Name : ______________________ Sec 4 ____ (   )

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2014
SECONDARY FOUR

Additional Mathematics — 4047/01

Paper 1 — 31 July 2014

2 hours

Additional Materials: Writing paper (8 sheets)
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiners' Use													
Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Marks													
													80

Parent's signature:
(date):

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

1 **(a)** Find the smallest value of the integer a for which $ax^2 - 8x + 3$ is positive for all values of x. [3]

(b) The equation of a curve is $y = x^3 + \frac{5}{2}x^2 - 2x + 1$.
Find the set of values of x for which y is a decreasing function. [4]

2 **Without using a calculator,**

(i) show that $\tan\left(\frac{5\pi}{12}\right) = \frac{1+\sqrt{3}}{\sqrt{3}-1}$, [2]

(ii) express $\tan\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right)$ in the form $k(2+\sqrt{3})$, where k is a constant. [4]

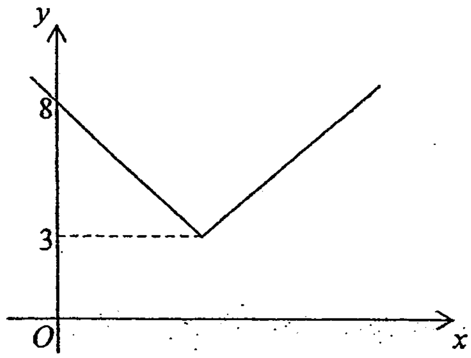
3 Show that $(\tan A + \cot A)\sec A = \frac{1}{\sin A - \sin^3 A}$ [4]

4 In the expansion of $\left(ax + \frac{b}{2x}\right)^n$, the fourth term is $-8437\frac{1}{2}$ and the coefficient of x^2 is $7593\frac{3}{4}$, where a and b are constants.

(i) Write down the value of n. [1]

(ii) Find the value of the positive constant a. [5]

5



The diagram above shows part of the graph of $y = |2x - p| + q$. The y–intercept of the graph is 8 and its minimum y value is 3.

(i) State the value of q. [1]

(ii) Find the value of p. [3]

Determine the number of intersections of the line $y = -2x + 10$ with the graph $y = |2x - p| + q$, justifying your answer. [1]

4

6 Express $\dfrac{3x(2x^2+3)-4}{3x^3-x^2}$ in partial fraction.

7 A population of a certain type of bacteria, P, at time t hours, is given by $P = P_0e^{kt}$, where P_0 is the initial number of bacteria and k is a constant.
The bacteria doubles in population every 7.5 hours. Given that there were approximately 100 bacteria to start with, find

(i) the value of k,

(ii) the approximate number of bacteria that will there be in 2.5 days.

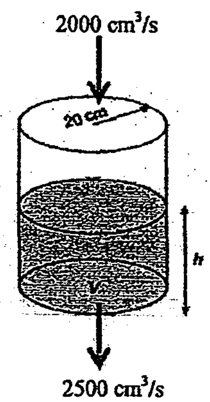
8 The roots of the equation $x^2 - 2x + 49 = 0$ are α^2 and β^2 where $\alpha > 0$ and $\beta > 0$.
Given that the roots of $x^2 + px + q = 0$ are α^3 and β^3, find the value of p and of q where p and q are constants.

9

(i) Differentiate $\dfrac{\sin 2x}{e^{2-x}}$. Hence find, in terms of π, the equation of the tangent to the curve $y = \dfrac{\sin 2x}{e^{2-x}}$ at the point when $x = \pi$.

(ii) Find the equation of the curve which passes through the point $(2, \pi)$ and has a gradient function of $\dfrac{dy}{dx} = \tan^2 x + 1$.

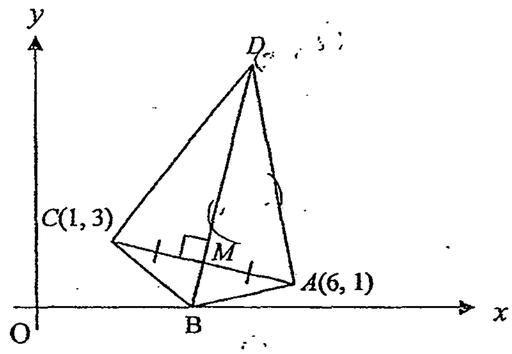
10 Water is leaking from a cylindrical tank of radius 20 cm at the rate of 2500 cm³/s and water is added at the rate of 2000 cm³/s. Find the rate at which the water level in the tank is decreasing.



11 **(i)** Find the value of k for which the line $y+4x=k$ is a tangent to the curve $y=x^4-k$. [4]

(ii) A curve has the equation $y=\cos 2x-3\sin x$ where $0\le x\le \pi$.
Find the x-coordinates of the stationary points. [3]

12 **Solutions to this question by accurate drawing will not be accepted.**
The diagram shows a quadrilateral $ABCD$ where A is (6, 1), B is on the x-axis and C is (1, 3). The diagonal BD bisects AC at right angles at M and $BD=\frac{7}{2}BM$. Find



(i) the equation of BD, [3]

(ii) the x-coordinate of B, [1]

(iii) the coordinates of D. [3]

13 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y=ab^x$, where a and b are constants.

x	1	2	3	4	5
y	42	120	365	920	2700

(i) Using graph paper, draw the graph of $\lg y$ against x. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) By drawing a suitable straight line on your graph, solve the equation $10^{2-2x}=ab^x$. [2]

End Of Paper

Name: ______________________ (   ) Class: 4 ____

Anglican High School
Secondary Four Preliminary Examination
Additional Mathematics Paper 2 (4047/02)

Additional Materials: 8 writing papers 05 August 2014 2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6	7	8
Marks								
Question	9	10	11	12	13	Total		
Marks							100	

Parent's Name and Signature Date

Mathematical Formulae

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For the equation $ax^2 + bx + c = 0$,

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

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Answer ALL Questions

1. The line $5y = 6x + 2$ meets the curve $3x^2 + 4xy - 4y^2 = 0$ at the points A and B. Find the coordinates of A and B. [5]

2. Solve the equation $3^{1-x} + 1 = 10(3^x)$. [5]

3. (i) Show that $\dfrac{d}{dx}\ln\left(\dfrac{x^2}{\sin x}\right) = \dfrac{2}{x} - \cot x$. [2]

(ii) Hence evaluate $\displaystyle\int_1^2 \left(\frac{3}{4}\cot x\right)\, dx$. [3]

4. Given that $(x - 1)$ and $(x + 2)$ are both factors of the expression $f(x)$ where $f(x) = 2x^3 + mx^2 - 7x + n$, find the value of m and of n. Factorise $f(x)$ completely. [5]
Hence, solve the equation $f(x) = 2\,(x - 1)(x + 2)$. [3]

5. Solve the following equations, where $0° < x < 360°$.

i) $2\operatorname{cosec}^2\frac{x}{2} = \sin\frac{x}{2}$ [4]

ii) $5\sin^2 x + 9\cos x - 9 = 0$ [5]

6. (a) If $\log_x \dfrac{p}{\sqrt{q}} - 3\log_x \sqrt{q} = \log_x(p - q)$, express p in terms of q. [3]

(b) Given that $a = \log_2 3$ and $b = \log_2 7$, express $\log_2 21 + \log_4 \dfrac{16}{7}$ in terms of a and b. [4]

(c) **Without using the calculator**, simplify the following

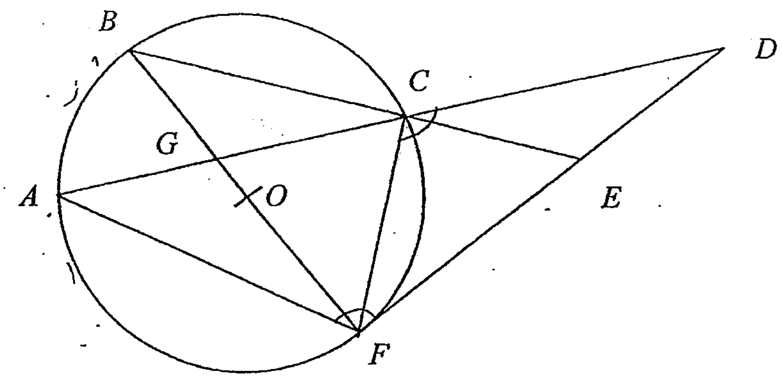
$$\frac{(\sqrt[10]{x} + 1)(x^{\frac{21}{10}} - x^2)}{\sqrt[5]{x} - 1}$$ [3]

7. In the diagram, not drawn to scale, A, B, C and F lie on a circle with centre, O. $BGOF$, $AGCD$, BCE and FED are straight lines where FED is a tangent to the circle at F.

