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BEDOK SOUTH SECONDARY SCHOOL END-OF-YEAR EXAMINATION 2023

3EXP

CANDIDATE
NAME

CLASS

INDEX
NUMBER

TEACHER Ms Lam YP / Mr Jeremy Ow / Miss Jacquelyn Wan / Mr Samuel Goh / Mr Sum TM

MATHEMATICS

4052/01

PAPER 1

4 October 2023

Candidates answer on the Question Booklet.
No additional materials are required.

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on **all** the work you hand in.
Write in **dark blue** or **black** pen on both sides of the paper.
You may use a soft **HB pencil** for any **diagrams** or **graphs**.
Do **not** use staples, paper clips, highlighters, glue or correction fluid.

Answer **all questions**. Write your answers and working on the spaces provided.
Show all your working on the **same page** as the rest of the answer.
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You are reminded of the need for **clear presentation** in your answers.

The number of marks is given in brackets [] at the end of each question or part question.
The total of the marks for this paper is **70**.

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If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

	Questions	Marks Deducted
Units		-1 / -2
Degree of Accuracy		
Presentation		

For Examiner's Use

Setter: Miss Jacquelyn Wan

This paper consists of **16** printed pages including this cover page.

Mathematical Formulae**Compound interest**

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curve surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

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$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

- 1 (a) Calculate $\frac{6.71}{\sqrt[3]{3.14 - 9.81^2}}$, giving your answer correct to 3 significant figures.

Answer [1]

- (b) The number of students in a school is given as 1200, correct to the nearest hundred. Write down the minimum and maximum possible number of students in the school.

Answer Minimum = , Maximum = [2]

- 2 (a) Express 2160 as a product of its prime factors.

Answer [1]

- (b) Find the smallest number n such that $2160n$ is a perfect cube, where n is a positive integer.

Answer [1]

- (c) x is a number between 100 and 150.
The highest common factor of x and 2160 is 10.
Find the possible values of x .

Answer and [2]

4

- 3 Given that $a = 5.45 \times 10^{54}$ and $b = 5.35 \times 10^{53}$.

Expressing your answers in standard form, find the value of

(a) $a - b$,

Answer [1]

(b) $\sqrt{ab}$.

Answer [1]

- 4 Simplify $\frac{6-3x}{4x+8} \div \frac{12x-24}{3x+6}$ into a single algebraic fraction.

Answer [3]

- 5 Simplify the following and leave your answer in positive index notation.

(a) $\left(\frac{y^8}{4y^6}\right)^{-\frac{3}{2}}$

Answer [2]

5

(b) $\frac{a^2b}{(ab)^{-1}} \div \frac{a^{23}}{b}$

Answer [2]

6 Solve the following equation.

(a) $\frac{1}{4y} = 2^{-6}$

Answer [2]

(b) $\frac{2^{5-n}}{8 \times 2^{4n}} = 2$

Answer [2]

6

7 (a) Solve the equation $\frac{x}{x-3} + \frac{2}{x^2-9} = 1$.

Answer [2]

(b) Given that $x = 1\frac{1}{2}$ is the only solution to the equation $4x^2 - ax + b = 0$, find the values of a and of b .

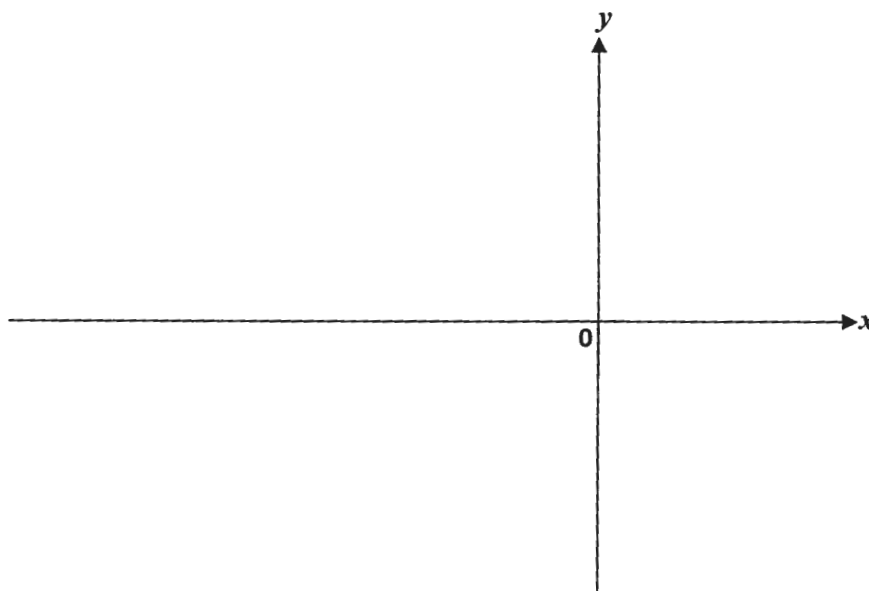
Answer $a = \dots\dots\dots$, $b = \dots\dots\dots$ [2]

7

- 8 (a) Factorise $y = -x^2 - 5x + 14$ into the form $y = -(x+a)(x-b)$.

Answer [1]

- (b) Hence, sketch the graph of $y = -x^2 - 5x + 14$ on the axes below, indicating clearly its intercept(s) and the coordinates of its turning point.



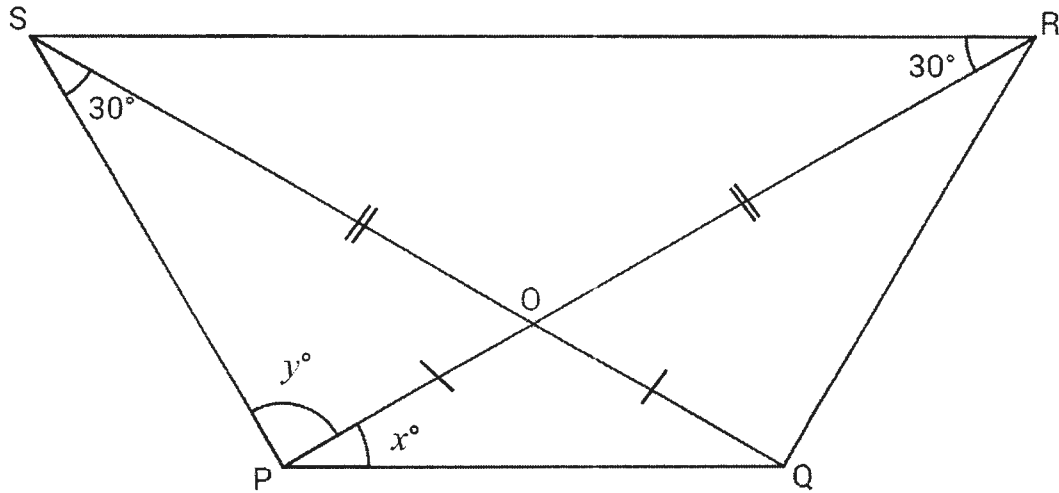
[3]

- (c) Write down the equation of the line of symmetry of the graph.

Answer [1]

8

- 9 In the quadrilateral $PQRS$, the diagonals PR and SQ intersect at O such that $OP = OQ$ and $OR = OS$.



- (a) Show that $\triangle SPR$ and $\triangle RQS$ are congruent.

Answer

.....

.....

.....

.....

..... [3]

- (b) Show that $\triangle POQ$ and $\triangle SOR$ are similar.

Answer

.....

.....

.....

.....

.....

..... [2]

9

- (c) Hence, if $RS = 9$ cm, $OS = 5.2$ cm and $PQ = 4.5$ cm, find the length of OP .

Answer [2]

- (d) If $\angle PSQ = \angle ORS = 30^\circ$, find the value of

(i) x ,

Answer [1]

(ii) y .

Answer [1]

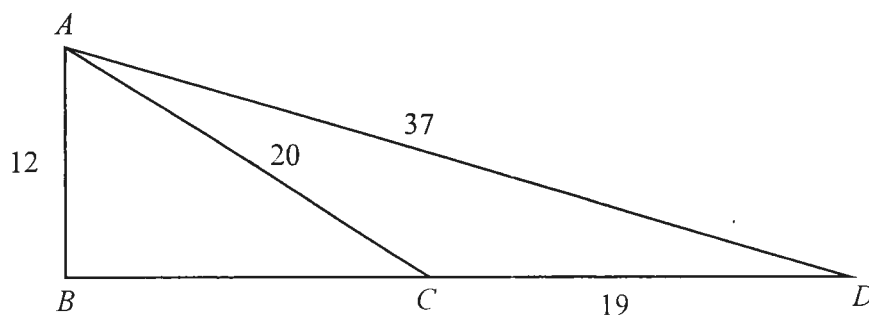
- (e) Hence, prove that the line PQ is parallel to RS .

.....

[1]

10

- 10 ABD is a triangle with $AB = 12$ cm, $AC = 20$ cm, $CD = 19$ cm, $AD = 37$ cm and $BD = 35$ cm.



- (a) Prove that $\triangle ABC$ is a right-angled triangle.

.....

.....

.....

.....

.....

.....

..... [2]

- (b) Express the following as a fraction in its lowest term.

(i) $\sin \angle ACD$

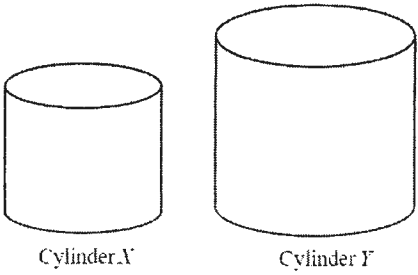
Answer [1]

(ii) $\cos \angle CAD$

Answer [2]

11

- 11 The diagram below shows two geometrically similar closed cylindrical solids, X and Y . The volume of Cylinder X is 216 cm^3 and the volume of Cylinder Y is 729 cm^3 .



- (a) Find $\frac{\text{height of Cylinder } X}{\text{height of Cylinder } Y}$.

Answer [1]

- (b) Find the surface area of Cylinder Y if the surface area of Cylinder X is 800 cm^2 .

Answer [2]

- (c) If the mass of Cylinder Y is 5832 grams, find the mass of Cylinder X .

Answer [2]

12

- 12 Tap A, when turned on alone, can fill a tank completely in y minutes.
 Tap B, when turned on alone, takes 12 minutes more to fill the same tank completely.
- (a) What fraction of the tank is filled after one minute if only Tap A is turned on?

Answer [1]

- (b) What fraction of the tank is filled after one minute if only Tap B is turned on?

Answer [1]

- (c) Both taps are turned on at the same time. The tank is completely filled after 40 minutes. Form an equation in y and show that it reduces to $y^2 - 68y - 480 = 0$.

