Calculator Model:

Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2014
SECONDARY 4

Additional Mathematics 4047/01

Paper 1

Tuesday 26 August 2014

2 hours

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

/80

This document consist of 6 printed pages

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^r,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 Given that α and β are the roots of a quadratic equation where $\alpha\beta = 4\frac{1}{4}$ and $\alpha^2 + \beta^2 = -2\frac{1}{4}$.

(i) Find the value of $\alpha + \beta$ where $\alpha + \beta < 0$. [2]

(ii) Show that $\alpha^3 + \beta^3 = 16\frac{1}{4}$. [1]

(iii) Find a quadratic equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [3]

2 Given that curve $y = x^2$ lies above the line $y = px - q^2$ for $-2 < p < 2$. Find the value(s) of constant q. [4]

3 The equation of a cuve is $y = 2xe^{3-x^2}$. Find the x-coordinates of the stationary points, leaving your answer in exact form and determine the nature of the stationary points. [5]

4 It is given that $2^{2x+1} + 4^{x-1} = 2(3^{1-x})$.

(i) Show that $12^x = 2\frac{2}{3}$. [3]

(ii) Find the value of x correct to 2 decimal places. [1]

5 Given that $\int_0^{\frac{\pi}{4}} \mathrm{f}(x)\,\mathrm{d}x = 5$ and $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \mathrm{f}(x)\,\mathrm{d}x = 2$, find the value of a given that $\int_{\frac{\pi}{2}}^{0} \left(\mathrm{f}(x) + a\sin 2x\right) = 10$. [4]

6 A particle travels in a straight line so that t seconds after passing a fixed point O with a velocity of 15 cms^{-1}, its acceleration a cms^{-2} is given by $a = k - 2t$ where k is a constant. The particle reaches maximum velocity in 1 second. Find

(i) the value of k, [1]

(ii) an expression for the velocity of the particle in terms of t, [2]

(iii) the distance travelled in the first 6 seconds. [4]

7 (i) Show that $\dfrac{2\cos^2\theta - \sin\theta\cos\theta + 1}{\sin^2\theta} = 3\cot^2\theta - \cot\theta + 1$. [3]

(ii) Hence solve the equation $2\cos^2\theta - \sin\theta\cos\theta + 1 = \sin^2\theta$ for $-\pi \le \theta \le \pi$. [4]

8 Variables x and y are connected by the equation $y = ax^2 + b\sqrt{x}$, where a and b are constants. When a graph of $\dfrac{y}{\sqrt{x}}$ against $x\sqrt{x}$ is plotted using experimental values of x and y, a straight line is obtained and passes through the point $(1,\ 5)$. The straight line makes an angle of 45° with the $x\sqrt{x}$-axis.

(i) Find the values of a and of b. [4]

(ii) Find the coordinates of the point on the line at which $y = 3\sqrt{x}$. [3]

9 (i) Sketch the graph $y = |x^2 - 4x|$ indicating the intercepts and coordinates of the turning point. [3]

(ii) In each of the following case, determine the number of solutions of the equation $|x^2 - 4x| = mx + c$ where $0 < c < 4$, justifying your answer.

(a) $m = 0$, [1]

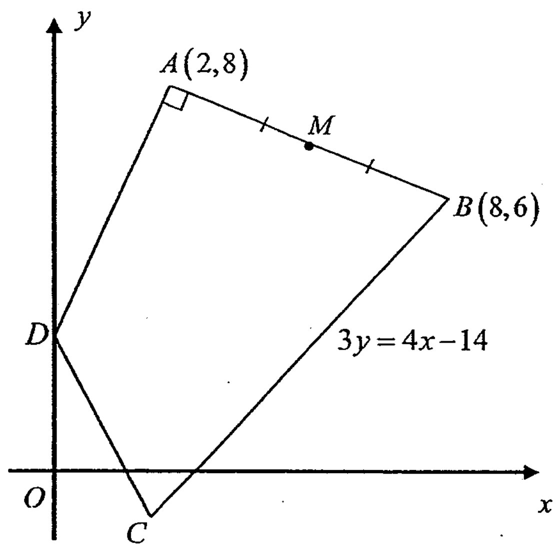
(b) $m = -1$. [2]

10 The diagram shows a quadrilateral $ABCD$ in which $A(2,\ 8)$ and $B(8,6)$. M is the midpoint of AB and CM is perpendicular to AB. The equation of BC is $3y = 4x - 14$. The point D lies on the y-axis and $\angle DAB = 90°$. Find

(i) the coordinates of D, [2]

(ii) the coordinates of C, [5]

(iii) the ratio of $AD : CM$. [2]



11 In the diagram, AB is a diameter of the circle with centre O. DE and BF are tangents to the circle at C and B respectively. DCE and BEF are straight lines. Prove that

(i) ΔABC and ΔAFB are similar, [3]

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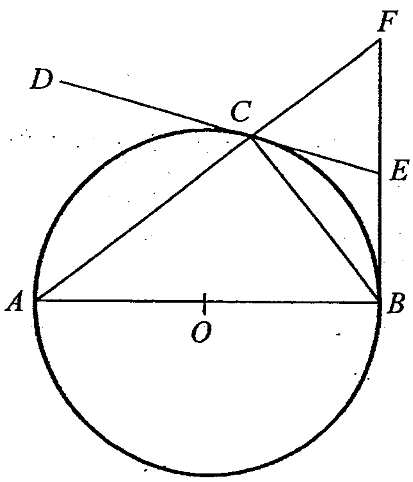
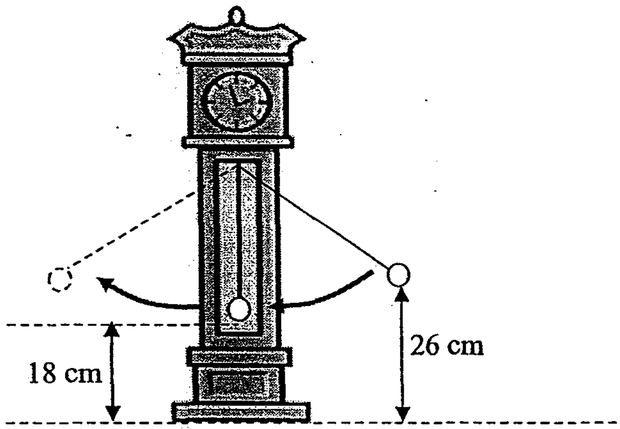


12 Sand is poured onto a flat surface at a rate of 2π cm^{3}s^{-1} and formed a right circular cone. The radius of the cone is always $\frac{1}{3}$ of its height.

(i) Find the rate of change of the radius 3 seconds after the start of pouring. [5]

(ii) State, with a reason, whether this rate will increase or decrease as t increases. [1]

13



The pendulum of a grandfather clock swings back and forth with a periodic motion that can be represented by the equation $h = a\cos kt + b$, where a, k and b are constants, h cm is the height of the pendulum above the base and t is the time in seconds after the pendulum is released 26 cm above the base. At rest, the pendulum is 18 cm above the base. It takes four seconds for a complete swing back and forth.

