Name: ________________________ (   )   Class: ____________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS   4047/01

Paper 1   **27 August 2014**

2 hours

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class, and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

FOR EXAMINER'S USE

Q1		Q6		Q11	
Q2		Q7		Q12	
Q3		Q8		Q13	
Q4		Q9			
Q5		Q10			

/ 80

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{\sqrt{5}}{5-2\sqrt{5}}+\sqrt{125}$ in the form $a+b\sqrt{5}$, where a and b are integers. [3]

2 Find the smallest value of the integer a for which $5x^2-ax+3$ is positive for all values of x. [3]

3 (i) Sketch the graph of $y=e^{3-x}-1$. [2]

(ii) Find the equation of the straight line which must be drawn on the graph $y=e^{3-x}-1$ to obtain the solution of the equation

$$\ln(3x+2)=\ln 2+(3-x).$$ [3]

4 The roots of the equation $x^2-3x+10=0$ are α and β. Find the quadratic equation whose roots are α^3 and β^3. [5]

5 A particle moves along the curve $y=\dfrac{e^{2x}}{x}$. Given that the y-coordinate of the particle is changing at a constant rate of 6 units per second, find the rate of change of the x-coordinate when $x=2$, giving your answer in terms of e. [5]

6 (i) In the binomial expansion of $\left(x^2+\dfrac{k}{x}\right)^6$, where k is a positive constant, the term independent of x is 240. Find the value of k. [3]

(ii) Using the same value of k found in part (i), find the coefficient of x^{12} in the expansion of $\left(1-5x^6\right)\left(x^2+\dfrac{k}{x}\right)^6$. [3]

7 Jim buys a new car. After t months its value $\$V$ is given by $V=120\,000e^{-kt}$, where k is a constant.

(i) Find the value of the car when Jim bought it. [1]

The value of the car after 12 months is expected to be \$ 100 000.

(ii) Calculate the expected value of the car after 4 years, giving your answer to 3 significant figures. [3]

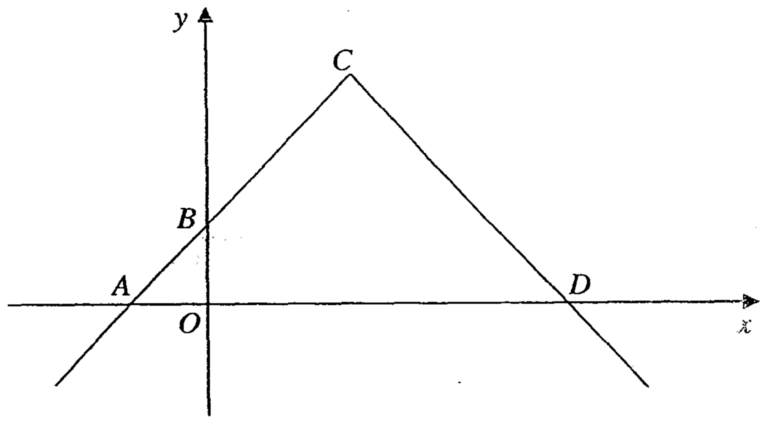
(iii) Find the age of the car, to the nearest month, when its expected value will be \$ 30 000. [2]

8 Given that $f(x) = \sin 2x - x$ for $0 \le x \le \pi$,

(i) sketch the graph represented by $y = f'(x)$,

(ii) find the range of values of x for which $f(x) = \sin 2x - x$ is an increa[illegible] function of x, giving your answer in terms of π.

9



The diagram shows part of the graph of $y = 9 - |2x - 6|$.

(a) Find the coordinates of A, B, C and D.

(b) The line $y = mx$, where $m > 0$, intersects the graph of $y = 9 - |2x - 6$[illegible] one point. Find the minimum value of m.

10 A particle moves in a straight line, so that, t seconds after leaving a fixed po[illegible] velocity v m/s, is given by $v = t(2 - 3t)^2$. Find

(i) an expression for the acceleration of the particle in terms of t,

(ii) the distance travelled by the particle before it comes to instantaneous re[illegible]

11 **(i)** By squaring $\sin^2\theta + \cos^2\theta$, or otherwise, show that

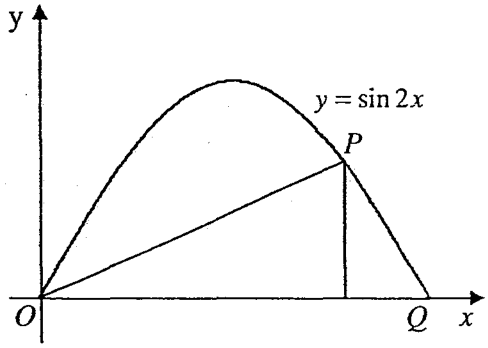
$$\sin^4\theta + \cos^4\theta = \frac{1}{4}(3 + \cos 4\theta).$$

Hence,

(ii) solve the equation $\sin^4\theta + \cos^4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$ for $0 \le \theta \le \pi$ radians, giving your answers in terms of π.

(iii) state, for $-\pi \le \theta \le \pi$, the number of solutions of the equation $\sin^4\theta + \cos^4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$.

12



The diagram shows part of the curve $y = \sin 2x$. P is a point on the curve and its y-coordinate is $\frac{\sqrt{3}}{2}$. The curve meets the x-axis at Q.

Find

(i) the coordinates of P and of Q in terms of π, [4]

(ii) the area of the shaded region. [5]

13 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = \frac{a}{e^{kx}}$, where a and k are constants.

x	1	2	3	4	5	6
y	5.58	7.79	10.87	15.17	21.18	29.56

(i) Plot $\ln y$ against x for the given data and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of k. [4]

(iii) On the same diagram, draw the line representing $y^2 = e^{-4x+8}$ and hence find the value of x for which $ae^{2x-4} = e^{kx}$. [3]

SNGS Additional Mathematics Prelim Paper 1

1.	$2+6\sqrt{5}$	8(ii)	$0\le x<\frac{\pi}{6},\ \frac{5\pi}{6}<x\le\pi$
2.	Smallest integer $a=-7$	9(a)	$A\left(-1\frac{1}{2},0\right),\ B\ (0,3)$ $C=(3,\ 9),\ D\left(7\frac{1}{2},0\right)$
3 (ii)	$y=\frac{3}{2}x$	(b)	minimum value of $m=2$
4.	$x^2+63x+1000=0$	10 (i)	$a=4-24t+27t^2$
5.	$\frac{8}{e^4}$ units per sec	(ii)	$\frac{4}{27}$ or $0.148m$
6 (i)	$k=2$	11(ii)	$\theta=\frac{\pi}{24},\frac{11\pi}{24},\frac{13\pi}{24},\frac{23\pi}{24}$
(ii)	-299	(iii)	8 solutions
7(i)	\$ 120 000	12(i)	$Q\left(\frac{\pi}{2},0\right),\ P\left(\frac{\pi}{3},\frac{\sqrt{3}}{2}\right)$
(ii)	\$ 57 900 (to 3 s.f.)	(ii)	0.547 sq units
(iii)	91 months	13(ii)	$a=4.06\ (3s.f)$ $k=-0.334$
		(iii)	$x=1.15$