Answer Shown in the space above [3]

13

- (d) Solve the equation $y^2 - 68y - 480 = 0$, giving your answers to 2 decimal places.

Answer [3]

- (e) Given that Tap A has a water flow rate of 20 ml per second, find the capacity of the tank in litres, correct to nearest litre.

Answer [2]

- 13** The variables x and y are connected by the equation $y = 1 - x^3 + 3x^2$.
 The table below shows some corresponding values of x and y , correct to 1 decimal place where appropriate.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	5	1.9	1	p	3	4.4	5	4.1	1

- (a) Calculate the value of p , giving your answer correct to 1 decimal place.

Answer [1]

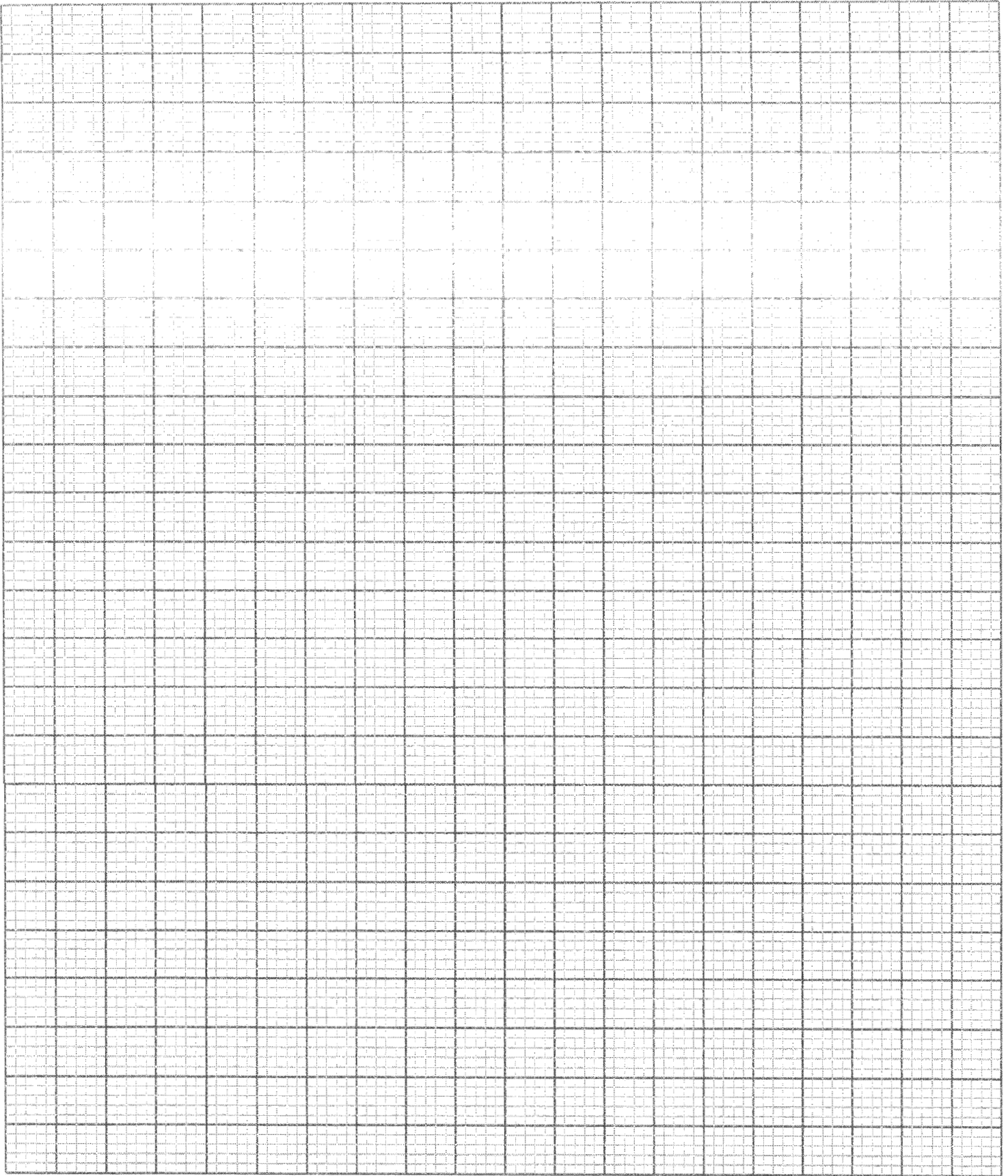
- (b) Using a scale of 2 cm to represent 0.5 units on a horizontal x -axis for $-1 \leq x \leq 3$ and 2 cm to represent 1 unit on a vertical y -axis for $-1 \leq y \leq 7$. On your axes, plot the points given in the table and join them with a smooth curve. [3]

- (c) Use your graph to find all the solutions of $1 - x^3 + 3x^2 = 2$.

Answer [2]

- (d) By drawing a suitable tangent, find the gradient of the curve $y = 1 - x^3 + 3x^2$ at the point where $x = 2.5$.

Answer [2]



16

(e) (i) On the same axes, draw the line $y = -2x + 5$ for $-1 \leq x \leq 3$. [2]

(ii) Hence, use your graph to find the range of values of x such that $1 - x^3 + 3x^2 \geq -2x + 5$ in the range $-1 \leq x \leq 3$.

Answer [1]

~ End of Paper ~



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3

- 1 (a) Calculate $\frac{6.71}{\sqrt[3]{3.14-9.81^2}}$, giving your answer correct to 3 significant figures.

Answer-1.48..... [1]

- (b) The number of students in a school is given as 1200, correct to the nearest hundred. Write down the minimum and maximum possible number of students in the school.

Answer Minimum =1150..... , Maximum =1249..... [2]

- 2 (a) Express 2160 as a product of its prime factors.

2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

$$2160 = 2^4 \times 3^3 \times 5 \quad \text{----- B1}$$

Answer $2160 = 2^4 \times 3^3 \times 5$ [1]

- (b) Find the smallest number n such that $2160n$ is a perfect cube, where n is a positive integer.

$$n = 2^2 \times 5^2$$

$$n = 100 \quad \text{----- B1}$$

Answer $n = 100$ [1]

- (c) x is a number between 100 and 150.
The highest common factor of x and 2160 is 10.
Find the possible values of x .

4

Answer110..... and130..... [2]

- 3 Given that $a = 5.45 \times 10^{54}$ and $b = 5.35 \times 10^{53}$.

Expressing your answers in standard form, find the value of

(a) $a - b$,

Answer 4.915×10^{54} [1]

(b) $\sqrt{ab}$.

Answer 1.71×10^{54} [1]

- 4 Simplify $\frac{6-3x}{4x+8} \div \frac{12x-24}{3x+6}$ into a single algebraic fraction.

$$\begin{aligned} & \frac{6-3x}{4x+8} \div \frac{12x-24}{3x+6} \\ &= \frac{6-3x}{4x+8} \times \frac{3x+6}{12x-24} && \text{----- M1 (Change division to multiplication)} \\ &= \frac{3(2-x)}{4(x+2)} \times \frac{3(x+2)}{12(x-2)} && \text{----- M1 (Factorisation)} \\ &= \frac{-3(x-2)}{4(x+2)} \times \frac{3(x+2)}{12(x-2)} \\ &= -\frac{3}{16} && \text{----- A1} \end{aligned}$$

Answer $-\frac{3}{16}$ [3]

- 5 Simplify the following and leave your answer in positive index notation.

(a) $\left(\frac{y^8}{4y^6}\right)^{-\frac{3}{2}}$

$$\begin{aligned} & \left(\frac{y^8}{4y^6}\right)^{-\frac{3}{2}} \\ &= \left(\frac{4y^6}{y^8}\right)^{\frac{3}{2}} && \text{----- M1} \\ &= \frac{8}{y^3} && \text{----- A1} \end{aligned}$$

5

$$\text{Answer} \dots\dots\dots \frac{8}{y^3} \dots\dots\dots [2]$$

$$(b) \quad \frac{a^2b}{(ab)^{-1}} \div \frac{a^{23}}{b}$$

$$\frac{a^2b}{(ab)^{-1}} \div \frac{a^{23}}{b}$$

$$= \frac{a^2b}{a^{-1}b^{-1}} \times \frac{b}{a^{23}} \quad \text{----- M1}$$

$$= \frac{b^3}{a^{20}} \quad \text{----- A1}$$

$$\text{Answer} \dots\dots\dots \frac{b^3}{a^{20}} \dots\dots\dots [2]$$

6 Solve the following equation.

$$(a) \quad \frac{1}{4y} = 2^{-6}$$

$$\frac{2^{-2}}{y} = 2^{-6}$$

$$\frac{1}{y} = \frac{2^{-6}}{2^{-2}}$$

$$\frac{1}{y} = 2^{-4} \quad \text{----- M1}$$

$$y = 16 \quad \text{----- A1}$$

$$\text{Answer} \dots\dots\dots y = 16 \dots\dots\dots [2]$$

$$(b) \quad \frac{2^{5-n}}{8 \times 2^{4n}} = 2$$

$$\frac{2^{5-n}}{8 \times 2^{4n}} = 2$$

$$\frac{2^{5-n}}{2^{3+4n}} = 2^1 \quad \text{----- M1}$$

$$5 - n - (3 + 4n) = 1$$

$$-5n = -1$$

$$n = \frac{1}{5} \quad \text{----- A1}$$

Alternative solution

$$\frac{2^{5-n}}{8 \times 2^{4n}} = 2$$

$$2^{5-n} = 16 \times 2^{4n}$$

$$2^{5-n} = 2^4 \times 2^{4n} \quad \text{----- M1}$$

$$5 - n = 4 + 4n$$

$$n = \frac{1}{5} \quad \text{----- A1}$$

6

Answer $n = \frac{1}{5}$ [2]

- 7 (a) Solve the equation $\frac{x}{x-3} + \frac{2}{x^2-9} = 1$.

$$\frac{x}{x-3} + \frac{2}{x^2-9} = 1$$

$$\frac{x}{x-3} + \frac{2}{(x+3)(x-3)} = 1$$

$$x(x+3)+2=(x+3)(x-3) \quad \text{----- M1}$$

$$x^2+3x+2=x^2-9$$

$$3x=-11$$

$$x=-\frac{11}{3} \quad \text{----- A1}$$

Answer $x = -\frac{11}{3}$ [2]

- (b) Given that $x = 1\frac{1}{2}$ is the only solution to the equation $4x^2 - ax + b = 0$, find the values of a and of b .

$$\left(x - \frac{3}{2}\right)^2 = 0 \quad \text{----- M1}$$

$$x^2 - 3x + \frac{9}{4} = 0$$

$$4x^2 - 12x + 9 = 0 \quad \text{----- A1}$$

Answer $a = 12$, $b = 9$ [2]

- 8 (a) Factorise $y = -x^2 - 5x + 14$ into the form $y = -(x+a)(x-b)$.

