Name:	Register No. :	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS

ADDITIONAL MATHEMATICS — 4047/01

Paper 1 — **19 Aug 2014**

Additional materials: Answer Paper
Mark Sheet

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 The equation of a straight line is $y = 7 + kx$, where k is a constant. Find the value of k for which this straight line is tangent to the curve $\left(\frac{y}{2}\right)^2 = y - x - 1$. [3]

2 Find the coordinates of the stationary point of the curve $y = \frac{3x^6 - x^3}{x^3}$, and determine the nature of this stationary point. [4]

3 It is given that $\int_0^{\pi} k\sin\theta \, d\theta = 9$, where k is a constant.

(i) Without the use of a calculator, find

(a) $\int_0^{2\pi} |k\sin\theta| \, d\theta$, [1]

(b) $\int_0^{\pi} (k\sin\theta + 3) \, d\theta$. [2]

(ii) State the smallest positive value of a and of b such that $\int_a^b k\cos\theta \, d\theta = 9$. [2]

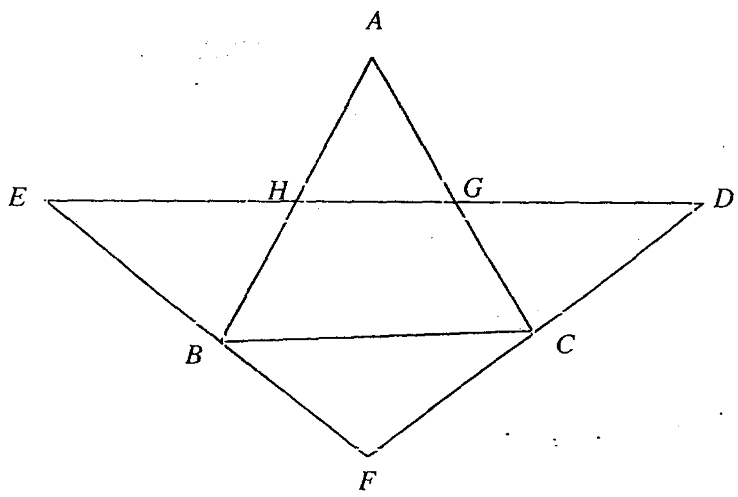
4 It is given that $3^{2x} = 2^{1-x}$. Find

(i) the exact value of 18^{1-x}, [3]

(ii) the value of x. [2]

[Turn over

5



The diagram above shows the triangles ABC and DEF, where $AB = AC$. H and G are the midpoints of AB and AC respectively, while B and C are the midpoints of EF and DF respectively.

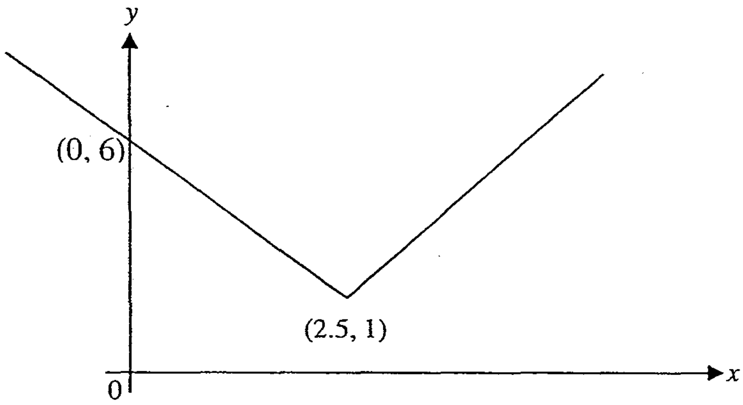
(i) Show that $DE = 4GH$. [2]

(ii) Given that $EH = GD$, show that ΔDEF is isosceles. [6]

6 The coordinates of points P and Q are (2, 5) and (3, 2) respectively. It is given that the shortest distance from point P to a line l_1 is three times the shortest distance from point Q to line l_1. Find the equation of the line l_1 if l_1 lies between the points P and Q, and is perpendicular to the line joining the points P and Q. [4]

7 (a) Solve the equation $x^2 - 11 = |x-9|$. [3]

(b)



The diagram above shows part of the graph of $y = |ax+b| + c$, where a, b and c are constants. The coordinates of the y-intercept and vertex are given as (0, 6) and (2.5, 1) respectively. Write down two possible functions of y that can be represented by the graph in the diagram. [3]

8 (i) Express $(\alpha-\beta)^2$ in terms of $\alpha+\beta$ and $\alpha\beta$. [1]

The roots of the quadratic equation $x^2 - 2x - 7 = 0$ are α and β, where $\alpha > \beta$.

(ii) Find the quadratic equation whose roots are α^3 and $-\beta^3$. [7]

9 (i) Solve the equation $\tan 2A = -3$ for $0° \le A \le 360°$. [4]

(ii) State the number of solutions of the equation $\tan 6A = -3$ for $0° \le A \le 360°$. [1]

10 A particle moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by $v = 12e^{-0.4t} - 1$. Find

(i) an expression for the acceleration of the particle in terms of t, [2]

(ii) the distance travelled by the particle before it comes to instantaneous rest. [5]

[Turn over

11 Variables x and y are connected by the equation $y = cx^2 + dx$, where c and d are constants.

(i) Express the given equation in a form suitable for drawing a straight line graph of $\frac{y}{x}$ against x. [1]

(ii) Given that the straight line passes through points $(-1, -3)$ and $(-3, -21)$, find the value of c and of d. [3]

(iii) Hence, find the coordinates of the point on the straight line at which $2y = x^2$. [3]

12 The equation of a curve is $y = \frac{8a^3}{x^2 + 4a^2}$, where a is a constant.

(i) Show that $\frac{dy}{dx} = \frac{-16a^3x}{\left(x^2 + 4a^2\right)^2}$. [2]

A particle moves along the curve. At the point P $(2, -1)$, the y-coordinate of the particle is decreasing at a rate of 0.3 units/sec.

(ii) Find the value of a. [5]

(iii) Find the corresponding rate of change of the x-coordinate of the particle at the point P. [2]

13 Scientists use trigonometric functions to model populations of predators and prey in the environment. In a particular study which stretched over a period of 4 months, the population of mousedeer is modeled by the equation $P = -300\cos\left(\frac{\pi}{2}t\right) + 500$.

The population of leopards in the same area is modeled by the equation $P = 50\cos\left(\frac{\pi}{2}t\right) + 100$. In each equation, P is the population of the animal and t is the time in months.

(i) On the same axes, sketch the graphs of $P = -300\cos\left(\frac{\pi}{2}t\right) + 500$ and $P = 50\cos\left(\frac{\pi}{2}t\right) + 100$ for $0 \le t \le 4$. [5]

(ii) Using the graphs of these two equations, state the relationship between the population of mousedeer and the population of leopards in the area. [1]

Leopards in the area are classified as vulnerable when their population is 80 or less.

