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SWISS COTTAGE SECONDARY SCHOOL
SECONDARY THREE EXPRESS
SECOND SEMESTRAL EXAMINATIONS

Name: _____( ) Class: Sec 3 E _____

MATHEMATICS

Paper 1

4048/01
Thursday 1 October 2015
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This question paper consists of **18** printed pages.

Setter: Mrs Chen Yen Wah
Vetter: Mr Ang Hanping

[Turn over 148

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved Surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Calculate $e^{\pi} - \left(\dfrac{-(0.25)^2 + 4.284}{3.87 - \dfrac{45}{4}} \right)^{\frac{1}{3}}$.

Give your answer correct to 3 significant figures.

Answer ..... [1]

---

2 Express as a single fraction in its simplest form $\dfrac{3}{3-s} + \dfrac{3s}{s^2 - 7s + 12}$.

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Answer ..... [3]

---

3 When written as the product of their prime factors,

$$a = 2^3 \times 3^2 \times 5$$

$$b = 2^2 \times 5^2 \times 11.$$

Find

(i) the largest integer which is a factor of a and b,

Answer ..... [1]

(ii) the smallest positive integer n for which nb is a multiple of a,

Answer $n =$ ..... [1]

(iii) the smallest positive integer k for which $\sqrt[3]{ak}$ is a whole number.

Answer $k =$ ..... [1]

---

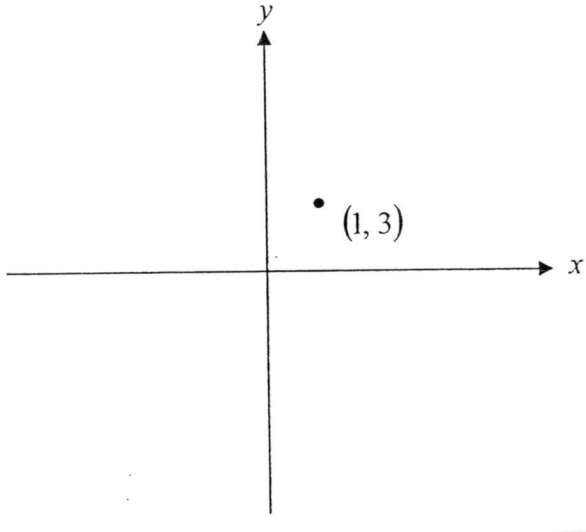
4 Sketch the graphs of the following equations in the same axes shown below.

The point $(1, 3)$ is marked on the graph.

(i) $y = 3^x$,

(ii) $y = \dfrac{3}{x^2}$.

Answer



[2]

---

5 The population of China is approximately 1.39×10^{9}. The population of the United States of America (USA) is approximately 3.25×10^{8}.

(i) How many more people live in China than in USA? Give your answer in standard form.

Answer ..... people [1]

(ii) Express the population of USA as a percentage of population of China.

Answer ..... % [2]

---

6 Factorise completely

(a) $6ax-12by-8bx+9ay$,

Answer ..... [2]

(b) $(s+2t)^2-2(s+2t)-15$.

150

Answer ..... [3]

---

7 An area of $25\,cm^2$ on a map represents an actual area of $6.25\,km^2$.

Find

(i) the area on the map, in square centimetres, which represents an actual area of $84\,km^2$.

Answer ..... cm^2 [1]

(ii) n, where $1 : n$ is the scale of the map.

Answer $n =$ ..... [2]

(iii) the actual distance, in kilometres, represented by a length of 8.5 cm.

Answer ..... km [1]

8 If $\frac{4a+5b}{3b+a}=2$, find the value of $\frac{a}{b}$.

Answer $\frac{a}{b}=$ ..... [2]

---

9 Solve the simultaneous equations

$$x-2y=9,$$

$$-2x-y=-8.$$

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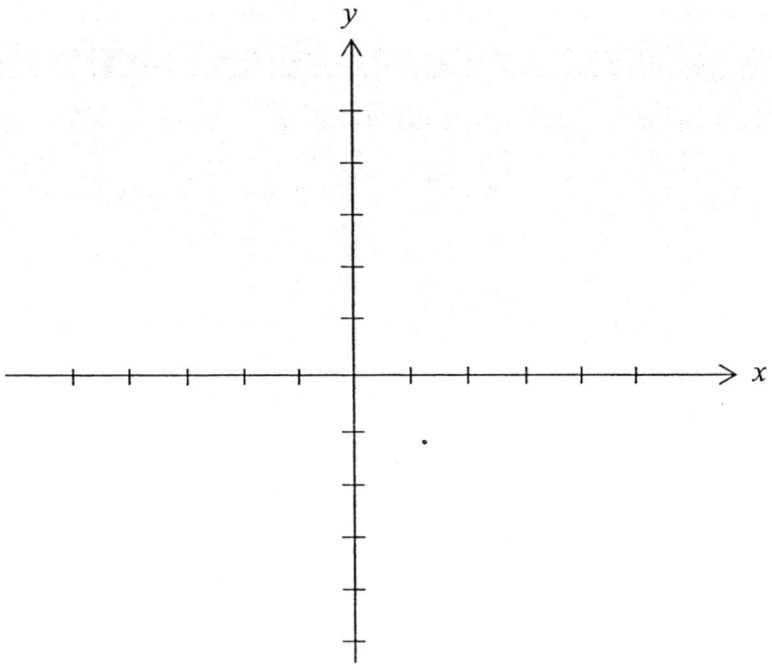
Answer $x=$ ..... $y=$ ..... [3]

---

10 **(i)** Sketch the graph of $y = (x+1)(x-3)$.

Answer



[2]

(ii) Write down the equation of the line of symmetry of the graph of $y = (x+1)(x-3)$.

Answer ..... [1]

---

11 **(i)** Express $x^2 - 4x - 1$ in the form $(x-a)^2 + b$.

Answer ..... [2]

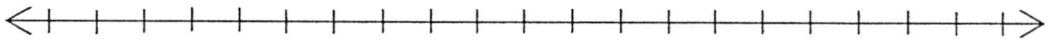
(ii) Hence, solve the equation $x^2 - 4x - 1 = 0$, giving your answers correct to two decimal places.

Answer $x =$ ..... or $x =$ ..... [2]

---

12 **(i)** Solve the inequality $\frac{7x+4}{6} < x-1 \leq \frac{x+17}{3}$.

Show your solution on the number line below.



[1]

Answer ..... [3]

(ii) Hence, write down the greatest integer value of x which satisfies

$\frac{7x+4}{6} < x-1 \leq \frac{x+17}{3}$.

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Answer $x =$ ..... [1]

13 Simplify the following, leaving your answers in positive index form.

(a) $\dfrac{\left(2p^4\right)^2}{16p^3q^2} \times \dfrac{\left(pq^{-1}\right)^4}{\left(p-3q^7\right)^0}$,

Answer ..... [2]

(b) $2a^3 \div a^{-1} \times \sqrt[3]{a^6}$.