Prove that

(a) $\angle FCD = \angle AFD$, and [3]

(b) $FC \times FE = BF \times CE$ [3]



8. On the same diagram, sketch the graphs of $y = \frac{1}{16}\sqrt{x^5}$ and $y = \frac{4}{\sqrt{x}}$ for $x \geq 0$. [4]

(i) Find the point of intersection of the graphs. [2]

(ii) Calculate the area enclosed by the curve $y = \frac{1}{16}\sqrt{x^5}$, $y = 3$, $x = 5$ and $x = 8$. [4]

9. (a) The function f is defined by $f(x) = c - a \sin bx$.

Given that the period of $f(x)$ is 720° and $-1 \leq f(x) \leq 3$,

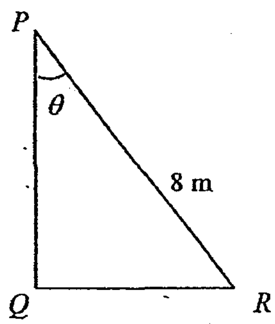
state the value of a, of b and of c.

(b) Sketch the graphs of $y = |3 \cos 2x|$ and $y = 2 - \frac{2}{\pi}x$ on the same axes for $0 \leq x \leq \pi$.

Hence, state the number of solution for $|3\pi \cos 2x| + 2x = 2\pi$. [4]

10. Given that the gradient of the tangent to the circle, C_1, at $A(5, 2)$ is $-\frac{4}{5}$, find the equation of the normal to the circle at A.
The circle, C_1, also passes through the point $B(5, 6)$.
Find the centre of the circle, C_1. [3]
Hence, find the equation of circle, C_1. [3]
A circle, C_2, is formed when the circle, C_1, is reflected along the line $y = 0$.
Write down the equation of the circle, C_2. [1]

11. A gardener is going to fence off part of his garden by using a piece of 19 metres long fence. The enclosed area is in the shape of a right-angled triangle PQR with $PR = 8$ m.
Given that θ is the included angle between the sides PQ and PR,



i) show that $8 \cos \theta + 8 \sin \theta = 11$. [2]

Hence, express the equation in the form $R \cos (\theta - \alpha) = C$, where C is an integer, $R > 0$ and $0° < \alpha < 90°$. [2]

ii) find the possible values of θ. [3]

12. A particle moves in a straight line with velocity, v m/s, given by $v = t^2 - 8t + 15$ where t is the time in seconds, measured from the start of the motion. Its initial displacement from a fixed point O is -6 metres.

(a) Find

(i) the values of t for which the particle is at instantaneous rest [2]

(ii) the minimum velocity of the particle [2]

(b) Determine the average speed of the particle in the first 7 seconds. [5]

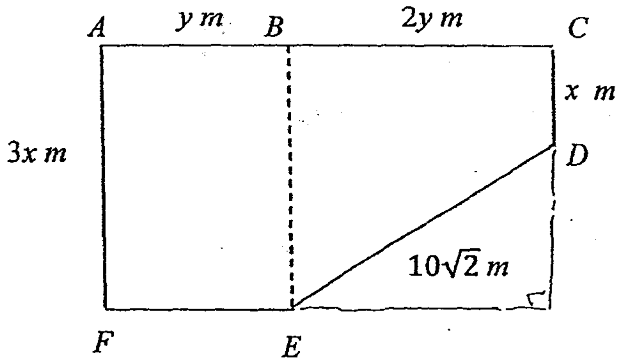
(c) Show that the particle does not return to its starting point. [1]

13. The diagram, not drawn to scale, shows the plan of a park, $ABCDEF$. $BCDE$ is a trapezium, with CD parallel to BE. $ABEF$ is a rectangle and ABC is a st
Given that $DE = 10\sqrt{2}$ metres, $AB = y$ metres, $BC = 2y$ metres, $CD = x$ metres
$AF = 3x$ metres.

(i) Express y in terms of x.

(ii) Show that the area of the park, A m^2, is given by $A = 7x\sqrt{50 - x^2}$

(iii) Determine the value of x that will give the maximum area.



***** END OF PAPER *****

Name : ______________________________ Sec 4 _____ (   )

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2014
SECONDARY FOUR

Additional Mathematics 4047/01
Paper 1
31 July 2014
2 hours

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$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
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$$\text{Area of }\Delta=\frac{1}{2}ab\sin C$$

1 (a) Find the smallest value of the integer a for which $ax^2 - 8x + 3$ is positive for all values of x. [3]

(b) The equation of a curve is $y = x^3 + \frac{5}{2}x^2 - 2x + 1$.

Find the set of values of x for which y is a decreasing function. [4]

(a) $ax^2 - 8x + 3 > 0$

$(-8)^2 - 12a < 0$

$12a > 64$

$a > 5\frac{1}{3}$

The smallest integer a is 6.

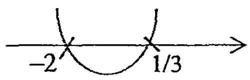
(b) $y = x^3 + \frac{5}{2}x^2 - 2x + 1$

$\frac{dy}{dx} = 3x^2 + 5x - 2$

For decreasing function, $\frac{dy}{dx} < 0$

$3x^2 + 5x - 2 < 0$

$(3x-1)(x+2) < 0$



$-2 < x < \frac{1}{3}$

---

2 **Without using a calculator,**

(i) show that $\tan\left(\frac{5\pi}{12}\right) = \frac{1+\sqrt{3}}{\sqrt{3}-1}$, [2]

(ii) express $\tan\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right)$ in the form $k(2+\sqrt{3})$, where k is a constant. [4]

(i) $\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{2\pi+3\pi}{12}\right)$

$= \tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$= \frac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\times\tan\frac{\pi}{4}}$

$= \frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}}$

$= \frac{1+\sqrt{3}}{\sqrt{3}-1}$

(ii) $\tan\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right) = \frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{1}{2}$

$= \frac{1}{2} \times \frac{(1+\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{1}{2} \times \frac{\sqrt{3}+1+3+\sqrt{3}}{3-1}$

$= \frac{1}{4}(4 + 2\sqrt{3})$

$= \frac{1}{2}(2 + \sqrt{3})$

---

3 Show that $(\tan A + \cot A)\sec A = \frac{1}{\sin A - \sin^3 A}$ [4]

$(\tan A + \cot A)\sec A = \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right) \times \frac{1}{\cos A}$

$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \times \frac{1}{\cos A}$

$= \frac{1}{\sin A \cos A} \times \frac{1}{\cos A}$

$= \frac{1}{\sin A \cos^2 A}$

$= \frac{1}{\sin A(1-\sin^2 A)}$

$= \frac{1}{\sin A - \sin^3 A}$

---

4 In the expansion of $\left(ax + \frac{b}{2x}\right)^n$, the fourth term is $-8437\frac{1}{2}$ and the coefficient of x^2 is $7593\frac{3}{4}$, where a and b are constants.

(i) Write down the value of n. [1]

(ii) Find the value of the positive constant a. [5]

(i) 4^{th} term, $r = 3$, $n = 6$

$\boxed{\text{or}}$ $T_{r+1} = \binom{n}{r}(ax)^{n-r}\left(\frac{b}{2x}\right)^r$

$T_4 = \binom{n}{3}(ax)^{n-3}\left(\frac{b}{2x}\right)^3$

$n - 3 - 3 = 0$

$\therefore n = 6$

5

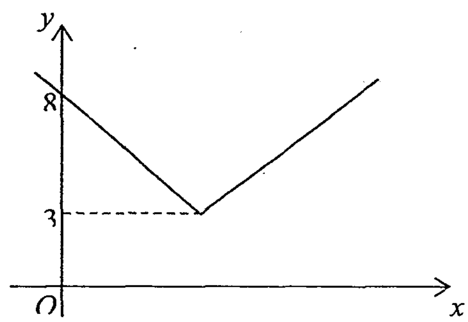
(ii)
$$\binom{6}{3}(ax)^3\left(\frac{b}{2x}\right)^3 = -8437\frac{1}{2}$$
$$20a^3\left(\frac{b^3}{8}\right) = -8437\frac{1}{2}$$
$$a^3b^3 = -3375$$
$$ab = -15$$

x^2 term: $6 - r - r = 2$
$$r = 2$$

$$\binom{6}{2}(ax)^4\left(\frac{b}{2x}\right)^2 = 7593\frac{3}{4}x^2$$
$$15a^4\left(\frac{b^2}{4}\right) = 7593\frac{3}{4}$$
$$a^4b^2 = 2025$$
$$a^2(-15)^2 = 2025$$
$$a^2 = 9$$

$\therefore a = 3$ or $a = -3$ (reject, $\because a > 0$)

5



The diagram above shows part of the graph of $y = |2x - p| + q$. The y-intercept of the graph is 8 and its minimum y value is 3.