SNGS Add Math P1 — 2014

1) $\dfrac{\sqrt{5}}{5-2\sqrt{5}} + \sqrt{125}$

$= \dfrac{\sqrt{5}}{5-2\sqrt{5}} \cdot \dfrac{5+2\sqrt{5}}{5+2\sqrt{5}} + 5\sqrt{5}$

$= \dfrac{\sqrt{5}(5+2\sqrt{5})}{25-20} + 5\sqrt{5}$

$= \dfrac{5\sqrt{5}+10}{5} + \dfrac{25\sqrt{5}}{5}$

$= \dfrac{10+30\sqrt{5}}{5}$

$= 2 + 6\sqrt{5}$

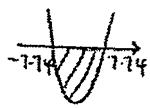
2) $5x^2 - ax + 3 > 0$ for all x,

$b^2 - 4ac < 0$

$(-a)^2 - 4(5)(3) < 0$

$a^2 - 60 < 0$

$(a+2\sqrt{15})(a-2\sqrt{15}) < 0$



$\therefore -7.74 < a < 7.74$

Smallest integer value of a is -7.

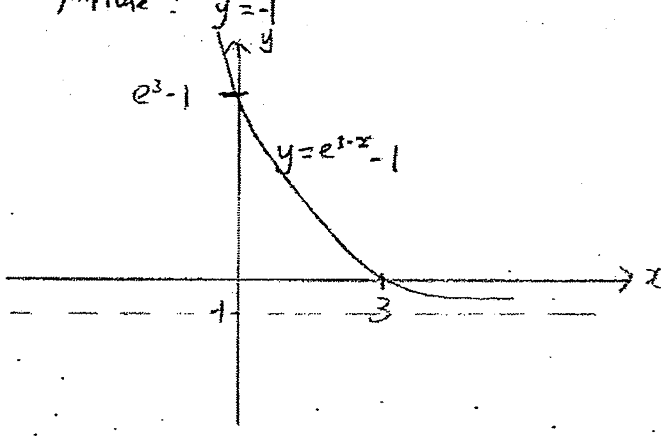
(i) $y = e^{3-x} - 1$

x-intercept : $y = 0$ $\quad e^{3-x} = 1$

$3 - x = 0 \Rightarrow x = 3$

y-intercept : $x = 0$ $\quad y = e^3 - 1 \approx 19.1$

Asymptote : $y = -1$



3 (ii) $\ln(3x+2) = \ln 2 + (3-x)$

$\ln(3x+2) - \ln 2 = 3 - x$

$\ln\left(\dfrac{3x+2}{2}\right) = 3 - x$

$\dfrac{3x+2}{2} = e^{3-x}$

$\dfrac{3x}{2} + 1 - 1 = e^{3-x} - 1$

$\therefore e^{3-x} - 1 = \dfrac{3}{2}x$

Equation of straight line to be drawn is $y = \dfrac{3}{2}x$.

4) $x^2 - 3x + 10 = 0$

Sum of roots, $\alpha + \beta = 3$

Product of roots, $\alpha\beta = 10$

For roots α^3 and β^3,

Sum of roots, $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$

$= 3(3^2 - 3(10))$

$= -63$

Product of roots, $(\alpha\beta)^3 = (10)^3$

$= 1000$

$\therefore$ Quadratic equation is $x^2 + 63x + 1000 = 0$.

5) $\dfrac{dy}{dt} = 6$ units/s

$y = \dfrac{e^{2x}}{x}$

$\dfrac{dy}{dx} = \dfrac{x(2e^{2x}) - e^{2x}}{x^2}$

$= \dfrac{e^{2x}(2x-1)}{x^2}$

When $x = 2$, $\dfrac{dy}{dx} = \dfrac{e^4(4-1)}{4} = \dfrac{3}{4}e^4$

$\dfrac{dx}{dt} = \dfrac{dx}{dy} \times \dfrac{dy}{dt}$

$= \dfrac{4}{3e^4} \times 6$

$= \dfrac{8}{e^4}$ units/s

$\therefore$ The rate of change of x is $\dfrac{8}{e^4}$ units/s.

(i) $6? (x^2+\frac{k}{x})^6$

$T_{r+1} = \binom{6}{r}(x^2)^{6-r}\cdot\left(\frac{k}{x}\right)^r$

$= \binom{6}{r}(x)^{12-2r}\cdot k^r\cdot x^{-r}$

$= \binom{6}{r}(k)^r\cdot x^{12-3r}$

For term independent of x, $12-3r=0$

$r=4$

$\therefore T_5 = \binom{6}{4}k^4 = 240$

$k^4 = 16$

$k = 2$ or -2 (rejected $\because k>0$)

$\therefore k=2$

ii) $(1-5x^6)(x^2+\frac{2}{x})^6$

For x^6 term in $(x^2+\frac{2}{x})^6$

$12-3r=6$

$r=2$

For x^{12} term in $(x^2+\frac{2}{x})^6$

$12-3r=12$

$r=0$

$\therefore (1-5x^6)(x^2+\frac{2}{x})^6$

$= (1-5x^6)\left(\dots+\binom{6}{0}(2)^0x^{12}+\binom{6}{2}(2)^2\cdot x^6+\dots\right)$

Coefficient of x^{12} is

$1(1)-5\binom{6}{2}(4)$

$= 1-300$

$= -299$

7(i) $V=120\,000e^{-kt}$

When $t=0$, $V=120\,000e^0$

The value of the car when Jim bought it is \$120 000.

(ii) $t=12$, $V=100\,000$

$100\,000 = 120\,000e^{-12k}$

$e^{-12k} = \frac{5}{6}$

$k = \frac{\ln(\frac{5}{6})}{-12}$

$= 0.015193$

$\therefore V=120\,000e^{-0.015193t}$

4 years $= 12(4) = 48$ months

Expected value of the car after 4 years is

$V = 120\,000e^{-0.015193(48)}$

≈ 57870.37

$= \$57900$ (to 3 s.f.)