(iii) Find the length of time for which the leopards became vulnerable during the study. [3]

END OF PAPER

Answer Key

1. $k=\frac{-1}{5}$	8. (i) $(\alpha+\beta)^2-4\alpha\beta$
2. (0, −1), point of inflexion	8. (ii) $x^2-44\sqrt{2}x+343=0$
3. (i) (a) 18, (b) $9+3\pi$	9. (i) 54.2°, 144.2°, 234.2°, or 324.2°
3. (ii) $a=\frac{3\pi}{2}, b=\frac{5\pi}{2}$	9. (ii) 12
4. (i) 9	10. (i) $-4.8e^{-0.4t}$
4. (ii) 0.240	10. (ii) 21.3 m
5. Proof	11. (i) $\frac{y}{x}=cx+d$
6. $y=\frac{x}{3}+\frac{11}{6}$	11. (ii) $c=9$, $d=6$
7. (a) 4, −5	11. (iii) $\left(\frac{-12}{17},\frac{-6}{17}\right)$
7. (b) $y=\|2x-5\|+1$, $y=\|5-2x\|+1$	12. (ii) $a=-1$
	12. (iii) −0.6 units/sec
	13. (iii) 1.48 months

Name: MARKING SCHEME	Register No. :	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS

ADDITIONAL MATHEMATICS — 4047/01

Paper 1 — **19 Aug 2014**

Additional materials: Answer Paper
Mark Sheet

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

1 The equation of a straight line is $y=7+kx$, where k is a constant. Find the value of k for which this straight line is tangent to the curve $\left(\frac{y}{2}\right)^2 = y-x-1$. [3]

$y = 7 + kx \quad —①$

$\left(\frac{y}{2}\right)^2 = y - x - 1 \quad —②$

sub ① into ②:

$\left(\frac{7+kx}{2}\right)^2 = 7 + kx - x - 1$ $\boxed{M1}$

$49 + 14kx + k^2x^2 = 28 + 4kx - 4x - 4$

$k^2x^2 + 10kx + 4x + 25 = 0$

$b^2 - 4ac = 0$

$(10k+4)^2 - 4k^2(25) = 0$ $\boxed{M1}$

$100k^2 + 80k + 16 - 100k^2 = 0$

$80k + 16 = 0$

$k = -\frac{1}{5}$ $\boxed{A1}$

2 Find the coordinates of the stationary point of the curve $y=\frac{3x^6-x^3}{x^3}$, and determine the nature of this stationary point. [4]

$\frac{dy}{dx} = 9x^2$ $\boxed{B1}$

when $\frac{dy}{dx} = 0$

$9x^2 = 0$

$\therefore x = 0$, $y = -1$ $\boxed{A1}$

x	-0.1	0	0.1
$\frac{dy}{dx}$	$+$	0	$+$
slope	/	—	/

$\boxed{M1}$

$\therefore$ $(0, -1)$ is a point of inflexion. $\boxed{A1}$ must agree with test result

award mark if spelling wrong but obvious what student meant

3 It is given that $\int_0^{\pi} k\sin\theta \, d\theta = 9$, where k is a constant.

(i) Without the use of a calculator, find

(a) $\int_0^{2\pi} |k\sin\theta| \, d\theta$, [1]

(b) $\int_0^{\pi} (k\sin\theta + 3) \, d\theta$. [2]

(ii) State the smallest positive value of a and of b such that $\int_a^b k\cos\theta \, d\theta = 9$. [2]

(i) (a) $\int_0^{2\pi} |k\sin\theta| \, d\theta = 2\int_0^{\pi} k\sin\theta \, d\theta$

$= 18$ $\boxed{B1}$

(b) $\int_0^{\pi} (k\sin\theta + 3) \, d\theta = 9 + [3\theta]_0^{\pi}$ $\boxed{M1}$

$= 9 + 3\pi$ $\boxed{A1}$

(ii) $a = \frac{3\pi}{2}$, $b = \frac{5\pi}{2}$ $\boxed{B2}$

4 It is given that $3^{2x} = 2^{1-x}$. Find

(i) the exact value of 18^{1-x}, [3]

(ii) the value of x. [2]

(i) $3^{2x} = 2^{1-x}$

$3^{2x} \times 9^{1-x} = 2^{1-x} \times 9^{1-x}$ $\boxed{M1}$

$3^{2x} \times \frac{9}{3^{2x}} = 18^{1-x}$ $\boxed{M1}$

$18^{1-x} = 9$ $\boxed{A1}$

Alt:

$9^x = \frac{2}{2^x}$

$\frac{1}{2} = 18^{-x}$

$\boxed{M1}$ $2^x \cdot 3^{2x} = 18$

$\frac{1}{2} \times 18 = 18^{-x} \times 18$ $\boxed{M1}$

$18^{1-x} = 9$ $\boxed{A1}$

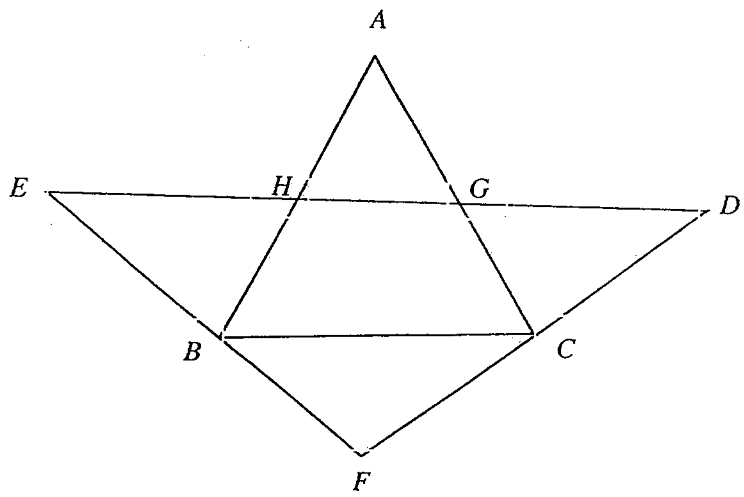
(ii) $18^{1-x} = 9$

$18^x = 2$

$x = \frac{\lg 2}{\lg 18}$ $\boxed{M1}$

$= 0.240$ $\boxed{A1}$

5



The diagram above shows the triangles ABC and DEF, where $AB = AC$. H and G are the midpoints of AB and AC respectively, while B and C are the midpoints of EF and DF respectively.

(i) Show that $DE = 4GH$. [2]

(ii) Given that $EH = GD$, show that ΔDEF is isosceles. [6]

(i) $GH = \frac{1}{2}BC$ (midpoint theorem) $\boxed{B1}$

$= \frac{1}{2}\left(\frac{1}{2}DE\right)$ (midpoint theorem) $\boxed{B1}$

$= \frac{1}{4}DE$

$\therefore DE = 4GH$ correct conclusion.