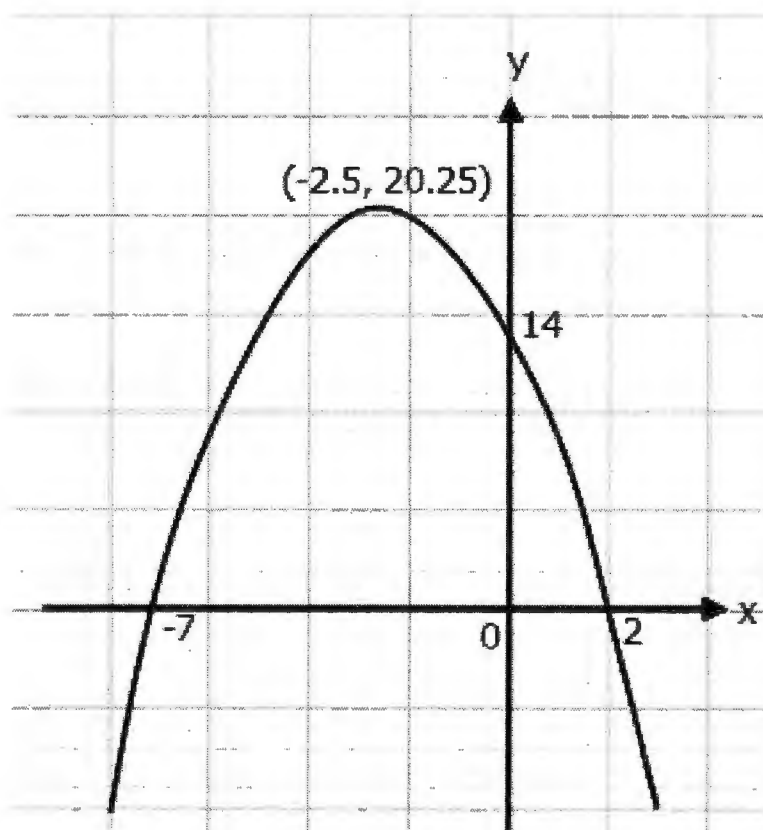
$$y = -x^2 - 5x + 14$$

$$y = -(x^2 + 5x - 14)$$

$$y = -(x+7)(x-2) \quad \text{----- A1}$$

Answer $y = -(x+7)(x-2)$ [1]

- (b) Hence, sketch the graph of $y = -x^2 - 5x + 14$ on the axes below, indicating clearly its intercept(s) and the coordinates of its turning point.



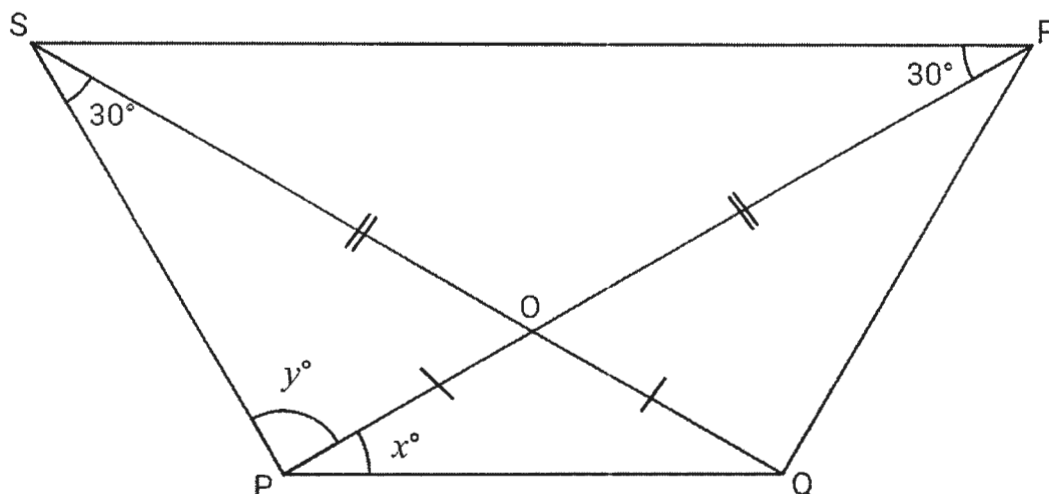
- B1: Shape of curve
- B1: x- and y- intercepts
- B1: Coordinates of turning point

[3]

- (c) Write down the equation of the line of symmetry of the graph.

Answer $x = -2.5$ [1]

- 9 In the quadrilateral $PQRS$, the diagonals PR and SQ intersect at O such that $OP = OQ$ and $OR = OS$.



- (a) Show that $\triangle SPR$ and $\triangle RQS$ are congruent.

Answer

In $\triangle SPR$ and $\triangle RQS$,

$SR = RS$ (common line)

$PR = QS$ (Given $OP = OQ$ and $OR = OS$)

$\angle QSR = \angle PRS = 30^\circ$ (base $\angle$ s of isos $\triangle OSR$)

$\triangle SPR \equiv \triangle RQS$ (SAS)

- M1: 2 correct reasons
- M2: 3 correct reasons
- A1: Correct congruence rule

[3]

- (b) Show that $\triangle POQ$ and $\triangle SOR$ are similar.

Answer

In $\triangle POQ$ and $\triangle SOR$,

$\angle POQ = \angle SOR$ (vert opp $\angle$ s)

$\frac{OP}{OS} = \frac{OQ}{OR}$ (Given $OP = OQ$ and $OR = OS$)

$\triangle POQ$ and $\triangle SOR$ are similar (SAS similarity test)

- M1: 2 correct reasons
- A1: Correct similarity test

[2]

- (c) Hence, if $RS = 9$ cm, $OS = 5.2$ cm and $PQ = 4.5$ cm, find the length of OP .

$$\frac{OP}{5.2} = \frac{4.5}{9} \quad \text{----- M1}$$

$$OP = 2.6 \text{ cm} \quad \text{----- A1}$$

Answer $OP = 2.6$ cm..... [2]

- (d) If $\angle PSQ = \angle ORS = 30^\circ$, find the value of

- (i) x ,

$$x = 30^\circ (\triangle POQ \text{ and } \triangle SOR \text{ are similar } \Delta / \text{isos } \Delta) \quad \text{----- B1}$$

- Reject: Alternate angle

Answer $x = 30^\circ$ [1]

- (ii) y .

$$y = 180^\circ - 30^\circ - 30^\circ - 30^\circ \text{ (sum of } \Delta = 180^\circ)$$

$$= 90^\circ \quad \text{----- A1}$$

- Reject: Interior angle

Answer $y = 90^\circ$ [1]

- (e) Hence, prove that the line PQ is parallel to RS .

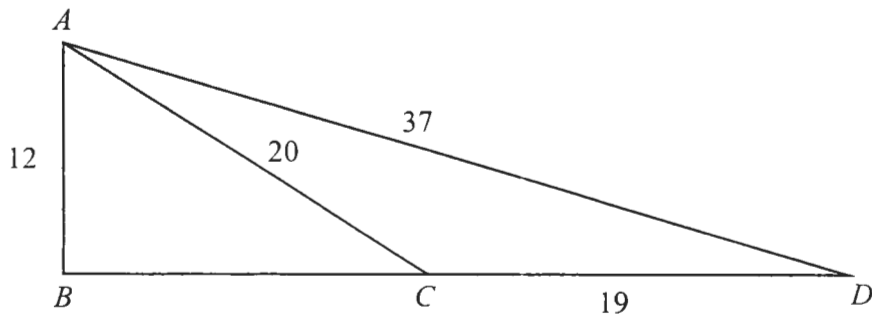
$$\text{Since } \angle SRP = \angle QPR = 30^\circ,$$

by **alternate angles**, PQ is parallel to RS . ----- B1

- Accept: Any properties of parallel lines

Answer Shown in the space above [1]

- 10 ABD is a triangle with $AB = 12$ cm, $AC = 20$ cm, $CD = 19$ cm, $AD = 37$ cm and $BD = 35$ cm.



- (a) Prove that $\triangle ABC$ is a right-angled triangle.

$$BC = 35 - 19$$

$$= 16 \text{ cm}$$

$$AB^2 + BC^2 = 12^2 + 16^2$$

$$= 400$$

$$AC^2 = 20^2$$

$$= 400$$

Since $AB^2 + BC^2 = AC^2$, [M1 – show working]

by converse of Pythagoras Theorem, $\triangle ABC$ is a right-angled triangle. [A1] [2]

- (b) Express the following as a fraction in its lowest term.

- (i) $\sin \angle ACD$

$$\sin \angle ACD = \sin \angle ACB = \frac{12}{20}$$

$$= \frac{3}{5} \quad \text{----- B1}$$

$$\text{Answer} \dots\dots\dots \frac{3}{5} \dots\dots\dots [1]$$

- (ii) $\cos \angle CAD$

By cosine rule,

$$19^2 = 20^2 + 37^2 - 2(20)(37)\cos \angle CAD \quad \text{----- M1}$$

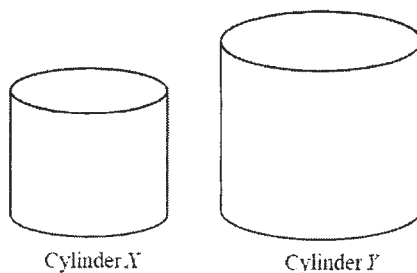
$$\cos \angle CAD = \frac{20^2 + 37^2 - 19^2}{2(20)(37)}$$

$$= \frac{176}{185} \quad \text{----- A1}$$

11

Answer $\frac{176}{185}$ [2]

- 11 The diagram below shows two geometrically similar closed cylindrical solids, X and Y . The volume of Cylinder X is 216 cm^3 and the volume of Cylinder Y is 729 cm^3 .



- (a) Find $\frac{\text{height of Cylinder } X}{\text{height of Cylinder } Y}$.

$$\begin{aligned} \frac{l_x}{l_y} &= \sqrt[3]{\frac{V_x}{V_y}} = \sqrt[3]{\frac{216}{729}} \\ &= \frac{2}{3} \quad \text{----- A1} \end{aligned}$$

Answer $\frac{2}{3}$ [1]

- (b) Find the surface area of Cylinder Y if the surface area of Cylinder X is 800 cm^2 .

$$\begin{aligned} \frac{A_y}{A_x} &= \left(\frac{l_y}{l_x}\right)^2 \\ \frac{A_y}{800} &= \left(\frac{3}{2}\right)^2 \quad \text{----- M1} \\ A_y &= \frac{9}{4}(800) \\ &= 1800 \text{ cm}^2 \quad \text{----- A1} \end{aligned}$$

Answer 1800 cm^2 [2]

- (c) If the mass of Cylinder Y is 5832 grams, find the mass of Cylinder X .

$$\begin{aligned} \frac{\text{Mass of } X}{\text{Mass of } Y} &= \left(\frac{2}{3}\right)^3 \\ \frac{\text{Mass of } X}{5832} &= \left(\frac{2}{3}\right)^3 \quad \text{----- M1} \\ \text{Mass of } X &= \left(\frac{2}{3}\right)^3 (5832) \\ &= 1728 \text{ g} \quad \text{----- A1} \end{aligned}$$

Answer 1728 g [2]

- 12** Tap A, when turned on alone, can fill a tank completely in y minutes.
Tap B, when turned on alone, takes 12 minutes more to fill the same tank completely.