Answer ..... [2]

14 **(a)** Solve $(2x-1)(x+2)=11$, giving your answers correct to 2 decimal places.

Answer $x =$ ..... or $x =$ ..... [3]

(b) Solve $\dfrac{1}{2(3y-1)}+\dfrac{3}{(1-3y)}=4$.

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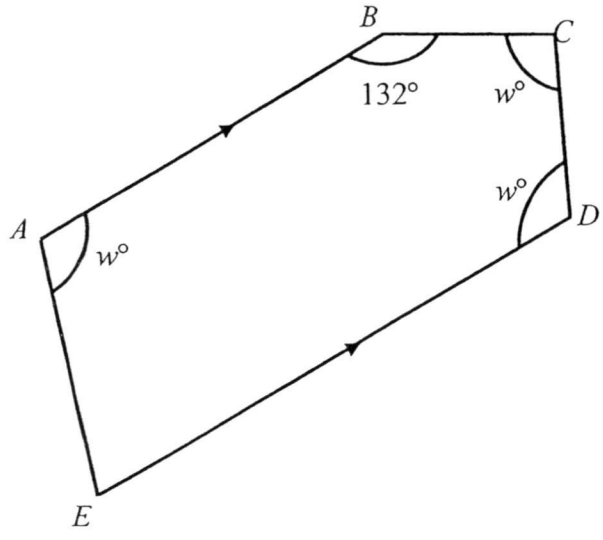
Answer y ..... [2]

15 **(a)** Each interior angle of a 20-sided regular polygon is 5.4 times each exterior angle of an n-sided polygon. Find the value of n.

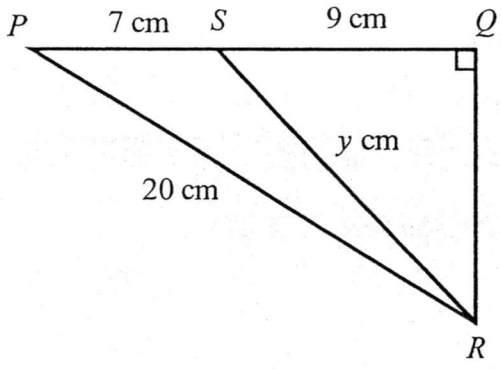
Answer n ..... [3]

(b) In the diagram, $ABCDE$ is a pentagon in which AB is parallel to ED. Given that angle $ABC = 132°$ and angle EAB = angle BCD = angle $CDE = w°$, calculate the value of w. State your reasons clearly.



Answer $w =$ ..... [3]

16



In the right-angled triangle ΔPSQ is a straight line. Given that $RS = y$ cm, $PS = 7$ cm, $SQ = 9$ cm and $PR = 20$ cm, find

(i) the value of y,

Answer ..... [2]

(ii) the exact value of $\cos \angle PSR$,

Answer ..... [1]

(iii) the perpendicular height from Q to PR.

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Answer ..... cm [2]

17 $\xi = \{x : x \text{ is an integer and } 1 \leq x \leq 14\}$

$A = \{x : x \text{ is a factor of } 12\}$

$B = \{x : x \text{ is divisble by } 3\}$

(i) Illustrate with a Venn diagram, the sets ξ, A and B.

[2]

(ii) List the elements contained in the set $(A \cup B)'$.

Answer ..... [1]

(iii) Write down $n(A \cap B)'$.

Answer ..... [1]

(iv) A number k is chosen at random from ξ. Find the probability that $k \in (A \cap B')$.

Answer ..... [1]

18 **(a)** p varies directly as the square root of q.

If q is increased by 300%, find the percentage increase in p.

Answer ..... % [3]

(b) A swimming pool is filled up with water by 4 taps. In order for the pool to be filled up 2 minutes faster, an additional tap must be used. If the time, t minutes, required to fill the pool varies inversely as the number of taps, n, used, how many additional taps must be used for the pool to be filled up 5 minutes faster?

155

Answer ..... taps [3]

19 (a) A group of students were surveyed on the distances they ran in a week. The results are represented in the table below.

Distance (km)	5	10	15	20	25
Number of boys	2	1	5	1	x

(i) Write down the greatest possible value of x given that the mode is 15 km.

Answer $x =$ ..... [1]

(ii) Write down the value of x given that the median is 20 km.

Answer $x =$ ..... [1]

(b) Fifteen boys and twenty girls took a Mathematics examination. The marks are shown in the stem-and-leaf diagram.

Boys									Girls					
					8	3	9							
						4	2	4	8					
				9	9	5	1	2	4	5				
		9	8	7	4	6	1	1	5	9	9			
9	8	6	5	3	1	7	1	3	5	6	9			
					2	8	2							
					0	9	0							

Key (Boys)
8| 3 means 38

Key (Girls)
3| 9 means 39

(i) Calculate the mean of the boys' marks.

Answer ..... marks [2]

(ii) Explain briefly whether the boys or the girls performed better in the examination.

.....

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..... [2]

20 The scale drawing shows an area of a city where town A is situated.

The scale is 1 cm to 2 km.

(i) Town B is on a bearing $038°$ from A and the distance of B from A is 10 km. Mark and label on the diagram the position of Town B. [1]

(ii) Town C is 18 km from Town B and angle ABC is $80°$. Mark and label on the diagram the position of Town C. [1]

(iii) A playground is to be built equidistant from AB and BC and equidistant from A and C. By making appropriate constructions, indicate clearly, on the diagram, the position of the playground. Label this point X. [3]

End of Paper

SCSS 2015 SA2 3E P1 Answer Key

1 24.0

2 $\dfrac{-12}{(3-s)(s-4)}$

3i 20

3ii 18

3iii 75

5i 1.065×10^{9}

5ii 23.4%

6a $(2x+3y)(3a-4b)$

6b $(s+2t-5)(s+2t+3)$

7i 336 cm^2

7ii $n=50000$

7iii 4.25 km

8 $\dfrac{a}{b}=\dfrac{1}{2}$

9 $y=-2,\ x=5$

10ii $x=1$

11i $(x-2)^2-5$

11ii $x=4.24$ (2dp) or $x=-0.24$ (2dp)

12i $x<-10$

12ii $x=-11$

13a $\dfrac{p^9}{4q^6}$

13b $2a^6$

14a $x=1.91$ (2dp) or $x=-3.41$ (2dp)

14b $y=\dfrac{1}{8}$

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15a $n=12$

15b $w=114$

16i $y=15\,\text{cm}$

16ii $\cos\angle PSR=-\frac{3}{5}$

16iii $QX=9.6$

17ii $(A\cup B)'=\{5,7,8,10,11,13,14\}$

17iii $n(A\cap B)'=11$

17iv $\frac{3}{14}$

18a 100%

18b 4

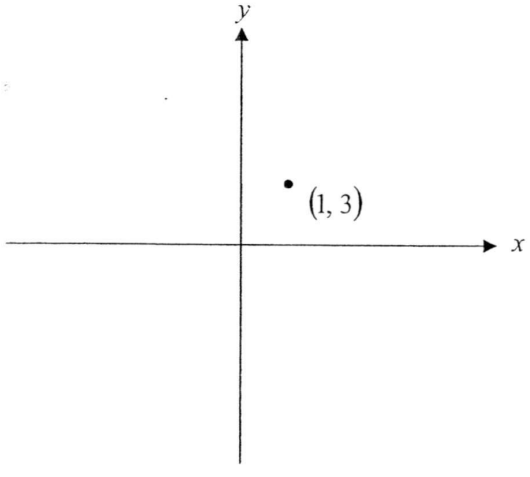
19ai $x=4$

19aii $x=8$

19bi 69.9

19bii 62.8. The boys performed better because they have a higher mean score.