(i) State the value of q. [1]

(ii) Find the value of p. [3]

Determine the number of intersections of the line $y = -2x + 10$ with the graph $y = |2x - p| + q$, justifying your answer. [1]

(i) $q = 3$

(ii) when $x = 0, y = 8$
$$8 = |0 - p| + 3$$
$$|-p| = 5$$
$$p = 5 \text{ or } p = -5\text{(reject)}$$

$y = -2x + 10$ is parallel to the Left-hand arm and y-intercept is $10 > 8$, it intersects RH arm once, $\therefore$ there is only 1 intersection.

6 Express $\frac{3x(2x^2+3)-4}{3x^3-x^2}$ in partial fraction. [7]

$$\frac{3x(2x^2+3)-4}{3x^3-x^2} = \frac{6x^3+9x-4}{3x^3-x^2}$$
$$= \frac{6x^3-2x^2+2x^2+9x-4}{3x^3-x^2}$$
$$= 2 + \frac{2x^2+9x-4}{x^2(3x-1)}$$

$\boxed{\text{or}}$

$$\begin{array}{r} 2 \\ 3x^3 - x^2 \,\overline{)\,6x^3 + \ 0 + 9x - 4} \\ 6x^3 - 2x^2 \\ \hline +2x^2 + 9x - 4 \end{array}$$

Let $\frac{2x^2+9x-4}{x^2(3x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x-1}$

$2x^2 + 9x - 4 = Ax(3x-1) + B(3x-1) + Cx^2$
$= (3A + C)x^2 + (-A + 3B)x - B$

By comparison: $B = 4$
$-A + 12 = 9, \quad \therefore A = 3$
$9 + C = 2, \quad \therefore C = -7$

$\boxed{\text{or}}$

when $x = 0,\ B = 4$
when $x = \frac{1}{3},\ \frac{2}{9} + 3 - 4 = \frac{C}{9}$
$\therefore C = -7$
when $x = 1,\ 7 = 2A + 8 - 7$
$\therefore A = 3$

Therefore $\frac{3x(2x^2+3)-4}{3x^3-x^2} = 2 + \frac{3}{x} + \frac{4}{x^2} - \frac{7}{3x-1}$

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7 A population of a certain type of bacteria, P, at time t hours, is given by $P = P_o e^{kt}$, where P_o is the initial number of bacteria and k is a constant.
The bacteria doubles in population every 7.5 hours. Given that there were approximately 100 bacteria to start with, find

(i) the value of k, [2]

(ii) the approximate number of bacteria that will there be in 2.5 days. [3]

(i)
$P = P_o e^{kt}$
$200 = 100e^{7.5k}$
$e^{7.5k} = 2$
$7.5k = ln2$
$k = 0.092419 \approx 0.0924$

(ii)
$P = 100e^{0.092419\times 60}$
$= 25599$
≈ 25600

There were approximately 25600 bacteria in 2.5 days.

8 The roots of the equation $x^2 - 2x + 49 = 0$ are α^2 and β^2 where $\alpha > 0$ and $\beta > 0$. Given that the roots of $x^2 + px + q = 0$ are α^3 and β^3, find the value of p and of q where p and q are constants. [5]

Product of roots: $\alpha^2\beta^2 = 49$

$\alpha\beta = 7$ or $\alpha\beta = -7$ (reject)

Sum of roots: $\alpha^2 + \beta^2 = 2$

$$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$
$$= 2 + 2(7)$$
$$(\alpha+\beta)^2 = 16$$

$\alpha + \beta = 4$ or $\alpha + \beta = -4$ (reject)

When $\alpha + \beta = 4$ and $\alpha\beta = 7$

$$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$$
$$= 4(2-7)$$
$$= -20$$

$$\alpha^3\beta^3 = 7^3 = 343$$

New equationn: $x^2 + 20x + 343 = 0$

$\therefore p = 20,\ q = 343.$

---

9 (i) Differentiate $\frac{\sin 2x}{e^{2-x}}$. Hence find, in terms of π, the equation of the tangent to the curve $y = \frac{\sin 2x}{e^{2-x}}$ at the point when $x = \pi$. [4]

(ii) Find the equation of the curve which passes through the point $(2, \pi)$ and has a gradient function of $\frac{dy}{dx} = \tan^2 x + 1$. [3]

(i) $$\frac{dy}{dx} = \frac{2e^{2-x}\cos 2x + (\sin 2x)e^{2-x}}{\left(e^{2-x}\right)^2}$$
$$= \frac{2\cos 2x + \sin 2x}{e^{2-x}}$$

When $x = \pi$, $y = \frac{\sin 2\pi}{e^{2-\pi}} = 0$

Gradient of tangent $= \frac{2\cos 2\pi + \sin 2\pi}{e^{2-\pi}}$

$$= \frac{2}{e^{2-\pi}}$$
$$= 2e^{\pi-2}$$

Equation of tangent: $y - 0 = 2e^{\pi-2}(x - \pi)$

$y = 2xe^{\pi-2} - 2\pi e^{\pi-2}$ o.e.

9 (ii) $\dfrac{dy}{dx} = \tan^2 x + 1$

$= \sec^2 x$

$y = \int \sec^2 x \, dx$

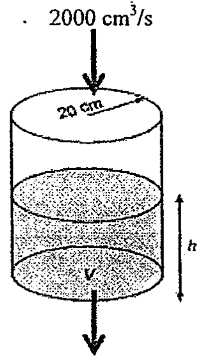
$= \tan x + c$

At point $(2, \pi)$: $\pi = \tan 2 + c$

$c = \pi - \tan 2$ or $c = 5.32663\ldots$

$\therefore \; y = \tan x + \pi - \tan 2$ or $y = \tan x + 5.33$

10 Water is leaking from a cylindrical tank of radius 20 cm at the rate of 2500 cm^3/s and fresh water is added at the rate of 2000 cm^3/s. Find the rate at which the water level in the tank is decreasing. [5]



Nett $\dfrac{dV}{dt} = 2000 - 2500$

$= -500 \quad cm^3/s$

Volume of water: $V = \pi r^2 h$

Since $r = 20$, $V = \pi(20)^2 h$

$= 400\pi h$

$\dfrac{dV}{dh} = 400\pi$

$\dfrac{dV}{dt} = \dfrac{dV}{dh} \times \dfrac{dh}{dt}$

$-500 = 400\pi \times \dfrac{dh}{dt}$

$\dfrac{dh}{dt} = -0.39788$

$\dfrac{dh}{dt} = -0.398$ cm/s

Ans: Water level in the tank is falling at 0.398 cm/s.