(iii) $30\,000 = 120\,000e^{-0.015193t}$

$e^{-0.015193t} = \frac{1}{4}$

$t = \frac{\ln(\frac{1}{4})}{-0.015193}$

≈ 91.25

$= 91$ months (to the nearest month)

The age of the car will be 91 months when its expected value is \$30 000.

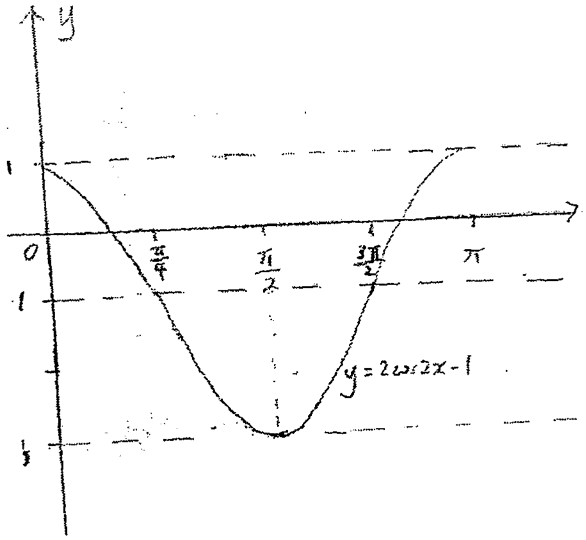
(i) $f(x) = \sin 2x - x \qquad 0 \le x \le \pi$

$f'(x) = 2\cos 2x - 1$

Amplitude $= 2$

Axis of graph is $y = -1$

Period $= \frac{2\pi}{2} = \pi$



ii) Let $f'(x) = 2\cos 2x - 1 = 0$

$\cos 2x = \frac{1}{2}$

$2x = \frac{\pi}{3}$ or $2x = 2\pi - \frac{\pi}{3}$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

For $f(x) = \sin 2x - x$ to be an increasing function, $f'(x) > 0$

$\therefore 0 \le x < \frac{\pi}{6}$ or $\frac{5\pi}{6} < x \le \pi$

9(a) $y = 9 - |2x - 6|$

At x-axis, $y = 0$

$9 - |2x-6| = 0$

$|2x-6| = 9$

$2x - 6 = 9$ or $2x - 6 = -9$

$x = 7\frac{1}{2}$ $\qquad x = -1\frac{1}{2}$

$\therefore A(-1\frac{1}{2}, 0)$

$D(7\frac{1}{2}, 0)$

At B, $x = 0$. $\therefore y = 9 - |-6|$
$= 9 - 6$
$= 3$

$\therefore B(0, 3)$

At vertex pt, $|2x-6| = 0 \Rightarrow x = 3$

$\therefore C(3, 9)$

9(b) $y = mx$ $(m > 0)$ is an upslope straight line passing through the origin.

Minimum m = gradient of AC

$= \frac{0-9}{-1\frac{1}{2} - 3}$

$= 2$

(Note: The line $y = mx$ must be parallel to AC so that m is minimum and intersects the modulus graph at one point)

10) $v = t(2-3t)^2$

(i) Acceleration, $a = \frac{dv}{dt}$

$= t(2)(2-3t)(-3) + (2-3t)^2$

$= (2-3t)(-6t + 2 - 3t)$

$a = (2-3t)(2-9t)$

(ii) At instanteneous rest, $v = 0$

$t(2-3t)^2 = 0$

$t = 0$ or $t = \frac{2}{3}$

$v = t(4 - 12t + 9t^2)$

$= 4t - 12t^2 + 9t^3$

$s = \int v\,dt = \int 4t - 12t^2 + 9t^3\,dt$

$= 2t^2 - 4t^3 + \frac{9}{4}t^4 + c$

when $t = 0$, $s = 0$, $c = 0$

$\therefore s = 2t^2 - 4t^3 + \frac{9}{4}t^4$

Distance travelled by the particle

$= 2(\frac{2}{3})^2 - 4(\frac{2}{3})^3 + \frac{9}{4}(\frac{2}{3})^4$

$= \frac{8}{9} - \frac{32}{27} + \frac{4}{9}$

$= \frac{4}{27}$

$= 0.148$ m

Q11 i)

$(\sin^2\theta + \cos^2\theta)^2 = \sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta$

$1 = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$

$\sin^4\theta + \cos^4\theta = \frac{1}{4}(3 + \cos 4\theta)$

LHS $= \sin^4\theta + \cos^4\theta$

$= 1 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\left(\frac{1-\cos 2\theta}{2}\right)\left(\frac{1+\cos 2\theta}{2}\right)$

$= 1 - \frac{1}{2}(1 - \cos^2 2\theta)$

$= 1 - \frac{1}{2} + \frac{1}{2}\left(\frac{1+\cos 4\theta}{2}\right)$

$= \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\cos 4\theta\right)$

$= \frac{1}{4}(3 + \cos 4\theta)$ (shown)

Note:

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\therefore \cos^2 2\theta = \frac{1+\cos 4\theta}{2}$ (change θ to 2θ)

ii) $\sin^4\theta + \cos^4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$

$\frac{3}{4} + \frac{1}{4}\cos 4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$

$\cos 4\theta = \frac{\sqrt{3}}{2}$

$0 \le \theta \le \pi$

$0 \le 4\theta \le 4\pi$

Basic $\angle = \frac{\pi}{6}$

$4\theta = \frac{\pi}{6}$

$4\theta = 2\pi - \frac{\pi}{6}$

$4\theta = 2\pi + \frac{\pi}{6}$

$4\theta = 4\pi - \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{24}, \frac{11\pi}{24}, \frac{13\pi}{24}$ or $\frac{23\pi}{24}$

iii) For $-\pi \le \theta \le \pi$, the number of solutions of the equation $\sin^4\theta + \cos^4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$ is $2(4) = \underline{8}$ #

12 (i)

At P, $\sin 2x = \frac{\sqrt{3}}{2}$

Basic $\angle = \frac{\pi}{3}$

$2x = \frac{\pi}{3}$ or $\pi - \frac{\pi}{3}$

$x = \frac{\pi}{6}$ (rejected) or $x = \frac{\pi}{3}$

$\therefore P\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

Note: You can draw a horizontal line $y = \frac{\sqrt{3}}{2}$. The x-coordinate of P is the second pt of intersection between $y = \sin 2x$ and $y = \frac{\sqrt{3}}{2}$.