SCSS 2015 SA2 3E P1 Marking Scheme

Que	Answer	Marks
1	$e^{\pi} - \left(\dfrac{-(0.25)^2 + 4.284}{3.87 - \dfrac{45}{4}} \right)^{\frac{1}{3}} = 24.0$ (1sf)	B1
2	$\dfrac{3}{3-s} + \dfrac{3s}{s^2 - 7s + 12}$ $= \dfrac{3}{3-s} + \dfrac{3s}{(s-4)(s-3)}$ $= \dfrac{3}{3-s} - \dfrac{3s}{(3-s)(s-4)}$ $= \dfrac{3(s-4) - 3s}{(3-s)(s-4)}$ $= \dfrac{3s - 12 - 3s}{(3-s)(s-4)}$ $= \dfrac{-12}{(3-s)(s-4)}$ or $= \dfrac{12}{(s-3)(s-4)}$	M1 – Factorise denominator M1 – Single fraction A1
3i	HCF $= 2^2 \times 5 = 20$	B1
3ii	$n = 2 \times 3^2 = 18$	B1
3iii	$k = 3 \times 5^2 = 75$	B1
4		B1 – $y = 3^x$ B1 – $y = \dfrac{3}{x^2}$

/8

Que	Answer	Marks
5i	Difference $= 1.39\times10^{9} - 3.25\times10^{8} = 1.065\times10^{9}$	B1
5ii	$\dfrac{3.25\times10^{8}}{1.39\times10^{9}}\times100$ $= 23.4\%$	M1 A1
6a	**Method 1** $6ax - 12by - 8bx + 9ay$ $= (6ax - 8bx) - (12by - 9ay)$ $= 2x(3a - 4b) + 3y(3a - 4b)$ $= 2x(3a - 4b) + 3y(3a - 4b)$ $= (2x + 3y)(3a - 4b)$ **Method 2** $6ax - 12by - 8bx + 9ay$ $= 6ax - 8bx + 9ay - 12by$ $= 2x(3a - 4b) + 3y(3a - 4b)$ $= (2x + 3y)(3a - 4b)$	M1 (2 Common factors) A1 M1 (2 Common factors) A1
6b	$(s + 2t)^2 - 2(s + 2t) - 15$ Let $a = (s + 2t)$ $a^2 - 2a - 15$ $= (a - 5)(a + 3)$ Substituting $a = (s + 2t)$, $(s + 2t)^2 - 2(s + 2t) - 15 \quad = (s + 2t - 5)(s + 2t + 3)$	 M1 M1 A1
7i	Area scale Map : Actual $25\ \text{cm}^2 : 6.25\ \text{km}^2$ $1\ \text{cm}^2 : 0.25\ \text{km}^2$ Area on map $= \dfrac{1}{0.25}\times84$ $= 336\ \text{cm}^2$	 A1

Que	Answer	Marks
7ii	<u>Area scale</u> Map : Actual $1\,\text{cm}^2 : 0.25\,\text{km}^2$ <u>Length scale</u> Map : Actual $\sqrt{1\,\text{cm}^2} : \sqrt{0.25\,\text{km}^2}$ $1\,\text{cm} : 0.5\,\text{km}$ $1 : 50000$ $n = 50000$	 M1 A1
7iii	Actual length $= 8.5 \times 0.5 = 4.25\,\text{km}$	A1
8	$\frac{4a+5b}{3b+a} = 2$ $4a+5b = 6b+2a$ $2a = b$ $\frac{a}{b} = \frac{1}{2}$	 M1 – Cross multiply A1
9	$x - 2y = 9$ - Eq (1) $-2x - y = -8$ - Eq (2) From (1), $x = 9 + 2y$ - Eq (3) Sub (3) into (2), $2(9+2y) + y = 8$ $18 + 4y + y = 8$ $5y = -10$ $y = -2$ $x = 9 + 2(-2) = 5$	 M1 – Substitution A1 A1

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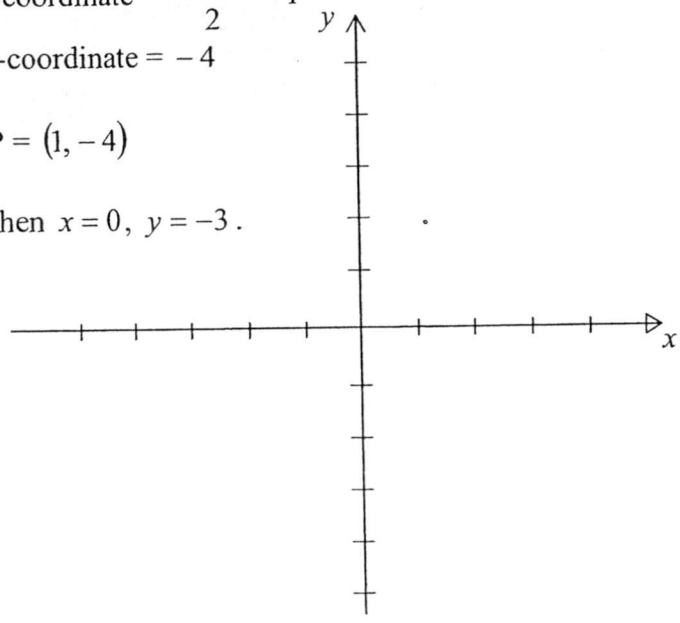
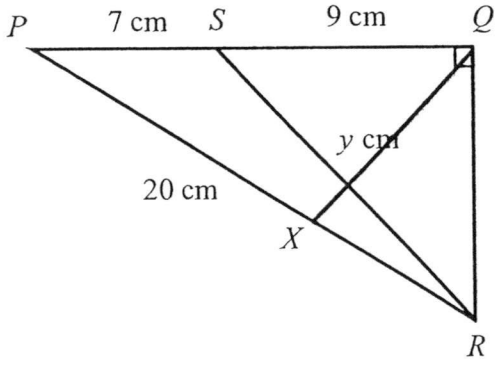
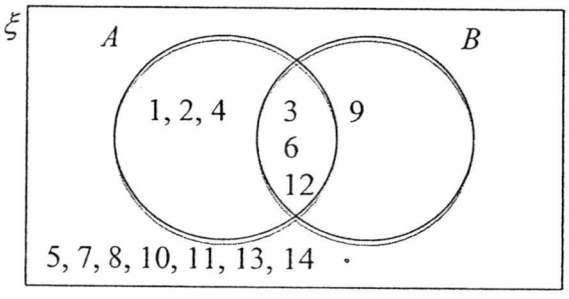
Que	Answer	Marks
10i	x-coordinate $= \dfrac{-1+3}{2} = 1$ y-coordinate $= -4$ TP $= (1, -4)$ When $x = 0$, $y = -3$. 	B1 – Intercepts B1 – Turning Point
10ii	$x = 1$	B1
11i	$x^2 - 4x - 1 = x^2 - 4x + \left(-\dfrac{4}{2}\right)^2 - \left(-\dfrac{4}{2}\right)^2 - 1$ $= (x-2)^2 - 5$	M1 A1
11ii	$(x-2)^2 - 5 = 0$ $(x-2)^2 = 5$ $x - 2 = \pm\sqrt{5}$ $x = \sqrt{5} + 2$ or $x = -\sqrt{5} + 2$ $x = 4.24$ (2dp) or $x = -0.24$ (2dp)	M1 A1
12i	$\dfrac{7x+4}{6} < x - 1$ or $x - 1 \le \dfrac{x+17}{3}$ $7x + 4 < 6x - 6$ $x < -10$ $3x - 3 \le x + 17$ $2x \le 20$ $x \le 10$ Therefore, $x < -10$.	M1 M1 A1 B1 – number line

