (b)(ii), (c) [3]



End of Paper

One Page Answer Secondary 3 Express Mathematics Paper 1

1.	-37.2838812 $=-37.284$
2(a)	$xy=(5)(-5)=-25$
2(b)	$y=-5$
3.	$(2+a^2)(y+3)(y-3)$
4(a)	$-3\frac{3}{4}\le x<1\frac{1}{2}$
4(b)	$-3.-2,-1,0,1$
5	$\frac{5x+12}{(x+3)(x-2)}$ or $\frac{-5x-12}{(x+3)(2-x)}$
6(a)	By pythagoras' theorem $AB^2=x^2-4^2$ $AB=\sqrt{x^2-16}$
6(b)(i)	$\frac{\sqrt{x^2-16}}{x}$
6(b)(ii)	$-\frac{4}{x}$
7	\$47.25
8(a)	1 : 1200
8(b)	13.2 mm
9(a)	$x^2-4x+7=(x-2)^2+3$
9(b)	Minimum Area = 3 cm^2
9(c)	
10(a)	$\frac{9x^5}{8}$

10(b)	20
11(a)	4.5
11(b)	53.1°
12(a)	31
12(b)	10
13(a)	\$9.05
13(b)	Loss = \$228 − \$226 = \$2 Percentage = 0.877%
14(a)	130
14(b)	9
15(a)	$AF = CE$ (given) $AF - EF = CE - EF$ $AE = CF$ $DE = BF$ (given) $\angle AED = \angle CFB$ ($DE // BF$) $\Delta ADE \equiv \Delta CBF$ (SAS)
15(b)	$\Delta ADE \equiv \Delta CBF$ $AD = BC$ $\angle DAE = \angle BCF$ $AD // BC$ Hence, $ABCD$ is a parallelogram.
16(a)	$-\frac{4}{5}$
16(b)	6.40 units^2
16(c)	$y = -\frac{4}{5}x - \frac{8}{5}$
16(d)	(8,0)
16(e)	20 units^2
16(f)	$D = \left(2, -3\frac{1}{5}\right)$
17(a)	227°

ANSWER SCHEME SEC 3 EXPRESS END OF YEAR PAPER 1 MATHEMATICS

1. Calculate $\frac{3.28^2}{\sqrt{36.36}+5}-(9.327-\pi)^2$, giving your answer correct to 5 significant figures.

-37.2838812 $=-37.284$	B2: Award B1 for -37.2838812

2. Given that $-2<x\leq 5$ and $-6<y<2$, find

(a) the least value of xy,

$xy=(5)(-5)=-25$	B1

(b) the value of y when $y^2=25$.

$y=-5$	B1

3. Factorise $2y^2-18+(ay)^2-9a^2$ completely.

$2(y^2-9)+a^2(y^2-9)$ $=(2+a^2)(y^2-9)$ $=(2+a^2)(y+3)(y-3)$	 M1 A1

4. (a) Solve the inequalities $\frac{x-10}{5}\leq x+1<\frac{1}{3}x+2$.

$\frac{x-10}{5}\leq x+1$ and $x+1<\frac{1}{3}x+2$ $x-10\leq 5x+5$ and $3x+3<x+6$ $x\geq -3\frac{3}{4}$ and $x<1\frac{1}{2}$ $-3\frac{3}{4}\leq x<1\frac{1}{2}$	 M1 A1

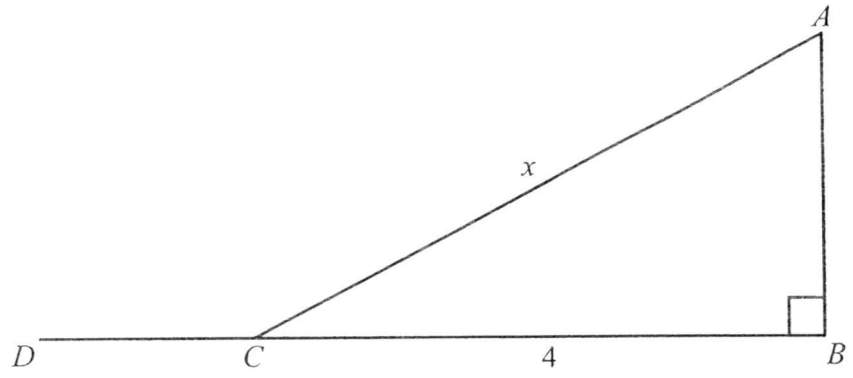
(b) Write down all the integers which satisfy $\frac{x-10}{5}\leq x+1<\frac{1}{3}x+2$.