(i) State the value of a and b. [2]

(ii) Show that the value of k is $\frac{\pi}{2}$ rad/s. [1]

(iii) Sketch the graph of $h = a\cos kt + b$ for $0 \le t \le 6$. [2]

(iv) Find the time interval in which the pendulum is 20 cm above the base during one revolution. [3]

Answer Key

1. (i) $-\frac{5}{2}$

   (iii) $x^2-\frac{65}{17}x+4\frac{1}{4}=0$

2. $q=\pm 1$

3. At $x=-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, the point is a minimum.

   At $x=\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$, the point is a maximum. ------------ A1

4. (ii) 0.39

5. $a=-17$

6. (i) $k=2$
   (ii) $v=2t-t^2+15$
   (iii) $62\frac{2}{3}$ cm

7. (ii) $\theta=-1.89$ or 1.25

8. (i) $a=1,\ b=4$
   (ii) $(-1,\ 3)$

10. (i) $D(0,2)$
    (ii) $C(2,-2)$
    (iii) $AD:CM=2:3$

12. (i) 0.202 cm/s

13. (i) $a=4,\ b=22$

    (iv) $0\le t\le\frac{2}{3}$ and $\frac{4}{3}\le t\le 2$

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Marking Scheme

(i) $(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$ --------- M1 c.a.o

$$= -2\frac{1}{4} + 2\left(4\frac{1}{4}\right)$$

$$= \frac{25}{4}$$

$\alpha + \beta = -\sqrt{\frac{25}{4}}$ (given that $\alpha + \beta < 0$)

$= -\frac{5}{2}$ ------- A1

(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ ---------- B1 c.a.o

$$= \left(-\frac{5}{2}\right)\left(-2\frac{1}{4} - 4\frac{1}{4}\right)$$

$= 16\frac{1}{4}$ ------- (A.G)

(iii) Sum of roots: $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$ ----- M1 c.a.o

$$= 16\frac{1}{4} \div 4\frac{1}{4}$$

$$= \frac{65}{17}$$

Product of roots: $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta$ ------- M1 c.a.o

$$= 4\frac{1}{4}$$

The quadratic equation is: $x^2 - \frac{65}{17}x + 4\frac{1}{4} = 0$ -------- A1

2 Given that curve $y = x^2$ lies above the line $y = px - q^2$ for $-2 < p < 2$. Find the value(s) of constant q.

Marking Scheme

$x^2 > px - q^2$ ---------- M1 c.a.o

$x^2 - px + q^2 > 0$

For no real roots, Discriminant < 0 -------- M1 c.a.o

$(-p)^2 - 4(1)(q^2) < 0$

$p^2 - 4q^2 < 0$

$-2 < p < 2 \Rightarrow (p-2)(p+2) < 0$ -------- M1 c.a.o

$p^2 - 4 < 0$

By comparison,

$4q^2 = 4$

$q = \pm 1$ --------- A1

3 The equation of a cuve is $y = 2xe^{3-x^2}$. Find the x-coordinates of the stationary points, leaving your answer in exact form and determine the nature of the stationary points. [5]

Marking Scheme

$y = 2xe^{3-x^2}$

$\frac{dy}{dx} = 2x\left(-2xe^{3-x^2}\right) + e^{3-x^2}(2)$ ------- M1 c.a.o

$= 2e^{3-x^2}\left(-2x^2+1\right)$

At stationary pt, $\frac{dy}{dx} = 0$.

$2e^{3-x^2}\left(-2x^2+1\right) = 0$ --------- M1

$x^2 = \frac{1}{2}$ or $2e^{3-x^2} = 0$ (N.A)

$x = \pm\frac{1}{\sqrt{2}}$ or $\pm\frac{\sqrt{2}}{2}$ ------- M1

x	-1	$-\frac{1}{\sqrt{2}}$	0
$\frac{dy}{dx}$	-14.8	0	40.2

At $x = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, the point is a minimum. ------------ A1

x	0	$\frac{1}{\sqrt{2}}$	1
$\frac{dy}{dx}$	40.2	0	-14.8

At $x = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$, the point is a maximum. ------------ A1

4 It is given that $2^{2x+1}+4^{x-1}=2\left(3^{1-x}\right)$.

(i) Show that $12^x=2\frac{2}{3}$. [3]

(ii) Hence find the value of x correct to 2 decimal places. [1]

Marking Scheme

(i) $2^{2x+1}+4^{x-1}=2\left(3^{1-x}\right)$

$2^{2x}\times 2+2^{2x}\times 2^{-2}=2\left(3\times 3^{-x}\right)$ --------- M1

$2^{2x}\left(2+\frac{1}{4}\right)=6\times 3^{-x}$ ----------- M1

$4^x\times 3^x=\frac{6}{2\frac{1}{4}}$ -------- M1

$12^x=2\frac{2}{3}$ (A.G)

(ii) $\lg 12^x=\lg 2\frac{2}{3}$

$x=\frac{\lg 2\frac{2}{3}}{\lg 12}$

$=0.39$ (2 d.p) --------- B1

5 Given that $\int_0^{\frac{\pi}{4}} \mathrm{f}(x)\ \mathrm{d}x = 5$ and $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \mathrm{f}(x)\ \mathrm{d}x = 2$, find the value of a given that $\int_{\frac{\pi}{2}}^{0} \left(\mathrm{f}(x) + a\sin 2x\right) = 10.$ [4]

Marking Scheme

$$\int_{\frac{\pi}{2}}^{0} \left(\mathrm{f}(x) + ax\right) = 10$$

$$\int_{\frac{\pi}{2}}^{0} \mathrm{f}(x)\ \mathrm{d}x + \int_{\frac{\pi}{2}}^{0} a\sin x\ \mathrm{d}x = 10$$

$$-\int_0^{\frac{\pi}{2}} \mathrm{f}(x)\ \mathrm{d}x + \left[-\frac{a\cos 2x}{2}\right]_{\frac{\pi}{2}}^{0} = 10 \text{ ----- M1 c.a.o , M1c.a.o}$$

$$-(5+2) + \left(-\frac{a}{2} - \frac{a}{2}\right) = 10 \text{ ----- M1 c.a.o}$$

$$-7 - a = 10$$

$$a = -17 \text{ -------- A1}$$

6 A particle travels in a straight line so that t seconds after passing a fixed point O with a velocity of 15 cms^{-1}, its acceleration a cms^{-2} is given by $a = k - 2t$ where k is a constant. The particle reaches maximum velocity in 1 second. Find

(i) the value of k, [1]

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(iii) the distance travelled in the first 6 seconds. [4]

Marking Scheme

(i) At maximum velocity, $a = 0$

$k - 2t = 0$ at $t = 1$

$k - 2(1) = 0$

$k = 2$ ---------- B1

(ii) $v = \int (2 - 2t)\ \mathrm{d}t$

$= 2t - t^2 + c$ where c is a constant ------ M1

Given that $v = 15$ when $t = 0$,

$c = 15$

$v = 2t - t^2 + 15$ -------- A1

(iii) $s = \int (2t - t^2 + 15)\ \mathrm{d}t$

$= t^2 - \frac{t^3}{3} + 15t + c_1$ where c_1 is a constant ------- M1

Since $s = 0$ when $t = 0$,

$c_1 = 0$

$s = t^2 - \frac{t^3}{3} + 15t$

When $v = 0$,

$2t - t^2 + 15 = 0$

$t^2 - 2t - 15 = 0$

$(t - 5)(t + 3) = 0$

$t = 5$ or -3 (N.A) ---------- M1

When $t = 0$, $s = 0$

When $t = 5$, $s = 58\frac{1}{3}$ ----------- M1

When $t = 6$, $s = 54$

Total distance travelled $= 58\frac{1}{3} + \left(58\frac{1}{3} - 54\right)$

$= 62\frac{2}{3}$ cm -------- A1

7 (i) Show that $\dfrac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=3\cot^2\theta-\cot\theta+1$. [3]

(ii) Hence solve the equation $2\cos^2\theta-\sin\theta\cos\theta+1=\sin^2\theta$ for $-\pi\le\theta\le\pi$. [4]

Marking Scheme

(i) $$\frac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=\frac{2\cos^2\theta}{\sin^2\theta}-\frac{\sin\theta\cos\theta}{\sin^2\theta}+\frac{1}{\sin^2\theta} \text{ ------ M1}$$
$$=2\cot^2\theta-\cot\theta+\operatorname{cosec}^2\theta \text{ ------- M1}$$
$$=2\cot^2\theta-\cot\theta+\left(1+\cot^2\theta\right) \text{ ------ M1}$$
$$=3\cot^2\theta-\cot\theta+1 \text{ ---- (A.G)}$$

Alternative Method

$$\frac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=\frac{2\cos^2\theta-\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta}{\sin^2\theta} \text{ ------ M1}$$
$$=\frac{3\cos^2\theta-\sin\theta\cos\theta+\sin^2\theta}{\sin^2\theta} \text{ --------- M1}$$
$$=\frac{3\cos^2\theta}{\sin^2\theta}-\frac{\sin\theta\cos\theta}{\sin^2\theta}+\frac{\sin^2\theta}{\sin^2\theta} \text{ --------- M1}$$
$$=3\cot^2\theta-\cot\theta+1 \text{ ------ (A.G)}$$

(ii) $2\cos^2\theta-\sin\theta\cos\theta+1=\sin^2\theta$

$$\frac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=1$$

$3\cot^2\theta-\cot\theta+1=1$ --------- M1

$3\cot^2\theta-\cot\theta=0$

$\cot\theta(3\cot\theta-1)=0$ ----------- M1

$\cot\theta=0$ (N.A) or $\cot\theta=\dfrac{1}{3}$

When $\tan\theta=3$, basic $\angle=1.249$ -------- M1

$\theta=-1.89$ or 1.25 ------- A1

8 Variables x and y are connected by the equation $y = ax^2 + b\sqrt{x}$, where a and b are constants. When a graph of $\frac{y}{\sqrt{x}}$ against $x\sqrt{x}$ is plotted using experimental values of x and y, a straight line is obtained and passes through the point $(1,\ 5)$. The straight line makes an angle of 45° with the $x\sqrt{x}$-axis.