(a) What fraction of the tank is filled after one minute if only Tap A is turned on?

y min fill 1 tank

1 min fill $\frac{1}{y}$ tank ----- B1

Answer $\frac{1}{y}$ [1]

(b) What fraction of the tank is filled after one minute if only Tap B is turned on?

$y+12$ min fill 1 tank

1 min fill $\frac{1}{y+12}$ tank ----- B1

Answer $\frac{1}{y+12}$ [1]

(c) Both taps are turned on at the same time. The tank is completely filled after 40 minutes. Form an equation in y and show that it reduces to $y^2 - 68y - 480 = 0$.

$$\frac{1}{y} + \frac{1}{y+12} = \frac{1}{40} \quad \text{----- M1}$$

$$40(y+12) + 40y = y(y+12) \quad \text{----- M1}$$

$$80y + 480 = y^2 + 12y \quad \text{----- A1}$$

$$y^2 - 68y - 480 = 0 \text{ (Shown)}$$

Answer Shown in the space above [3]

- (d) Solve the equation $y^2 - 68y - 480 = 0$, giving your answers to 2 decimal places.

$$a = 1, b = -68, c = -480$$

$$y = \frac{-(-68) \pm \sqrt{(-68)^2 - 4(1)(-480)}}{2(1)} \quad \text{----- M1}$$

$$y = \frac{68 \pm \sqrt{6544}}{2}$$

$$y = 74.4474 \quad \text{or} \quad y = -6.4474$$

$$y = 74.45 \text{ (2 dp)} \quad \text{or} \quad y = -6.45 \text{ (2 dp)}$$

----- A1

----- A1

Answer $y = 74.45$ or $y = -6.45$ [3]

- (e) Given that Tap A has a water flow rate of 20 ml per second, find the capacity of the tank in litres, correct to nearest litre.

$$\begin{aligned} \text{Capacity} &= 74.4474 \left(\frac{20}{1000} \right) (60) \quad \text{-- M1} \\ &= 89.33688 \\ &= 89 \text{ l (nearest litre)} \quad \text{-- A1} \end{aligned}$$

Alternative solution

$$\begin{aligned} \text{Capacity} &= 74.4474 (20) (60) \quad \text{--} \\ &\text{M1} \\ &= 89336.88 \text{ ml} \\ &= 89 \text{ l (nearest litre)} \quad \text{--} \end{aligned}$$

A1

Answer 89 l [2]

- 13 The variables x and y are connected by the equation $y = 1 - x^3 + 3x^2$.

The table below shows some corresponding values of x and y , correct to 1 decimal place where appropriate.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	5	1.9	1	p	3	4.4	5	4.1	1

- (a) Calculate the value of p , giving your answer correct to 1 decimal place.

Sub $x = 0.5$, $y = 1.625$

$p = 1.6$ (1dp) ----- B1

Answer $p = 1.6$ [1]

- (b) Using a scale of 2 cm to represent 0.5 units on a horizontal x -axis for $-1 \leq x \leq 3$ and 2 cm to represent 1 unit on a vertical y -axis for $-1 \leq y \leq 7$. On your axes, plot the points given in the table and join them with a smooth curve. [3]

- B1: Correct scale and axes labelled
- B1: Correct points plotted
- B1: Smooth curve

- (c) Use your graph to find all the solutions of $1 - x^3 + 3x^2 = 2$.

$x = -0.53$ or $x = 0.65$ or $x = 2.88$ (± 0.05)

- B1: 2 correct values
- B2: 3 correct values

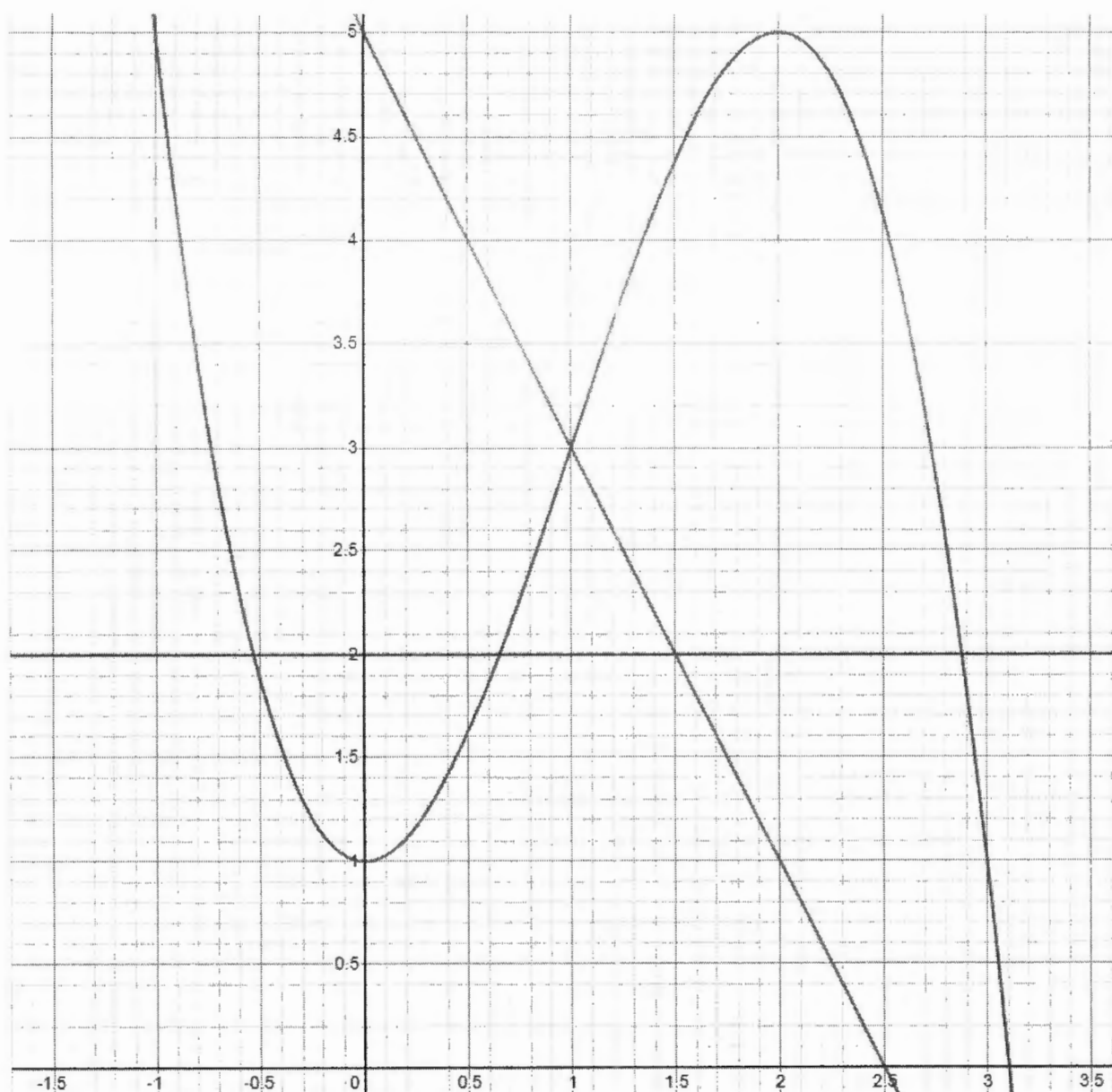
Answer $x = -0.53$ or $x = 0.65$ or $x = 2.88$ [2]

- (d) By drawing a suitable tangent, find the gradient of the curve $y = 1 - x^3 + 3x^2$ at the point where $x = 2.5$.

grad = -3.75 (± 0.5)

- M1: Correct tangent drawn
- A1: Correct calculation of tangent

Answer-3.75..... [2]



(e) (i) On the same axes, draw the line $y = -2x + 5$ for $-1 \leq x \leq 3$. [2]

- B1: Correct gradient
- B1: Correct y-intercept

(ii) Hence, use your graph to find the range of values of x such that $1 - x^3 + 3x^2 \geq -2x + 5$ in the range $-1 \leq x \leq 3$.

Answer $1 \leq x \leq 3$ [1]



BEDOK SOUTH SECONDARY SCHOOL END-OF-YEAR EXAMINATION 2023

3EXP

CANDIDATE
NAME

CLASS

INDEX
NUMBER

TEACHER Ms Lam YP / Mr Jeremy Ow / Miss Jacquelyn Wan / Mr Samuel Goh / Mr Sum TM

MATHEMATICS

4052/02

PAPER 2

10 October 2023

Candidates answer on the Question Booklet.
No additional materials are required.

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on **all** the work you hand in.
Write in **dark blue** or **black** pen on both sides of the paper.
You may use a soft **HB pencil** for any **diagrams** or **graphs**.
Do **not** use staples, paper clips, highlighters, glue or correction fluid.

Answer **all questions**. Write your answers and working on the spaces provided.
Show all your working on the **same page** as the rest of the answer.
Omission of essential working will result in loss of marks.
You are reminded of the need for **clear presentation** in your answers.

The number of marks is given in brackets [] at the end of each question or part question.
The total of the marks for this paper is **70**.

The use of an **approved scientific calculator** is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

	Questions	Marks Deducted
Units		-1 / -2
Degree of Accuracy		
Presentation		

For Examiner's Use
70

Setter: Miss Jacquelyn Wan

This paper consists of **16** printed pages including this cover page.

Mathematical Formulae**Compound interest**

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curve surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

- 1 (a) Express $\frac{4}{x-6} - \frac{3x}{x^2-12x+36}$ as a single fraction in its simplest form.

Answer [2]

- (b) Simplify $\frac{2^{n+1} - 2^n}{2^5}$.

Answer [2]

4

- (c) Given that $\frac{3x+2y}{3x-2y} = \frac{4}{3}$, find $\frac{x}{y}$.

Answer [3]

- 2 Ahmad, Bala and Caleb each decided to buy a new car which was priced in the showroom at \$98 000.

- (a) By selling each new car for \$98 000, the sales representative would make a profit of 10% on the cost price. Calculate the cost price of each new car.