Que	Answer	Marks
12ii	Greatest integer value of $x = -11$	B1
13a	$\frac{(2p^4)^2}{16p^3q^2} \times \frac{(pq^{-1})^4}{(p-3q^7)^0}$ $= \frac{4p^8}{16p^3q^2} \times \frac{p^4q^{-4}}{1}$ $= \frac{p^9}{4q^6}$	M1 – Simplification A1
13b	$2a^3 \div a^{-1} \times \sqrt[3]{a^6}$ $= 2a^3 \div a^{-1} \times a^{\frac{6}{3}}$ $= 2a^{3+1+2}$ $= 2a^6$	M1 – Simplification A1
14a	$(2x-1)(x+2) = 11$ $2x^2 + 4x - x - 2 - 11 = 0$ $2x^2 + 3x - 13 = 0$ $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-13)}}{2(2)}$ $x = 1.91$ (2dp) or $x = -3.41$ (2dp)	M1 A1/A1
14b	$\frac{1}{2(3y-1)} + \frac{3}{(1-3y)} = 4$ $\frac{1}{2(3y-1)} - \frac{3}{(3y-1)} = 4$ $1 - 6 = 8(3y-1)$ $8(3y-1) = -5$ $3y - 1 = -\frac{5}{8}$ $y = \frac{1}{8}$	M1 A1
15a	One interior angle $= \frac{(20-2)\times 180}{20} = 162°$ One exterior angle $= \frac{360}{n}$ $162 = 5.4 \times \frac{360}{n}$	M1 M1 160

Que	Answer	Marks
	$\frac{360}{n} = 30$ $n = 12$	A1
15b	Sum of interior angles of pentagon $= (5-2)\times 180$ $= 540°$ $\angle AED = 180° - w°$ $132 + 3w + 180 - w = 540$ $2w = 228$ $w = 114$	M1 M1 A1
16i	$QR = \sqrt{20^2 - 16^2} = 12$ cm $y = \sqrt{12^2 + 9^2} = 15$ cm	M1 A1
16ii	$\cos \angle PSR = -\cos \angle QSR = -\frac{9}{15} = -\frac{3}{5}$	B1
16iii	 Area of $\Delta PQR = \frac{1}{2}\times 16\times 12$ $= 96$ cm^2 $\frac{1}{2}\times QX \times 20 = 96$ $QX = 9.6$ cm	M1 A1
17i	$\xi = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$ $A = \{1,2,3,4,6,12\}$ $B = \{3,6,9,12\}$	

Que	Answer	Marks
		
17ii	$(A \cup B) = \{1,2,3,4,6,9,12\}$ $(A \cup B)' = \{5,7,8,10,11,13,14\}$	B1
17iii	$(A \cap B) = \{3,6,12\}$ $(A \cap B)' = \{1,2,4,5,7,8,9,10,11,13,14\}$ $n(A \cap B)' = 11$	B1
17iv	$B' = \{1,2,4,5,7,8,10,11,13,14\}$ $A = \{1,2,3,4,6,12\}$ $(A \cap B') = \{1,2,4\}$ Probability $= \frac{3}{14}$	B1
18a	$p = k\sqrt{q}$ Let original value of q be a. $p = k\sqrt{a}$ New $q = 4a$, new $p = k\sqrt{4a}$ Percentage increase $= \frac{k\sqrt{4a} - k\sqrt{a}}{k\sqrt{a}} \times 100$ $= \frac{2k\sqrt{a} - k\sqrt{a}}{k\sqrt{a}} \times 100$ $= \frac{k\sqrt{a}}{k\sqrt{a}} \times 100$ $= 100\%$	M1 M1 A1
18b	Let number of taps be n and time taken be t. $n = \frac{k}{t}$ When $n = 4$, $t = x$. Amount of work $k = 4x$	161

Que	Answer	Marks
	When $n=5$, $t=x-2$ $k=5(x-2)$ $5(x-2)=4x$ $5x-10=4x$ $x=10$ min $k=40$ When $t=x-5$, $n=\frac{40}{5}=8$ taps Additional taps = 4	 M1 M1 A1
19ai	$x=4$	B1
19ai	Median position $\frac{10+x}{2}=9$ $x=8$	A1
19bi	Mean of boys $=\frac{38+59+59+64+67+68+69+...}{15}$ $=69.9$	M1 A1
19bii	Mean of girls $=\frac{39+42+44+48+...}{20}$ $=62.8$ The boys performed better because they have a higher mean score.	M1 A1

Que	Answer	Marks
20	The scale drawing shows an area of a city and where town A is situated. The scale is 1 cm to 2 km.  	

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SWISS COTTAGE SECONDARY SCHOOL
SECONDARY THREE EXPRESS
SECOND SEMESTRAL EXAMINATIONS

Name: _____( ) Class: Sec 3E____

MATHEMATICS

Paper 2

4048/02
Monday 5 October 2015
2 hours 30 minutes

Additional materials: Answer Paper (8 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Paper provided.
Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together with the Summary Page in front of your Answer Script. Submit Section A and B separately.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of **11** printed pages.

Setter: Ms Tan Hui Lan
Vetter: Mr Ang Han Ping

[Turn over

163

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area } \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions.