(i) Find the values of a and of b. [4]

(ii) Find the coordinates of the point on the line at which $y = 3\sqrt{x}$. [3]

Marking Scheme

(i) $y = ax^2 + b\sqrt{x}$

$\frac{y}{\sqrt{x}} = a\frac{x^2}{\sqrt{x}} + b$

$\frac{y}{\sqrt{x}} = ax\sqrt{x} + b$ --------- M1 c.a.o

$\tan 45° = a$

$a = 1$ ----------- A1

Sub. $(1,5)$,

$5 = 1 + b$ ---------- M1

$b = 4$ --------- A1

(ii) $y = 3\sqrt{x} \Rightarrow \frac{y}{\sqrt{x}} = 3$ ------ M1

$3 = x\sqrt{x} + 4$ ------ M1

$x\sqrt{x} = -1$

The point is $(-1,\ 3)$ --------- A1

9 (i) Sketch the graph $y=|x^2-4x|$ indicating the intercepts and coordinates of the turning point. [3]

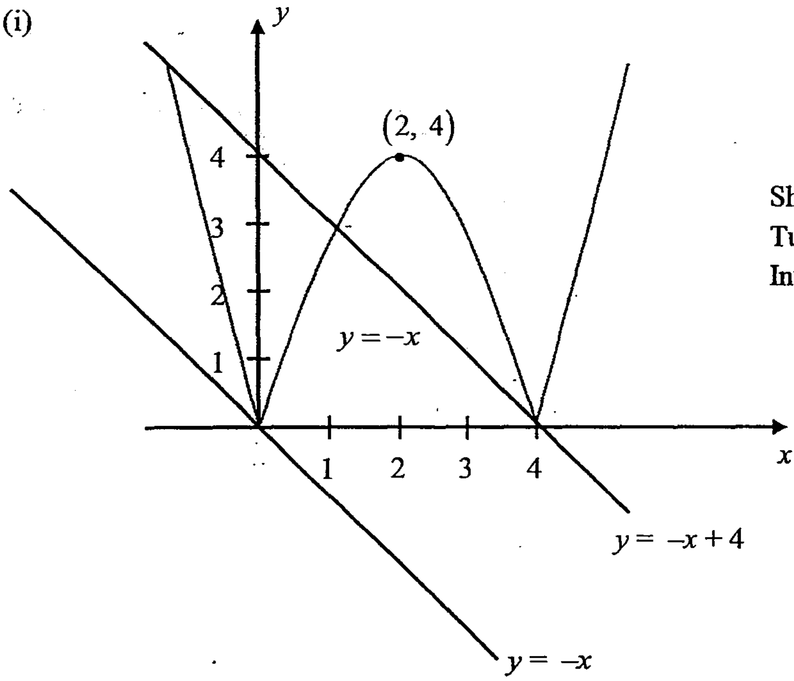
(ii) In each of the following case, determine the number of solutions of the equation $|x^2-4x|=mx+c$ where $0<c<4$, justifying your answer .

(a) $m=0$, [1]

(b) $m=-1$. [2]

Marking Scheme

(i)



Shape ------- B1
Turning pt ------- B1
Intercepts ------- B1

Alternative, draw these two lines to infer ---- M1

(iia) For $0<c<4$, the line $y=c$ cuts the graph $y=|x^2-4x|$ at 4 points, thus there are 4 solutions. --------------- B1

(iib) For $0<c<4$, the line $y=-x$ only intersect the <u>left-hand side half of the graph</u> $y=|x^2-4x|$ at 2 points, thus there are 2 solutions. --------------- A1

M1

10 The diagram shows a quadrilateral $ABCD$ in which $A(2,\ 8)$ and $B(8,6)$. M is the midpoint of AB and CM is perpendicular to AB. The equation of BC is $3y=4x-14$. The point D lies on the y-axis and $\angle DAB=90°$. Find

(i) the coordinates of D, [2]

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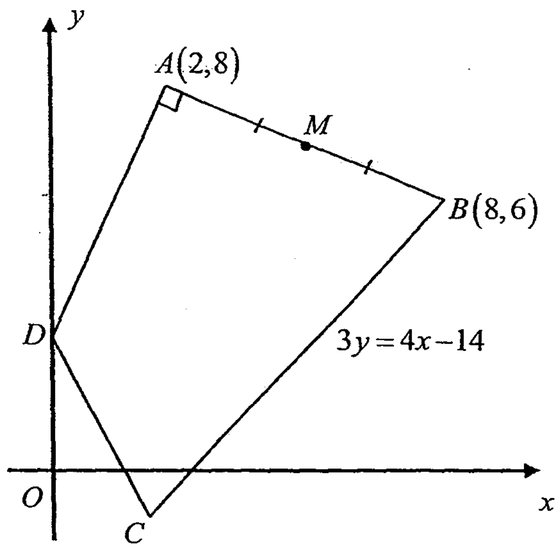
(iii) the ratio of $AD : CM$. [2]



(i) Let $D(0,\ a)$

Gradient of $AB=\dfrac{8-6}{2-8}$

$=-\dfrac{1}{3}$

Gradient of $AD\times$ Gradient of $AB=-1$

$\dfrac{8-a}{2-0}\times\left(-\dfrac{1}{3}\right)=-1$ -------- M1

$a=2$

$\therefore D(0,2)$ ------------ A1

(ii) Gradient of $CM=3$ -------- M1

$M=\left(\dfrac{2+8}{2},\ \dfrac{8+6}{2}\right)$ --------- M1 c.a.o

$=(5,\ 7)$

Eq. of CM,

$y-7=3(x-5)$ ------ M1

$y=3x-8$ ------- (1)

Sub. (1) into $3y=4x-14$,

$3(3x-8)=4x-14$ ----------- M1

$9x-24=4x-14$

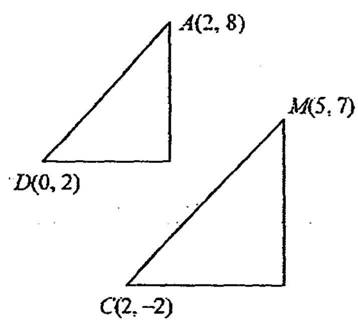
$5x=10$

$x=2$

$y=-2$

$\therefore C(2,-2)$ ---------- A1

(iii) Since AD // CM, by similar triangles:



------------ M1

$\dfrac{AD}{CM}=\dfrac{2-0}{5-2}$

$AD:CM=2:3$ ---------- A1

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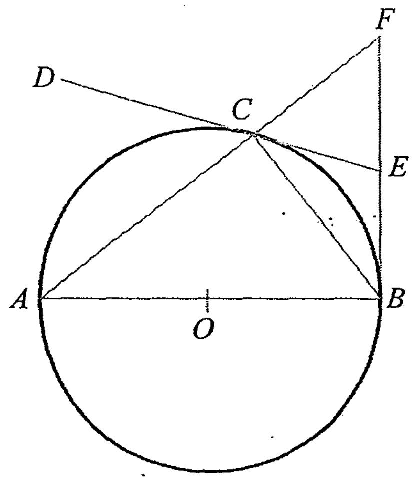


Marking Scheme

(i) $\angle ACB = 90°$ (rt. $\angle$ in a semi-circle) ---- M1
$\angle FBA = 90°$ (tan. $\perp$ rad.) ----- M1
$\therefore \angle ACB = \angle FBA$
$\angle FAB = \angle CAB$ (common $\angle$) ------ M1
Hence ΔABC and ΔAFB are similar.