At Q, $\sin 2x = 0$

$2x = 0$ (rejected) or $2x = \pi$

$x = \frac{\pi}{2}$

$\therefore Q\left(\frac{\pi}{2}, 0\right)$

(ii) Area of shaded region is

$\int_0^{\frac{\pi}{2}} \sin 2x \, dx - \frac{1}{2}\left(\frac{\pi}{3}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}} - \frac{\sqrt{3}\pi}{12}$

$= -\frac{1}{2}\cos\pi - \left(-\frac{1}{2}\right) - \frac{\sqrt{3}\pi}{12}$

$= 1 - \frac{\sqrt{3}\pi}{12}$

$= 0.547$ sq units

2014 SNG3 Prelim P1 01

13

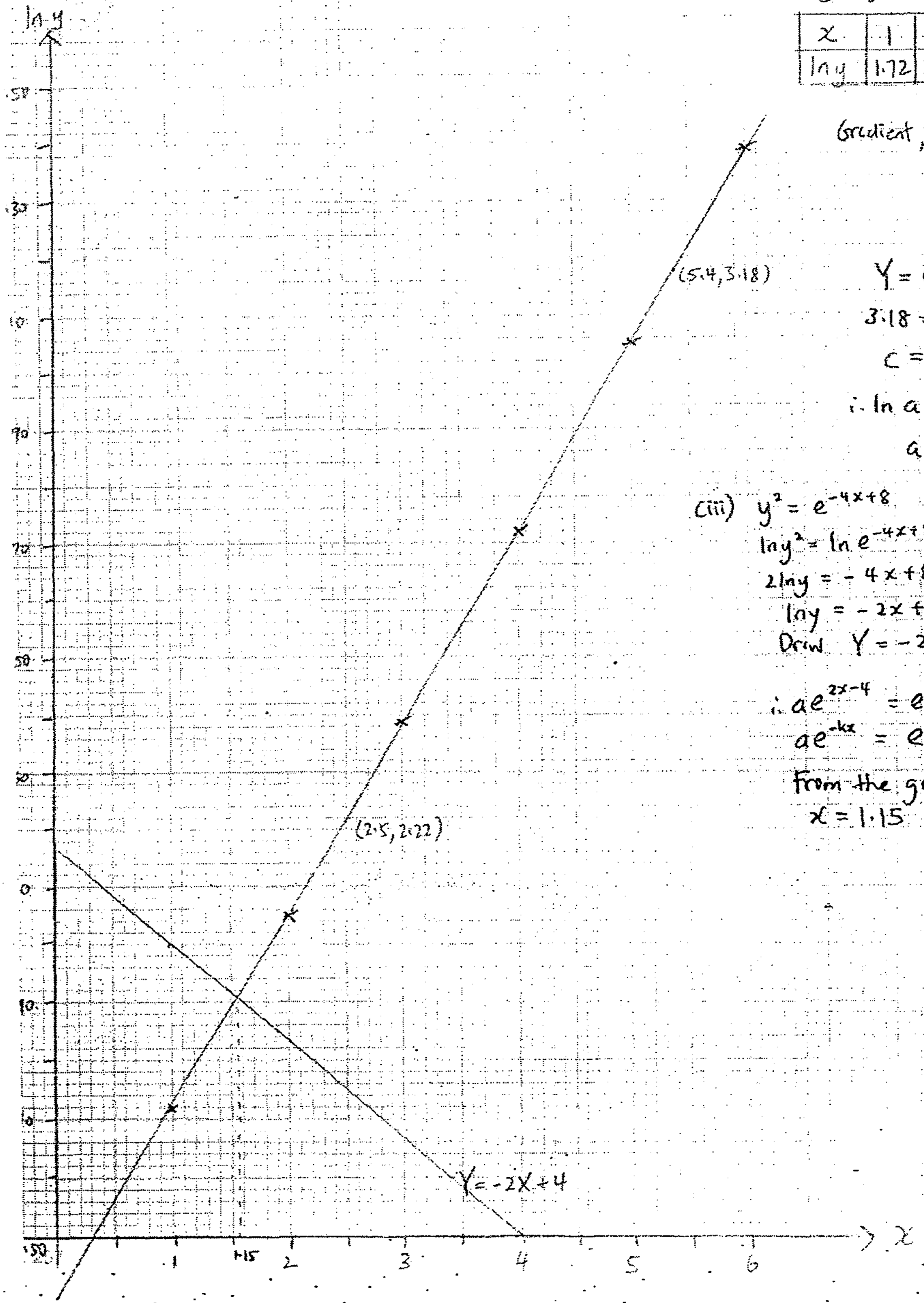
$y = \frac{a}{e^{kx}} = ae^{-kx}$

(i) $\ln y = \ln e^{-kx} + \ln a$

$= -kx + \ln a$

Plot $\ln y$ against x

x	1	2	3	4	5	6
$\ln y$	1.72	2.05	2.39	2.72	3.05	3.39



Gradient, $-k = \frac{3.18 - 2.22}{5.4 - 2.5}$

$= 0.331$

$k = -0.331$

$Y = 0.33103X + c$

$3.18 = 0.33103(5.4) + c$

$c = 1.3924$

$\therefore \ln a = 1.3924$

$a = 4.02$

(iii) $y^2 = e^{-4x+8}$

$\ln y^2 = \ln e^{-4x+8}$

$2\ln y = -4x + 8$

$\ln y = -2x + 4$

Draw $Y = -2X + 4$

$\therefore ae^{2x-4} = e^{kx}$

$ae^{-kx} = e^{-2x+4}$

From the graph,

$x = 1.15$

Name: ______________________ (   )   Class: ____________

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ADDITIONAL MATHEMATICS

4047/02

Paper 2

28 August 2014

2 hours 30 minutes

Additional Materials: Answer Paper

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FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8		Q12	

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Mathematical Formulae

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Quadratic Equation

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$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!\,r!}=\frac{n(n-1)\ldots\ldots..(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$
$$\sec^2 A=1+\tan^2 A.$$
$$\operatorname{cosec}^2 A=1+\cot^2 A.$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 Using a **separate** diagram for each part, represent on the number line the solution set of

(i) $x^2 + 2x > 3,$ [2]

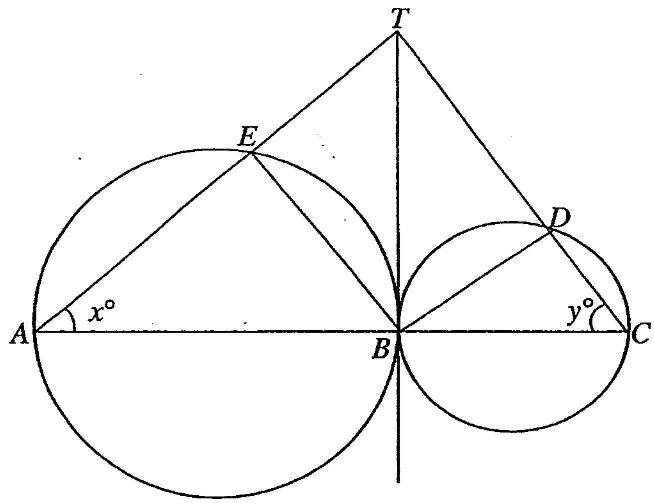
(ii) $-3 < 2(2x + 1) - x^2.$ [2]

State the set of values of x which satisfy both of these inequalities. [1]

2 **(i)** Differentiate $\sqrt{\cos 2x}$ with respect to x. [2]

(ii) Use your answer to part **(i)** to evaluate $\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{2\sqrt{\cos 2x}}\,dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are constants. [3]

3



The diagram shows two circles and BT is a tangent to the circles at B.
AE produced and CD produced meet the tangent at T.
$\angle TAB = x^\circ$ and $\angle TCB = y^\circ$. ABC, AET and CDT are straight lines.