Answer [1]

- (b) Ahmad managed to negotiate with the sales representative and was given a discount. Given that he paid \$92 500 for his new car, calculate the percentage discount he received.

Answer [1]

5

- (c) Under a hire purchase scheme, Bala was allowed to pay 50% of the showroom price of the car as a deposit and the balance in equal monthly instalments of \$1430 over a period of 3 years.

Calculate the difference between what Bala has to pay under hire purchase scheme and the price of the car at the showroom.

Answer [2]

- (d) Caleb traded in his old car and the sales representative gave him a \$28 000 discount towards the cost of the new car. He decided to take up a loan with a finance company at a compound interest of 3.25% per annum, compounded monthly for 3 years. Calculate the monthly instalment he has to pay.

Answer [3]

6

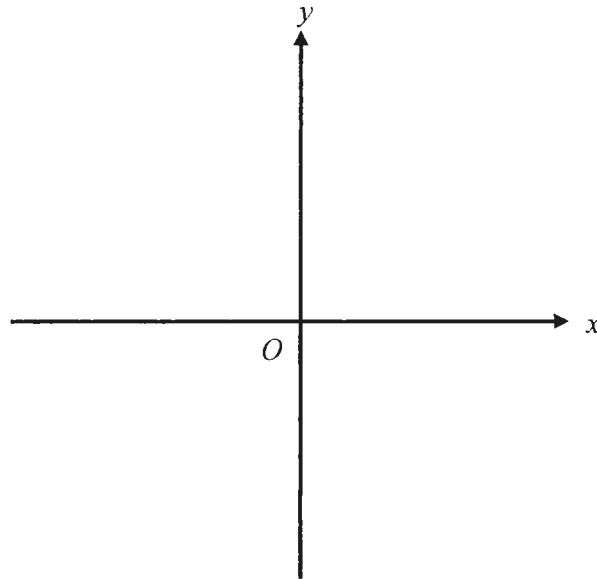
- 3 (a) In the same axes below, sketch the graphs of

(i) $y = 2x^3$,

[1]

(ii) $y = \frac{1}{x}$.

[1]



- (b) Hence, state the number of solutions for the equation $2x^3 - \frac{1}{x} = 0$.

Answer [1]

- 4 (a) These are the first four terms in a sequence.

11, 7, 3, -1, ...

- (i) Find an expression, in terms of n , for the n th term, R_n , of this sequence.

Answer [1]

- (ii) Explain why it is not possible for a term in the sequence to be a multiple of 4.

Answer Shown in the space above [2]

7

- (b) The first four terms in another sequence of numbers, $T_1, T_2, T_3, T_4, T_5 \dots$, are given below.

$$T_1 = 4^2 + 4 = 20$$

$$T_2 = 4^3 + 16 = 80$$

$$T_3 = 4^4 + 36 = 292$$

$$T_4 = 4^5 + 64 = 1088$$

- (i) Find T_5 .

Answer [1]

- (ii) Find an expression, in terms of n , for the n th term, T_n , of this sequence.

Answer [2]

- (iii) Evaluate T_{20} . Leave your answer in standard form.

Answer [1]

- (iv) Find and simplify an expression, in terms of n , for $T_{n+1} - T_n$.

Answer [2]

8

- 5 (a) A construction firm has been employed to construct 30 pavilions in a park. The firm has 10 workers who can build 25 pavilions in 15 days.

Assuming if all the workers work at equal rate, calculate

- (i) the number of days the firm need to build the 30 pavilions,

Answer [2]

- (ii) the additional number of workers the firm need to employ if it is given only 12 days to build the 30 pavilions.

Answer [2]

9

- (b) It is given that y is directly proportional to x^3 .
- (i) When $x = 2, y = 72$, find the formula connecting y and x .

Answer [2]

- (ii) Find the percentage increase in the value of y when the value of x is doubled.

Answer [2]

- (c) A map is drawn to a scale of 1 : 50 000.
- (i) Two towns are 28 km apart. Calculate, in cm, their distance apart on the map.

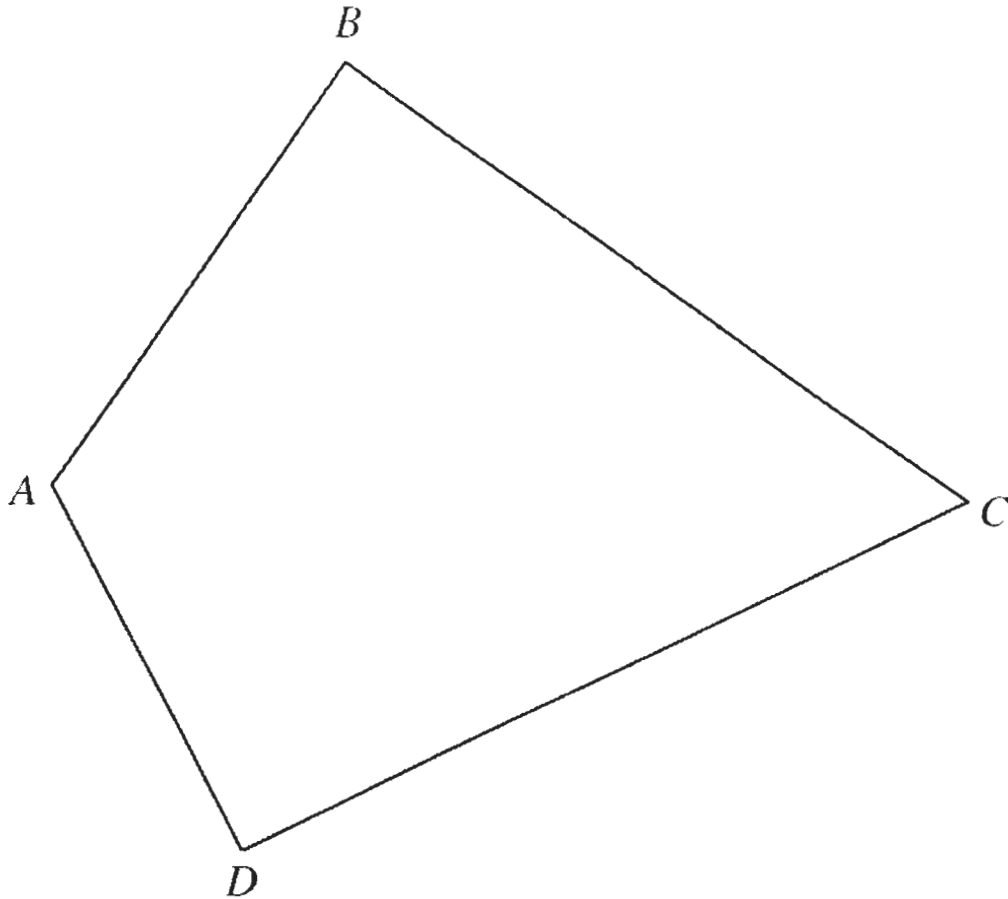
Answer [1]

- (ii) A forest on the map has an area of 18 cm^2 . Calculate, in square kilometres, the actual area of the forest.

Answer [2]

10

- 6 The diagram shows a plot of land $ABCD$ in the shape of a quadrilateral. A point Q is inside the quadrilateral $ABCD$ such that it is equidistant from B and C , and equidistant from BC and DC .

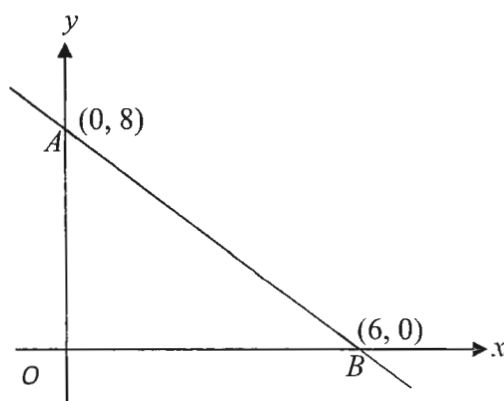


On the diagram,

- (a) construct the bisector of angle BCD , [1]
- (b) construct the perpendicular bisector of BC , [1]
- (c) find and label the point Q . [1]

11

- 7 A straight line crosses the y -axis at the point $A(0, 8)$ and the x -axis at the point $B(6, 0)$.



- (a) Find the equation of the line AB .

Answer [2]

- (b) Calculate the length of the line segment AB .

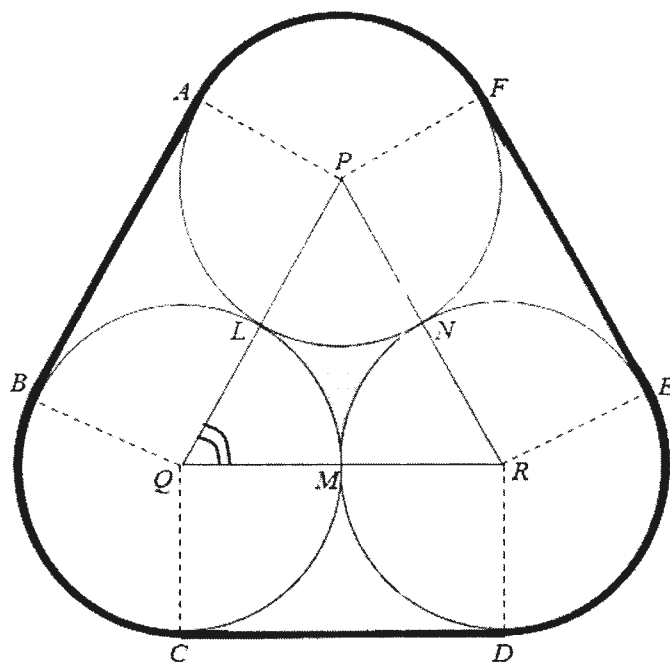
Answer [1]

- (c) Another line, which is parallel to the y -axis and which passes through the point $(9, 0)$, meets AB produced at C . Calculate the coordinates of the point C .

Answer [2]

12

- 8 The diagram shows the cross-section of 3 circular tins of the same size with centres P , Q and R . The tins touch one another externally at L , M and N .



- (a) (i) Show that $\angle PQR = \frac{\pi}{3}$ radians.

Answer Shown in the space above [2]

- (ii) Given that the area of $\triangle PQR = 6.93 \text{ cm}^2$, show that the length QL is approximately 2.00 cm.

Answer Shown in the space above [2]

13

- (b) Calculate the shaded area.