OR

$DE = 2BC$

$= 2(2GH)$

$= 4GH$

(ii) ΔAHG is Isosceles ($AH = AG$)

$\angle AHG = \angle AGH$ (base $\angle$s of isos Δ)

$\left.\begin{array}{l}\angle AHG = \angle EHB \text{ (vert opp } \angle\text{s)} \\ \angle AGH = \angle DGC \text{ (vert opp } \angle\text{s)}\end{array}\right\}$ $\boxed{M1}$ either

$\therefore \angle EHB = \angle DGC$ $\boxed{A1}$

$EH = GD$ (given) $\boxed{M1}$

$HB = \frac{1}{2}AB$ (H is midpoint)

$= \frac{1}{2}AC$ (given)

$= GC$ (G is mid point) $\boxed{M1}$

$\therefore \Delta EHB \equiv \Delta DGC$ (SAS) $\boxed{A1}$

$\angle HEB = \angle GDC$ $\boxed{A1}$

(corr $\angle$s of congruent Δs)

$\therefore \Delta DEF$ is isosceles.

correct conclusion

6 The coordinates of points P and Q are $(2,\ 5)$ and $(3, 2)$ respectively. It is given that the shortest distance from point P to a line l_1 is three times the shortest distance from point Q to line l_1. Find the equation of the line l_1 if l_1 is perpendicular to the line joining the points P and Q. [4]

gradient of PQ

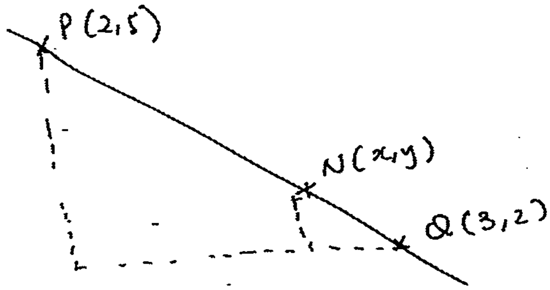
$= \dfrac{5-2}{2-3}$

$= -3$

gradient of $l_1 = \dfrac{1}{3}$ $\boxed{M1}$

Let the point of intersection between PQ and l_1 be $N(x, y)$

$\dfrac{PN}{QN} = 3$



$\dfrac{3-x}{3-2} = \dfrac{1}{4}$

$3 - x = \dfrac{1}{4}$

$x = \dfrac{11}{4}$

$\boxed{M1}$ either or

$\dfrac{y-2}{5-2} = \dfrac{1}{4}$

$y - 2 = \dfrac{3}{4}$

$y = \dfrac{11}{4}$

$\therefore N\left(\dfrac{11}{4}, \dfrac{11}{4}\right)$ $\boxed{A1}$

$l_1: \ y - \dfrac{11}{4} = \dfrac{1}{3}\left(x - \dfrac{11}{4}\right)$

$y = \dfrac{x}{3} - \dfrac{11}{12} + \dfrac{11}{4}$

$y = \dfrac{x}{3} + \dfrac{11}{6}$ // $\boxed{A1}$

7 (a) Solve the equation $x^2 - 11 = |x - 9|$. [3]

$x - 9 = x^2 - 11$ or $x - 9 = -x^2 + 11$ $\boxed{M1}$ 2 cases

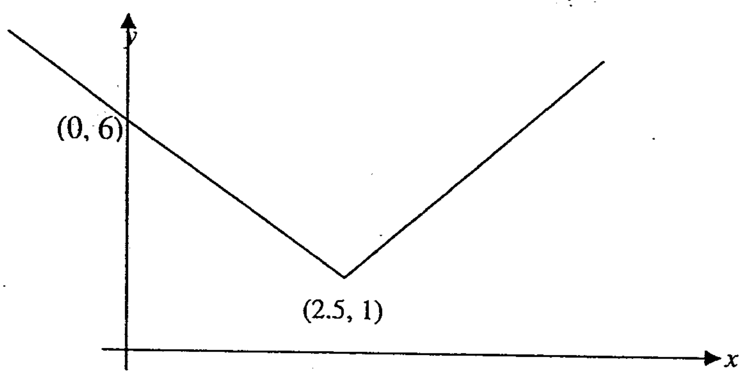
$x^2 - x - 2 = 0$ $\quad$ $x^2 + x - 20 = 0$

$(x-2)(x+1) = 0$ $\quad$ $(x+5)(x-4) = 0$ $\boxed{M1}$ factorisation

$x = 2$ (rej) or $x = -1$ (rej) $\quad$ $x = -5$ // or $x = 4$ //

$\boxed{A1}$ all four values but reject two

(b)



The diagram above shows part of the graph of $y = |ax + b| + c$, where a, b and c are constants. The coordinates of the y-intercept and vertex are given as (0, 6) and (2.5, 1) respectively. Write down two possible functions of y that can be represented by the graph in the diagram. [3]

$c = 1$ $\boxed{B1}$

$\therefore$ The two possible functions of y are:

$y = |2x - 5| + 1$ $\boxed{B1}$ or $y = |5 - 2x| + 1$ $\boxed{B1}$

8 **(i)** Express $(\alpha-\beta)^2$ in terms of $\alpha+\beta$ and $\alpha\beta$. [1]

The roots of the quadratic equation $x^2-2x-7=0$ are α and β, where $\alpha>\beta$.

(ii) Find a quadratic equation whose roots are α^3 and $-\beta^3$. [7]

(i) $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2 = (\alpha+\beta)^2 - 4\alpha\beta$ $\boxed{B1}$

(ii) $\left.\begin{aligned}\alpha+\beta &= 2\\ \alpha\beta &= -7\end{aligned}\right\}$ $\boxed{B1}$ both

$\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$ $\boxed{B1}$

$\therefore \alpha^3-\beta^3 = (\alpha-\beta)\left[(\alpha+\beta)^2-\alpha\beta\right]$ $\boxed{M1}$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= 4+28$

$= 32$ $\boxed{M1}$

$\alpha-\beta = 4\sqrt{2}$ $(\because \alpha>\beta)$

$\alpha^3-\beta^3 = 4\sqrt{2}(4+7)$

$= 44\sqrt{2}$ $\boxed{M1}$

$-\alpha^3\beta^3 = -(\alpha\beta)^3$

$= 343$ $\boxed{M1}$

$\therefore$ required equation:

$x^2 - 44\sqrt{2}x + 343 = 0$ $\boxed{A1}$

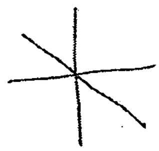
only in exact form

9 (i) Solve the equation $\tan 2A = -3$ for $0° \le A \le 360°$. [4]

(ii) State the number of solutions of the equation $\tan 6A = -3$ for $0° \le A \le 360°$. [1]

(i) $\tan 2A = -3$

Basic $\angle = 71.565°$ $\boxed{M1}$



$0° \le 2A \le 720°$ $\boxed{M1}$

$\therefore\ 2A = 108.435°,\ 288.435°,\ 468.435°,\ 648.435°$ $\boxed{M1}$ dependent on range stated

$A = 54.2°,\ 144.2°,\ 234.2°$ or $324.2°$ $\boxed{A1}$ all

(ii) no of solutions $= 4 \times 3$ or 6×2

$= 12$ $\boxed{B1}$

10 A particle moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by $v = 12e^{-0.4t} - 1$. Find

(i) an expression for the acceleration of the particle in terms of t, [2]

(ii) the distance travelled by the particle before it comes to instantaneous rest. [5]