$-3.-2,-1,0,1$	B1

5. Simplify $\frac{2x+3}{x^2+x-6}-\frac{3}{2-x}$.

$\frac{2x+3}{x^2+x-6}-\frac{3}{2-x}$ $=\frac{2x+3}{(x+3)(x-2)}-\frac{3}{2-x}$ $=\frac{2x+3}{(x+3)(x-2)}+\frac{3}{x-2}$ $=\frac{2x+3}{(x+3)(x-2)}+\frac{3(x+3)}{(x+3)(x-2)}$ $=\frac{5x12}{(x+3)(x-2)}$	$\frac{2x+3}{x^2+x-6}-\frac{3}{2-x}$ $=\frac{2x+3}{(x+3)(x-2)}-\frac{3}{2-x}$ or $=\frac{-(2x+3)}{(x+3)(2-x)}-\frac{3}{2-x}$ $=\frac{-2x-3}{(x+3)(x-2)}-\frac{3(x+3)}{(x+3)(2-x)}$ $=\frac{-5x-12}{(x+3)(2-x)}$	M1: Factorises x^2+x-6 M1: Creates common denominator and forms one fraction A1

6. In the triangle ABC, angle $ABC = 90°$, $AC = x$ cm, $BC = 4$ cm and BC is produced to D.



(a) Show that $AB = \sqrt{x^2-16}$.

By pythagoras' theorem $AB^2 = x^2 - 4^2$ $AB = \sqrt{x^2-16}$	B1

(b) Find in terms of x

(i) $\sin A\hat{C}B$,

(ii) $\cos A\hat{C}D$.

(b)(i) $\sin A\hat{C}B = \frac{\sqrt{x^2-16}}{x}$	B1
(b)(ii) $\cos A\hat{C}D = -\frac{4}{x}$	B1

7. Leonardo owns 540 cubes each of side 1cm, each costing \$0.25.
He decides to build a large cube using these existing cubes and additional cubes that he would purchase.
What is the least amount of money he has to spend to complete the large cube?

Next smallest possible cube = 729	M1
He needs $729 - 540 = 189$ more cubes	M1
Least amount to spend $= 189 \times \$0.25 = \$47.25$	A1

8. On a diagram, the length of a cruise ship is 300 mm.
It's actual length is 360 meters.
(a) Find the scale of the drawing in the form $1 : n$.

$300 \times 10^{-3} : 360$ $1 : 1200$	B1

(b) The ship has a swimming pool in the shape of a square on its deck.
The area of the pool is 250 m^2.
Find the length of one side of the pool on the diagram, giving your answer in mm.

1 mm : 1.2 m $1\ mm^2 : 1.44\ m^2$	
Area of pool on diagram $= \frac{250}{1.44}\ mm^2$	M1
Length of pool $= \sqrt{\frac{250}{1.44}} = 13.2$ mm	A1

9. The area of a rectangle, A cm^2, is given by $A = x^2 - 4x + 7$.
(a) Express $x^2 - 4x + 7$ in the form $(x - a)^2 + b$.