Section A (39 marks)

1 **(a)** Simplify $25a^2 + 4 - (5a - 4)^2$. [2]

(b) It is given that $p = \dfrac{q - r^2}{5q^2 + 7r^2}$.

(i) Find p when $q = \dfrac{1}{2}$ and $r = -\dfrac{1}{3}$. [1]

(ii) Express r in terms of p and q. [3]

(c) Simplify $\dfrac{7m^2 - 28}{m^2 + 2m}$. [2]

(d) Given that $8^6 \div 4^{2-n} = 16^n$, find the value of n. [2]

---

2 Anthony is planning to drive during his trip in Australia. He researches and found three possible routes.

(a) If Anthony travels on route A, which is 325 km long, he expects to cover x km per day.

Route B, which is the same distance as route A, has more difficult driving conditions and Anthony expects to cover $\left(x - \dfrac{1}{2}\right)$ km per day.

Route C, which is 15 km longer than Routes A and B, has the easiest driving conditions and Anthony expects to cover $(x + 15)$ km per day.

Write down an expression, in terms of x, for the number of days Anthony expects to take to complete

(i) route A,

(ii) route B,

(iii) route C. [2]

(b) Anthony estimates that he will reach his destination 2 days earlier if he travels on route C rather than route B.

Form an equation in x, and show that it reduces to $x^2 + 22x - 2530 = 0$. [3]

(c) Solve the equation $x^2 + 22x - 2530 = 0$, giving your answers to 2 decimal places. [4]

(d) Calculate the number of days Anthony will take if he travels on route B. [1]

---

3 **(a)** The cash price of a vase is \$2399. Patricia bought the vase on hire purchase. She pays a deposit of \$1500 followed by 12 monthly instalments of \$165.20.

(i) What is the total amount Patricia will pay for the vase? [2]

(ii) Express the extra cost of buying the vase on hire purchase as a percentage of the cash price. [2]

(b) Patricia sold the vase to Amanda at a profit of 125% of the amount she paid for it.

(i) Find the selling price. [1]

(ii) To pay for the vase, Amanda took a loan on the selling price for 3 years at a compound interest rate of 1.5% per annum compounded half-yearly. Calculate how much interest Amanda will pay. [2]

(c) Mr Chia has an annual income of \$42 000. He is entitled to a total tax relief of \$2300. Using the income tax rates table below, calculate the total amount of tax Mr Chia has to pay.

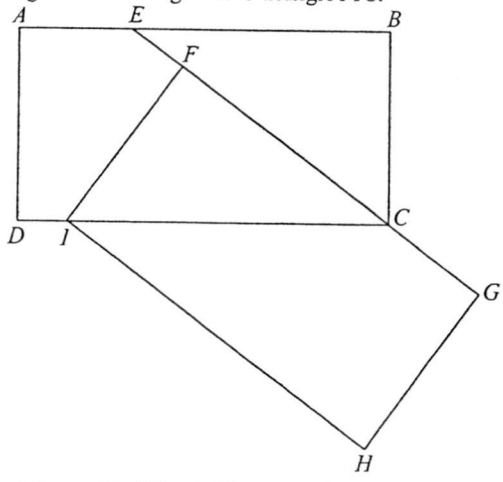
Chargeable Income	Rate (%)	Gross Tax Payable
On the first \$20 000	0	0
On the next \$10 000	3.5	350
On the first \$30 000		350
On the next \$10 000	5.5	550
On the first \$40 000		900
On the next \$40 000	8.5	3400

[3]

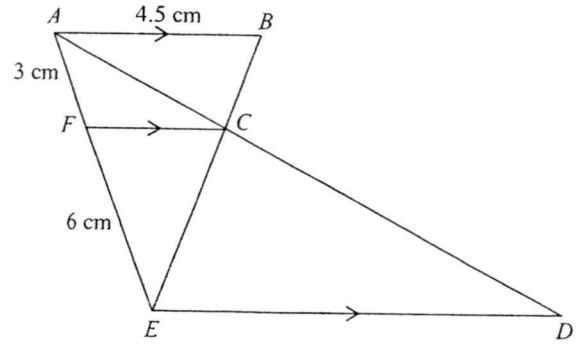
---

4 **(a)** *ABCD* and *FGHI* are two identical rectangles. *EFCG* is a straight line.

Prove that triangle *BCE* is congruent to triangle *FIC*. [3]



(b) In the diagram below, *AB*, *FC* and *ED* are parallel to one another. *ACD*, *AFE* and *BCE* are straight lines. $AB = 4.5$ cm, $AF = 3$ cm and $FE = 6$ cm.



(i) Triangles *ABE* and *FCE* are similar. Name two other pairs of similar triangles. [2]

(ii) Find the length of

(a) *CF*, [1]

(b) *DE*. [1]

(iii) Write down the numerical value of

(a) $\dfrac{\text{area of } \Delta ABE}{\text{area of } \Delta FCE}$, [1]

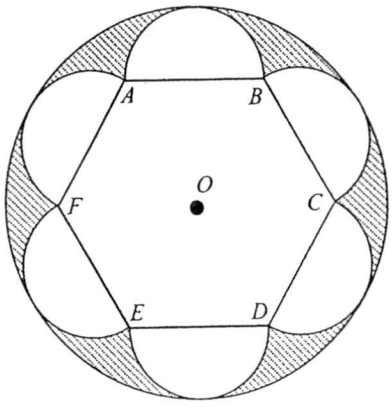
(b) $\dfrac{\text{area of } \Delta AFC}{\text{area of } \Delta AEC}$. [1]

Answer **all** questions.

Section B (61 marks)

Begin Section B on a fresh sheet of paper.

5 The diagram below shows a pendant made of wire. It consists of a regular hexagon $ABCDEF$, six identical semicircles with diameter 6 cm and an outer circle, centre O. Each semicircle touches the outer circle as shown.