(ii) $\angle ABC = \angle DCA$ (tangent-chord theorem) ----- M1
$\angle FCE = \angle DCA$ (vertically opp. $\angle$s) ------ M1
$\angle ABC = \angle CFE$ (similar Δs) ------ M1
Since $\angle CFE = \angle FCE$, ΔCFE is isosceles (base $\angle$s are equal) ----- A1

12 Sand is poured onto a flat surface at a rate of 2π cm^{3}s^{-1} and formed a right circular cone. The radius of the cone is always $\frac{1}{3}$ of its height.

(i) Find the rate of change of the radius 3 seconds after the start of pouring. [5]

(ii) State, with a reason, whether this rate will increase or decrease as t increases. [1]

Marking Scheme

(i) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (3r)$

$= \pi r^3$ ------------ M1 c.a.o

After 3 seconds, $\pi r^3 = 6\pi$

$r = \sqrt[3]{6}$ ------- M1

$\frac{dV}{dr} = 3\pi r^2$ --------- M1

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ ------- M1 c.a.o

$= \frac{2}{3r^2}$

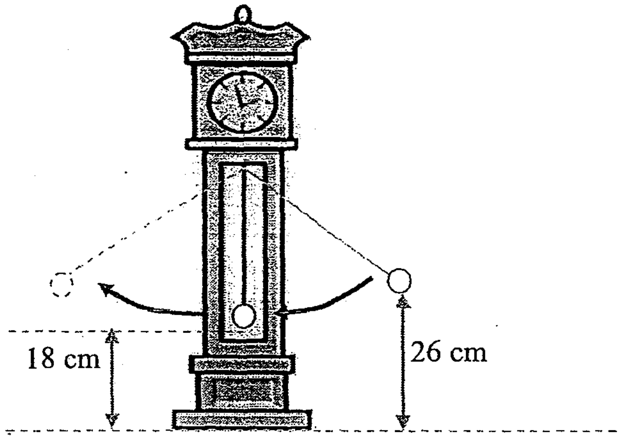
$= \frac{1}{3\pi\left(\sqrt[3]{6}\right)^2} \times 2\pi$ at $r = \sqrt[3]{6}$

$= 0.202$ (3 s.f)

The rate of change of radius is 0.202 cm/s. -------- A1

(ii) As $\frac{2}{3r^2}$ is inversely proportional to r, thus as t increases, r increases and $\frac{2}{3r^2}$ decreases.
This rate will decrease. ------ B1

13



The pendulum of a grandfather clock swings back and forth with a periodic motion that can be represented by the equation $h = a\cos kt + b$, where a, k and b are constants, h cm is the height of the pendulum above the base and t is the time in seconds after the pendulum is released 26 cm above the base. At rest, the pendulum is 20 cm above the base. It takes four seconds for a complete swing back and forth.

(i) State the value of a and b. [2]

(ii) Show that the value of k is $\frac{\pi}{2}$ rad/s. [1]

(iii) Sketch the graph of $h = a\cos kt + b$ for $0 \le t \le 6$. [2]

(iv) Find the time interval in which the pendulum is 20 cm above the base during one revolution. [3]

Marking Scheme

(i) $a = \frac{26-18}{2}$

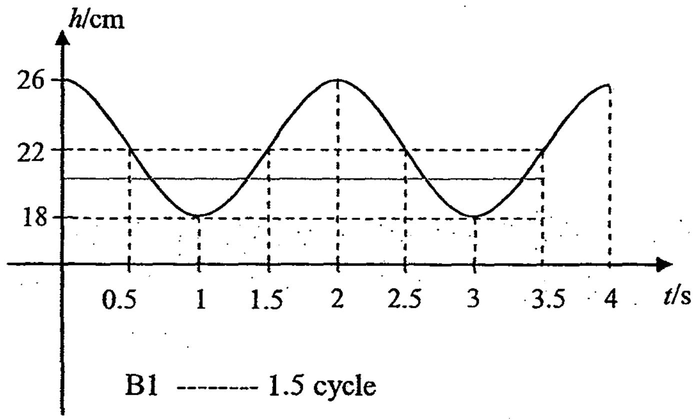
$= 4$ —— B1

$b = 22$ ——— B1

(ii) $k = \frac{2\pi}{4 \div 2}$ ------ B1

$= \pi$ rad/s —— (A.G)

(iii)



B1 -------- 1.5 cycle

B1 ---------- Max. and Min. values

(iii) $h = 4\cos \pi t + 22$

$4\cos \pi t + 22 = 20$

$\cos \dfrac{\pi}{2} t = -\dfrac{1}{2}$ -------- M1

Basic $\angle = \dfrac{\pi}{3}$

$\pi t = \dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$

$t = \dfrac{2}{3}$ or $\dfrac{4}{3}$ ------- M1

Thus the time interval is $0 \le t \le \dfrac{2}{3}$ and $\dfrac{4}{3} \le t \le 2$ -------- A1

Calculator Model:

Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2014
SECONDARY 4

Additional Mathematics — 4047/02

Paper 2 — **Friday, 29th August 2014**

2 hours 30 mins

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams, graphs or rough working.
Do not use highlighters or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/100

This document consist of 7 printed pages and 1 blank page

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \tfrac{1}{2}bc\sin A$$

1 (i) Calculate the coordinates of the point of intersection of $y = \frac{1}{4}x^{\frac{5}{2}}$ and $y = 256x^{-\frac{5}{2}}$, for $x > 0$ [2]

(ii) Sketch, on the same diagram, the graphs of $y = \frac{1}{4}x^{\frac{5}{2}}$ and $y = 256x^{-\frac{5}{2}}$ for $x > 0$, clearly indicating the point of intersection. [3]

(iii) Determine, with explanation, whether the tangents to the two graphs at the point of intersection are perpendicular. [4]

2 (a) The independent term in the expansion of $(2+x)^n$ and the independent term in the expansion of $(2-ax)^{2n+1}$ are in the ratio 1 : 8.

(i) Show that $n = 2$. [2]

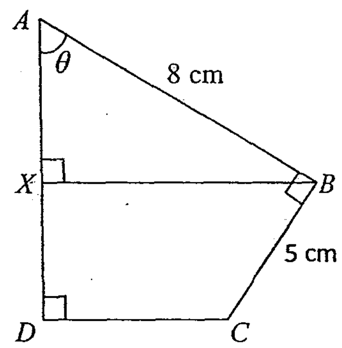
(ii) Given that the coefficient of x^2 in the expansion of $(1+x)(2-ax)^{2n+1}$ is 160, find the value(s) of a. [3]

(b) Given that the coefficient of the $\frac{1}{x^2}$ term in the expansion of $\left(\frac{2}{x}-x\right)^6 - \left(1+\frac{2}{x}\right)^n$ is 128, find the value of n. [5]

3 (i) Differentiate $x\cos 3x$ with respect to x. [2]

(ii) Using your answer to part (i), show that $\int_0^{\frac{\pi}{3}} x\sin 3x \, dx = \frac{\pi}{9}$. [4]

4 The diagram shows a quadrilateral $ABCD$ with $AB = 8$ cm, $BC = 5$ cm. $\angle ABC = 90°$ $\angle ADC = 90°$ and $\angle BAD = \theta$ where $0° < \theta < 90°$. BX is perpendicular to AD.