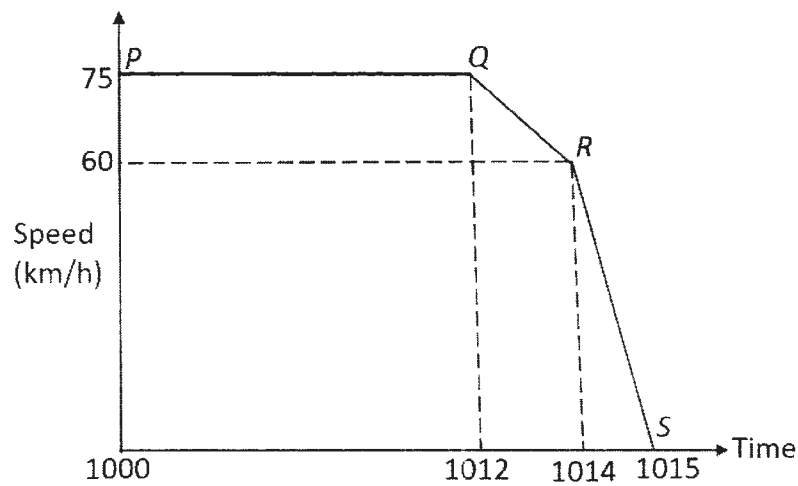
Answer [3]

- (c) The circular tins are bounded together by a piece of rope $ABCDEF$.
Calculate the total length of the rope used.

Answer [4]

14

- 9 The diagram shows the speed-time graph of a car which travelled from P to S .



- (a) How long was the car travelling at constant speed?

Answer [1]

- (b) What is the distance travelled by the car from P to S ?

Answer [2]

- (c) What is the deceleration from Q to R in km/h^2 ?

Answer [1]

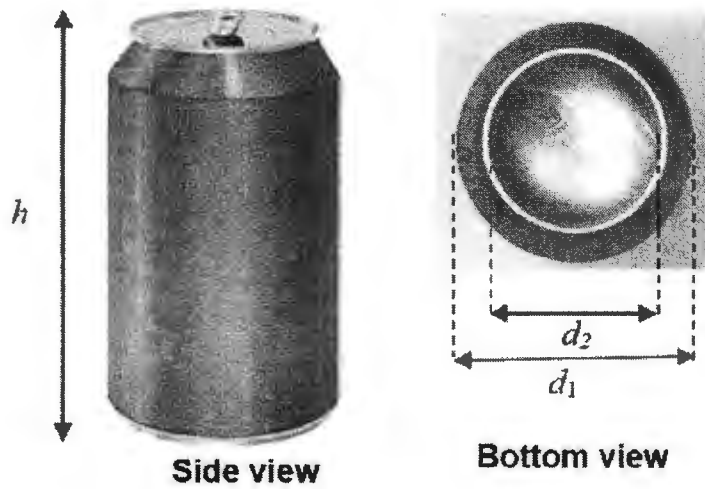
- 10 The following are the information about a soda can.

Soda Can

Height (h): 11.4 cm

Outer Diameter (d_1): 6.4 cm

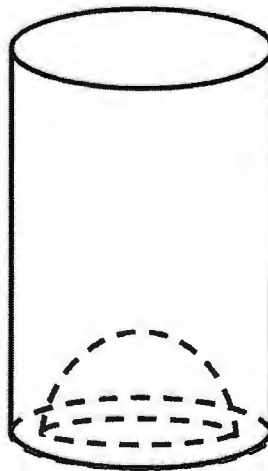
Inner Diameter (d_2): 5.0 cm



Density of soda = 1.2 g/cm^3

Safety information: The soda can is to be filled to a maximum of 95% of its total volume.

The soda can (above) can be modelled as a cylinder with an inner hemisphere that is hollowed inwards (concave) at the base of the can.



- (a) (i) Find the base area, in square centimetres, of the soda can.

Answer [2]

16

- (i) Find the total volume, in cubic centimetres, of the soda can.

Answer [2]

- (b) The material used to make the wall of the soda can must be carefully chosen such that the total mass of each filled soda can is less than 620 g. The manufacture manager of the soda can proposed to use an alloy which has a mass of 0.8 g for every 1 cm^2 to make the can, If the thickness of the soda can is negligible, will you accept his proposal? Justify your answer with suitable calculations.

Answer Shown in the space above [6]

~ End of Paper ~



BEDOK SOUTH SECONDARY SCHOOL

END-OF-YEAR EXAMINATION 2023

MARKING SCHEME

3EXP

CANDIDATE
NAME

CLASS

INDEX
NUMBER

TEACHER Ms Lam YP / Mr Jeremy Ow / Miss Jacquelyn Wan / Mr Samuel Goh / Mr Sum TM

MATHEMATICS

4052/02

PAPER 2

10 October 2023

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2 hours

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	Questions	Marks Deducted
Units		-1 / -2
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For Examiner's Use

Setter: Miss Jacquelyn Wan

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$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

- 1 (a) Express $\frac{4}{x-6} - \frac{3x}{x^2-12x+36}$ as a single fraction in its simplest form.

$$\begin{aligned}
 & \frac{4}{x-6} - \frac{3x}{x^2-12x+36} \\
 &= \frac{4}{x-6} - \frac{3x}{(x-6)^2} \\
 &= \frac{4(x-6)-3x}{(x-6)^2} \quad \text{----- M1} \\
 &= \frac{4x-24-3x}{(x-6)^2} \\
 &= \frac{x-24}{(x-6)^2} \quad \text{----- A1}
 \end{aligned}$$

Answer $\frac{x-24}{(x-6)^2}$ [2]

- (b) Simplify $\frac{2^{n+1}-2^n}{2^n 5}$.

$$\begin{aligned}
 & \frac{2^{n+1}-2^n}{2^n 5} \\
 &= \frac{2^n(2-1)}{2^n 5} \quad \text{----- M1 (Factorise out } 2^n, \text{ apply law of indices)} \\
 &= \frac{1}{5} \quad \text{----- A1}
 \end{aligned}$$

Answer $\frac{1}{5}$ [2]

4

- (c) Given that $\frac{3x+2y}{3x-2y} = \frac{4}{3}$, find $\frac{x}{y}$.

$$\frac{3x+2y}{3x-2y} = \frac{4}{3}$$

$$3(3x+2y) = 4(3x-2y) \quad \text{----- M1 (Cross multiply)}$$

$$9x+6y=12x-8y$$

$$14y=3x \quad \text{----- M1 (Simplify like terms)}$$

$$\frac{x}{y} = \frac{14}{3} \quad \text{----- A1}$$

$$\text{Answer} \dots\dots\dots \frac{x}{y} = \frac{14}{3} \dots\dots\dots [3]$$

- 2 Ahmad, Bala and Caleb each decided to buy a new car which was priced in the showroom at \$98 000.

- (a) By selling each new car for \$98 000, the sales representative would make a profit of 10% on the cost price. Calculate the cost price of each new car.

$$\begin{aligned} \text{Cost price} &= \frac{100}{110}(98\,000) \\ &= 89\,090.9090 \\ &= \$89\,090.91 \text{ (2dp)} \quad \text{----- A1} \end{aligned}$$

$$\text{Answer} \dots\dots\dots \$89\,090.91 \dots\dots\dots [1]$$

- (b) Ahmad managed to negotiate with the sales representative and was given a discount. Given that he paid \$92 500 for his new car, calculate the percentage discount he received.

$$\begin{aligned} \% \text{ discount} &= \frac{98\,000 - 92\,500}{98\,000} \times 100 \\ &= 5.61224 \\ &= 5.61 \% \text{ (3sf)} \quad \text{----- A1} \end{aligned}$$

$$\text{Answer} \dots\dots\dots 5.61 \% \dots\dots\dots [1]$$

5

- (c) Under a hire purchase scheme, Bala was allowed to pay 50% of the showroom price of the car as a deposit and the balance in equal monthly instalments of \$1430 over a period of 3 years.

Calculate the difference between what Bala has to pay under hire purchase scheme and the price of the car at the showroom.

$$\begin{aligned}\text{Total amt paid} &= \frac{50}{100} \times 98\,000 + 1430(12)(3) && \text{----- M1} \\ &= \$100\,480\end{aligned}$$

$$\begin{aligned}\text{Difference} &= \$100\,480 - \$98\,000 && \text{----- A1} \\ &= \$2480\end{aligned}$$

Answer\$2480..... [2]

- (d) Caleb traded in his old car and the sales representative gave him a \$28 000 discount towards the cost of the new car. He decided to take up a loan with a finance company at a compound interest of 3.25% per annum, compounded monthly for 3 years. Calculate the monthly instalment he has to pay.

$$\begin{aligned}\text{Amt loaned} &= \$98\,000 - \$28\,000 \\ &= \$70\,000\end{aligned}$$

$$r = \frac{3.25}{12}$$

$$r = \frac{13}{48}$$

$$n = 12(3)$$

$$n = 36$$

$$\begin{aligned}\text{Amt owed} &= 70\,000 \left(1 + \frac{\left(\frac{13}{48} \right)}{100} \right)^{36} && \text{----- M1} \\ &= \$77\,158.6312\end{aligned}$$

$$\begin{aligned}\text{Monthly instalment} &= \frac{77\,158.6312}{36} && \text{----- M1} \\ &= \$2143.30 \text{ (2dp)} && \text{----- A1}\end{aligned}$$

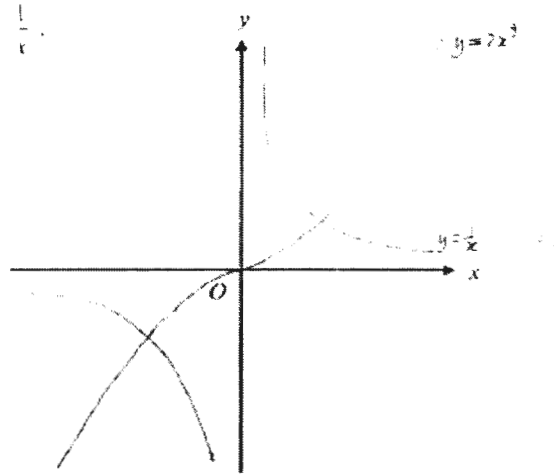
Answer\$2143.30..... [3]

6

- 3 (a) In the same axes below, sketch the graphs of

(i) $y = 2x^3$, [1]

(ii) $y = \frac{1}{x}$. [1]



- (b) Hence, state the number of solutions for the equation $2x^3 - \frac{1}{x} = 0$.

Answer2..... [1]

- 4 (a) These are the first four terms in a sequence.

11, 7, 3, -1, ...