(i) $a = 12e^{-0.4t}(-0.4)$ $\boxed{M1}$

$= -4.8e^{-0.4t}$ $\boxed{A1}$

(ii) when $v = 0$

$12e^{-0.4t} - 1 = 0$ $\boxed{M1}$

$\dfrac{12}{e^{0.4t}} = 1$

$12 = e^{0.4t}$

$0.4t = \ln 12$

$t = 6.2123$ $\boxed{A1}$

$\therefore$ distance travelled

$= \int_0^{6.2123} (12e^{-0.4t} - 1)\,dt$ $\boxed{M1}$

$= \left[\dfrac{12e^{-0.4t}}{-0.4} - t\right]_0^{6.2123}$ $\boxed{M1}$

$= \left[-30e^{-0.4t} - t\right]_0^{6.2123}$

$= \left[-8.7123 - (-30)\right]$

$= 21.3$ m // $\boxed{A1}$

OR Alt method:

$s = \int v\,dt$

$= \int 12e^{-0.4t} - 1\,dt$

$= \dfrac{12e^{-0.4t}}{-0.4} - t + c$ $\boxed{M1}$

$= -30e^{-0.4t} - t + c$

when $t = 0, s = 0$

$\therefore c = 30$ $\boxed{M1}$ value of c

$\therefore s = -30e^{-0.4t} - t + 30$

Distance travelled

$= -30e^{-0.4(6.2123)} - 6.2123 + 30$

$= 21.3$ m $\boxed{A1}$

11 Variables x and y are connected by the equation $y = cx^2 + dx$, where c and d are constants.

(i) Express the given equation in a form suitable for drawing a straight line graph of $\frac{y}{x}$ against x. [1]

(ii) Given that the straight line passes through points $(-1, -3)$ and $(-3, -21)$, find the value of c and of d. [3]

(iii) Hence, find the coordinates of the point on the straight line at which $2y = x^2$. [3]

(i) $\frac{y}{x} = cx + d$ $\boxed{B1}$

(ii) $c =$ gradient

$= \frac{-21+3}{-3+1} = \frac{-18}{-2}$

$= 9$ $\boxed{B1}$

At $(-1, -3)$,

$-3 = -9 + d$ $\boxed{M1}$

$d = 6$ $\boxed{A1}$

(iii) $\frac{y}{x} = 9x + 6$ — (1)

$2y = x^2$

$\frac{y}{x} = \frac{x}{2}$ — (2)

(1) = (2)

$\frac{x}{2} = 9x + 6$ $\boxed{M1}$

$x = 18x + 12$

$17x = -12$

$x = \frac{-12}{17}$ $\boxed{A1}$ value of one coordin.

$\therefore \frac{y}{x} = \frac{-12}{17} \div 2$

$= \frac{-6}{17}$

$\therefore$ point is $\left(\frac{-12}{17}, \frac{-6}{17}\right)$ $\boxed{A1}$

12 The equation of a curve is $y=\dfrac{8a^3}{x^2+4a^2}$, where a is a constant.

(i) Show that $\dfrac{dy}{dx}=\dfrac{-16a^3x}{\left(x^2+4a^2\right)^2}$. [2]

$y = 8a^3(x^2+4a^2)^{-1}$

$\dfrac{dy}{dx} = 8a^3(-1)(x^2+4a^2)^{-2}(2x)$ $\boxed{M1}$

$= \dfrac{-16a^3x}{(x^2+4a^2)^2}$ // $\boxed{A1}$

A particle moves along the curve. At the point P (2, –1), the y-coordinate of the particle is decreasing at a rate of 0.3 units/sec.

(ii) Find the value of a. [5]

(iii) Find the corresponding rate of change of the x-coordinate of the particle at the point P. [2]

[Turn over

(ii) Given $\dfrac{dy}{dt} = -0.3$

At P, $-1 = \dfrac{8a^3}{(4+4a^2)}$ $\boxed{M1}$

$-4-4a^2 = 8a^3$

$2a^3 + a^2 + 1 = 0$ $\boxed{M1}$

Let $f(a) = 2a^3 + a^2 + 1$

$f(-1) = 0$

$\therefore$ By factor theorem,

$a+1$ is a factor of $f(a)$ $\boxed{M1}$

$(a+1)(2a^2 + Ba + 1) = 2a^3 + a^2 + 1$

By comparing coefficient of a:

$1 + B = 0$

$B = -1$

$\therefore f(a) = (a+1)(2a^2-a+1)$ $\boxed{M1}$

when $f(a)=0$

$a=-1$ or $2a^2-a+1=0$ $\boxed{A1}$

(no solution $\because$ $b^2-4ac<0$)

(iii) $\therefore y = \dfrac{-8}{x^2+4}$

$\dfrac{dy}{dx} = \dfrac{16x}{(x^2+4)^2}$

$\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$

At P,

$-0.3 = 0.5 \times \dfrac{dx}{dt}$ $\boxed{M1}$

$\dfrac{dx}{dt} = -0.6$ unit/s $\boxed{A1}$

13 Scientists use sinusoidal functions to model populations of predators and prey in the environment. In a particular study which stretched over a period of 4 months, the population of mousedeer is modeled by the equation $P=-300\cos\left(\frac{\pi}{2}t\right)+500$. The population of leopards in the same area is modeled by the equation $P=50\cos\left(\frac{\pi}{2}t\right)+100$. In each formula, P is the population of the animal and t is the time in months .

(i) On the same axes, sketch the graphs of $P=-300\cos\left(\frac{\pi}{2}t\right)+500$ and $P=50\cos\left(\frac{\pi}{2}t\right)+100$ for $0\le t\le 4$. [5]

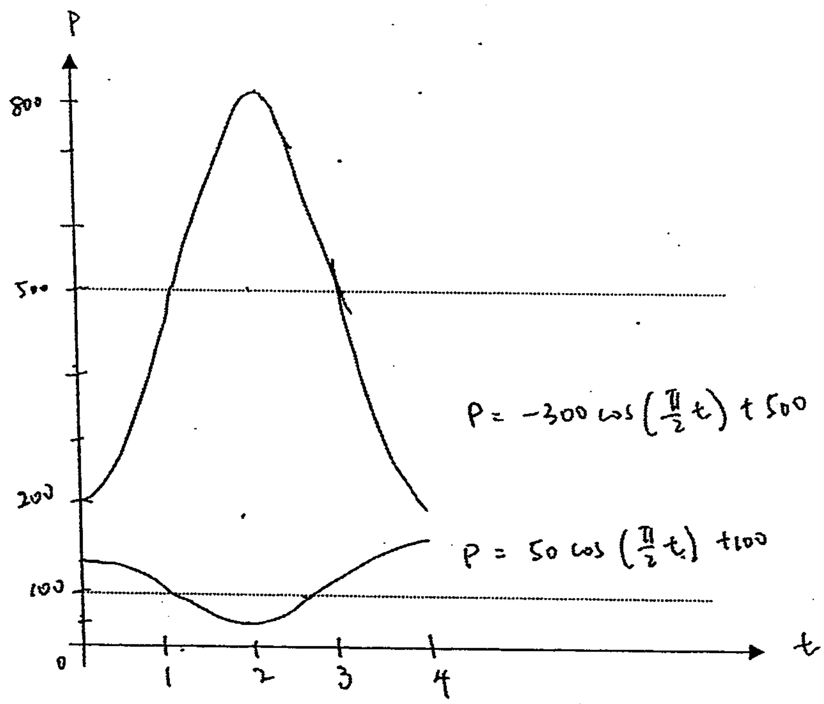
Mousedeer
$\left\{\begin{array}{l}\text{max : } 800\\ \text{min : } 200\end{array}\right.$ $\boxed{B1}$

Leopard
$\left\{\begin{array}{l}\text{max : } 150\\ \text{min : } 50\end{array}\right.$ $\boxed{B1}$

1 cycle each $\boxed{B1}$

axes of curves $\boxed{B1}$

shape is inversion of each other $\boxed{B1}$



(ii) Using the graphs of these two equations, state the relationship between the population of mousedeer and the population of leopards in the area. [1]

when the population of mousedeer is at its maximum, the number of leopards is at its minimum, and vice versa. $\boxed{B1}$ only if description matches (i)

OR

when population of leopards dropped, the population of mousedeer thrived

Leopards in the area are classified as vulnerable when their population is 80 or less.