(i) Show that the perimeter of $ABCD$, P cm is given by $P = 13 + 13\sin\theta + 3\cos\theta$. [3]

(ii) Express P in the form $k + R\sin(\theta + \alpha)$, where k is a constant, $R > 0$ and α is an acute angle. [3]

(iii) Find the value of θ for which $P = 25$. [2]

(iv) Find the maximum value of P and its corresponding value of θ. [2]

5 A function is defined by $y = \dfrac{1-4x}{x+1}$ where $x > 0$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [3]

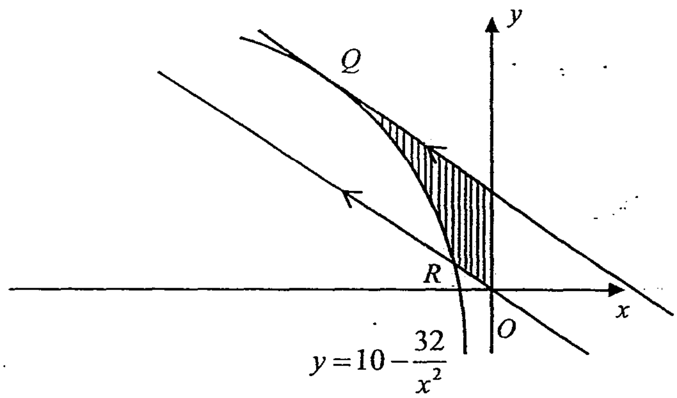
(iii) Determine, with explanation, whether $y = \dfrac{1-4x}{x+1}$ is an increasing or decreasing function. [2]

(iv) Find the range of x for which the gradient of the curve is an increasing function. [3]

6 (i) Express $\dfrac{x^3-2x^2-x+3}{x^2-2x+1}$ as partial fractions. [5]

(ii) Hence, find $\displaystyle\int \frac{x^3-2x^2-x+3}{x^2-2x+1}\,dx$. [2]

7



The diagram shows part of the curve $y=10-\dfrac{32}{x^2}$ and two parallel lines OR and PQ. The line OR intersects the curve at the point $R(-2,\ 2)$ and the line PQ is a tangent to the curve at the point Q. Find

(i) the gradient of OR, [1]

(ii) an expression for $\dfrac{dy}{dx}$ and hence, find the coordinates of Q, [3]

(iii) the area of the shaded region $OPQR$. [4]

8 **(a)** **(i)** Prove that $\cot 2A = \dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A}$ [3]

(ii) Hence solve the equation $\dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A} = 1$ for $0 \le A \le 2\pi$, leaving your answers in terms of π. [3]

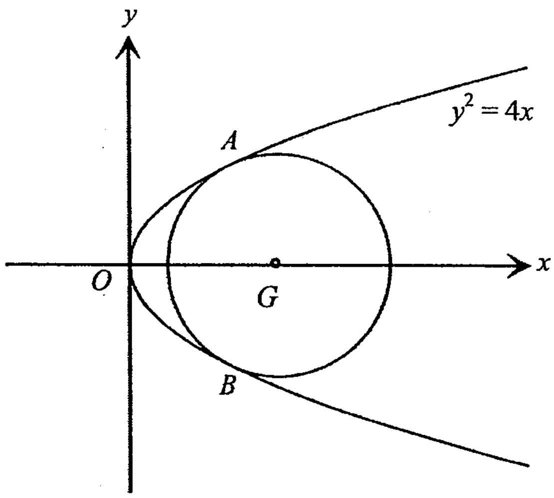
(b) A and B are acute angles such that $\sin(A+B) = \dfrac{7}{9}$ and $\sin A \cos B = \dfrac{4}{9}$. Without using a calculator, find the value of

(i) $\cos A \sin B$, [1]

(ii) $\sin(A-B)$, [1]

(iii) $\dfrac{\tan A}{\tan B}$. [2]

9



In the diagram, the circle with centre G touches the curve $y^2 = 4x$ at the points A and B. The equation of the common tangent to both the circle and the curve at the point A is $y = \dfrac{1}{2}x + 2$. Find

(i) the coordinates of the point A, [3]

(ii) the equation of AG, [2]

(iii) the equation of the circle, [3]

(iv) the equation of a second circle whose centre is G and area is 5 times that of the original circle. [2]

10 (a) Solve $\lg(x-8)+2\lg 3 = 2+\lg\left(\dfrac{x}{20}\right)$. [3]

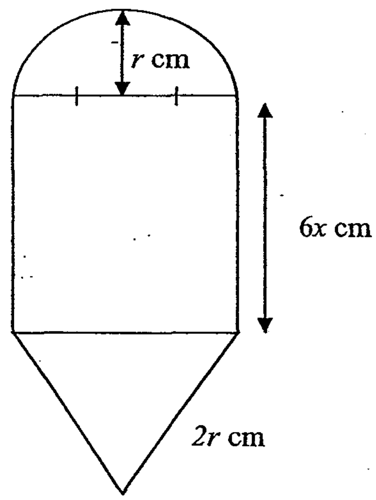
(b) Given that $\log_9 p = x$ and $\log_2 q = y$, express $q\log_3 p$ in terms of x and/or y. [3]

(c) The value, V, of a piece of rare jewel at the beginning of 1990 was \$20000. This value increased continuously such that, after a period of 10 years, the value of the jewel increased exponentially to \$100000. Given that $V = A(1.08)^{kt}$, where A and k are constants and t is the time in years from the beginning of 1990, find

(i) the value of A and k, [3]

(ii) the year in which the value of the stone first reached \$150000. [3]

11 A piece of wire, 400 cm long is cut to form the shape as shown in the diagram below. The shape consists of a semi-circular arc of radius r cm, a rectangle of length of $6x$ cm and an equilateral triangle.



(i) Express x in terms of r. [1]

(ii) Show that the area enclosed by the shape, A cm^2, given by [2]
$A = 400r + r^2\left(\sqrt{3} - 8 - \dfrac{\pi}{2}\right)$.

(iii) Given that r and x can vary, find the value of r for which A has a stationary value. [3]

(iv) Determine whether this value of r makes A a maximum or a minimum. [2]

~End of Paper~

Answer Key

1. (i) $(4, 8)$

2. (aii) $a=-1$ or 2
   (b) $n=8$

3. (i) $\cos 3x - 3x\sin 3x$

4. (ii) $P=13+\sqrt{178}\sin(\theta+13.0°)$
   (iii) $\theta=51.1°$
   (iv) Max. value of $P=26.3$, $\theta=77.0°$

5. (i) $\dfrac{-5}{(x+1)^2}$
   (ii) $y=\dfrac{5}{16}x-\dfrac{5}{64}$
   (iv) $x>-1$

6. (i) $x-\dfrac{2}{x-1}+\dfrac{1}{(x-1)^2}$
   (ii) $\dfrac{1}{2}x^2-2\ln(x-1)-\dfrac{1}{(x-1)}+C$

7. (i) -1
   (ii) $Q(-4,8)$
   (iii) 10units^2

8. (aii) $A=\dfrac{\pi}{8},\dfrac{5\pi}{8},\dfrac{9\pi}{8} or \dfrac{13\pi}{8}$
   (b) (i) $\dfrac{1}{3}$
   (ii) $\dfrac{1}{9}$
   (iii) $\dfrac{4}{3}$

9. (i) $(4, 4)$
   (ii) $y=-2x+12$
   (iii) $(x-6)^2+y^2=20$
   (iv) $(x-6)^2+y^2=100$

10. (i) $x=18$
   (ii) $2^{y+1}x$
   (iii) (a) $A=20000, k=2.09$
   (b) 2002

11. (i) $x = \dfrac{400 - 8r - \pi r}{12}$

(iii) $r = 25.5$

(iv) A is a maximum when $r = 25.5$

Calculator Model:

Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2014
SECONDARY 4

Additional Mathematics **4047/02**

Paper 2 *(Marking Scheme)* **Friday, 29th August 2014**

2 hours 30 mins

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 100.

/100

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Mathematical Formulae

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$.