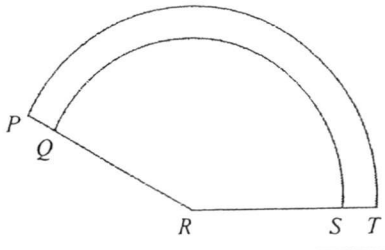
(a) Find the length of the perpendicular from O to ED. [3]

(b) Show that the radius of the outer circle is 8.196 cm, correct to 3 decimal places. [1]

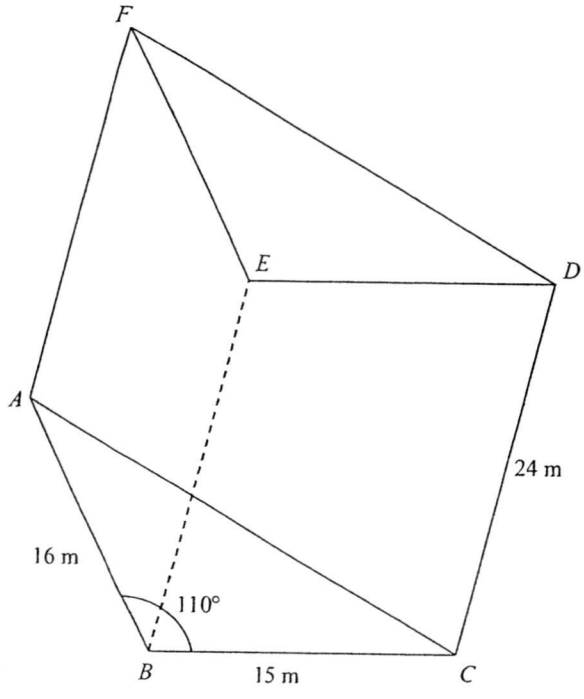
(c) Find the total length of wire required to make the pendant. [3]

(d) Find the area of the shaded region. [3]

(e) The wire is then recrafted to form the two sectors PRT and QRS with centre R as shown below. It is given that $QR = 17$ cm and $PQ = 5$ cm. Find the value of $\angle PRT$ in degrees. [2]



6 The diagram shows a ramp $ABCDEF$ in the form of a triangular prism. $AB = 16\,\text{m}$, $BC = 15\,\text{m}$, $CD = 24\,\text{m}$, $\angle ABC = 110°$ and $BCDE$ lies on horizontal ground.



(a) Find the length of AC. [3]

(b) Find $\angle ACB$. [2]

(c) Calculate the volume of the ramp. [3]

(d) A pole of 18 m stands vertically at point D. Calculate the angle of elevation of the top of the pole from E. [2]

(e) All faces of the ramp, including its base, are covered with a layer of paint which costs \$30.25 per litre. Given that each litre of paint can cover an area of 450 cm^2. calculate the total amount of money required to paint the ramp. [3]

7 The variables x and y are connected by the equation $y = 16 - x - 2x^2$. The table below shows some values of x and their corresponding values of y.

x	-3	-2	-1	0	1	2	2.5
y	1	p	15	16	q	6	1

(a) Find the values of p and q. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-3 \le x \le 2.5$.

Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $0 \le y \le 16$.

On your axes, plot the points in the table and join them with a smooth curve. [3]

Comment [M1]: I changed the range of y values to avoid confusion in exam. Hope it is ok.

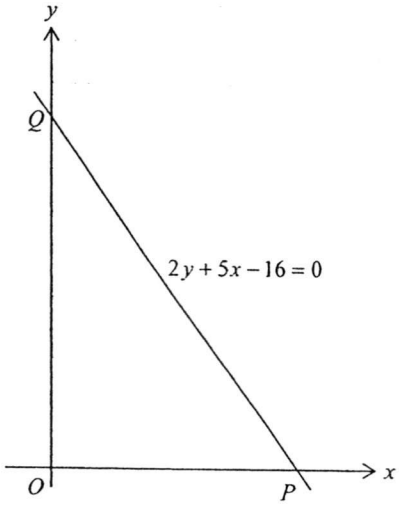
(c) Use your graph to find the value of y when $x = -1.5$. [1]

(d) By drawing a tangent, find the gradient of the curve at $(-1, 15)$. [2]

(e) Use your graph to find the maximum value of y and the value of x for which this occurs. [2]

(f) By drawing a suitable line on the same axis, use your graph to find the solutions of the equation $0 = 11 - 2x^2$ [2]

8 In the diagram below, a straight line, l_1, with the equation $2y + 5x - 16 = 0$ cuts the x-axis at P and the y-axis at Q.



Find

(a) the coordinates of P and Q, [2]

(b) the area of ΔPOQ, [2]

(c) the length of PQ, [2]

(d) the shortest distance from O to PQ, [2]

(e) the equation of another line, l_2 which passes through the origin and is parallel to line l_1. [1]

9 The table below shows the number of cups of coffee, tea and milo sold by three different stalls in a span of an hour.

	Coffee	Tea	Milo
Stall *A*	40	35	15
Stall *B*	28	31	7
Stall *C*	11	16	23

Each stall sold the beverages at a common price of \$1.10 for a cup of coffee, \$0.90 for a cup of tea and \$1.30 for a cup of milo.

(a) Write down a matrix **X** to represent the number of different types of beverages sold by the three stalls in a span of an hour. [1]

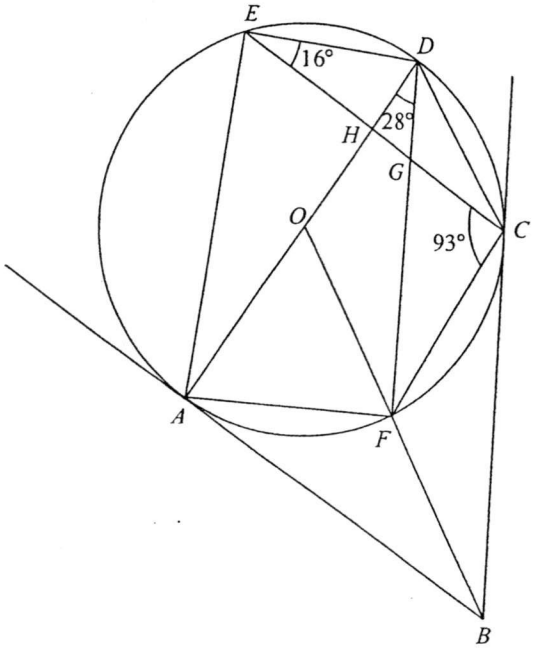
(b) Write down a column matrix **Y** representing the cost of each type of beverage. [1]

(c) Evaluate **Z** = **XY**. [1]

(d) Describe what the elements of **Z** represent. [1]

(e) In one day, Stall *A* operates for 8 hours, Stall *B* operates for 6 hours and Stall *C* operates for 12 hours. If each stall sells the same number of beverages per hour as shown in the table above, use matrix multiplication to calculate the total amount of money collected by all three stalls in that day. [2]

10\ In the diagram below, AB and CB are tangents to the circle with centre O at A and C respectively. A, F, C, D and E are points on the circumference of the circle and $EHGC$, AOD and OFB are straight lines.