11 **(i)** Find the value of k for which the line $y+4x=k$ is a tangent to the curve $y=x^4-k$. [4]

(ii) A curve has the equation $y=\cos 2x-3\sin x$ where $0\le x\le \pi$.
Find the x-coordinates of the stationary points. [3]

(i)

(i) Given the curve $y=x^4-k$ ....①

$$\frac{dy}{dx}=4x^3$$

Given tangent of the curve: $y=-4x+k$ ....②

Gradient of tangent $=-4$

$$\therefore \quad 4x^3=-4$$
$$x=-1$$

When $x=-1$,

①=②, $(-1)^4-k=-4(-1)+k$

$$2k=-3$$
$$k=-\frac{3}{2}$$

(ii)

(ii) Given $y=\cos 2x-3\sin x$

$$\frac{dy}{dx}=-2\sin 2x-3\cos x$$

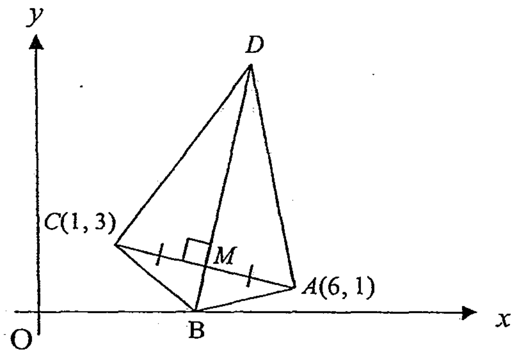
For stationary points, $\frac{dy}{dx}=0$

$$-2\sin 2x-3\cos x=0$$
$$4\sin x\cos x+3\cos x=0$$
$$\cos x(4\sin x+3)=0$$
$$\cos x=0 \quad \text{or} \quad (4\sin x+3)=0$$
$$x=\frac{\pi}{2} \quad \text{or} \quad \sin x=-\frac{3}{4}\text{ (reject)}$$

12 **Solutions to this question by accurate drawing will not be accepted.**

10

The diagram shows a quadrilateral $ABCD$ where A is (6, 1), B is on the x-axis and C is (1, 3). The diagonal BD bisects AC at right angles at M and $BD = \frac{7}{2}BM$. Find



(i) the equation of BD, [3]

(ii) the x-coordinate of B, [1]

(iii) the coordinates of D. [3]

(i) Coordinates of $M = \left(\frac{1+6}{2}, \frac{3+1}{2}\right) = \left(\frac{7}{2}, 2\right)$

Gradient of $AC = \frac{3-1}{1-6} = -\frac{2}{5}$

Gradient of $BD = \frac{5}{2}$

Equation of BD:

$$y - 2 = \frac{5}{2}\left(x - \frac{7}{2}\right)$$

$$y - 2 = \frac{5}{2}x - \frac{35}{4}$$

$$4y = 10x - 27$$

(ii) When $y = 0$,

$$0 = 10x - 27$$

$$x = 2\frac{7}{10}$$

(iii) $\overrightarrow{BD} = \frac{7}{2}\overrightarrow{BM}$

$$\overrightarrow{OD} - \overrightarrow{OB} = \frac{7}{2}\overrightarrow{OM} - \frac{7}{2}\overrightarrow{OB}$$

$$\overrightarrow{OD} = \frac{7}{2}\overrightarrow{OM} - \frac{5}{2}\overrightarrow{OB}$$

$$= \frac{7}{2}\begin{pmatrix} \frac{7}{2} \\ 2 \end{pmatrix} - \frac{5}{2}\begin{pmatrix} \frac{27}{10} \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5\frac{1}{2} \\ 7 \end{pmatrix}$$

$\therefore\ D$ is $\left(5\frac{1}{2}, 7\right)$

`Alternative by similar triangle:
Let the coordinates of D be (a, b).

$$\frac{b}{2} = \frac{7}{2}$$

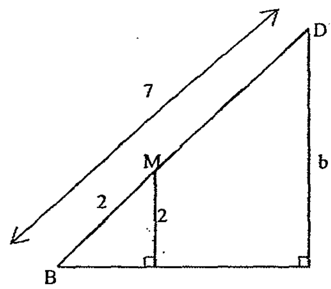
$\therefore\ b = 7$

$4y = 10x - 27$

$28 = 10a - 27$

$10a = 55$

$a = 5\frac{1}{2}$



$\therefore\ D$ is $\left(5\frac{1}{2}, 7\right)$ o.e.

---

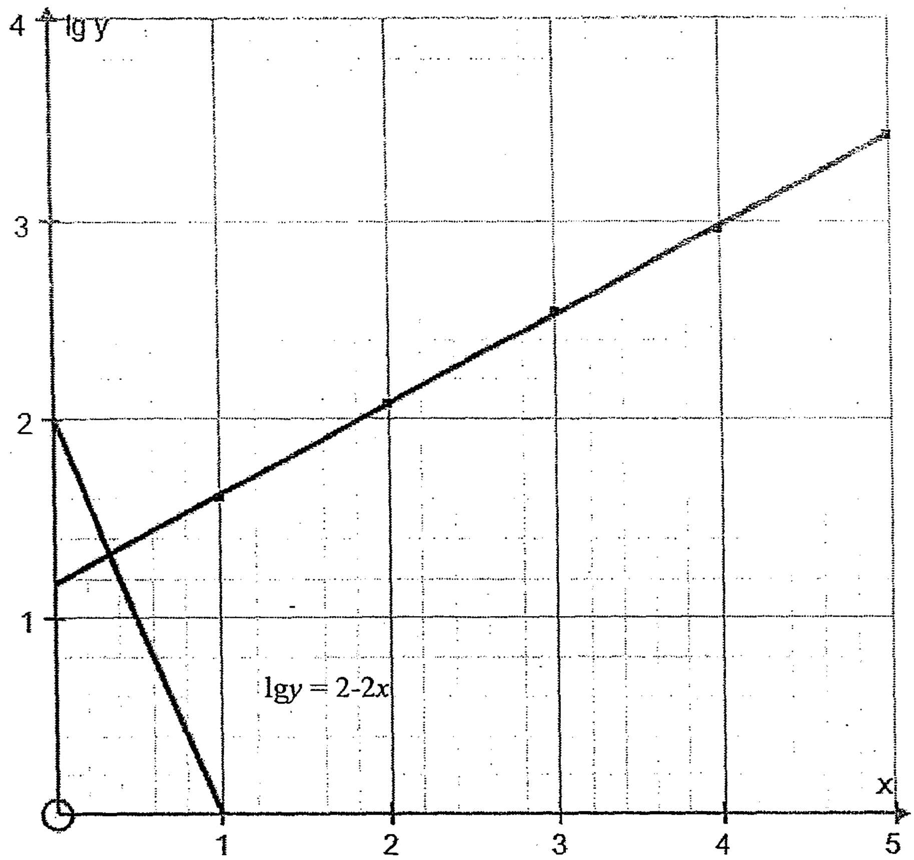
13 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = ab^x$, where a and b are constants.

x	1	2	3	4	5
y	42	120	365	920	2700

(i) Using graph paper, draw the graph of $\lg y$ against x. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) By drawing a suitable straight line on your graph, solve the equation $10^{2-2x} = ab^x$. [2]



$y = ab^x$
$\lg y = \lg a + x \lg b$

Plot $\lg y$ against x.

x	1	2	3	4	5
y	1.62	2.07	2.56	2.96	3.43

From the graph,

$\lg a = 1.15$
$a = 14.1$

$\lg b = \dfrac{2.96 - 1.15}{4 - 0}$
$= 0.4525$
$b = 2.83$

$y = 10^{2-2x}$
$\lg y = 2 - 2x$

From the graph, $x = 0.3$

End Of Paper

ANGLICAN HIGH SCHOOL

SECONDARY FOUR PRELIMINARY EXAMINATION

Additional Mathematics Paper 2 (4047/02)

Additional Materials: 8 writing papers | 05 August 2014 | 2 hours 30 minutes

Solutions

1. The line $5y = 6x + 2$ meets the curve $3x^2 + 4xy - 4y^2 = 0$ at the points A and B. Find the coordinates of A and B. [5]

Solution:

Given: $5y = 6x + 2$

$$y = \frac{6}{5}x + \frac{2}{5} \quad \text{............①}$$

sub ① into $\quad 3x^2 + 4xy - 4y^2 = 0$

$$3x^2 + 4x(\frac{6}{5}x + \frac{2}{5}) - 4(\frac{6}{5}x + \frac{2}{5})^2 = 0$$

$$3x^2 + \frac{24}{5}x^2 + \frac{8}{5}x - 4(\frac{36}{25}x^2 + \frac{24}{25}x + \frac{4}{25}) = 0$$

$$3x^2 + \frac{24}{5}x^2 + \frac{8}{5}x - \frac{144}{25}x^2 - \frac{96}{25}x - \frac{16}{25} = 0$$

$$\frac{51}{25}x^2 - \frac{56}{25}x - \frac{16}{25} = 0$$

$$51x^2 - 56x - 16 = 0$$

$$x = \frac{-(-56) \pm \sqrt{(-56)^2 - 4(51)(-16)}}{2(51)}$$

$$= \frac{56 + 80}{102} \quad \text{or} \quad = \frac{56 - 80}{102}$$

$$= \frac{136}{102} \quad \text{or} \quad = -\frac{24}{102}$$

$$= \frac{4}{3} \quad \text{or} \quad = -\frac{4}{17}$$

Sub $x = \frac{4}{3}$ into①

$$y = \frac{6}{5}(\frac{4}{3}) + \frac{2}{5}$$

$$= 2$$

or sub $x = -\frac{4}{17}$ into①

$$y = \frac{6}{5}(-\frac{4}{17}) + \frac{2}{5}$$

$$= \frac{2}{17}$$

Ans: $A(\frac{4}{3}, 2)$ and $B(-\frac{4}{17}, \frac{2}{17})$

$\boxed{\textbf{Accept } A(1.33, 2) \text{ and } B(-0.235, 0.118)}$

2. Solve the equation $3^{1-x}+1=10(3^x)$. [5]

Solution

$$3^{1-x}+1=10(3^x)$$
$$\frac{3}{3^x}+1=10\ (3^x)$$
$$3+1(3^x)=10\ (3^x)(3^x)$$
$$3+3^x=10\ (3^{2x})$$
$$10\ (3^{2x})-3^x-3=0$$

Let $u=3^x$

$$10u^2-u-3=0$$
$$(5u-3)(2u+1)=0$$

$$u=\frac{3}{5}$$
$$3^x=\frac{3}{5}$$
$$\ln 3^x=\ln\frac{3}{5}$$
$$x\ln 3=\ln\frac{3}{5}$$
$$x=\ln\frac{3}{5}\div\ln 3$$
$$=-0.46497$$
$$=-0.465$$

$$u=-\frac{1}{2}$$
$$3^x=-\frac{1}{2}\ \text{(No solution)}$$

3. (i) Show that $\frac{d}{dx}\ln\left(\frac{x^2}{\sin x}\right)=\frac{2}{x}-\cot x$. [2]

(ii) Hence evaluate $\int_1^2 \frac{3}{4}\cot x \, dx$. [3]

Solution

(i)

$$\frac{d}{dx}\ln\left(\frac{x^2}{\sin x}\right)=\frac{d}{dx}\left(\ln x^2-\ln\sin x\right)$$
$$=\frac{d}{dx}(2\ln x-\ln\sin x)$$
$$=2\left(\frac{1}{x}\right)-\frac{1}{\sin x}(\cos x)$$
$$=\frac{2}{x}-\cot x$$

ALTERNATIVE

$$\frac{d}{dx}\ln\left(\frac{x^2}{\sin x}\right)=\frac{1}{\left(\frac{x^2}{\sin x}\right)}\left(\frac{(\sin x)(2x)-(x^2)(\cos x)}{(\sin x)^2}\right)$$
$$=\frac{\sin x}{x^2}\left(\frac{2x\sin x-x^2\cos x}{\sin^2 x}\right)$$
$$=\frac{\sin x}{x^2}\left(\frac{2x\sin x}{\sin^2 x}\right)-\frac{\sin x}{x^2}\left(\frac{x^2\cos x}{\sin^2 x}\right)$$
$$=\frac{2}{x}-\frac{\cos x}{\sin x}$$
$$=\frac{2}{x}-\cot x$$

(ii)

$$\int_1^2\left(\frac{2}{x}-\cot x\right)dx=\left[\ln\frac{x^2}{\sin x}\right]_1^2$$
$$\int_1^2\frac{2}{x}\,dx-\int_1^2\cot x\,dx=\left(\ln\frac{(2)^2}{\sin 2}\right)-\ln\frac{(1)^2}{\sin 1}$$
$$[2\ln x]_1^2-\int_1^2\cot x\,dx=1.3087$$
$$(2\ln 2-2\ln 1)-\int_1^2\cot x\,dx=1.3088$$
$$\int_1^2\cot x\,dx=2\ln 2-1.3087$$
$$=0.077594$$

$$\int_1^2\frac{3}{4}\cot x\,dx=\frac{3}{4}\times 0.077594$$
$$=0.0581955$$
$$=0.0582$$

4. Given that $(x-1)$ and $(x+2)$ are both factors of the expression $f(x)$ where $f(x)=2x^3+mx^2-7x+n$, find the value of m and of n. Factorise $f(x)$ completely. [5]

Hence, solve the equation $f(x)=2(x-1)(x+2)$. [3]

Solution:

Let $2x^3+mx^2-7x+n=(x-1)(x+2)(2x+c)$

$=(x^2+x-2)(2x+c)$

coeff of x: $-7=c-4$

$c=-3$

coeff of x^2: $m=c+2$

$=-3+2$

$=-1$

constant: $n=-2c$

$=-2(-3)$

$=6$

Therefore,

$f(x)=(x-1)(x+2)(2x-3)$

OR

Given $f(x)=2x^3+mx^2-7x+n$

and $f(1)=0$

$2(1)^3+m(1)^2-7(1)+n=0$

$m+n=5$ ....①

Given $f(-2)=0$

$2(-2)^3+m(-2)^2-7(-2)+n=0$

$4m+n=2$ ....②

②–①, $3m=-3$

$m=-1$

Sub $m=-1$ into ①,

$-1+n=5$

$n=6$

By inspection

$\therefore f(x)=2x^3-x^2-7x+6$

$=(x-1)(x+2)(2x-3)$

Given: $f(x)=2(x-1)(x+2)$

$$(x-1)(x+2)(2x-3)=2(x-1)(x+2)$$

$$(x-1)(x+2)(2x-3)-2(x-1)(x+2)=0$$

$$(x-1)(x+2)[(2x-3)-2]=0$$

$$(x-1)(x+2)(2x-5)=0$$

$$(x-1)=0,\quad (x+2)=0,\quad (2x-5)=0$$

$$x=1,\qquad x=-2,\qquad x=\frac{5}{2}$$

5. Solve the following equations, where $0^\circ < x < 360^\circ$.

i) $2\ \text{cosec}^2 \frac{x}{2} = \sin \frac{x}{2}$ [4]

ii) $5 \sin^2 x + 9 \cos x - 9 = 0$ [5]

Solution:

i) $2\ \text{cosec}^2 \frac{x}{2} = \sin \frac{x}{2}$ $\quad [0^\circ < \frac{x}{2} < 180^\circ]$

$\frac{2}{\sin^2 \frac{x}{2}} = \sin \frac{x}{2}$

$\sin^3 \frac{x}{2} = 2$

$\sin \frac{x}{2} = \sqrt[3]{2}$

no solution $[\because \sqrt[3]{2} > 1]$

ii) $5 \sin^2 x + 9 \cos x - 9 = 0$ $\quad [0^\circ < x < 360^\circ]$

$5(1 - \cos^2 x) + 9 \cos x - 9 = 0$

$5 - 5\cos^2 x + 9 \cos x - 9 = 0$

$5 \cos^2 x - 9 \cos x + 4 = 0$

$(5\cos x - 4)(\cos x - 1) = 0$

$\cos x = \frac{4}{5}$ or $\cos x = 1$

$x = 36.869^\circ, 360^\circ - 36.869^\circ$ $\quad x = 0^\circ, 360^\circ$ (all rejected)

$\approx 36.869^\circ, 323.131^\circ$

$\approx 36.9^\circ \quad , 323.1^\circ$

6 (a) If $\log_x \dfrac{p}{\sqrt{q}} - 3\log_x \sqrt{q} = \log_x (p-q)$, express p in terms of q. [3]

Solution

6a)

$$\log_x \frac{p}{\sqrt{q}} - 3\log_x \sqrt{q} = \log_x (p-q)$$

$$\log_x \frac{p}{q^{\frac{1}{2}}} - \log_x q^{\frac{3}{2}} = \log_x (p-q)$$

$$\log_x \left(\frac{p}{q^{\frac{1}{2}}} \div q^{\frac{3}{2}} \right) = \log_x (p-q)$$

$$\log_x \left(\frac{p}{q^{\frac{1}{2}} \times q^{\frac{3}{2}}} \right) = \log_x (p-q)$$

$$\frac{p}{q^2} = p - q$$

$$p = pq^2 - q^3$$

$$p - pq^2 = -q^3$$

$$p\left(1-q^2\right) = -q^3$$

$$p = \frac{-q^3}{1-q^2}$$

$$= \frac{q^3}{q^2-1}$$

6 (b) Given that $a = \log_2 3$ and $b = \log_2 7$, express $\log_2 21 + \log_4 \frac{16}{7}$ in terms of a and b, [4]