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$$\sin^2 A + \cos^2 A = 1$$

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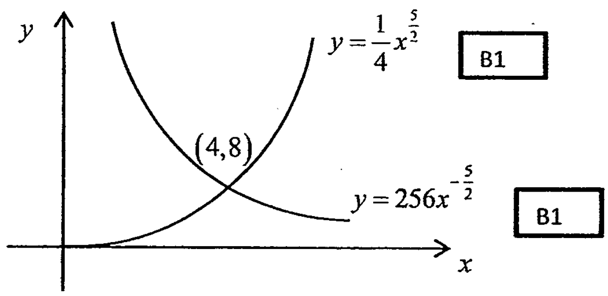
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

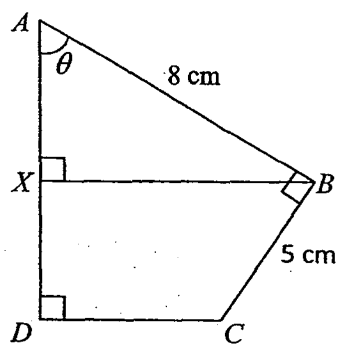
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1	**(i)** Calculate the coordinates of the point of intersection of $y=\frac{1}{4}x^{\frac{5}{2}}$ and $y=256x^{-\frac{5}{2}}$, for $x>0$	[2]
	Subst $y=\frac{1}{4}x^{\frac{5}{2}}$ into $y=256x^{-\frac{5}{2}}$ $\frac{1}{4}x^{\frac{5}{2}}=256x^{-\frac{5}{2}}$ $x^5=1024$ -M1 $x=4$ $y=\frac{1}{4}(4)^{\frac{5}{2}}$ $=8$ Coordinates of the point of intersection is $(4,\ 8)$. -A1	
	(ii) Sketch, on the same diagram, the graphs of $y=\frac{1}{4}x^{\frac{5}{2}}$ and $y=256x^{-\frac{5}{2}}$ for $x>0$, clearly indicating the point of intersection.	[3]
	 Labelling of graphs, axis and intersection – M1 cao	

	(iii) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular.	[4]
	$y=\frac{1}{4}x^{\frac{5}{2}}$ $\frac{d}{dx}\left(\frac{1}{4}x^{\frac{5}{2}}\right)=\frac{1}{4}x^{\frac{3}{2}}\times\frac{5}{2}$ $=\frac{5}{8}x^{\frac{3}{2}}$ –M1 cao $y=256x^{-\frac{5}{2}}$ $\frac{d}{dx}\left(256x^{-\frac{5}{2}}\right)=256x^{-\frac{7}{2}}\times\left(-\frac{5}{2}\right)$ $=-640x^{-\frac{7}{2}}$ –M1 cao When $x=4$, product of gradients $=\frac{5}{8}(4)^{\frac{3}{2}}\times\left[-640(4)^{-\frac{7}{2}}\right]$ -M1 $=5\times(-5)$ $\neq -1$ $\therefore$ The lines are not perpendicular. -A1	
2	**(a)** The independent term in the expansion of $(2+x)^n$ and the independent term in the expansion of $(2-ax)^{2n+1}$ are in the ratio 1 : 8.	
	(i) Show that $n=2$.	[2]
	$\frac{2^n}{2^{2n+1}}=\frac{1}{8}$ --------- M1 c.a.o (Applying ratio) $2^{2n+1-n}=2^3$ ------ M1 By comparison, $n+1=3$ $n=2$ ------- A.G	

	(ii) Given that the coefficient of x^2 in the expansion of $(1+x)(2-ax)^{2n+1}$ is 160, find the value(s) of a.	[3]
	$(1+x)(2-ax)^5=(1+x)\left[2^5+\binom{5}{1}(2^4)(-ax)+\binom{5}{2}(2^3)(-ax)^2+...\right]$ ---- M1 c.a.o $=(1+x)(32-80ax+80a^2x^2+...)$ $80a^2-80a=160$ ------ M1 $a^2-a=2$ $a^2-a-2=0$ $(a-2)(a+1)=0$ $a=-1$ or 2 ---------- A1	
	(b) Given that the coefficient of the $\frac{1}{x^2}$ term in the expansion of $\left(\frac{2}{x}-x\right)^6-\left(1+\frac{2}{x}\right)^n$ is 128, find the value of n.	[5]
	T_{r+1} of $\left(\frac{2}{x}-x\right)^6=\binom{6}{r}\left(\frac{2}{x}\right)^{6-r}(-1)^r(x)^r$ Let $\frac{x^r}{x^{6-r}}=x^{-2}$ -------- M1 $x^{2r-6}=x^{-2}$ By comparison, $2r-6=-2$ $r=2$ Coeff. of $\frac{1}{x^2}$ term $=\binom{6}{2}(2)^4(-1)^2$ $=240$ ------------ M1 c.a.o $\frac{1}{x^2}$ term of $\left(1+\frac{2}{x}\right)^n=\binom{n}{2}\left(\frac{2}{x}\right)^2$ -------- M1 c.a.o $\therefore\binom{n}{2}(4)=240-128$ $\frac{n(n-1)(4)}{2}=112$ ---------- M1 c.a.o $n(n-1)=56$ By inspection: $(8)(7)=56$ $\therefore n=8$ ------------ A1	

3	(i) Differentiate $x\cos 3x$ with respect to x.
	$\frac{d}{dx}(x\cos 3x)=\cos 3x+x(-3\sin 3x)$ –M1 cao $=\cos 3x-3x\sin 3x$ –A1
	(ii) Using your answer to part (i), show that $\int_0^{\frac{\pi}{3}} x\sin 3x\,dx=\frac{\pi}{9}$.
	$\frac{d}{dx}(x\cos 3x)=\cos 3x-3x\sin 3x$ Integrating both sides, $[x\cos 3x]_0^{\frac{\pi}{3}}=\int_0^{\frac{\pi}{3}}\cos 3x-3x\sin 3x\,dx$ –M1 (Integrate both sides) $-\frac{\pi}{9}-0=\int_0^{\frac{\pi}{3}}\cos 3x\,dx-\int_0^{\frac{\pi}{3}}3x\sin 3x\,dx$ –M1 cao (Subst of values) $\int 3x\sin 3x\,dx=\int_0^{\frac{\pi}{3}}\cos 3x\,dx+\frac{\pi}{9}$ $\int 3x\sin 3x\,dx=\left[\frac{1}{3}\sin 3x\right]_0^{\frac{\pi}{3}}+\frac{\pi}{9}$ –M1 (Correct integration) $=0-0+\frac{\pi}{9}$ –M1 cao (Subst of values) $=\frac{\pi}{9}$ –AG
4	The diagram shows a quadrilateral $ABCD$ with $AB=8$ cm, $BC=5$ cm. $\angle ABC=90°$ $\angle ADC=90°$ and $\angle BAD=\theta$ where $0°<\theta<90°$. BX is perpendicular to AD. 

	(i) Show that the perimeter of $ABCD$, P cm is given by $P = 13 + 13\sin\theta + 3\cos\theta$.	[3]
	$\cos\theta = \frac{AX}{8}$ $AX = 8\cos\theta$ –B1 (For AX, DX or BX) Since $\angle XBC = 90° - \angle XAB$ $= 90° - (90° - \theta)$ $= \theta$ $\sin\theta = \frac{DX}{5}$ $DX = 5\sin\theta$ $\frac{BY}{5} = \cos\theta$ $BY = 5\cos\theta$ $\sin\theta = \frac{BX}{8}$ $BX = 8\sin\theta$ $DC = 8\sin\theta - 5\cos\theta$ Perimeter of $ABCD = AD + DC + CB + BA$ $= (8\cos\theta + 5\sin\theta) + (8\sin\theta - 5\cos\theta) + 5 + 8$ –M1 (Finding AD or DC) $= 13 + 13\sin\theta + 3\cos\theta$ –A1	
	(ii) Express P in the form $k + R\sin(\theta + \alpha)$, where k is a constant, $R > 0$ and α is an acute angle.	[3]
	$P = 13 + 13\sin\theta + 3\cos\theta$ For $13\sin\theta + 3\cos\theta$, $R = \sqrt{13^2 + 3^2}$ –M1 cao $= \sqrt{178}$ $\alpha = \tan^{-1}\frac{3}{13}$ –M1 cao $= 12.995°$ (3dec.pl.) $\therefore P = 13 + \sqrt{178}\sin(\theta + 13.0°)$ (1dec.pl.) –A1	

	(iii) Find the value of θ for which $P = 25$.	[2]
	$$P = 25$$ $$13+\sqrt{178}\sin(\theta+12.995°) = 25$$ $$\sqrt{178}\sin(\theta+12.995°) = 12$$ $$\sin(\theta+12.995°) = \frac{12}{\sqrt{178}} \qquad \text{–M1}$$ $$\theta+12.995° = 64.084°$$ $$\theta = 51.1° \qquad \text{–A1}$$	
	(iv) Find the maximum value of P and its corresponding value of θ.	[2]
	$$\text{Max. value of } P = 13+\sqrt{178}$$ $$= 26.3 \qquad \text{–B1}$$ $$\text{When } \sin(\theta+12.995°) = 1$$ $$\theta+12.995° = 90°$$ $$\theta = 77.0° \qquad \text{(1dec.pl.)} \qquad \text{–B1}$$	
5	A function is defined by $y = \dfrac{1-4x}{x+1}$ where $x > 0$.	
	(i) Find an expression for $\dfrac{dy}{dx}$.	[2]
	$$y = \frac{1-4x}{x+1}$$ $$\frac{dy}{dx} = \frac{-4(x+1)-(1-4x)}{(x+1)^2} \qquad \text{–M1 cao}$$ $$= \frac{-5}{(x+1)^2} \qquad \text{–A1}$$	