(iii) Find the length of time for which the leopards became vulnerable during the study. [3]

$p = 50\cos\left(\frac{\pi}{2}t\right) + 100$

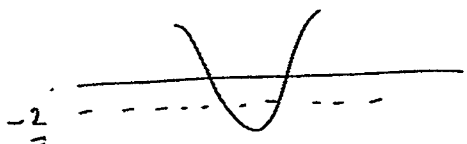
$50\cos\left(\frac{\pi}{2}t\right) + 100 \leq 80$

$\cos\left(\frac{\pi}{2}t\right) \leq -\frac{2}{5}$ $\boxed{M1}$

Basic α = 1.1593

$0 \leq t \leq 4$

$0 \leq \frac{\pi}{2}t \leq 2\pi$



$\therefore \frac{\pi}{2}t = 1.9823$ or 4.3008 $\boxed{M1}$

$t = 1.2619$ or 2.7379

END OF PAPER

$\therefore$ Length of time

$= 2.7379 - 1.2619$

$= 1.48$ months $\boxed{A1}$

Name:	Register No:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONALMATHEMATICS4047/02

Paper 2 **21August 2014**

Additional materials: Answer Paper
Mark Sheet

2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

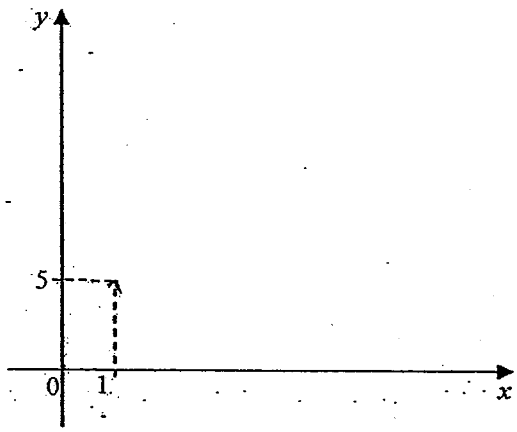
At the end of the examination, fasten all your work and mark sheetsecurely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

1 The curves $y = m\sqrt[3]{x}$ and $y = \dfrac{2m}{kx^2}$ meet at the point (1, 5), where m and k are constants.

(i) Find the value of m and of k. [2]

(ii) Sketch the two curves on the same axes, for $x > 0$. [3]

(iii) The normal to the curve $y = m\sqrt[3]{x}$ at (1, 5) meet the x-axis at P. Find the coordinates of P. [3]



2 (i) Given that coefficient of x^{-3} in the expansion of $\left(ax^2 - \dfrac{1}{x}\right)^{12}$ is -1760, where a is a constant, find the value of a. [4]

AD

(ii) Using the value of a found in part (i), find the coefficient of x^{-3} in the expansion of

$\left(1+5x^3\right)\left(ax^2-\dfrac{1}{x}\right)^{12}$. [4]

3 Given the graph $f(x)=hx^3+\dfrac{k}{x^2}$ for $x>0$, has a gradient function of $f'(x)=3x^2-\dfrac{96}{x^3}$, where h and k are constants.

(i) Find the value of h and of k. [3]

(ii) Showing clear working, determine whether $f(x)$ is an increasing or decreasing function for $0<x<2$. [3]

(iii) Determine the nature of the turning point of the graph. [3]

4 (i) Show that $\frac{d}{dx}\left(\cos^3 3x\right) = -9\cos^2 3x \sin 3x$. [2]

Hence evaluate each of the following.

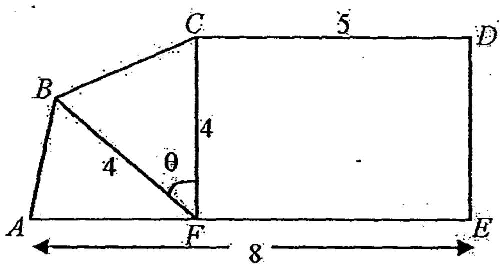
(ii) $\int_0^{\frac{\pi}{6}} \cos^2 3x \sin 3x \, dx$. [3]

(iii) $\int_0^{\frac{\pi}{6}} \sin^3 3x \, dx$. [4]

5 (i) Express $\frac{2x^2-3x+1}{9x^3-6x^2+x}$ as partial fractions. [6]

(ii) Hence find $\int \frac{2x^2-3x+1}{9x^3-6x^2+x}\,dx$. [3]

6



In the diagram, AFE is a straight line, $BF = CF = 4$ m, $CD = 5$ m and $AE = 8$ m. Given also that $CDEF$ is a rectangle and $\angle BFC = \theta$. The area of the pentagon $ABCDE$ is T m^2.

(i) Show that T can be expressed as $a + b\sin\theta + c\cos\theta$, where a, b and c are constants to be found. [3]

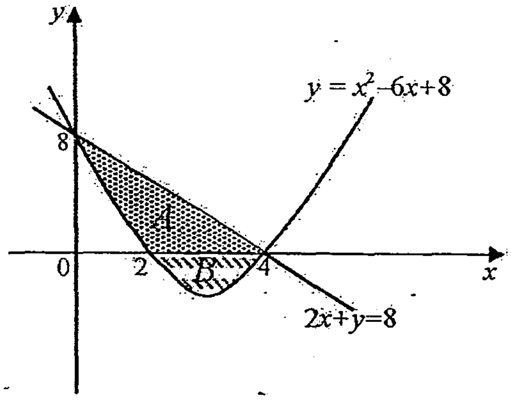
(ii) Express T in the form $a + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Find the maximum value of T and the corresponding value of θ. [2]

(iv) Explain whether T can be equal to 20 m^2. [2]

7 (a) Find the area bounded by the curve $y = \ln(x-1)$, the lines $y = -2$, $y = 2$ and the y-axis. [4]

(b) The diagram shows part of the curve $y = x^2 - 6x + 8$ and the line $2x + y = 8$. Find the ratio of shaded region A to the shaded region B. [7]



8 It is given that $\cos 2\theta = a + b$ and $\sin 2\theta = a - b$.

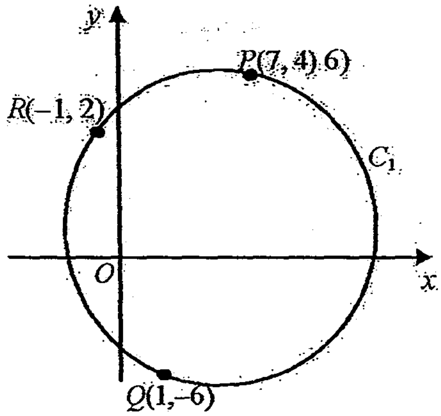
(i) Show that $2(a^2 - b^2) = \sin 4\theta$. [2]

(ii) Find the value of $a^2 + b^2$. [4]

(iii) Deduce that the value of $\tan 2\theta = -\frac{4}{3}$ if $\frac{a}{b} = -\frac{1}{7}$. [3]

(iv) Hence, without the use of a calculator, find the possible values of $\tan\theta$. [3]

9



In the diagram, which is not drawn to scale, P, Q and R are points on the circle C_1.