Solution

6b)

$$\log_2 21 + \log_4 \frac{16}{7} = \log_2 (3 \times 7) + \left(\log_4 2^4 - \log_4 7\right)$$

$$= \log_2 3 + \log_2 7 + \left(\frac{\log_2 2^4}{\log_2 4} - \frac{\log_2 7}{\log_2 4}\right)$$

$$= \log_2 3 + \log_2 7 + \left(\frac{4\log_2 2}{\log_2 2^2} - \frac{\log_2 7}{\log_2 2^2}\right)$$

$$= a + b + \left(\frac{4(1)}{2\log_2 2} - \frac{b}{2\log_2 2}\right)$$

$$= a + b + \left(\frac{4}{2(1)} - \frac{b}{2(1)}\right)$$

$$= a + b + 2 - \frac{1}{2}b$$

$$= 2 + a + \frac{1}{2}b$$

---------------------------------- Or ----------------------------------

$$\log_2 21 + \log_4 \frac{16}{7} = \log_2 21 + \frac{\log_2 \frac{16}{7}}{\log_2 4}$$

$$= \log_2 3 + \log_2 7 + \frac{1}{2}(\log_2 2^4 - \log_2 7)$$

$$= \log_2 3 + \log_2 7 + 2 - \frac{1}{2}\log_2 7$$

$$= \log_2 3 + 2 + \frac{1}{2}\log_2 7$$

$$= a + 2 + \frac{1}{2}b$$

6. (c) **Without using the calculator**, simplify the following

$$\frac{(\sqrt[10]{x}+1)(x^{\frac{21}{10}}-x^2)}{\sqrt[5]{x}-1}$$ [3]

Solution:

6c) $$\frac{(\sqrt[10]{x}+1)(x^{\frac{21}{10}}-x^2)}{\sqrt[5]{x}-1}=\frac{(\sqrt[10]{x}+1)(x^2)(x^{\frac{1}{10}}-1)}{\sqrt[5]{x}-1}$$

$$=\frac{((\sqrt[10]{x})^2-1)(x^2)}{\sqrt[5]{x}-1}$$

$$=\frac{(\sqrt[5]{x}-1)(x^2)}{\sqrt[5]{x}-1}$$

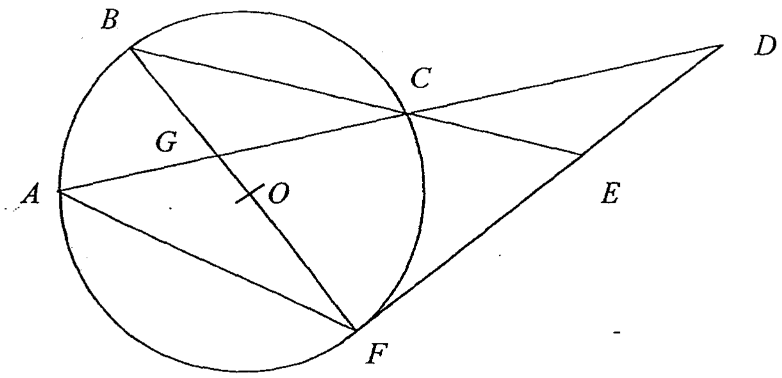
$$=x^2$$

7. In the diagram, not drawn to scale, A, B, C and F lie on a circle with centre, O. $BGOF$, $AGCD$, BCE and FED are straight lines, where FED is a tangent to the circle at F.

Prove that

(a) $\angle FCD = \angle AFD$, and [3]

(b) $FC \times FE = BF \times CE$ [3]



Solution

(a) $\angle DFC = \angle FAD$ (Alternate Segment Theorem / Tangent Chord Theorem)

$\angle CDF = \angle ADF$ (Common angle)

Triangle CDF and triangle FDA are similar (AA property).

Hence $\angle FCD = \angle AFD$.

(b) $\angle BCF = 90°$ (Angle in a semicircle)

$\angle FCE = 90°$ (Sum of angles on a straight line)

$\angle BCF = \angle FCE$

$\angle FBC = \angle EFC$ (Alternate Segment Theorem / Tangent Chord Theorem)

Triangle BFC and triangle FEC are similar (AA property).

$$\frac{BF}{FC} = \frac{FE}{EC}$$

$BF \times EC = FE \times FC$

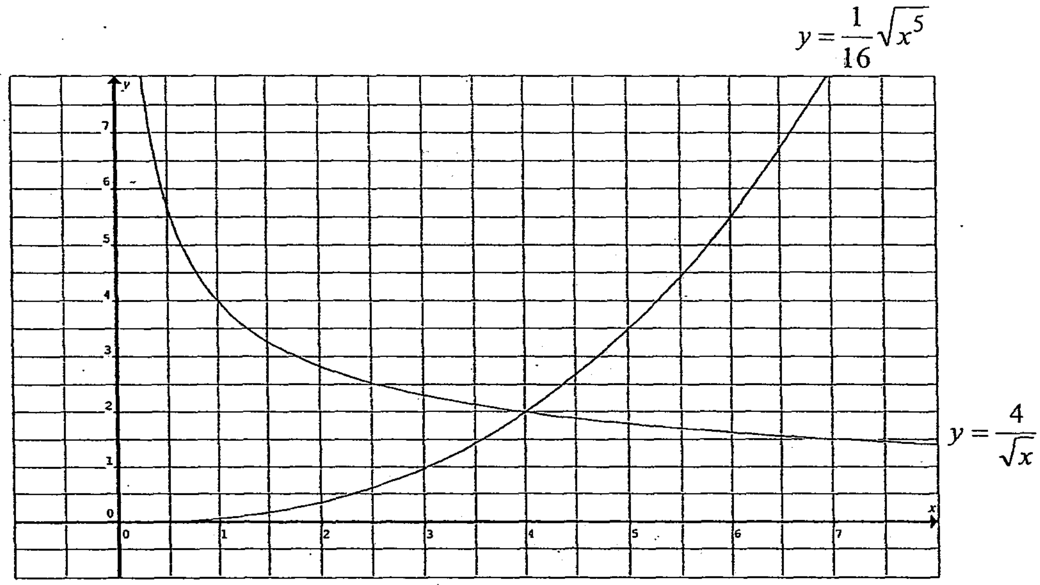
Hence $FC \times FE = BF \times CE$.

8. On the same diagram, sketch the graphs of $y=\frac{1}{16}\sqrt{x^5}$ and $y=\frac{4}{\sqrt{x}}$ for $x\geq 0$. [4]

(i) Find the point of intersection of the graphs. [2]

(ii) Calculate the area enclosed by the curve $y=\frac{1}{16}\sqrt{x^5}$, $y=3$, $x=5$ and $x=8$. [4]

Solution



(i) $\frac{1}{16}\sqrt{x^5}=\frac{4}{\sqrt{x}}$

$$\frac{1}{16}x^{\frac{5}{2}}=4x^{-\frac{1}{2}}$$

$$\left(x^{\frac{5}{2}}\right)\left(x^{\frac{1}{2}}\right)=(4)(16)$$

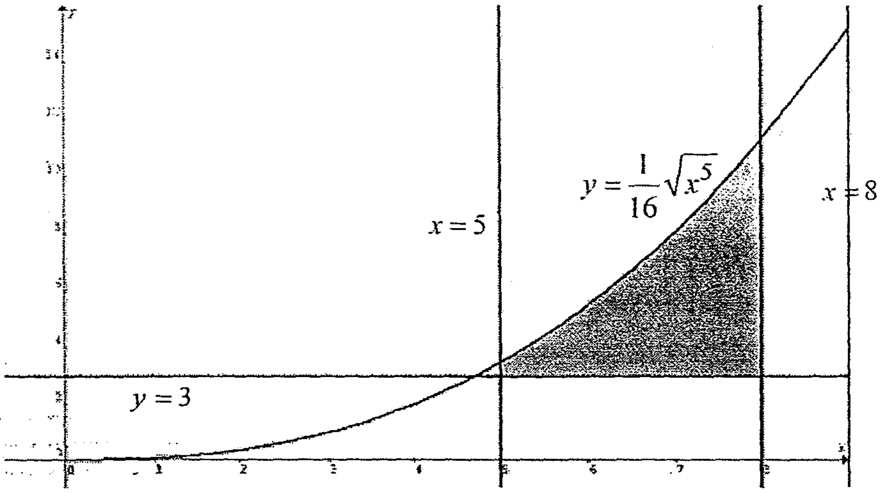
$$x^3=4^3$$

$$x=4$$

When $x=4$, $y=\frac{4}{\sqrt{4}}=2$.