$x^2 - 4x + 7 = (x - 2)^2 + 3$	B1: -2 B1: 3

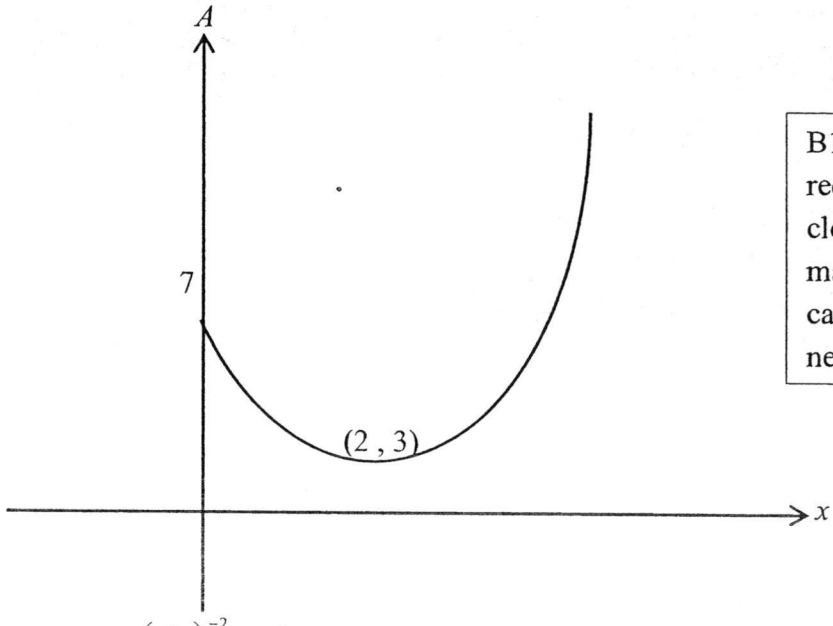
(b) State the minimum area of the rectangle.

Minimum Area = 3 cm^2	B1

(27

(c) Sketch the graph of $A = x^2 - 4x + 7$ on the axes below, clearly stating the turning point and the vertical intercept.

Answer (c)



B1: Curve and require points clearly labelled, no mark awarded if candidate draws in negative x region.

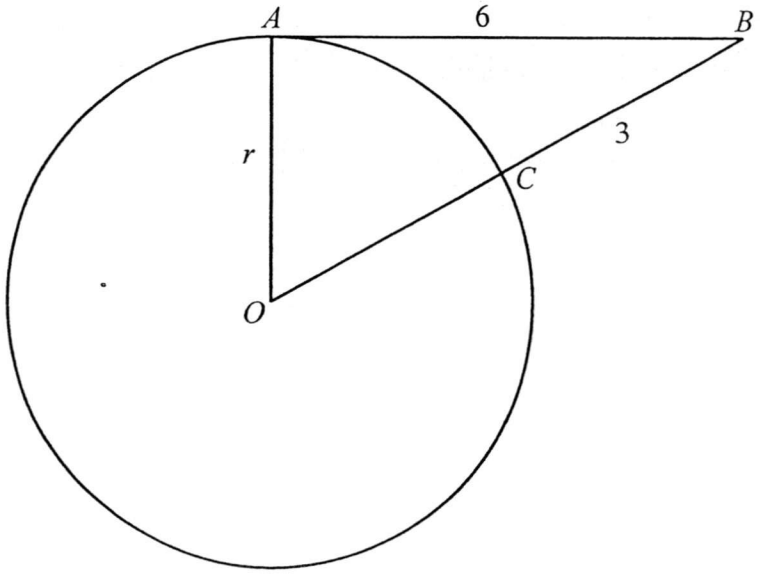
10. (a) Simplify $\left(\frac{2}{3x}\right)^{-2} \div \frac{2}{x^3}$.

$\left(\frac{9x^2}{4}\right) \times \frac{x^3}{2}$	M1: Obtains $\frac{9x^2}{4}$
$= \frac{9x^5}{8}$	A1

(b) Given that $3^{0.2x} - 3^3 = 54$, find x.

$3^{0.2x} = 81$	M1
$3^{0.2x} = 3^4$	
$0.2x = 4$	
$x = 20$	A1

11. The diagram shows a circle with centre O and radius r cm.
AB is tangent to the circle, C lies on the circumference of the circle, AB = 6 cm and BC= 3cm .



(a) Find the radius, r, of the circle.

By pythagoras' theorem $(r+3)^2 = r^2 + 36$ $r^2 + 6r + 9 = r^2 + 36$ $r = 4.5$	M1 A1

(b) Find angle AOB.

$\tan \hat{AOB} = \frac{6}{4.5}$ $\hat{AOB} = 53.1°$	M1: Forms appropriate trigo ratio. Accept cosine and sine. A1

12. There are 45 tickets in a lucky draw, some golden, some silver and the rest bronze.

A ticket is picked at random. The probability of drawing a golden ticket is $\frac{1}{9}$ and the probability of drawing a silver ticket is $\frac{1}{5}$.

(a) Find the number of bronze tickets in the lucky draw.