(i) Show that PQ is the diameter of the circle C_1 and hence find the centre of C_1. [5]

(ii) Find the equation of C_1 in the form $x^2+y^2+px+qy+r=0$, where p, q and r areintegers. [3]

(iii) Given that C_2 is a reflection of the circle C_1 in the line $x=-1$, find the centre of C_2. [2]

(iv) Detemine whether Q lies inside or outside the circle C_2. [2]

[Turn over

10 (a) Solve the following equations.

(i) $\log_4 x - \log_4 0.5 = \log_{16}(7x-3)$. [4]

(ii) $\log_a e^{2x} + \log_a 2 = \log_a\left(15 - e^x\right)$. [4]

(b) The population, P, of a certain type of bacteria, given a favourable growth medium, can be modelled by the equation $P = Ae^{kt}$, where A and k are constants and t is the time in hours. It is known that the population, P, doubles every 6.5 hours andthere were approximately 1000 bacteria at the start, how many bacteria will there be in a day and a half? Leave your answer correct to 4 significant figures? [4]

Name:	Register No:	Class:

3IRLS' SCHOOL
(FOUR
IY EXAMINATION

Marking Scheme

ADDITIONALMATHEMATICS4047/02

Paper 2 **21August 2014**

Additional materials: Answer Paper
Mark Sheet

2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheetsecurely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

[Turn over

This document consists of 6printed pages.

1 The curves $y = m\sqrt[3]{x}$ and $y = \dfrac{2m}{kx^2}$ meet at the point (1, 5), where m and $k$are constants.

(i) Find the value of m and of k. [2]

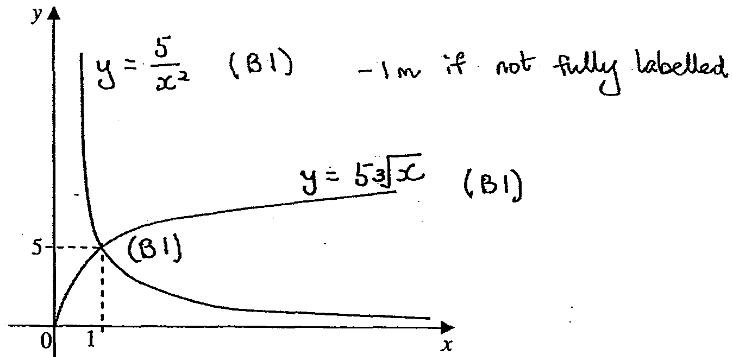
(ii) Sketch the two curves on the same axes, for $x > 0$. [3]

(iii) The normal to the curve $y = m\sqrt[3]{x}$ at (1, 5) meet the x-axis at P.Find the coordinates of P. [3]

(i) sub (1, 5) into $y = m\sqrt[3]{x}$, $\underline{m = 5}$ (B1)

sub (1, 5) into $y = \dfrac{2m}{kx^2}$, $k = \dfrac{2(5)}{(5)(1)^2} = \underline{2}$ (B1)

(ii)



(iii) $y = 5x^{\frac{1}{3}}$

$\dfrac{dy}{dx} = \dfrac{5}{3}x^{-\frac{2}{3}}$ (B1)

At $x = 1$, $\dfrac{dy}{dx} = \dfrac{5}{3}$

$\therefore$ grad of normal $= -\dfrac{3}{5}$ (M1)

eqn' of normal is $y - 5 = -\dfrac{3}{5}(x - 1)$

when $y = 0$, $x = \dfrac{28}{5}$ $\therefore$ coord of $P = \underline{\left(\dfrac{28}{5}, 0\right)}$ (A1)

2 (i) Given that coefficient of x^{-3} in the expansion of $\left(ax^2 - \dfrac{1}{x}\right)^{12}$ is -1760, where a is a constant, find the value of a. [4]

$T_{r+1} = \binom{12}{r}(ax^2)^{12-r}\left(-\dfrac{1}{x}\right)^r$ (M1)

$= \binom{12}{r}a^{12-r}(-1)^r x^{24-3r}$

$\therefore 24 - 3r = -3 \Rightarrow r = 9$ (A1)

$\therefore$ coeff of $x^{-3} = \binom{12}{9}a^3(-1)^9$

$a^3 = \dfrac{-1760}{-220} \Rightarrow \underline{a = 2}$ (M1, A1)

(ii) Using the value of a found in part (i), find the coefficient of x^{-3} in the expansion of

$(1+5x^3)\left(ax^2-\frac{1}{x}\right)^{12}$. [4]

When $24-3r=-6 \Rightarrow r=10$ (M1)

$\therefore T_{11} = \binom{12}{10}(2)^{12-10}(-1)^{10}x^{-6}$

$= 264x^{-6}$ (A1)

hence $(1+5x^3)(\ldots -1760x^{-3}+264x^{-6}+\ldots)$

coeff of $x^{-3} = -1760+5(264)$ (M1)

$= -440$ (A1)

3 Given the graph $f(x)=hx^3+\frac{k}{x^2}$ for $x>0$, has a gradient function of $f'(x)=3x^2-\frac{96}{x^3}$, where h and k are constants.

(i) Find the value of h and of k. [3]

(ii) Showing clear working, determine whether $f(x)$ is an increasing or decreasing function for $0<x<2$. [3]

(iii) Determine the nature of the turning point of the graph. [3]

(i) $f'(x) = 3hx^2 - \frac{2k}{x^3}$ (M1)

$\therefore h=1$, $k=48$ (A1, A1)

(ii) $f'(x) = \frac{3x^5-96}{x^3}$

$= \frac{3(x^5-32)}{x^3}$ (M1)

For $0<x<2$, $x^3>0$, $x^5-32<0$ (M1)

$\therefore f'(x)<0$ ie. $f(x)$ is a decreasing fn for $0<x<2$ (A1)

(iii) For turning pt $f'(x)=0$

$3x^5-96=0$ (M1)

$x=2$

$f''(x) = 6x + \frac{288}{x^4}$ (M1)

$f''(2)>0$

$\therefore$ turning pt. is a min pt. (A1)

4 (i) Show that $\frac{d}{dx}\left(\cos^3 3x\right) = -9\cos^2 3x \sin 3x$. [2]

Hence evaluate each of the following.