Given that $\angle DEC = 16°$, $\angle ADF = 28°$ and $\angle ECF = 93°$, find with clearly stated reasons

(a) $\angle OAB$, [1]

(b) $\angle DOF$, [1]

(c) $\angle OBC$, [2]

(d) $\angle DCF$, [2]

(e) $\angle AEC$, [1]

(f) $\angle EAF$, [1]

(g) the length of BC, given that the radius of the circle is 8 cm. [2]

End of paper

2015 SA2 3 Express Elementary Mathematics
Paper 2 Answer Key

1(a)	$40a-12$
1(b)(i)	$p=\frac{14}{73}$
1(b)(ii)	$r=\pm\sqrt{\frac{q-5pq^2}{7p+1}}$
1(c)	$\frac{7(m-2)}{m}$
1(d)	$n=7$
2(a)(i)	$\frac{325}{x}$ days
2(a)(ii)	$\frac{325}{x-\frac{1}{2}}$ days
2(a)(iii)	$\frac{340}{x+15}$ days
2(c)	$x=40.49$, $x=-62.49$
2(d)	8.13 days
3(a)(i)	\$3482.40
3(a)(ii)	45.2%
3(b)(i)	\$7835.40
3(b)(ii)	\$359.27
3(c)	\$883.50
4(a)	AAS test
4(b)(i)	ΔAFC and ΔAED ΔABC and ΔDEC
4(b)(ii)(a)	3 cm
4(b)(ii)(b)	9 cm
4(b)(iii)(a)	$\frac{9}{4}$
4(b)(iii)(b)	$\frac{1}{3}$

5(a)	5.20 cm
5(c)	144 cm
5(d)	$32.7\ \text{cm}^2$
5(e)	$\theta=147.0°$
6(a)	25.4 m
6(b)	36.3°
6(c)	$2710\ \text{m}^3$
6(d)	50.2°
6(e)	\$106.15
8(a)	$P=\left(3\frac{1}{5},0\right)$
	$Q=(0,8)$
8(b)	$12\frac{4}{5}\ \text{unit}^2$
6(d)	50.2°
6(e)	\$106.15
8(a)	$P=\left(3\frac{1}{5},0\right)$
	$Q=(0,8)$
8(b)	$12\frac{4}{5}\ \text{unit}^2$
8(c)	2.97 units
8(e)	$y=-2\frac{1}{2}x$
9(a)	$X=\begin{pmatrix}40 & 35 & 15\\ 28 & 31 & 7\\ 11 & 16 & 23\end{pmatrix}$
9(b)	$Y=\begin{pmatrix}1.1\\ 0.9\\ 1.3\end{pmatrix}$

9(c)	$Z = \begin{pmatrix} 95 \\ 67.8 \\ 56.4 \end{pmatrix}$
9(d)	It represents the total selling price for beverages sold in stall A, B and C respectively.
9(e)	$1843.60
10(a)	90°
10(b)	124°
10(c)	34°
10(d)	118°
10(e)	74°
10(f)	87°
10(g)	11.9 cm

2015 SA2 3 Express Elementary Mathematics
Paper 2 Marking Scheme

1(a)	$25a^2+4-(5a-4)^2$ $=25a^2+4-(25a^2-40a+16)$ $=25a^2+4-25a^2+40a-16$ $=40a-12$	[M1] [A1]
1(b)(i)	$p=\dfrac{\frac{1}{2}-\left(-\frac{1}{3}\right)^2}{5\left(\frac{1}{2}\right)^2+7\left(-\frac{1}{3}\right)^2}$ $p=\dfrac{14}{73}$ (0.191 also accepted)	[B1]
1(b)(ii)	$p=\dfrac{q-r^2}{5q^2+7r^2}$ $p(5q^2+7r^2)=q-r^2$ $5pq^2+7pr^2=q-r^2$ $7pr^2+r^2=q-5pq^2$ $r^2(7p+1)=q-5pq^2$ $r^2=\dfrac{q-5pq^2}{7p+1}$ $r=\pm\sqrt{\dfrac{q-5pq^2}{7p+1}}$	[M1] [M1] [A1]
1(c)	$\dfrac{7m^2-28}{m^2+2m}=\dfrac{7(m^2-4)}{m(m+2)}$ $=\dfrac{7(m+2)(m-2)}{m(m+2)}$ $=\dfrac{7(m-2)}{m}$ $\left(7-\dfrac{2}{m}\text{ also accepted}\right)$	[M1] [A1]
1(d)	$8^6\div4^{2-n}=16^n$ $(2^3)^6\div(2^2)^{2-n}=(2^4)^n$ $2^{18}\div2^{4-2n}=2^{4n}$ $2^{14+2n}=2^{4n}$ $14+2n=4n$ $14=2n$ $n=7$	[M1] [A1]
2(a)(i)	$\dfrac{325}{x}$ days	
2(a)(ii)	$\dfrac{325}{x-\frac{1}{2}}$ days $\left(\dfrac{650}{2x-1}\text{ days also accepted}\right)$	

2(a)(iii)	$\frac{340}{x+15}$ days	[B2]
2(b)	$\frac{325}{x-\frac{1}{2}} = \frac{340}{x+15} + 2$ $\frac{325}{x-\frac{1}{2}} = \frac{340+2(x+15)}{x+15}$ $\frac{325}{x-\frac{1}{2}} = \frac{340+2x+30}{x+15}$ $325(x+15) = (370+2x)\left(x-\frac{1}{2}\right)$ $325x+4875 = 370x-185+2x^2-x$ $0 = 2x^2+44x-5060$ $0 = x^2+22x-2530$	[M1] [M1] [A1]
2(c)	$x = \frac{-22 \pm \sqrt{22^2-4(1)(-2530)}}{2(1)}$ $x = \frac{-22 \pm \sqrt{10604}}{2}$ $x = 40.49$ or $x = -62.49$	[M2] [A2]
2(d)	$\frac{325}{40.49-\frac{1}{2}} = 8.13$ days	[B1]
3(a)(i)	$1500 + (165.20 \times 12)$ = \$3482.40	[M1] [A1]
3(a)(ii)	$\frac{3482.40-2399}{2399} \times 100\%$ = 45.2% (3 s.f.)	[M1] [A1]
3(b)(i)	$\frac{225}{100} \times 3482.40$ = \$7835.40	[B1]
3(b)(ii)	$A = 7835.40\left(1+\frac{0.75}{100}\right)^6$ = \$8194.671 $I = 8194.671 - 7835.40$ = \$359.27 (2 d.p.)	[M1] [A1]
3(c)	Chargeable income = $42000 - 2300$ = \$39700 Income tax = $350 + \frac{5.5}{100} \times 9700$ = \$883.50	[M1] [M1] [A1]