- (i) Find an expression, in terms of n , for the n th term, R_n , of this sequence.

$$\begin{aligned} R_n &= a + (n-1)d \\ &= 11 + (n-1)(-4) \\ &= 15 - 4n \quad \text{----- A1} \end{aligned}$$

Answer $R_n = 15 - 4n$ [1]

- (ii) Explain why it is not possible for a term in the sequence to be a multiple of 4.

$$\begin{aligned} R_n &= 15 - 4n \\ &= 4(3-n) + 3 \quad \text{----- M1} \end{aligned}$$

$4(3-n) + 3$ is not divisible by 4, therefore it is not possible for the term to be a multiple of 4. [B1]

Alternative solution

Since 15 is not a multiple of 4, but $4n$ is, then the sequence is not a multiple of 4. **OR** Since 15 cannot be divisible by 4, but $4n$ can, then $15 - 4n$ is not a multiple of 4. [2]

7

- (b) The first four terms in another sequence of numbers, $T_1, T_2, T_3, T_4, T_5 \dots$, are given below.

$$T_1 = 4^2 + 4 = 20$$

$$T_2 = 4^3 + 16 = 80$$

$$T_3 = 4^4 + 36 = 292$$

$$T_4 = 4^5 + 64 = 1088$$

- (i) Find T_5 .

$$T_5 = 4^6 + 100 = 4196 \text{ ----- A1}$$

$$\text{Answer } T_5 = 4^6 + 100 = 4196 \text{ [1]}$$

- (ii) Find an expression, in terms of n , for the n th term, T_n , of this sequence.

$$4, 16, 36, 64$$

$$4(1^2), 4(2^2), 4(3^2), 4(4^2)$$

$$\text{Sequence} = 4n^2 \text{ ----- B1}$$

$$\therefore T_n = 4^{n+1} + 4n^2 \text{ ----- A1}$$

$$\text{Answer } T_n = 4^{n+1} + 4n^2 \text{ [2]}$$

- (iii) Evaluate T_{20} . Leave your answer in standard form.

$$T_{20} = 4^{20+1} + 4 \times 20^2$$

$$T_{20} = 4.3980 \times 10^{12}$$

$$= 4.40 \times 10^{12} \text{ (3sf) ----- A1}$$

$$\text{Answer } 4.40 \times 10^{12} \text{ [1]}$$

- (iv) Find and simplify an expression, in terms of n , for $T_{n+1} - T_n$.

$$T_{n+1} - T_n$$

$$= [4^{n+1+1} + 4(n+1)^2] - (4^{n+1} + 4n^2) \text{ ----- M1}$$

$$= 4^{n+2} + 4(n+1)^2 - 4^{n+1} - 4n^2$$

$$= 4^{n+2} + 4(n^2 + 2n + 1) - 4^{n+1} - 4n^2$$

$$= 4^{n+2} - 4^{n+1} + 8n + 4 \text{ ----- A1}$$

$$\text{Answer } 4^{n+2} - 4^{n+1} + 8n + 4 \text{ [2]}$$

8

- 5 (a) A construction firm has been employed to construct 30 pavilions in a park. The firm has 10 workers who can build 25 pavilions in 15 days.

Assuming if all the workers work at equal rate, calculate

- (i) the number of days the firm need to build the 30 pavilions,

<u>workers</u>	<u>pavilion</u>	<u>days</u>
10	25	15
10	1	$\frac{15}{25}$ [M1]
10	30	$\frac{15}{25} \times 30$ [A1] = 18

Answer18 days..... [2]

- (ii) the additional number of workers the firm need to employ if it is given only 12 days to build the 30 pavilions.

<u>workers</u>	<u>pavilion</u>	<u>days</u>
10	25	15
10	$\frac{25}{15}$	1
10	$\frac{25}{15} \times 12$ = 20	12
$\frac{10}{2}$ = 5	$\frac{20}{2}$ = 10	12
$5 \times 3 = 15$ [M1]	$10 \times 3 = 30$	12

Additional workers = 15 – 10
= 5 workers [A1]

Answer5 workers..... [2]

(b) It is given that y is directly proportional to x^3 .

(i) When $x = 2, y = 72$, find the formula connecting y and x .

$$y = k(x)^3$$

$$72 = k(2)^3 \quad \text{----- M1}$$

$$k = 9$$

$$y = 9x^3 \quad \text{----- A1}$$

Answer $y = 9x^3$ [2]

(ii) Find the percentage increase in the value of y when the value of x is doubled.

$$\text{Sub } x_{\text{new}} = 2x,$$

$$y = k(2x)^3$$

$$y = k(8x^3) \quad \text{----- M1}$$

$$\% \text{ increase} = \frac{y_{\text{new}} - y_{\text{old}}}{y_{\text{old}}} \times 100$$

$$= \frac{k(8x^3) - k(x^3)}{k(x^3)} \times 100$$

$$= 700\% \quad \text{----- A1}$$

Answer 700% [2]

(c) A map is drawn to a scale of 1 : 50 000.

(i) Two towns are 28 km apart. Calculate, in cm, their distance apart on the map.

$$1 \text{ cm} : 50\,000 \text{ cm}$$

$$1 \text{ cm} : 0.5 \text{ km}$$

$$\text{Distance apart} = \frac{28}{0.5}$$

$$= 56 \text{ cm} \quad \text{----- A1}$$

Answer 56 cm [1]

(ii) A forest on the map has an area of 18 cm^2 . Calculate, in square kilometres, the actual area of the forest.

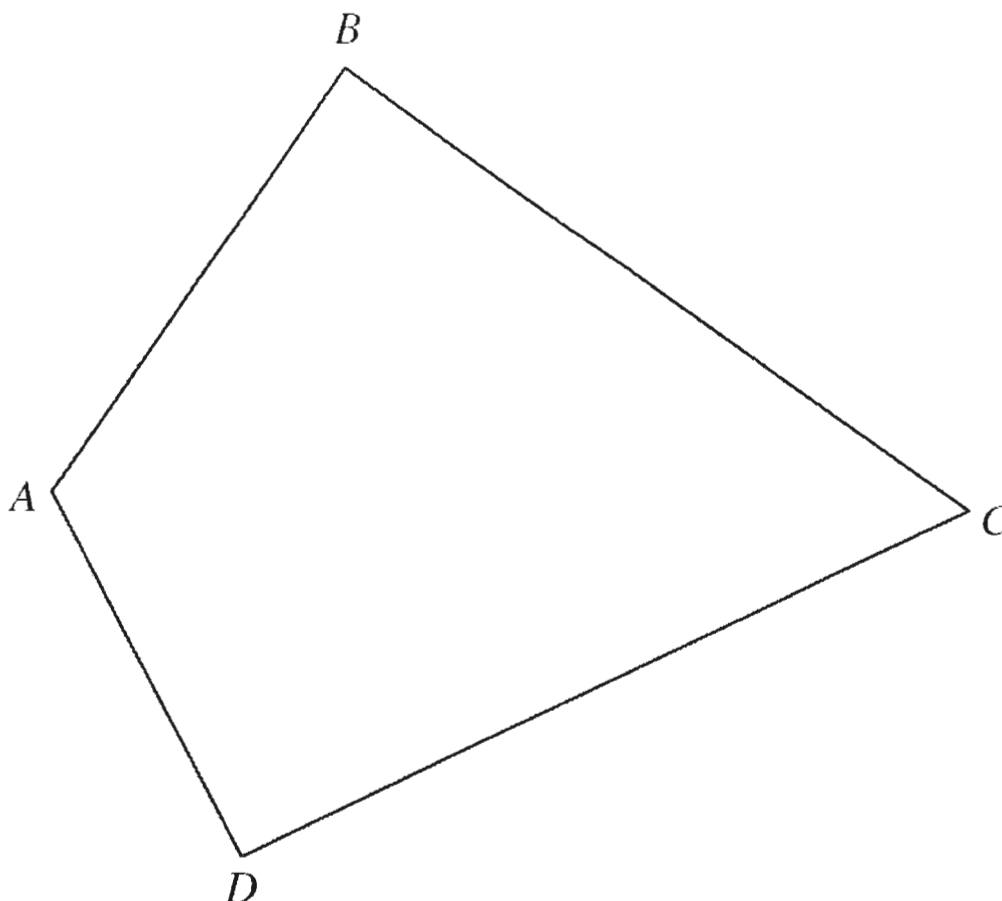
$$1 \text{ cm}^2 : 0.25 \text{ km}^2 \quad \text{----- M1}$$

$$\text{Actual area} = 18(0.25)$$

$$= 4.5 \text{ km}^2 \quad \text{----- A1}$$

Answer 4.5 km^2 [2]

- 6 The diagram shows a plot of land $ABCD$ in the shape of a quadrilateral. A point Q is inside the quadrilateral $ABCD$ such that it is equidistant from B and C , and equidistant from BC and DC .

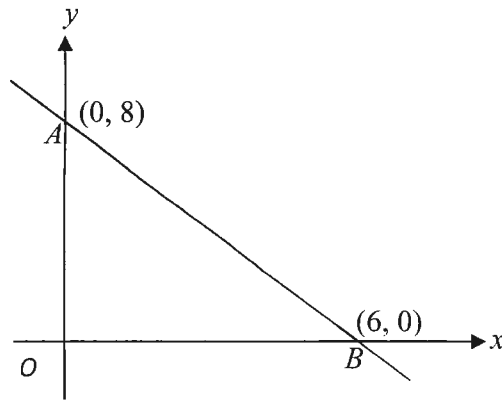


On the diagram,

- (a) construct the bisector of angle BCD , [1]
- (b) construct the perpendicular bisector of BC , [1]
- (c) find and label the point Q . [1]

11

- 7 A straight line crosses the y -axis at the point $A(0,8)$ and the x -axis at the point $B(6,0)$.