(ii) $\int_0^{\frac{\pi}{6}} \cos^2 3x \sin 3x \, dx$. [3]

(iii) $\int_0^{\frac{\pi}{6}} \sin^3 3x \, dx$. [4]

(i) $\frac{d}{dx}(\cos 3x)^3 = 3(\cos 3x)^2 (-3 \sin 3x)$ (B1) (B1)

$= -9 \cos^2 3x \sin 3x$

(ii) $-9\int_0^{\pi/6} (\cos^2 3x \sin 3x)\, dx = \left[\cos^3 3x\right]_0^{\frac{\pi}{6}}$ (M1)

$= 0^3 - 1^3$ (M1)

$\therefore \int_0^{\frac{\pi}{6}} (\cos^2 3x \sin 3x)\, dx = \frac{1}{9}$ (A1)

(iii) $\cos^2 3x \sin 3x = (1 - \sin^2 3x)(\sin 3x)$ (M1)

$= \sin 3x - \sin^3 3x$

$\therefore \int_0^{\frac{\pi}{6}} (\sin 3x - \sin^3 3x)\, dx = \frac{1}{9}$

$\int_0^{\frac{\pi}{6}} \sin 3x \, dx - \int_0^{\frac{\pi}{6}} \sin^3 3x \, dx = \frac{1}{9}$ (M1)

$\therefore \int_0^{\frac{\pi}{6}} \sin^3 3x \, dx = \left[-\frac{\cos 3x}{3}\right]_0^{\frac{\pi}{6}} - \frac{1}{9}$

$= \frac{1}{3} - \frac{1}{9}$ (M1)

$= \frac{2}{9}$ (A1)

5 (i) Express $\frac{2x^2 - 3x + 1}{9x^3 - 6x^2 + x}$ as partial fractions. [6]

(i) $9x^3 - 6x^2 + x = x(9x^2 - 6x + 1)$

$= x(3x-1)^2$ (B1)

$\therefore \frac{2x^2 - 3x + 1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$ (M1, M1)

$\therefore 2x^2 - 3x + 1 = A(3x-1)^2 + Bx(3x-1) + Cx$

When $x = 0$, $A = 1$ (M1, A 2, 1, 0)

When $x = \frac{1}{3}$, $C = \frac{2}{3}$

When $x = 1$, $B = -\frac{7}{3}$ $\therefore \frac{2x^2 - 3x + 1}{x(3x-1)^2} = \frac{1}{x} - \frac{7}{3(3x-1)} + \frac{2}{3(3x-1)}$

(-1M if final ans not shown)

(ii) Hence find $\int \frac{2x^2-3x+1}{9x^3-6x^2+x}dx$. [3]

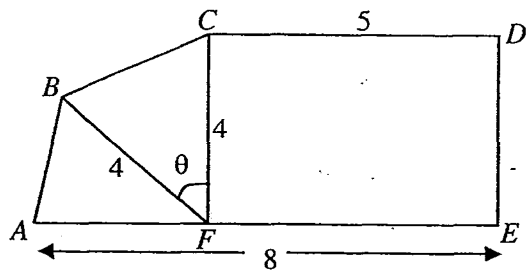
(ii) $\int \frac{2x^2-3x+1}{9x^3-6x^2+x}dx = \int \frac{1}{x} - \frac{7}{3(3x-1)} + \frac{2}{3(3x-1)^2}\,dx$

$= \ln x - \frac{7}{3} \times \frac{\ln(3x-1)}{3} + \frac{2}{3} \times \frac{(3x-1)^{-1}}{3(-1)} + c$

$= \ln x - \frac{7}{9}\ln(3x-1) - \frac{2}{9(3x-1)} + c$

(B1) (B1) (B1)

6



In the diagram, AFE is a straight line, $BF = CF = 4$ m, $CD = 5$ m and $AE = 8$ m. Given also that $CDEF$ is a rectangle and $\angle BFC = \theta$. The area of the pentagon $ABCDE$ is T m^2.

(i) Show that T can be expressed as $a + b\sin\theta + c\cos\theta$, where a, b and c are constants to be found. [3]

(ii) Express T in the form $a + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Find the maximum value of T and the corresponding value of θ. [2]

(iv) Explain whether T can be equal to 20 m^2. [2]

(i) $T = \frac{1}{2}AF.BF\sin(90° - \theta) + \frac{1}{2}BF.CF\sin\theta + (CF.CD)$ (M1)

$= \frac{1}{2}(8-5)(4)\cos\theta + \frac{1}{2}(4)(4)\sin\theta + (4 \times 5)$

(M1)

$= 6\cos\theta + 8\sin\theta + 20$ (A1)

(ii) $T = 20 + R\cos(\theta - \alpha)$

where $R = \sqrt{6^2 + 8^2} = 10$

$\tan\alpha = \frac{8}{6}$

$\alpha = 53.130°$

(M1, A1, A1)

$\therefore T = 20 + 10\cos(\theta - 53.1°)$ (-1 if final ans not shown)

(iii) max $T = 10 + 20 = 30$ (A1)

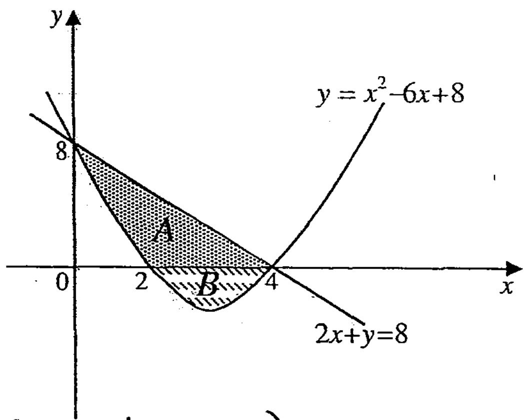
when $\cos(\theta - 53.130°) = 1 \Rightarrow \theta \approx 53.1°$ (A1)

(iv) when $T = 20$ $10\cos(\theta - 53.130°) = 0 \Rightarrow \theta = 143.130°$ (NA)

$\therefore T \neq 20$ (M1, A1)

7 **(a)** Find the area bounded by the curve $y = \ln(x-1)$, the lines $y=-2$, $y=2$ and the y-axis.[4]

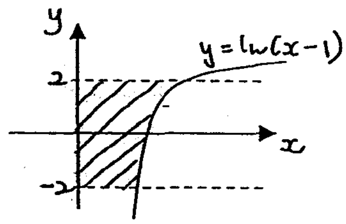
(b) The diagram shows part of the curve $y = x^2 - 6x + 8$ and the line $2x + y = 8$. Find the ratio of shaded region A to the shaded region B. [7]



7 a) $y = \ln(x-1)$

$x = e^y + 1$ (M1)

$\therefore$ Area $= \int_{-2}^{2} e^y + 1 \, dy$ (M1)



$= \left[e^y + y\right]_{-2}^{2}$ (M1)

$= (e^2 + 2) - (e^{-2} - 2)$

$= 4 + e^2 - \frac{1}{e^2}$ or 11.3 unit^2 (A1)