	(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis.	[3]
	$y=\frac{1-4x}{x+1}$ When $y=0$, $\frac{1-4x}{x+1}=0$ $x=\frac{1}{4}$ —M1 cao When $x=\frac{1}{4}$, $\frac{dy}{dx}=\frac{-5}{\left(\frac{1}{4}+1\right)^2}$ $=-\frac{16}{5}$ Gradient of normal $=\frac{5}{16}$ —M1 Equation of normal: $y-0=\frac{5}{16}\left(x-\frac{1}{4}\right)$ $y=\frac{5}{16}x-\frac{5}{64}$ —A1	
	(iii) Determine, with explanation, whether $y=\frac{1-4x}{x+1}$ is an increasing or decreasing function.	[2]
	For $x>0$, $\therefore\ (x+1)^2>0$, —M1 $\frac{1}{(x+1)^2}>0$ $\frac{-5}{(x+1)^2}<0$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}<0$ Therefore, $y=\frac{1-4x}{x+1}$ is a decreasing function. —A1	

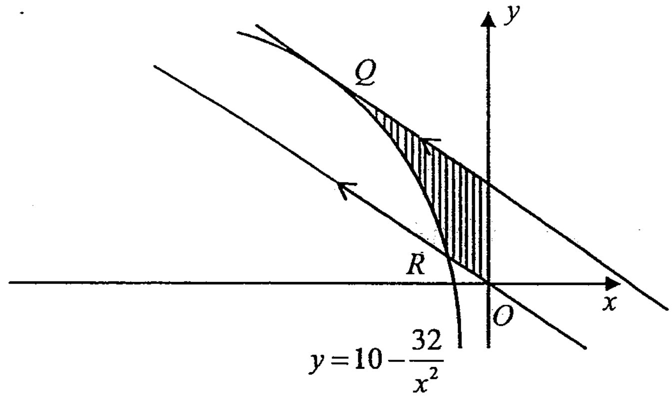
	(iv) Find the range of x for which the gradient of the curve is an increasing function.	[3]
	$\frac{dy}{dx}=\frac{-5}{(x+1)^2}$ $=-5(x+1)^{-2}$ $\frac{d^2y}{dx^2}=-5(x+1)^{-3}\times(-2)$ $=\frac{10}{(x+1)^3}$ –M1 For gradient to be increasing function, $\frac{d^2y}{dx^2}>0$ –M1cao $\frac{10}{(x+1)^3}>0$ $\therefore\ (x+1)^3>0$ $(x+1)(x+1)^2>0$ Since $(x+1)^2>0$, $x+1>0$ $x>-1$ –A1	
6	**(i)** Express $\frac{x^3-2x^2-x+3}{x^2-2x+1}$ as partial fractions.	[5]
	$(x^2-2x+1)\overline{)\ x^3-2x^2-x+3\ }$ quotient x; $-(x^3-2x^2+x)$; remainder $-2x+3$ –M1 Long Division $\frac{x^3-2x^2-x+3}{x^2-2x+1}=x+\frac{(-2x+3)}{x^2-2x+1}$ Let $\frac{3-2x}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}$, where A and B are constants. –M1cao $3-2x=A(x-1)+B$ –M1(*Comparing or substitution*) $-2x+3=Ax+(B-A)$ By comparing coefficients, $A=-2, B=1$ –M1 cao $\therefore\frac{x^3-2x^2-x+3}{x^2-2x+1}=x-\frac{2}{x-1}+\frac{1}{(x-1)^2}$ –A1	

(ii) Hence, find $\int \frac{x^3-2x^2-x+3}{x^2-2x+1}\,dx$. [2]

$$\int \frac{x^3-2x^2-x+3}{x^2-2x+1}\,dx = \int x-\frac{2}{x-1}+\frac{1}{(x-2)^2}\,dx \qquad \text{–M1}$$

$$=\frac{1}{2}x^2-2\ln(x-1)-\frac{1}{(x-1)}+C, \text{ where } C \text{ is a constant.} \qquad \text{–A1}$$

7



The diagram shows part of the curve $y=10-\frac{32}{x^2}$ and two parallel lines OR and PQ. The line OR intersects the curve at the point $R(-2, 2)$ and the line PQ is a tangent to the curve at the point Q. Find

(i) the gradient of OR, [1]

$$\text{Gradient of } OR = \frac{2-0}{-2-0}$$

$$=-1 \qquad \text{–B1}$$

(ii) an expression for $\dfrac{dy}{dx}$ and hence, find the coordinates of Q, [3]

$$y = 10 - \frac{32}{x^2}$$

$$\frac{dy}{dx} = -\frac{32}{x^3}(-2)$$

$$= \frac{64}{x^3} \qquad \text{–A1}$$

Since $OR // PQ$,

$$\frac{64}{x^3} = -1 \qquad \text{–M1}$$

$$x = -4$$

When $x = -4$, $y = 10 - \dfrac{32}{(-4)^2}$

$$= 8$$

$\therefore Q(-4, 8)$ –A1

(iii) the area of the shaded region $OPQR$, [4]

Equation of line PQ:

$$y - 8 = (-1)(x + 4)$$

$$y = -x + 4$$

$\therefore P(0, 4)$ –M1

Area of trapezium below line $PQ = \dfrac{1}{2}(4+8)(4)$

$= 24\text{units}^2$

Area of □ below $OR = \dfrac{1}{2}(2)(2)$

$= 2\text{units}^2$

–M1 cao M1 cao

Area below curve $RQ = \displaystyle\int_{-4}^{-2} 10 - \frac{32}{x^2}\,dx$

$$= \left[10x + \frac{32}{x}\right]_{-4}^{-2}$$

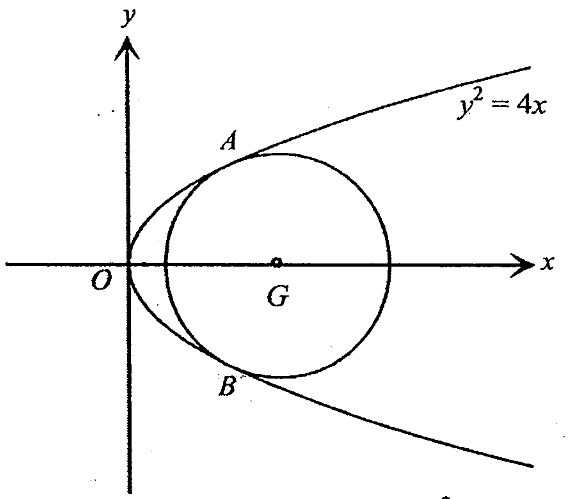
$$= (-20 - 16) - (-40 - 8)$$

$$= 12\text{units}^2 \qquad \text{–M1}$$

Area of shaded region $= 24 - 2 - 12$

$$= 10\text{units}^2 \qquad \text{–A1}$$

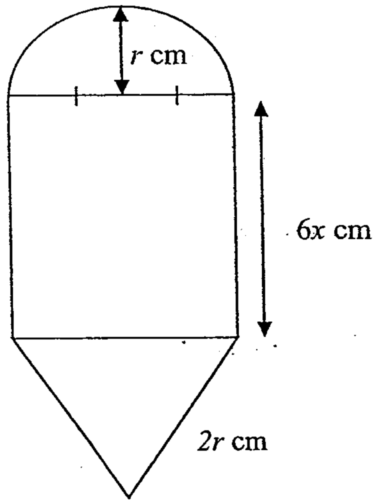
8	**(a)** **(i)** Prove that $\cot 2A = \dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A}$.	[3]
	$\text{RHS} = \dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A}$ $= \dfrac{\frac{1}{\sin A} - 2\sin A}{2\cos A}$ –M1 cao (Replace $\operatorname{cosec} A$) $= \dfrac{1-2\sin^2 A}{2\sin A\cos A}$ –M1 $\left(\text{Multiply } \dfrac{\sin \text{A}}{\sin \text{A}}\right)$ $= \dfrac{\cos 2A}{\sin 2A}$ –M1cao $= \cot 2A$ (Proven) –AG	
	(ii) Hence solve the equation $\dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A} = 1$ for $0 \le A \le 2\pi$, leaving your answers in terms of π.	[3]
	$\dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A} = 1$ $\cot 2A = 1$ –M1 cao $\tan 2A = 1$ Basic angle of $2A = \dfrac{\pi}{4}$ –M1 $0 \le A \le 2\pi$ $2A = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4} \text{ or } \dfrac{13\pi}{4}$ $0 \le 2A \le 4\pi$ $A = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8} \text{ or } \dfrac{13\pi}{8}$ –A1	
	(b) A and B are acute angles such that $\sin(A+B) = \dfrac{7}{9}$ and $\sin A\cos B = \dfrac{4}{9}$. Without using a calculator, find the value of	
	(i) $\cos A\sin B$,	[1]
	Given $\sin(A+B) = \dfrac{7}{9}$ and $\sin A\cos B = \dfrac{4}{9}$, $\sin A\cos B + \cos A\sin B = \dfrac{7}{9}$ $\dfrac{4}{9} + \cos A\sin B = \dfrac{7}{9}$ $\cos A\sin B = \dfrac{1}{3}$ –B1	

	(ii) $\sin(A-B)$,	[1]
	$\sin(A-B)=\sin A\cos B-\cos A\sin B$ $=\frac{4}{9}-\frac{1}{3}$ $=\frac{1}{9}$ –B1	
	(iii) $\frac{\tan A}{\tan B}$.	[2]
	$\frac{\tan A}{\tan B}=\frac{\sin A\cos B}{\cos A\sin B}$ –M1 cao $=\frac{\frac{4}{9}}{\frac{1}{3}}$ $=\frac{4}{3}$ –A1	
9	 In the diagram, the circle with centre G touches the curve $y^2=4x$ at the points A and B. The equation of the common tangent to both the circle and the curve at the point A is $y=\frac{1}{2}x+2$. Find	

	(i) the coordinates of the point A,	[3]
	$y = 2\sqrt{x}$ $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$ –M1 cao (Either method) At A, $\frac{dy}{dx} = \frac{1}{2}$ $\frac{1}{\sqrt{x}} = \frac{1}{2}$ –M1cao $x = 4$ $\therefore y = 4$ Coordinates of A is $(4,\ 4)$. –A1 OR Subst. $y = \frac{1}{2}x + 2$ into $y^2 = 4x$, –M1 cao $\left(\frac{1}{2}x+2\right)^2 = 4x$ $\frac{1}{4}x^2 + 2x + 4 - 4x = 0$ $x^2 - 8x + 16 = 0$ $(x-4)^2 = 0$ – M1 (Factorisation) $x = 4$ $\therefore y = 4$ Coordinates of A is $(4,\ 4)$. –A1	
	(ii) the equation of AG,	[2]
	Gradient of $AG = -2$ –M1 cao Equation of AG, $\frac{y-4}{x-4} = -2$ $y = -2x + 12$ –A1	
	(iii) the equation of the circle,	[3]
	G lies on the line AG and the x–axis, hence G is $(6,\ 0)$. –M1 Radius of circle $= \sqrt{(4-6)^2 + (4-0)^2}$ –M1 $= \sqrt{20}$ $= 2\sqrt{5}$ units Equation of circle: $(x-6)^2 + y^2 = 20$ –A1 *or* $x^2 + y^2 - 12x + 16 = 0$	

	(iv) the equation of a second circle whose centre is G and area is 5 times that of the original circle.	[2]
	Let r cm be the radius of the second circle. Area of second circle = 5 × Area of first circle $\pi r^2 = 5\pi\left(\sqrt{20}\right)^2$ –M1 $r^2 = 100$ Equation of second circle: $(x-6)^2 + y^2 = 100$ –A1	
10	**(i)** Solve $\lg(x-8) + 2\lg 3 = 2 + \lg\left(\frac{x}{20}\right)$.	[3]
	$\lg(x-8) + 2\lg 3 = 2 + \lg\left(\frac{x}{20}\right)$ $\lg(x-8) + \lg 9 = \lg 100 + \lg\left(\frac{x}{20}\right)$ –M1 cao (Power law) $\lg[9(x-8)] = \lg\left[100\left(\frac{x}{20}\right)\right]$ –M1 (Product Law) By comparison, $9x - 72 = 5x$ $4x = 72$ $x = 18$ –A1	
	(ii) Given that $\log_9 p = x$ and $\log_2 q = y$, express $q\log_3 p$ in terms of x and/or y.	[3]
	$\log_2 q = y$ $q = 2^y$ –M1 cao $q\log_3 p = 2^y\left(\frac{\log_9 p}{\log_9 3}\right)$ –M1 cao (for change of base or for forming $\log_9 9$) $= 2^y\left(\frac{x}{\log_9 9^{\frac{1}{2}}}\right)$ $= 2^y(2x)$ $= 2^{y+1}x$ –A1	

	(iii) The value, V, of a piece of rare jewel at the beginning of 1990 was \$20000. This value increased continuously such that, after a period of 10 years, the value of the jewel increased exponentially to \$100000. Given that $V = A(1.08)^{kt}$, where A and k are constants and t is the time in years from the beginning of 1990, find	
	(a) the value of A and k,	[3]
	$V = A(1.08)^{kt}$ When $t = 0$, $V = 20000$ $\therefore A = 20000$ —B1 When $t = 10$, $V = 100000$ $100000 = 20000(1.08)^{10k}$ $(1.08)^{10k} = 5$ $\ln(1.08)^{10k} = \ln 5$ —M1 $10k = \dfrac{\ln 5}{\ln 1.08}$ $k = 2.0912...$ $= 2.09 \text{(3sig.fig.)}$ —A1	
	(b) the year in which the value of the stone first reached \$150000.	[3]
	$V = 20000(1.08)^{2.0912t}$ When $V = 150000$, $150000 = 20000(1.08)^{2.0912t}$ $(1.08)^{2.0912t} = 7.5$ $\ln(1.08)^{2.0912t} = \ln 7.5$ —M1 $2.0912t = \dfrac{\ln 7.5}{\ln 1.08}$ $t = 12.519$ —M1 Year in which value exceeded \$150000 = 1990+12 = 2002 —A1	

11	A piece of wire, 400 cm long is cut to form the shape as shown in the diagram below. The shape consists of a semi-circular arc of radius r cm, a rectangle of length of $6x$ cm and an equilateral triangle. 	
	(i) Express x in terms of r.	[1]
	Total Length $= 400$ $12x+8r+\pi r=400$ $x=\dfrac{400-8r-\pi r}{12}$ –B1	
	(ii) Show that the area enclosed by the shape, A cm^2, given by $A=400r+r^2\left(\sqrt{3}-8-\dfrac{\pi}{2}\right)$.	[2]
	$A=\dfrac{1}{2}\pi r^2+12rx+\dfrac{1}{2}\times 2r\times 2r\times \sin 60°$ –M1 cao $=\dfrac{1}{2}\pi r^2+12r\left(\dfrac{400-8r-\pi r}{12}\right)+2r^2\times\dfrac{\sqrt{3}}{2}$ –M1 $=\dfrac{1}{2}\pi r^2+400r-8r^2-\pi r^2+\sqrt{3}r^2$ $=400r+r^2\left(\sqrt{3}-8-\dfrac{\pi}{2}\right)$ –AG	

	(iii) Given that r and x can vary, find the value of r for which A has a stationary value.	[3]
	$\frac{dA}{dr}=400+2r\left(\sqrt{3}-8-\frac{\pi}{2}\right)$ –M1 cao For stationary value, $400+2r\left(\sqrt{3}-8-\frac{\pi}{2}\right)=0$ –M1 $r=\frac{400}{2\left(\frac{\pi}{2}+8-\sqrt{3}\right)}$ $=25.5$ –A1	
	(iv) Determine whether this value of r makes A a maximum or a minimum.	[2]
	$\frac{d^2A}{dr^2}=2\left(\sqrt{3}-8-\frac{\pi}{2}\right)$ $=-15.677<0$ –M1 cao A is a maximum when $r=25.5$ –A1	

~End of Paper~