- (a) Find the equation of the line AB .

$$\begin{aligned}\text{Grad} &= \frac{8-0}{0-6} \\ &= -\frac{4}{3} \quad \text{----- M1} \\ \therefore y &= -\frac{4}{3}x + 8 \quad \text{----- A1}\end{aligned}$$

$$\text{Answer } y = -\frac{4}{3}x + 8 \text{ [2]}$$

- (b) Calculate the length of the line segment AB .

$$\begin{aligned}\text{By Pythagoras Theorem,} \\ \text{Length} &= \sqrt{(0-6)^2 + (8-0)^2} \\ &= \sqrt{(-6)^2 + (8)^2} \\ &= 10 \text{ units} \quad \text{----- A1}\end{aligned}$$

$$\text{Answer } 10 \text{ units [1]}$$

- (c) Another line, which is parallel to the y -axis and which passes through the point $(9,0)$, meets AB produced at C . Calculate the coordinates of the point C .

parallel to the y -axis, grad = undefined

$$\text{Sub } x=9 \text{ into } y = -\frac{4}{3}x + 8,$$

$$y = -\frac{4}{3}(9) + 8$$

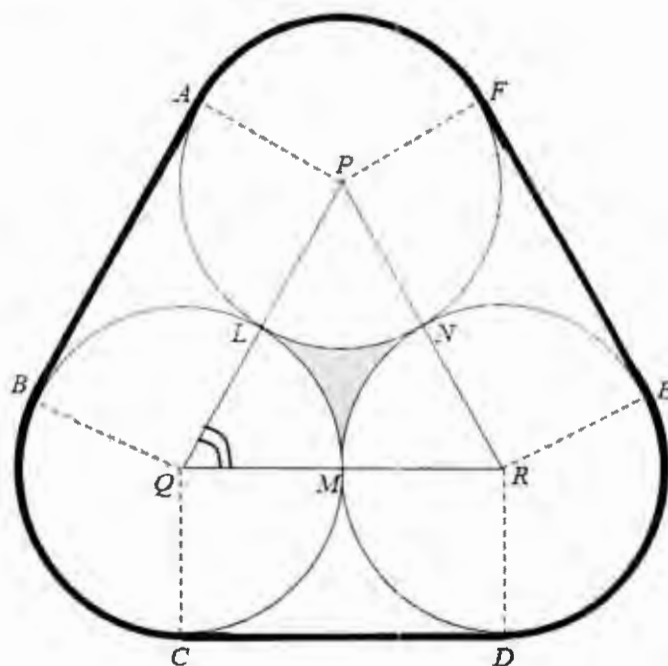
$$y = -4 \quad \text{----- M1}$$

$$\therefore C(9, -4) \quad \text{----- A1}$$

- Award B2 if student use counting method to get answer.

$$\text{Answer } (9, -4) \text{ [2]}$$

- 8 The diagram shows the cross-section of 3 circular tins of the same size with centres P , Q and R . The tins touch one another externally at L , M and N .



- (a) (i) Show that $\angle PQR = \frac{\pi}{3}$ radians.

$$PQ = QR = RP$$

$$PL + LQ = QM + MR = RN + NP \text{ (Sum of radii)} \quad \text{----- M1}$$

$$\therefore PQR \text{ is an equilateral triangle,} \quad \text{----- A1}$$

$$\angle PQR = \frac{\pi}{3} \text{ radians (Shown)}$$

Answer Shown in the space above [2]

- (ii) Given that the area of $\triangle PQR = 6.93 \text{ cm}^2$, show that the length QL is approximately 2.00 cm.

$$\frac{1}{2}(2r)^2 \sin\left(\frac{\pi}{3}\right) = 6.93 \quad \text{----- M1}$$

$$4r^2 = \frac{2(6.93)}{\sin\left(\frac{\pi}{3}\right)}$$

$$4r^2 = 16.004149$$

$$r^2 = 4.001037$$

$$r = 2.0002593$$

$$r = 2.00 \text{ cm (3sf) (Shown)} \quad \text{----- A1}$$

Answer Shown in the space above [2]

13

- (b) Calculate the shaded area.

$$\begin{aligned}\text{Area of sector QML} &= \frac{1}{2}(2^2)\left(\frac{\pi}{3}\right) && \text{----- M1} \\ &= 2.094395 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Shaded area} &= \text{Area of } \triangle PQR - \text{Area of sector} \\ &= 6.93 - (2.094395 \times 3) && \text{----- M1} \\ &= 0.646815 \\ &= 0.647 \text{ cm}^2 \text{ (3sf)} && \text{----- A1}\end{aligned}$$

Answer 0.647 cm² [3]

- (c) The circular tins are bounded together by a piece of rope *ABCDEF*.
Calculate the total length of the rope used.

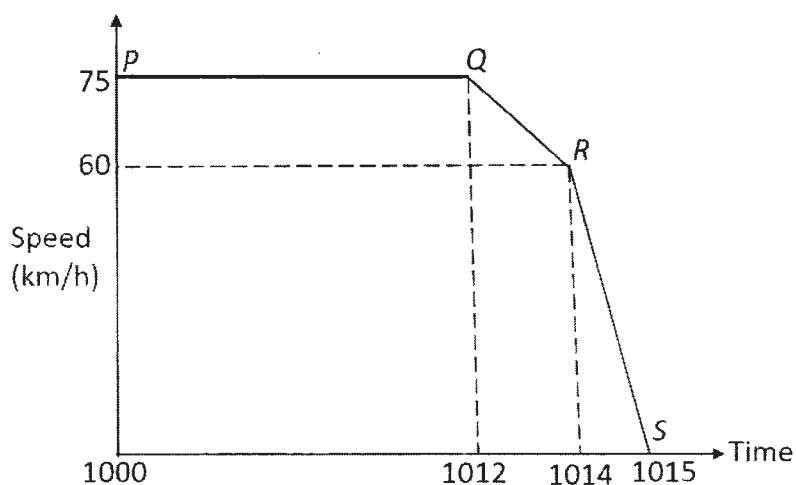
$$\begin{aligned}\angle BQC &= 2\pi - 2\left(\frac{\pi}{2}\right) - \frac{\pi}{3} && \text{----- M1} \\ &= \frac{2}{3}\pi\end{aligned}$$

$$\begin{aligned}\text{Length of minor arc } BC &= 2\left(\frac{2}{3}\pi\right) && \text{----- M1} \\ &= \frac{4\pi}{3}\end{aligned}$$

$$\begin{aligned}\text{Length of rope used} &= 3\left(\frac{4\pi}{3}\right) + 3(4) && \text{----- M1} \\ &= 24.5663 \\ &= 24.6 \text{ cm (3sf)} && \text{----- A1}\end{aligned}$$

Answer 24.6 cm [4]

- 9 The diagram shows the speed-time graph of a car which travelled from P to S .



- (a) How long was the car travelling at constant speed?

Answer12 minutes..... [1]

- (b) What is the distance travelled by the car from P to S ?

Distance travelled = Area under graph

$$= \frac{1}{2} \left(\frac{15}{60} + \frac{14}{60} \right) (60) + \frac{1}{2} \left(\frac{14}{60} + \frac{12}{60} \right) (15) \text{ ---- M1}$$

$$= 17.75 \text{ km} \text{ ---- A1}$$

Answer17.75 km..... [2]

- (c) What is the deceleration from Q to R km/h^2 ?

$$\text{Grad} = - \frac{15}{\left(\frac{2}{60} \right)}$$

$$= -450$$

$$\therefore \text{deceleration} = 450 \text{ km/h}^2 \text{ ---- A1}$$

Answer450 km/h^2 [1]

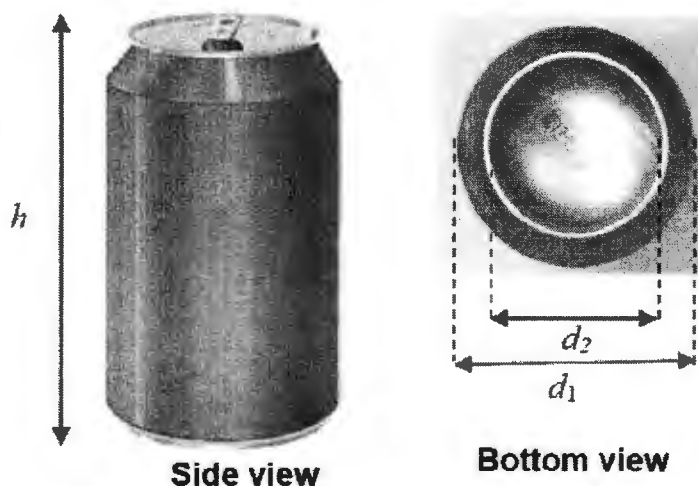
- 10 The following are the information about a soda can.

Soda Can

Height (h): 11.4 cm

Outer Diameter (d_1): 6.4 cm

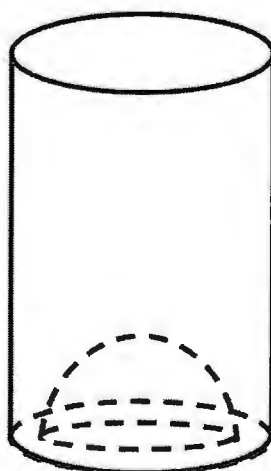
Inner Diameter (d_2): 5.0 cm



Density of soda = 1.2 g/cm^3

Safety information: The soda can is to be filled to a maximum of 95% of its total volume.

The soda can (above) can be modelled as a cylinder with an inner hemisphere that is hollowed inwards (concave) at the base of the can.



- (a) (i) Find the base area, in square centimetres, of the soda can.

BA = (Area of big circle – Area of small circle) + curved SA of hemisphere

$$= \left[(\pi) \left(\frac{6.4}{2} \right)^2 - (\pi) \left(\frac{5.0}{2} \right)^2 \right] + 2(\pi)(2.5)^2 \quad \text{----- M1 (hemisphere)}$$

$$= (32.169908 - 19.63495) + 39.269908$$

$$= 51.8048$$

$$= 51.8 \text{ cm}^2 \text{ (3sf)} \quad \text{----- A1}$$

Answer51.8 cm²..... [2]

16

- (ii) Find the total volume, in cubic centimetres, of the soda can.

Total volume = Vol of cylinder – Vol of hemisphere

$$\begin{aligned}
 &= (\pi) \left(\frac{6.4}{2} \right)^2 (11.4) - \frac{2}{3} (\pi) \left(\frac{5.0}{2} \right)^3 \quad \text{----- M1} \\
 &= 366.7369 - 32.72492 \\
 &= 334.0119 \\
 &= 334 \text{ cm}^3 \text{ (3sf)} \quad \text{----- A1}
 \end{aligned}$$

Answer334 cm³..... [2]

- (b) The material used to make the wall of the soda can must be carefully chosen such that the total mass of each filled soda can is less than 620 g. The manufacture manager of the soda can proposed to use an alloy which has a mass of 0.8 g for every 1 cm² to make the can, If the thickness of the soda can is negligible, will you accept his proposal? Justify your answer with suitable calculations.

$$\begin{aligned}
 \text{Vol of soda can} &= \left(\frac{95}{100} \right) (334.0119) \\
 &= 317.3113 \text{ cm}^3 \quad \text{----- M1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Mass of soda} &= \text{Density} \times \text{Volume} \\
 &= 1.2 \times 317.3113 \\
 &= 380.77356 \text{ g} \quad \text{----- M1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of material used} &= \text{Curved SA} + \text{BA} + \text{Area of top circle} \\
 &= 2\pi(3.2)(11.4) + 51.8048 + \pi(3.2)^2 \\
 &= 229.2106 + 51.8048 + 32.1699 \\
 &= 313.1853 \text{ cm}^2 \quad \text{----- M1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Mass of material used} &= 313.1853 \times 0.8 \\
 &= 250.54824 \text{ g} \quad \text{----- M1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Total mass} &= 380.77356 + 250.54824 \\
 &= 631.3218 \text{ g} \quad \text{----- M1}
 \end{aligned}$$

Since total mass 631.3218 g > 620 g, therefore the proposal is not acceptable. -- A1

Answer Shown in the space above [6]

~ End of Paper ~