(i) Prove that $\angle EBD + \angle ETD = 180^\circ$. [3]

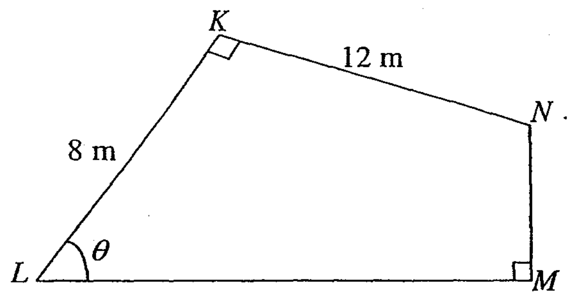
(ii) Prove that $AT \times ET = BT^2$. [3]

4 The function $f(x) = x^3 + ax^2 + bx + 4$, where a and b are constants, is exactly divisible by $(x - 2)$. Given that $f(x)$ leaves a remainder of -3 when divided by $x + 1$,

(i) find the value of a and of b. [4]

(ii) express $f(x)$ in the form $(x-2)(x-1-\sqrt{d})(x-1+\sqrt{d})$, where d is an integer. [3]

5



The diagram shows a pond $KLMN$ in a school. $LK = 8$ cm and $KN = 12$ m. $L\hat{K}N = L\hat{M}N = 90°$ and $K\hat{L}M = \theta$, where $0° < \theta < 90°$.

Given that the perimeter of the pond is P m,

(i) find the values of the integers a, b and of d for which

$$P = a + b\cos\theta + d\sin\theta.$$ [3]

Using the values of a, b and d found in part (i),

(ii) express P in the form of $a + R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

Hence

(iii) find the value of θ when $P = 38$. [2]

6 The function f is defined, for all values of x, by $f(x) = 2\cos\left(\frac{x}{2}\right) - 1$.

(i) State the maximum and minimum values of f (x). [2]

(ii) State the amplitude of f. [1]

(iii) State the period of f in terms of π. [1]

(iv) Find, in terms of π, the smallest positive value of x for which $f(x) = 0$. [2]

(v) Sketch the graph of $y = f(x)$ for $-2\pi \le x \le 2\pi$. [3]

7 The equation of a curve is $y = 3x^2 \ln x$. The tangent to the curve at the point $x = e^2$ meets the x-axis at A and the y-axis at B.

(i) Show that the coordinates of A are $\left(\frac{3}{5}e^2, 0\right)$. [7]

(ii) Hence find the area of triangle AOB in terms of e. [2]

8 (a) Solve the equation

(i) $\log_5\left(x^2 - 5x + 20\right) = \frac{1}{\log_2 5} + \frac{1}{\log_7 5}$, [3]

(ii) $\lg y^2 = (\lg y)^2$. [3]

(b) Solve the equation

$8e^{4x} - 15e^{2x} - 2 = 0$. [3]

9 The point $A(-8,16)$ lies on the curve $y = kx^{\frac{5}{3}}$, where k is a constant.

(i) Show that $k = -\frac{1}{2}$. [1]

(ii) Sketch the graph of $y = kx^{\frac{5}{3}}$ for $x \geq 0$. [1]

(iii) On the same diagram, sketch the graph of $y = kx^{-\frac{5}{3}}$ for $x > 0$. [1]

(iv) Calculate the coordinates of the point of intersection of the two graphs. [2]

(v) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

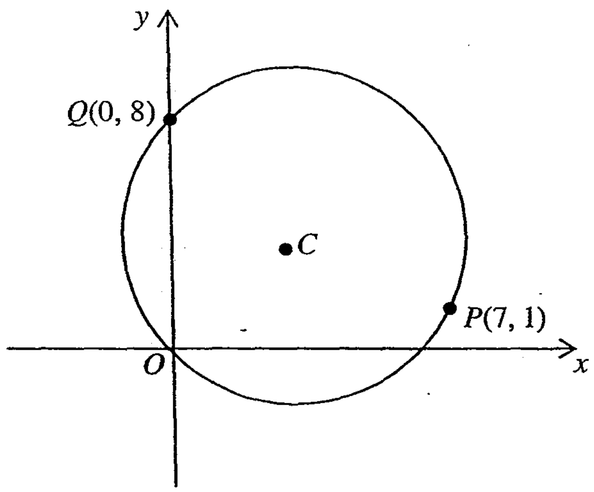
10 (i) Find $\frac{d}{dx}\left[\ln\left(x^2 + 9\right)\right]$. [2]

(ii) Express $\frac{x^2 + 45}{x^3 + 9x}$ in partial fractions. [4]

(iii) Find $\int \frac{x^2 + 45}{x^3 + 9x}\,dx$ and hence evaluate $\int_1^3 \frac{x^2 + 45}{x^3 + 9x}\,dx$. [4]

11 A curve is such that $\dfrac{d^2y}{dx^2}=\dfrac{4}{(2x-3)^2}$. The gradient of the curve at the point (2,1) is $\dfrac{1}{2}$.
Find the coordinates of the stationary point of the curve and determine the nature of this stationary point. [11]

12



The diagram shows a circle, centre C, that passes through the origin O.
The points $P(7, 1)$ and $Q(0, 8)$ lie on the circle.

(i) Find the equation of the perpendicular bisector of OP and of OQ. [4]

(ii) Hence show that the coordinates of C are (3, 4). [1]

(iii) Find the equation of the circle. [2]

The points R and T, which lie on the circle, are the same distance from the x-axis as the point P.