4(a)	$\angle CBE = \angle IFC$ (right angle of rectangle) $\angle BEC = \angle FCI$ (alt. $\angle$s) $BC = FI$ (equal sides of rectangle) $\therefore \Delta BCE \equiv \Delta FIC$ (AAS test)	[M2] [A1]
4(b)(i)	ΔAFC and ΔAED ΔABC and ΔDEC	[B1] [B1]
4(b)(ii)(a)	$\frac{6}{9} = \frac{CF}{4.5}$ $CF = \frac{6 \times 4.5}{9}$ $= 3$ cm	[B1]
4(b)(ii)(b)	$\frac{3}{9} = \frac{3}{DE}$ $DE = \frac{3 \times 9}{3}$ $= 9$ cm	[B1]
4(b)(iii)(a)	$\frac{\text{area of } \Delta ABE}{\text{area of } \Delta FCE} = \left(\frac{9}{6}\right)^2$ $= \frac{9}{4}$	[B1]
4(b)(iii)(b)	$\frac{\text{area of } \Delta AFC}{\text{area of } \Delta EFC} = \frac{3}{9}$ $= \frac{1}{3}$	[B1]
5(a)	$\angle EOD = \frac{360°}{6}$ $= 60°$ Let the midpoint of ED be Z $\angle EOZ = \frac{60°}{2}$ $= 30°$ $EZ = 3$ cm (radius of circle) $\tan 30° = \frac{3}{OZ}$ $OZ = 5.19615$ cm $= 5.20$ cm (3 s.f.)	[M1] [M1] [A1]
5(b)	Radius $= 3 + 5.19615$ $= 8.196$ cm (shown)	[B1]
5(c)	Total length of pentagon $ABCDEF = 6 \times 6$ $= 36$ cm Circumference of 6 semicircles $= 3 \times 2\pi(3)$ $= 18\pi$ cm Circumference of big circle $= 2\pi(8.196)$ $= 16.392\pi$ cm Total length of wire $= 36 + 18\pi + 16.392\pi$ $= 144$ cm (3 s.f.)	[M1] [M1] [A1]

5(d)	Area of big circle $= \pi(8.196)^2$ $= 67.174416\pi \text{ cm}^2$ Area of 6 semicircles $= 3\times\pi(3)^2$ $= 27\pi \text{ cm}^2$ Area of pentagon $= 6\left(\frac{1}{2}\times 6\times 5.19615\right)$ $= 93.5307 \text{ cm}^2$ Area of shaded region $= 67.174416\pi - 27\pi - 93.5307$ $= 32.7 \text{ cm}^2$ (3 s.f.)	[M1] [M1] [A1]
5(e)	Total arc length of PT and QS $= 144.04565 - 22 - 22$ $= 100.04565$ cm Let reflex angle PRT be θ $\frac{\theta}{360}\times 2\pi(17) + \frac{\theta}{360}\times 2\pi(22) = 100.04565$ $\frac{17\pi\theta}{180} + \frac{11\pi\theta}{90} = 100.04565$ $\frac{17\pi\theta}{180} + \frac{22\pi\theta}{180} = 100.04565$ $\frac{39\pi\theta}{180} = 100.04565$ $39\pi\theta = 18008.217$ $\theta = 147.0°$ (1 d.p.)	[M1] [A1]
6(a)	$AC = \sqrt{16^2 + 15^2 - 2(16)(15)\cos 110°}$ $= 25.4002$ m $= 25.4$ m (3 s.f.)	[M2] [A1]
6(b)	$\frac{\sin\angle ACB}{16} = \frac{\sin 110°}{25.4002}$ $\angle ACB = \sin^{-1}\left(\frac{16\sin 110°}{25.4002}\right)$ $= 36.2939°$ $= 36.3°$ (1 d.p.)	[M1] [A1]
6(c)	Area of triangle $ABC = \frac{1}{2}(16)(15)\sin(110°)$ $= 112.7631 \text{ m}^2$ Volume of ramp $= 112.7631\times 24$ $= 2706.3147 \text{ m}^3$ $= 2710 \text{ m}^3$ (3 s.f.)	[M1] [M1] [A1]
6(d)	Let the angle be θ $\tan\theta = \frac{18}{15}$ $\theta = \tan^{-1}\frac{18}{15}$ $= 50.1944°$ $= 50.2°$ (1 d.p.)	[M1] [A1]

6(e)	Surface area of ramp $= (16+15+25.4002)\times 24 + 2(112.7631)$ $= 1579.131 \text{ cm}^2$ Amount of money $= \dfrac{1579.131}{450}\times 30.25$ $= \$106.15$ (2 d.p.)	[M1] [M1] [A1]
8(a)	$2(0)+5x-16=0$ $x = 3\dfrac{1}{5}$ Coordinates of $P = \left(3\dfrac{1}{5}, 0\right)$ $2y+5(0)-16=0$ $y=8$ Coordinates of $Q = (0, 8)$	[B1] [B1]
8(b)	$\dfrac{1}{2}\times 3\dfrac{1}{5}\times 8$ $= 12\dfrac{4}{5} \text{ unit}^2$	[M1] [A1]
8(c)	$PQ = \sqrt{\left(0-3\dfrac{1}{5}\right)^2 + (8-0)^2}$ $= \sqrt{72.24}$ $= 8.62$ units (3 s.f.)	[M1] [A1]
8(d)	$12\dfrac{4}{5} \div \dfrac{1}{2} \div \sqrt{72.24}$ $= 2.97$ units (3 s.f.)	[M1] [A1]
8(e)	Gradient of $l_1 = \dfrac{8-0}{0-3\dfrac{1}{5}}$ $= -2\dfrac{1}{2}$ $y = -2\dfrac{1}{2}x$	[B1]
9(a)	$X = \begin{pmatrix} 40 & 35 & 15 \\ 28 & 31 & 7 \\ 11 & 16 & 23 \end{pmatrix}$	[B1]
9(b)	$Y = \begin{pmatrix} 1.1 \\ 0.9 \\ 1.3 \end{pmatrix}$	[B1]
9(c)	$Z = \begin{pmatrix} 95 \\ 67.8 \\ 56.4 \end{pmatrix}$	[B1]
9(d)	It represents the total selling price for beverages sold in stall A, B and C respectively.	[B1]

9(e)	$\begin{pmatrix}8 & 6 & 12\end{pmatrix}\begin{pmatrix}95\\67.8\\56.4\end{pmatrix}$ $= (1843.6)$ Total amount of money collected = \$1843.60 (2 d.p.)	[M1] [A1]
10(a)	$\angle OAB = 90°$ (rad $\perp$ tan)	[B1]
10(b)	$\angle DOF = 180° - (28° \times 2)$ ($\angle$ sum of isos. Δ) $= 124°$	[B1]
10(c)	$\angle AOB = 180° - 124°$ (adj. $\angle$s on str. line) $= 56°$ $\angle ABO = 180° - 90° - 56°$ ($\angle$ sum of Δ) $= 34°$ $\angle OBC = 34°$ (tangent from ext. point)	[M1] [A1]
10(d)	Reflex $\angle DOF = 360° - 124°$ ($\angle$s at a point) $= 236°$ $\angle DCF = 236° \div 2$ ($\angle$ at centre = 2 $\angle$ at circum) $= 118°$	[M1] [A1]
10(e)	$\angle AEC = 90° - 16°$ (right $\angle$ in semicircle) $= 74°$	[B1]
10(f)	$\angle EAF = 180° - 93°$ ($\angle$ in opp. segment) $= 87°$	[B1]
10(g)	$\tan 56° = \frac{AB}{8}$ $= 11.9$ cm $BC = 11.9$ cm (tangent from ext. pt)	[M1] [A1]

END.