Probability of drawing a bronze ticket $=1-\frac{1}{9}-\frac{1}{5}$ $=\frac{31}{45}$ Number of bronze tickets $=\frac{31}{45}\times 45$ $=31$	M1 A1

(b) Before the draw begins, a sponsor adds an equal number of bronze and silver tickets to the box and the probability of drawing a golden ticket becomes $\frac{1}{13}$.

Find the number of bronze tickets added to the box.

Let the number of bronze and silver tickets each be x. $\frac{5}{2x+45}=\frac{1}{13}$ $2x+45=65$ $x=10$ 10 bronze tickets were added.	 M1 A1

13. A shopkeeper sells fruits at the prices shown in the table below.

Apples	40 cents each
Durians	\$3 per kg
Kiwis	\$2.20 each

(a) Adrian buys $2\frac{3}{4}$ kg of durians and 2 apples.

Calculate how much he pays.

Amount paid $= 2\frac{3}{4} \times \$3 + 2 \times \$0.40$ $= \$9.05$	B1

(b) The shopkeeper buys

100 apples for \$30,

40 kg of durians at \$2.20 per kg,

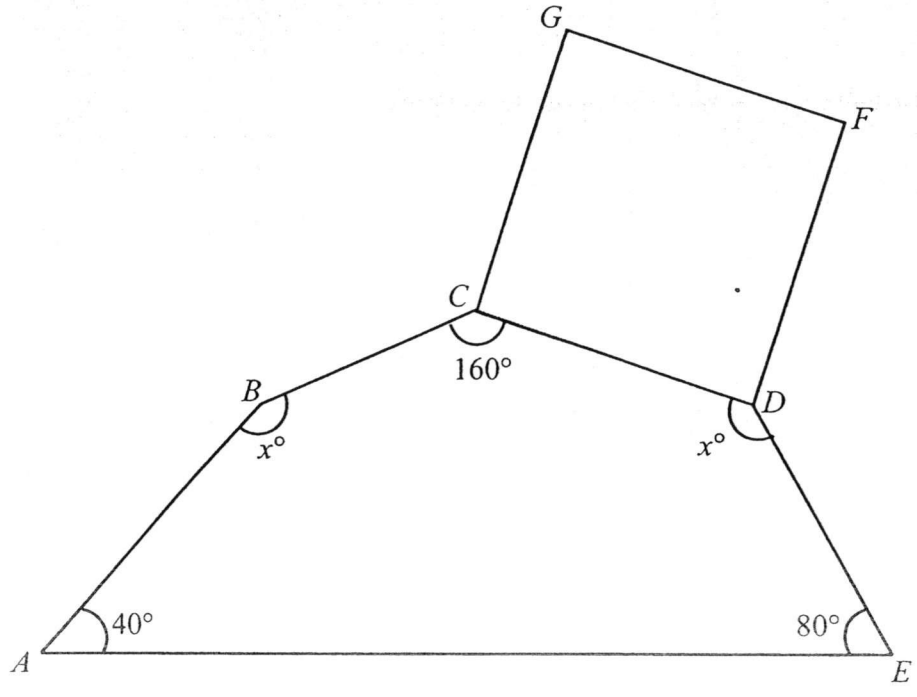
30 kiwis for \$60.

He sells all of the fruits at the prices shown in the table.

Express his profit as a percentage of his cost price.

Selling Price $= \$0.40 \times 100 + 40 \times \$3 + \$2.20 \times 30$ $= \$226$	
Cost Price $= \$30 + \$3.20 \times 40 + \$70$ $= \$228$	B1: Cost Price
Loss $= \$228 - \$226 = \$2$	B1: States that seller makes a loss of \$2
Percentage $= \frac{2}{228} \times 100\%$ $= 0.877\%$	A1