(iv) Find the equation of the tangent to the circle at R and at T. [4]

SNGS Additional Mathematics Prelim Paper 2

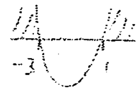
1.	$1 < x < 5$	7(ii)	Area $\Delta AOB = \frac{27}{10}e^6$ / $2.7e^6$ sq units
2(i)	$\dfrac{-\sin 2x}{\sqrt{\cos 2x}}$	8(a)(i)	$x = 2$ or $x = 3$
(ii)	$\frac{1}{2} - \frac{1}{4}\sqrt{2}$	(ii)	$x = 1$ or $x = 100$
4(i)	$a = -4, \quad b = 2$	(b)	$e^{2x} = -\dfrac{1}{8}$ (no solution), $x = 0.347$ (3 s.f.)
(ii)	$f(x) = (x-2)(x-1-\sqrt{3})(x-1+\sqrt{3})$	9(iv)	$\left(1, -\dfrac{1}{2}\right)$
5(i)	$20 - 4\cos\theta + 20\sin\theta$	10(i)	$\dfrac{2x}{x^2+9}$
(ii)	$P = 20 + 4\sqrt{26}\,\sin(\theta - 11.3°)$	(ii)	$\dfrac{5}{x} - \dfrac{4x}{x^2+9}$
(iii)	$\theta = 73.3°$ (1 *d.p.*)	(iii)	$5\ln x - 2\ln(x^2+9) + c$ $\ln 75$ *or* 4.32
6(i)	Max value = 1 Min value– 3	11.	min $(1.9, 0.973)$
(ii)	Amplitude = 2 Period = 4π	12(i)	$y = -7x + 25$
(iii)	$x = \dfrac{2\pi}{3}$	(iii)	$(x-3)^2 + (y-4)^2 = 25$
		(iv)	$y = -\dfrac{4}{3}x - \dfrac{1}{3}$ $y = -1$

SNGS Add Math P2 — 2014

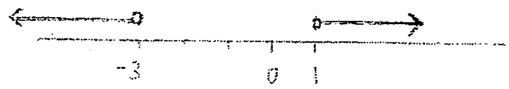
1i) $x^2 + 2x > 3$

$x^2 + 2x - 3 > 0$

$(x+3)(x-1) > 0$



$x < -3$ or $x > 1$



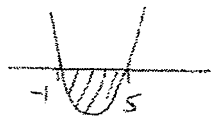
ii) $-3 < 2(2x+1) - x^2$

$0 < 4x + 2 - x^2 + 3$

$x^2 - 4x - 5 < 0$

$(x-5)(x+1) < 0$

$-1 < x < 5$



Set of values of x which satisfy both inequalities is $\{x : 1 < x < 5\}$

2i) Let $y = (\cos 2x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(\cos 2x)^{-\frac{1}{2}} \cdot (-2)\sin 2x$$

$$= \frac{-\sin 2x}{\sqrt{\cos 2x}}$$

ii) $$\frac{d}{dx}\sqrt{\cos 2x} = \frac{-\sin 2x}{\sqrt{\cos 2x}}$$

$$-\frac{1}{2}\frac{d}{dx}\sqrt{\cos 2x} = \frac{\sin 2x}{2\sqrt{\cos 2x}}$$

$$\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{2\sqrt{\cos 2x}}\,dx = -\frac{1}{2}\Big[\sqrt{\cos 2x}\Big]_0^{\frac{\pi}{6}}$$

$$= -\frac{1}{2}\left(\sqrt{\cos\frac{\pi}{3}} - 1\right)$$

$$= -\frac{1}{2}\left(\frac{\sqrt{2}}{2} - 1\right)$$

$$= \frac{1}{2} - \frac{1}{4}\sqrt{2}$$

3(i) $\angle EBT = x^\circ$ (alternate segment theorem)

$\angle DBT = y^\circ$ (alternate segment theorem)

Let $\angle AEB = \theta$

$\angle ETB = \theta - x^\circ$ (exterior $\angle$ of Δ)

Let $\angle BDC = \alpha$

$\angle BTD = \alpha - y^\circ$ (exterior $\angle$ of Δ)

$x^\circ + \theta = x^\circ + y^\circ + (180^\circ - \alpha - y^\circ)$ (ext. $\angle$ of Δ)

$\theta = 180^\circ - \alpha$

$\therefore \angle EBD + \angle ETD$

$= (x^\circ + y^\circ) + (\theta - x^\circ) + (\alpha - y^\circ)$

$= \theta + \alpha$

Since $\theta = 180^\circ - \alpha$

$\angle EBD + \angle ETD = 180^\circ - \alpha + \alpha$

$= 180^\circ$ (proven)

(ii) Consider ΔABT and ΔBET

$\angle BAT = \angle EBT = x^\circ$ (alternate segment theorem)

$\angle ATB = \angle BTE = \theta - x^\circ$ (common angle)

$\therefore \Delta ABT$ and ΔBET are similar.

$$\frac{AT}{BT} = \frac{BT}{ET}$$ (ratio of corresponding sides are equal)

$AT \times ET = BT^2$ (proven)

68

4. $f(x) = x^3 + ax^2 + bx + 4$

i) $f(2) = 0$

$\therefore 8 + 4a + 2b + 4 = 0$

$4a + 2b = -12$

$2a + b = -6$ —— (1)

$f(-1) = -3$

$-1 + a - b + 4 = -3$

$a - b = -6$ —— (2)

(1)+(2): $3a = -12$

$a = -4$

Subst $a = -4$ into (2): $-4 - b = -6$

$b = 2$

$\therefore a = -4,\ b = 2$

(ii) $f(x) = x^3 - 4x^2 + 2x + 4 = (x-2)(x^2 + px - 2)$

Compare coeff of x: $2 = -2 - 2p$

$p = -2$

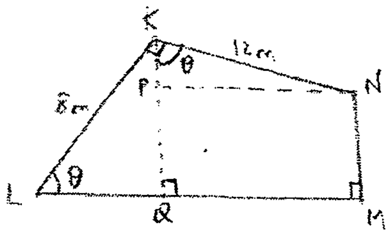
$\therefore f(x) = (x-2)(x^2 - 2x - 2)$

$= (x-2)(\underbrace{x^2 - 2x + (1)^2} - (1)^2 - 2)$

$= (x-2)[(x-1)^2 - 3]$

$= (x-2)(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})$

5)



(i) $KQ = 8\sin\theta$

$KP = 12\cos\theta$

$NM = 8\sin\theta - 12\cos\theta$

$LQ = 8\cos\theta$

$PN = 12\sin\theta$

$LM = 8\cos\theta + 12\sin\theta$

Perimeter, $P = 8 + 12 + 8\sin\theta - 12\cos\theta + 8\cos\theta + 12\sin\theta$