7b) Area of A $= \frac{1}{2}(4)(8) - \int_0^2 (x^2 - 6x + 8)\,dx$ (M1)

$= 16 - \left[\frac{x^3}{3} - 3x^2 + 8x\right]_0^2$ (M1) – integration

$= \frac{28}{3}$ unit^2 (A1)

Area of B $= \left|\int_2^4 (x^2 - 6x + 8)\,dx\right|$ (M1)

$= \left|\left[\frac{x^3}{3} - 3x^2 + 8x\right]_2^4\right|$ (M1)

$= \frac{4}{3}$ unit^2 (A1)

$\therefore$ Ratio of A to ratio of B $= 7:1$ (A1)

8 It is given that $\cos 2\theta = a+b$ and $\sin 2\theta = a-b$.

(i) Show that $2(a^2-b^2)=\sin 4\theta$. [2]

(ii) Find the value of a^2+b^2. [4]

(iii) Deduce that the value of $\tan 2\theta = -\frac{4}{3}$ if $\frac{a}{b}=-\frac{1}{7}$. [3]

(iv) Hence, without the use of a calculator, find the possible values of $\tan\theta$. [3]

8 i) $2(a^2-b^2)$

$= 2(a+b)(a-b)$ (M1)

$= 2\sin 2\theta \cos 2\theta$

$= \sin 4\theta$ (B1)

ii) $(a+b)^2 = (\cos 2\theta)^2$

$a^2+b^2+2ab = \cos^2 2\theta$ .... (1) (M1)

$(a-b)^2 = (\sin 2\theta)^2$

$a^2+b^2-2ab = \sin^2 2\theta$ .... (2) (M1)

(1)+(2) $2(a^2+b^2) = \sin^2 2\theta + \cos^2 2\theta$ (M1)

$\therefore a^2+b^2 = \frac{1}{2}$ (A1)

(iii) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

$= \frac{a-b}{a+b}$ (M1)

$= \frac{\frac{a}{b}-1}{\frac{a}{b}+1}$ (M1)

$= \frac{-\frac{1}{7}-1}{-\frac{1}{7}+1}$ (A1)

$= -\frac{4}{3}$

(iv) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

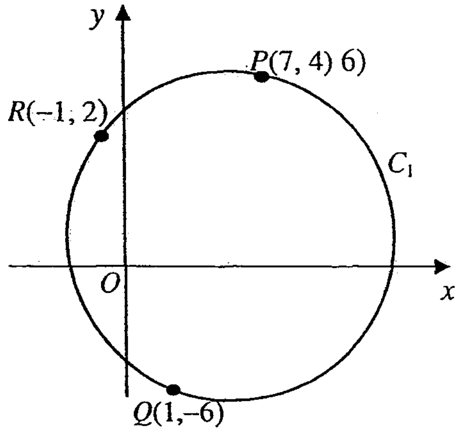
$\therefore \frac{2\tan\theta}{1-\tan^2\theta} = -\frac{4}{3}$ (M1)

$4\tan^2\theta - 6\tan\theta - 4 = 0$

$2(2\tan\theta+1)(\tan\theta-2)=0$ (M1)

$\therefore \tan\theta = -\frac{1}{2}$ or 2 (A1)

9



In the diagram, which is not drawn to scale, P, Q and R are points on the circle C_1.

(i) Show that PQ is the diameter of the circle C_1 and hence find the centre of C_1. [5]

(ii) Find the equation of C_1 in the form $x^2+y^2+px+qy+r=0$, where p, q and r areintegers. [3]

(iii) Given that C_2 is a reflection of the circle C_1 in the line $x=-1$, find the centre of C_2. [2]

(iv) Determine whether Q lies inside or outside the circle C_2. [2]

(i) $\text{grad}_{RQ} = \dfrac{2-(-6)}{-1-1} = -4$ (B1)

$\text{grad}_{RP} = \dfrac{4-2}{7-(-1)} = \dfrac{1}{4}$ (B1)

since $\text{grad}_{RQ} \times \text{grad}_{RP} = -4 \times \dfrac{1}{4} = -1$ (M1)

$\therefore RQ \perp RP$

$\therefore \angle QRP = 90°$ ($\angle$ in semi-circle) (M1)

$\therefore PQ$ is the diameter of C_1

$\therefore$ Centre of $C_1 = \left(\dfrac{1+7}{2}, \dfrac{-6+4}{2}\right) = (4, -1)$ (A1)

(ii) radius of $C_1 = \sqrt{(4-1)^2+(-1-(-6))^2} = \sqrt{34}$ (M1)

eqn of C_1: $(x-4)^2 + (y+1)^2 = (\sqrt{34})^2$ (M1)

$x^2+y^2-8x+2y-17=0$ (A1)

(iii) Centre of $C_2 = (-6, -1)$, radius $= \sqrt{34}$ (M1)

$\therefore$ eqn of C_2: $(x+6)^2 + (y+1)^2 = 34$ (A1)

(iv) dist of Q fr centre of $C_2 = \sqrt{(1+6)^2+(-6+1)^2} = \sqrt{74}$ (M1)

Since $\sqrt{74} > \sqrt{34}$, Q lies outside C_2 (A1)

[Turn over

10 **(a)** Solve the following equations.

(i) $\log_4 x - \log_4 0.5 = \log_{16}(7x-3)$. [4]

(ii) $\log_a e^{2x} + \log_a 2 = \log_a(15-e^x)$. [4]

(b) The population, P,of a certain type of bacteria, given a favourable growth medium, can be modelled by the equation $P = Ae^{kt}$, where A and k are constants and t is the time in hours.It is known that the population, P,doubles every 6.5 hours andthere were approximately 1000 bacteria at the start, how many bacteria will there be in a day and a half?Leave your answer correct to 4 significant figures? [4]

10 a) i)

$\log_4\left(\frac{x}{\frac{1}{2}}\right) = \frac{\log_4(7x-3)}{\log_4 4^2}$ (M1)

$\log_4(2x) = \frac{1}{2}\log_4(7x-3)$ (M1)

$\log_4(4x^2) = \log_4(7x-3)$

$\therefore 4x^2 = 7x-3$

$4x^2 - 7x + 3 = 0$ (M1)

$(4x-3)(x-1) = 0$

$\therefore x = 1$ or $\frac{3}{4}$ (A1)

ii) $\log_a(2e^{2x}) = \log_a(15-e^x)$ (M1)

$\therefore 2e^{2x} + e^x - 15 = 0$ (M1)

$(2e^x - 5)(e^x + 3) = 0$

$\therefore e^x = \frac{5}{2}$ or $e^x = -3$ (rej) (A1)

$\therefore x = \ln\left(\frac{5}{2}\right)$ or 0.916 (A1)

10b) $P = Ae^{kt}$

When $t = 0$, $P = 1000$

$\therefore A = 1000$ (B1)

When $t = 6.5$, $P = 2A$,

$\therefore 2A = Ae^{k(6.5)}$ (M1)

$\therefore 6.5k = \ln 2$

$k = \frac{\ln 2}{6.5}$ (A1)

When $t = 36$, $P = 1000e^{\left(\frac{\ln 2}{6.5}\right)(36)} \approx 46480$ bacteria (A1)