Therefore the point of intersection is $(4, 2)$

8(ii)



$$\text{Area} = \int_5^8 \left(\frac{1}{16}x^{\frac{5}{2}} - 3\right)\,dx$$

$$= \left[\frac{1}{16}\left(\frac{2}{7}\right)x^{\frac{7}{2}} - 3x\right]_5^8$$

$$= \left(\frac{1}{56}(8)^{\frac{7}{2}} - 3(8)\right) - \left(\frac{1}{56}(5)^{\frac{7}{2}} - 3(5)\right)$$

$$= 11.868681$$

$$\approx 11.9 \quad \text{units}^2$$

Or $\quad \text{Area} = \int_5^8 \left(\frac{1}{16}\,x^{\frac{5}{2}}\right)\,dx - 3(8-5)$

$$= \left[\frac{1}{56}\,x^{\frac{7}{2}}\right]_5^8 - 9$$

$$= \left(\frac{1}{56}(8)^{\frac{7}{2}} - \frac{1}{56}(5)^{\frac{7}{2}}\right) - 9$$

$$= 20.868681 - 9$$

$$= 11.868681$$

$$\approx 11.9 \quad \text{units}^2$$

9. (a) The function f is defined by $f(x) = c - a \sin bx$.
Given that the period of $f(x)$ is 720° and $-1 \le f(x) \le 3$.
State the value of a, of b and of c.

(b) Sketch the graphs of $y = |3 \cos 2x|$ and $y = 2 - \frac{2}{\pi}x$ on the same axes.
Hence, state the number of solution for $|3\pi \cos 2x| + 2x = 2\pi$.

Solution:

(a) $a = \pm 2$,

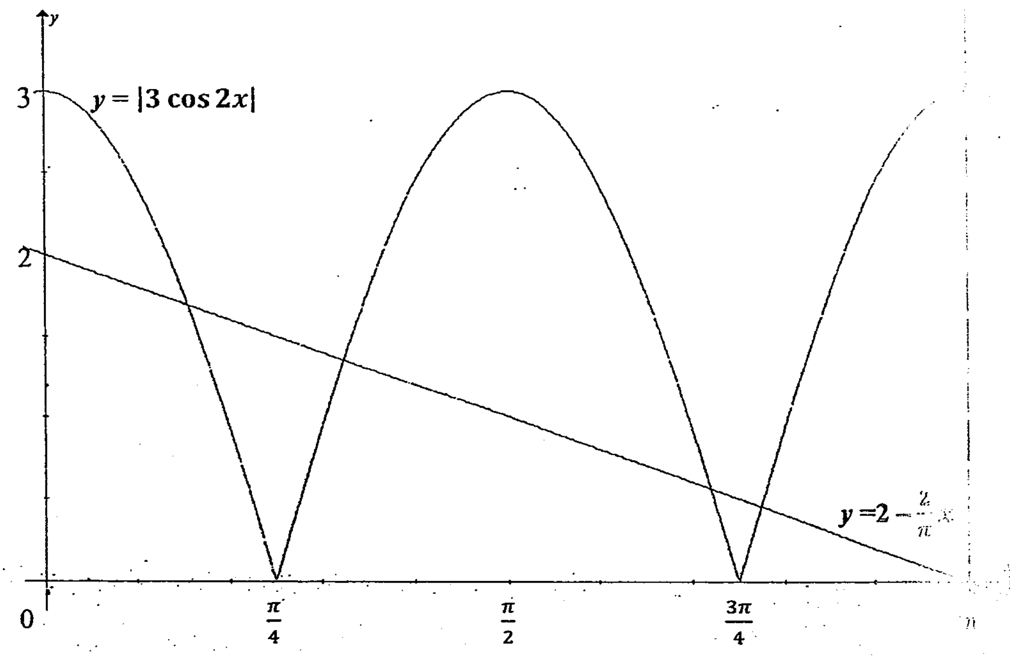
$b = \frac{1}{2}$

$c = 1$

$f(x) = 1 - 2 \sin \frac{x}{2}$

For $f(x) = c - a \sin bx$
Given period of $f(x) = 720^\circ$ $\therefore$ $b = 360$
$= \frac{1}{2}$
Given: $-1 \le f(x) \le 3$. $\therefore$ $a =$ amplitude
$= \frac{1}{2}(3 - (-1))$
$= 2$
$c =$ max value
$= 3 - 2$
$= 1$

(b) Given: $y = |3 \cos 2x|$ and $y = 2 - \frac{2}{\pi}x$ for $0 \le x \le \pi$.



$|3\pi \cos 2x| + 2x = 2\pi$.

$|3 \cos 2x| + \frac{2}{\pi}x = 2$

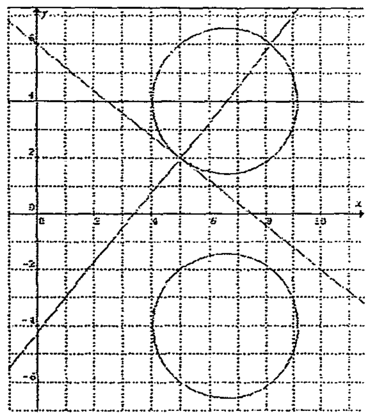
$|3 \cos 2x| = 2 - \frac{2}{\pi}x$

Number of solution = 4

10. Given that the gradient of the tangent to the circle, C_1, at $A(5,2)$ is $-\frac{4}{5}$, find the equation of the normal to the circle at A. [2]
The circle, C_1, also passes through the point $B(5, 6)$. Find the centre of the circle, C_1. [3]
Hence, find the equation of circle, C_1. [3]
A circle, C_2, is formed when the circle, C_1, is reflected along the line $y = 0$.
Write down the equation of the circle, C_2. [1]

Solution:

Gradient of normal $= -\frac{1}{\left(-\frac{4}{5}\right)} = \frac{5}{4}$



Equation of normal:

$$y-2=\frac{5}{4}(x-5)$$

$$y=\frac{5}{4}x-\frac{17}{4}$$

Mid-point of (5, 2) and (5, 6) $= (5, \frac{2+6}{2})$

$= (5, 4)$

Perpendicular bisector of AB is $y=4$

The perpendicular bisector will intersect the normal at the centre of the circle.

Consider the intersection of $y=4$ and $y=\frac{5}{4}x-\frac{17}{4}$

$$4=\frac{5}{4}x-\frac{17}{4}$$

$$\frac{5}{4}x=\frac{33}{4}$$

$$x=\frac{33}{5}$$

Centre of the circle is $\left(\frac{33}{5}, 4\right)$

$$\text{Radius of the circle} = \sqrt{\left(5-\frac{33}{5}\right)^2+(2-4)^2}$$

$$=\sqrt{\frac{164}{25}}$$

Equation of Circle, C_1:

$$\left(x-\frac{33}{5}\right)^2+(y-4)^2=\left(\sqrt{\frac{164}{25}}\right)^2$$

$$\left(x-\frac{33}{5}\right)^2+(y-4)^2=\frac{164}{25}$$

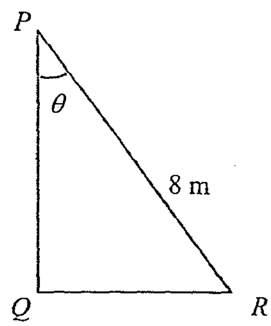
Accept $5x^2+5y^2-66x-40y+265=0$

Equation of Circle, C_2:

$$\left(x-\frac{33}{5}\right)^2+(y+4)^2=\frac{164}{25}$$

Accept $5x^2+5y^2-66x+40y+265=0$

11. A gardener is going to fence off part of his garden by using a piece of 19 metres long fence. The enclosed area is in the shape of a right-angled triangle PQR with $PR = 8$ m.