$P = 20 - 4\cos\theta + 20\sin\theta$

$\therefore a = 20$

$b = -4$

$d = 20$

(ii) Let $20\sin\theta - 4\cos\theta = R\sin(\theta - \alpha)$

$R = \sqrt{20^2 + 4^2}$

$= \sqrt{416}$

$= 4\sqrt{26}$

$\alpha = \tan^{-1}\left(\frac{4}{20}\right)$

$\approx 11.310°$

$= 11.3°$ (to 1dp)

$\therefore P = 20 + 4\sqrt{26}\sin(\theta - 11.3°)$

(iii) $P = 38$

$38 = 20 + 4\sqrt{26}\sin(\theta - 11.310°)$

$\sin(\theta - 11.310°) = \frac{18}{4\sqrt{26}}$

$= \frac{9}{2\sqrt{26}}$

Basic $\angle = 61.948°$

$\theta - 11.310° = 61.948°$

$\theta = 73.3°$ (to 1dp)

6 (i) $f(x) = 2\cos\left(\frac{x}{2}\right) - 1$

Maximum value of $f(x) = 2(1) - 1 = 1$

Minimum value of $f(x) = 2(-1) - 1 = -3$

(ii) Amplitude $= 2$

(iii) Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

(iv) $f(x) = 0 \Rightarrow 2\cos\left(\frac{x}{2}\right) - 1 = 0$

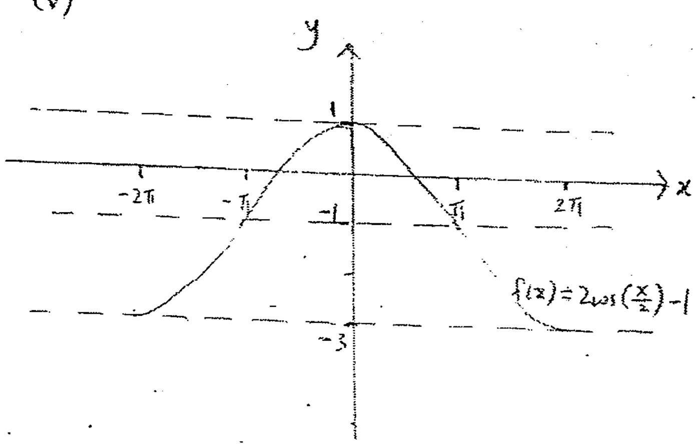
$\cos\frac{x}{2} = \frac{1}{2}$

Basic $\angle = \frac{\pi}{3}$

$\frac{x}{2} = \frac{\pi}{3}$

$x = \frac{2\pi}{3}$

(v)



1) $y = 3x^2 \ln x$

(i) $\frac{dy}{dx} = 3x^2 \cdot \frac{1}{x} + \ln x\,(6x)$

$= 3x + 6x\ln x$

At $x = e^2$

$\frac{dy}{dx} = 3e^2 + 6e^2 \ln e^2$

$= 3e^2 + 6e^2(2)$

$= 15e^2$

$y = 3(e^2)^2 \ln e^2$

$= 3e^4(2)$

$= 6e^4$

Equation of tangent is

$y - 6e^4 = 15e^2(x - e^2)$

$= 15e^2x - 15e^4$

$y = 15e^2x - 9e^4$

At A, $y = 0$

$15e^2x = 9e^4$

$x = \frac{3}{5}e^2$

$\therefore$ Coordinates of A are $(\frac{3}{5}e^2, 0)$

(ii) At B, $x = 0$

$y = -9e^4$

Coordinates of B are $(0, -9e^4)$

Area of ΔAOB is

$\frac{1}{2}(\frac{3}{5}e^2)(9e^4)$

$= \frac{27}{10}e^6$

$= \underline{2.7e^6}$ sq units

8(a)(i)

$\log_5(x^2 - 5x + 20) = \frac{1}{\log_2 5} + \frac{1}{\log_7 5}$

$= \log_5 2 + \log_5 7$

$= \log_5 14$

$\therefore x^2 - 5x + 20 = 14$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ or 2

(ii) $\lg y^2 = (\lg y)^2$

$2\lg y = (\lg y)^2$

Let $p = \lg y$

$2p = p^2$

$p(2-p) = 0$

$p = 0$ or $p = 2$

$\lg y = 0$ $\quad$ $\lg y = 2$

$y = 10^0$ $\quad$ $y = 10^2$

$= 1$ $\quad$ $= 100$

$\therefore y = 1$ or 100

(b) $8e^{4x} - 15e^{2x} - 2 = 0$

Let $q = e^{2x}$

$\therefore 8q^2 - 15q - 2 = 0$

$(8q + 1)(q - 2) = 0$

$q = -\frac{1}{8}$ or $q = 2$

$e^{2x} = -\frac{1}{8}$ or $e^{2x} = 2$

(rejected $\because e^{2x} > 0$)

$2x = \ln 2$

$x = \frac{1}{2}\ln 2$

$= 0.347$

(i) 7 $y = kx^{\frac{5}{3}}$

When $x = -8$, $y = 16$

$16 = k(-8)^{\frac{5}{3}}$

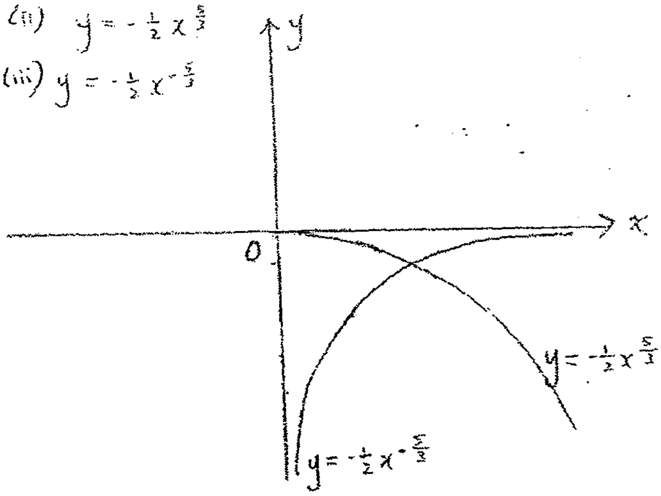
$= k(-2^3)^{\frac{5}{3}}$

$= -32k$

$k = -\frac{1}{2}$ (shown)

(ii) $y = -\frac{1}{2}x^{\frac{5}{3}}$

(iii) $y = -\frac{1}{2}x^{-\frac{5}{3}}$



Note : For $y = -\frac{1}{2}x^{-\frac{5}{3}}$

$= -\frac{1}{2x^{\frac{5}{3}}}$

Note that x cannot be zero.