14. In the diagram, *ABCDE* is a pentagon, *CDFG* is a square and *FDE* is part of a regular polygon with n sides.



Find

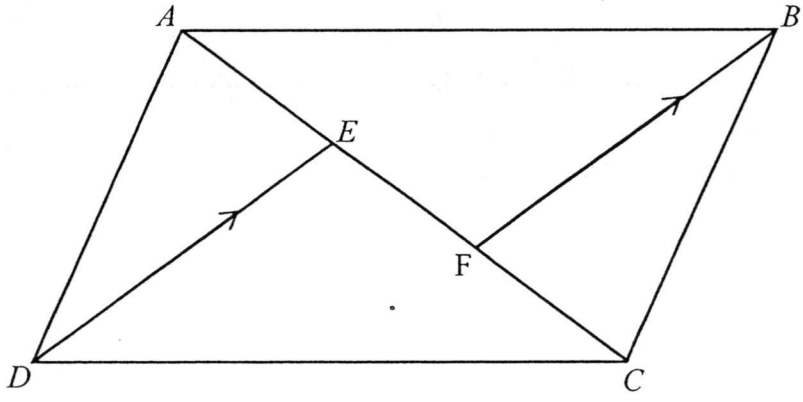
(a) the value of x.

Sum of interior angles of pentagon $=(5-2)\times 180°$ $=540°$	M1
$40° + x° + 160° + x° + 80° = 540°$ $2x° = 260°$ $x° = 130°$ $x = 130$	A1

(b) the value of n.

Angle FDE $= 360° - 90° - 130° = 140°$	M1
Exterior angle $= 180° - 140° = 40°$ $n = \dfrac{360°}{40°} = 9$	A1

15. In the diagram, $ABCD$ is a quadrilateral, $AF = CE$, $DE = BF$, $DE \parallel BF$ and E and F are points on AC.



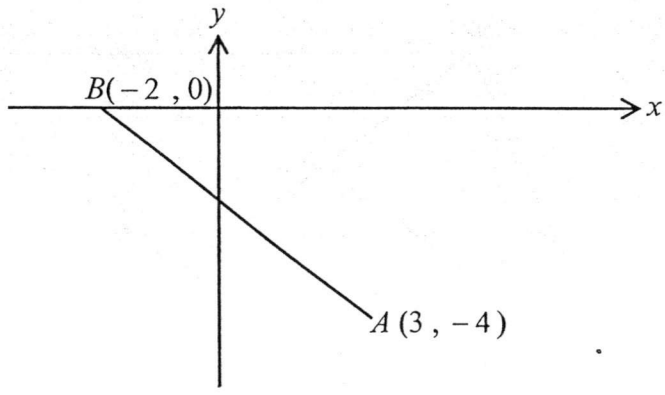
(a) Prove that triangle ADE is congruent to triangle CBF.

$AF = CE$ (given) $AF - EF = CE - EF$ $AE = CF$	B1: Proving $AE = CF$
$DE = BF$ (given) $\angle AED = \angle CFB$ ($DE \parallel BF$)	B1:Stating $DE = BF$ and $\angle AED = \angle CFB$ ($DE \parallel BF$)
$\Delta ADE \equiv \Delta CBF$ (SAS)	A1

(b) Using your result from part (a), show that $ABCD$ is a parallelogram.

$\Delta ADE \equiv \Delta CBF$ $AD = BC$ $\angle DAE = \angle BCF$	B1: Stating $AD = BC$
$AD \parallel BC$ Hence, $ABCD$ is a parallelogram.	B1: Showing $AD \parallel$ BC

16. The diagram shows the points $A\ (3\ ,\ -4)$ and $B(-2\ ,\ 0)$.



Find

(a) the gradient of AB,

$m = \dfrac{0-(-4)}{-2-3} = -\dfrac{4}{5}$	B1

(b) the length of AB,

Length $= \sqrt{(0-(-4))^2 + (-2-3)^2}$ $= \sqrt{41}$ $= 6.40$ units^2	B1

(c) the equation of the line AB,

$y = -\dfrac{4}{5}x + c$ Substitute $(-2,0)$ $0 = \dfrac{8}{5} + c$ $c = -\dfrac{8}{5}$ $y = -\dfrac{4}{5}x - \dfrac{8}{5}$	B1: o.e.

(d) The point D lies on the x-axis such that $AB = AC$.

Write down the coordinates of point C.

(8,0)	B1

(e) Find the area of triangle ABC.

$\dfrac{1}{2} \times 10 \times 4 = 20$ units^2	B1

(f) The line $x = 2$ intersects AB at the point D.

Find the coordinates of D.

Sub $x = 2$ into $y = -\frac{4}{5}x - \frac{8}{5}$ $y = -\frac{16}{5}$ $D = \left(2, -3\frac{1}{5}\right)$	B1