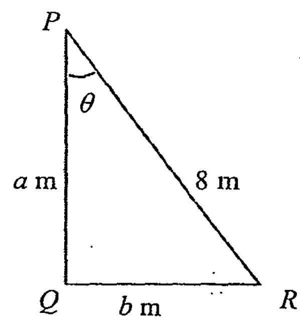
Given that θ is the included angle between the sides PQ and PR,

i) show that $8 \cos \theta + 8 \sin \theta = 11$. [2]

Hence, express the equation in the form $R \cos(\theta - \alpha) = C$, where C is an integer, $R > 0$ and $0° < \alpha < 90°$. [2]

ii) find the possible values of θ. [3]

Solution:



i) $a = 8 \cos \theta$

$b = 8 \sin \theta$

$8 \cos \theta + 8 \sin \theta + 8 = 19$

$8 \cos \theta + 8 \sin \theta = 11$ (shown)

$(\sqrt{8^2 + 8^2}) \cos(\theta - \tan^{-1} \frac{8}{8}) = 11$ $[-45° < \theta - 45° < 45°]$

$\sqrt{128} \cos(\theta - 45°) = 11$

$8\sqrt{2} \cos(\theta - 45°) = 11$

ii) $\cos(\theta - 45°) = \frac{11}{\sqrt{128}}$

$\theta - 45° \approx -13.524°, 13.524°$

$\theta \approx -13.524° + 45°, 13.524° + 45°$

$\approx 31.476°, 58.524°$

$\approx 31.5°, 58.5°$

Ans: the possible values of $\theta \approx 31.5°, 58.5°$

12 A particle moves in a straight line with velocity, v m/s, given by $v=t^2-8t+15$ where t is the time in seconds, measured from the start of the motion. Its initial displacement from a fixed point O is -6 metres.

(a) Find

(i) the values of t for which the particle is at instantaneous rest [2]

(ii) the minimum velocity of the particle [2]

(b) Determine the average speed of the particle in the first 7 seconds. [5]

(c) Show that the particle does not return to its starting point. [1]

Solution

(a)(i) At instantaneous rest, $v=0$

$$t^2-8t+15=0$$
$$(t-3)(t-5)=0$$
$$t=3 \text{ s} \quad \text{or } t=5\text{ s}$$

(a)(ii) At maximum / minimum velocity, $a=0$

$$a=\frac{dv}{dt}$$
$$=2t-8$$

$$a=0$$
$$2t-8=0$$
$$t=4$$

Check for max / min, $\frac{d^2v}{dt^2}=2>0$, $t=4$ s gives minimum velocity.

Minimum velocity,

$$v=(4)^2-8(4)+15=-1 \text{ m/s}$$

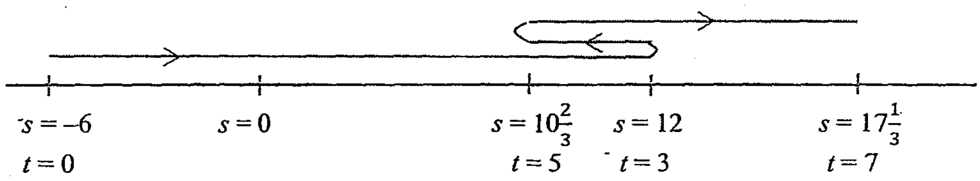
(b) Displacement, $s=\int v\,dt$

$$s=\int(t^2-8t+15)dt$$
$$=\frac{1}{3}t^3-4t^2+15t+c$$

When $t=0$, $s=-6$, therefore $-6=\frac{1}{3}(0)^3-4(0)^2+15(0)+c$, and $c=-6$

Hence $s=\frac{1}{3}t^3-4t^2+15t-6$

Time, t (sec)	Displacement, s (m)	Distance travelled (m)
0	-6	
3	12	$12-(-6)=18$
5	$10\frac{2}{3}$	$12-\left(10\frac{2}{3}\right)=1\frac{1}{3}$
7	$17\frac{1}{3}$	$17\frac{1}{3}-\left(10\frac{2}{3}\right)=6\frac{2}{3}$



$$\text{Total distance travelled} = 18+1\frac{1}{3}+6\frac{2}{3}$$
$$= 26 \text{ m}$$

$$\text{Average Speed} = \frac{26}{7}$$
$$= 3\frac{5}{7} \qquad \text{or} \qquad 3.7143 \approx 3.71 \text{ m/s}$$

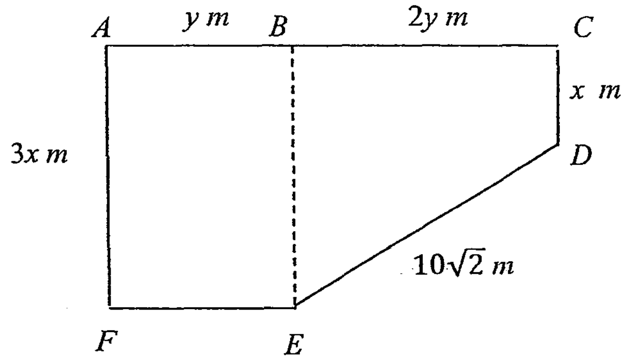
(c) At the second turning point, the displacement, $s = 1\frac{1}{3}$ m. The displacement increases after that, and never becomes -6 m. Hence, the particle never returns to its starting point.

13. The diagram, not drawn to scale, shows the plan of a park, $ABCDEF$. $BCDE$ is a trapezium, with CD parallel to BE. $ABEF$ is a rectangle and ABC is a straight line. Given that $DE = 10\sqrt{2}$ metres, $AB = y$ metres, $BC = 2y$ metres, $CD = x$ metres and $AF = 3$ metres.

(i) Express y in terms of x. [2]

(ii) Show that the area of the park, A m^2, is given by $A = 7x\sqrt{50-x^2}$ [2]

(iii) Determine the value of x that will give the maximum area. [5]



Solution

(i) $(2y)^2 + (2x)^2 = \left(10\sqrt{2}\right)^2$ (Pythagoras' Theorem)

$$4y^2 + 4x^2 = 200$$
$$4y^2 = 200 - 4x^2$$
$$y = \sqrt{50 - x^2} \quad (y > 0)$$

(ii) $A = (3x)(y) + \frac{1}{2}(x + 3x)(2y)$

$$A = 3xy + 4xy$$
$$= 7xy$$
$$= 7x\sqrt{50 - x^2}$$

13(iii) $\dfrac{dA}{dx} = 7\sqrt{50-x^2} + 7x\left(\dfrac{1}{2}\right)\left(50-x^2\right)^{-\frac{1}{2}}(-2x)$

When $\dfrac{dA}{dx} = 0$

$$7\sqrt{50-x^2} + 7x\left(\frac{1}{2}\right)\left(50-x^2\right)^{-\frac{1}{2}}(-2x) = 0$$

$$7\sqrt{50-x^2} - \frac{7x^2}{\sqrt{50-x^2}} = 0$$

$$7\sqrt{50-x^2} = \frac{7x^2}{\sqrt{50-x^2}}$$

$$\sqrt{50-x^2} = \frac{x^2}{\sqrt{50-x^2}}$$

$$50-x^2 = x^2$$

$$2x^2 = 50$$

$$x^2 = 25$$

$x = 5$ or $x = -5$ (NA)

Not necessary to prove

x	Slightly <5	5	Slightly >5
$\dfrac{dA}{dx}$	>0	0	<0
Or sketch of $\dfrac{dA}{dx}$	/	—	\

-------- Or ----------

13(iii) $\dfrac{dA}{dx} = 7\sqrt{50-x^2} + 7x\left(\dfrac{1}{2}\right)\left(50-x^2\right)^{-\frac{1}{2}}(-2x)$

$= 7(50-x^2)^{-\frac{1}{2}}(50-2x^2)$

When $\dfrac{dA}{dx} = 0$

$7(50-x^2)^{-\frac{1}{2}}(50-2x^2) = 0$

$\Rightarrow \quad (50-2x^2) = 0$

$2x^2 = 50$

$x^2 = 25$

$x = 5$ or $x = -5$ (NA)

**** END OF PAPER *****