$\therefore$ the y-axis is the asymptote.

y cannot be zero because $x^{\frac{5}{3}} > 0$

$\therefore$ the x-axis (i.e. $y=0$) is the asymptote

iv) $-\frac{1}{2}x^{\frac{5}{3}} = -\frac{1}{2}x^{-\frac{5}{3}}$

$\frac{x^{\frac{5}{3}}}{x^{-\frac{5}{3}}} = 1$

$x^{\frac{10}{3}} = 1$

$x = 1$

When $x = 1$, $y = -\frac{1}{2}(1)^{\frac{5}{3}} = -\frac{1}{2}$

Coordinates of the point of intersection are $(1, -\frac{1}{2})$.

v) $y = -\frac{1}{2}x^{\frac{5}{3}}$

$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{5}{3}\right)x^{\frac{2}{3}}$

$= -\frac{5}{6}x^{\frac{2}{3}}$

When $x = 1$, $\frac{dy}{dx} = -\frac{5}{6}$

$y = -\frac{1}{2}x^{-\frac{5}{3}}$

$\frac{dy}{dx} = -\frac{1}{2}\left(-\frac{5}{3}\right)x^{-\frac{8}{3}}$

$= \frac{5}{6}x^{-\frac{8}{3}}$

When $x = 1$, $\frac{dy}{dx} = \frac{5}{6}$

Since the product of the gradients $-\frac{5}{6} \times \frac{5}{6} \neq -1$, the tangents to the graph at the point of intersection are <u>not</u> perpendicular.

10 i) $\frac{d}{dx}\left[\ln(x^2+9)\right]$

$= \frac{1}{x^2+9} \cdot 2x$

$= \frac{2x}{x^2+9}$

(ii) $\frac{x^2+45}{x^3+9x} = \frac{x^2+45}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$

$x^2+45 = A(x^2+9) + x(Bx+C)$

Compare constant, $45 = 9A \Rightarrow A = 5$

Compare coeff of x^2 : $1 = A + B \Rightarrow B = -4$.

Compare coeff of x : $0 = C$

$\therefore \frac{x^2+45}{x^3+9x} = \frac{5}{x} - \frac{4x}{x^2+9}$

(iii) $\int \frac{x^2+45}{x^3+9x}\,dx = \int \frac{5}{x}\,dx - 2\int \frac{2x}{x^2+9}\,dx$

$= 5\ln x - 2\ln(x^2+9) + C$

$\int_1^3 \frac{x^2+45}{x^3+9x}\,dx = \left[5\ln x - 2\ln(x^2+9)\right]_1^3$

$= 5\ln 3 - 2\ln 18 - (0 - 2\ln 10)$

$= \ln 3^5 - \ln 18^2 + \ln 10^2$

$= \ln\left(\frac{243 \times 100}{324}\right)$

$= \ln 75$ #

<u>Note</u> : $\int \frac{g'(x)}{g(x)}\,dx = \ln[g(x)] + C$

1) $\frac{d^2y}{dx^2} = \frac{4}{(2x-3)^2} = 4(2x-3)^{-2}$

$\frac{dy}{dx} = \int 4(2x-3)^{-2}\,dx$

$= 4 \cdot \frac{(2x-3)^{-1}}{(-1)(2)} + c$

$= -\frac{2}{2x-3} + c$

when $x=2$, $\frac{dy}{dx} = \frac{1}{2}$

$\frac{1}{2} = -\frac{2}{4-3} + c$

$c = \frac{5}{2}$

$\therefore \frac{dy}{dx} = -\frac{2}{2x-3} + \frac{5}{2}$

At stationary point, $\frac{dy}{dx} = 0$

$-\frac{2}{2x-3} + \frac{5}{2} = 0$

$\frac{2}{2x-3} = \frac{5}{2}$

$4 = 10x - 15$

$x = 1.9$

$y = \int -\frac{2}{2x-3} + \frac{5}{2}\,dx$

$= -2\int \frac{1}{2x-3}\,dx + \frac{5}{2}\int 1\,dx$

$= -2 \cdot \frac{\ln(2x-3)}{2} + \frac{5}{2}x + C_2$

$= -\ln(2x-3) + \frac{5}{2}x + C_2$

hen $x=2$, $y=1$

$1 = -\ln 1 + 5 + C_2$

$C_2 = -4$

$\therefore y = -\ln(2x-3) + \frac{5}{2}x - 4$

en $x=1.9$, $y = -\ln(0.8) + \frac{5}{2}(1.9) - 4$

$= 0.973$

$\left.\frac{d^2y}{dx^2}\right|_{x=1.9} = 4(3.8-3)^{-2} = 6.25 < 0$

$\therefore (1.9, 0.973)$ is a minimum point.

12i) Gradient of OP $= \frac{1}{7}$

Gradient of $\perp$ bisector of OP $= -7$

Midpoint of OP $= \left(\frac{0+7}{2}, \frac{0+1}{2}\right) = (3.5, 0.5)$

Equation of $\perp$ bisector of OP is

$y - \frac{1}{2} = -7\left(x - \frac{7}{2}\right)$

$y - \frac{1}{2} = -7x + \frac{49}{2}$

$y = -7x + 25$

Midpoint of OQ $= \left(\frac{0+0}{2}, \frac{0+8}{2}\right) = (0, 4)$

Equation of $\perp$ bisector of OQ is $y = 4$

(ii) $-7x + 25 = 4$

$x = 3$

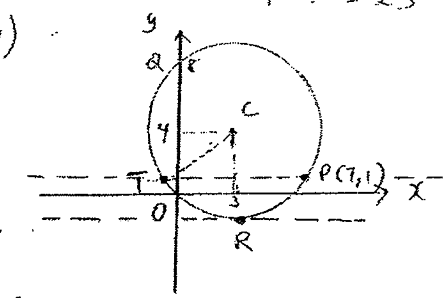
$\therefore$ Coordinates of C are $(3, 4)$

(iii) Radius $= \sqrt{(3-0)^2 + (4-8)^2} = 5$ units

Equation of circle is

$(x-3)^2 + (y-4)^2 = 25$

(iv)



Note: Red dotted lines are equidistant fr. x-axis as P.

Since centre of circle is $(3,4)$ and radius is 5 units,

Equation of tangent is $y = -1$ #

When $y = 1$; $(x-3)^2 + (1-4)^2 = 25$

$(x-3)^2 = 16$

$x - 3 = 4$ or $x - 3 = -4$

$x = 7$ or -1

When $x = -1$, $y = 1$. $T(-1, 1)$

Gradient of CT $= \frac{4-1}{3+1} = \frac{3}{4}$

Gradient of tangent at T $= \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$ (tan $\perp$ rad)

Equation of tangent at T is $y - 1 = -\frac{4}{3}(x+1)$

$y = -\frac{4}{3}x - \frac{1}{3}$ #