NAME: ______________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

Comment [CLYA1]: Insert header on every page for students to write their name, register no. and class

Comment [CLYA2]: Use Arial 12 pt font as this is similar to the font used by UCLES. Where appropriate, follow the O-Level or N-Level format.

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS   4047/01

Comment [CLYA3]: Insert paper no. only for Sec 4/5 papers to familiarise students with O-Level format

Paper 1

Date: 26 August 2014   **Duration: 2 hours**

Comment [SZ4]: Insert Date of Exam Paper

Comment [CLYA5]: Just indicate duration of the paper, not specific time

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use anHB pencil for any diagrams or graphs.
Do not use staples, paper clips,glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

At the end of the examination, fasten all your work securely together.

Comment [SZ6]: Insert instructions by following specific Subject requirement. Font: Arial Size 12

For Examiner's Use	
Paper 1	/ 80

Setter: MdmToh SL and Miss Lee CP

This question paper consists of 7 printed pagesincluding the cover page.

Name: ______________________________ (   )   Class: _________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

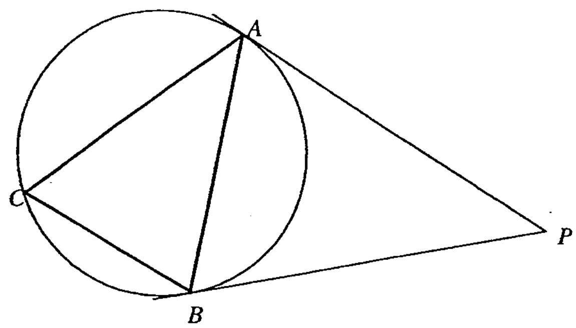
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Name: ______________________________ (   )   Class: __________

1 Given that the line $y = \frac{1}{2}x + 3$ is a tangent to the curve $y^2 = kx$, where k is a positive constant, prove that k is a multiple of 3. [4]

2



The diagram shows point A, B and C lying on a circle. The point P is such that the lines PA and PB are tangents to the circle. Given that $AB = AC$, show that $\angle APB = \angle BAC$. [4]

3 It is given that $3^{x+1} \times 2^{2x+1} = 2^{x+2}$.

(i) Find the exact value of 6^x. [3]

(ii) Hence find the value of x correct to 2 decimal places. [2]

4 Find the coordinates of the stationary point of the curve $y = 4x^2 - \frac{1}{x}, x \neq 0$ and determine the nature of this stationary point. [5]

5 It is given that $\int_1^3 px^2 dx = 7$, where p is a constant.

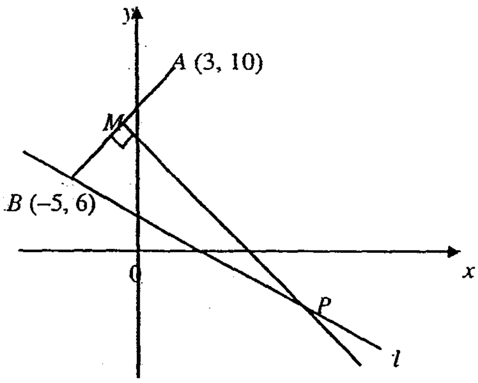
(i) Find the value of $\int_3^1 px^2 dx$. [1]

(ii) Find the value of $\int_3^0 px^2 dx$. [3]

(iii) Express $\int_1^3 (px^2 + q)\, dx$ in terms of the constant q. [2]

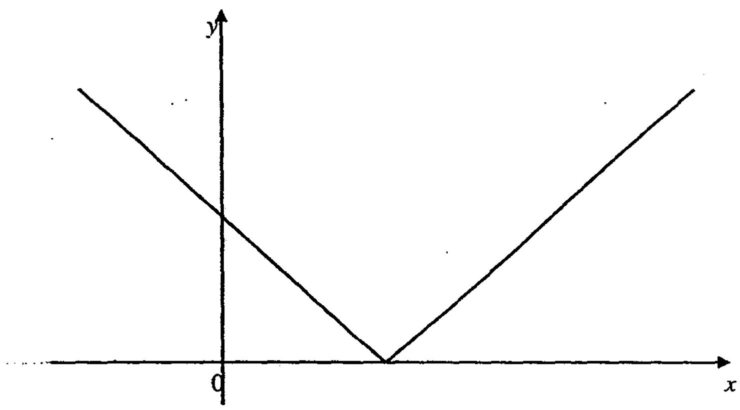
Name: ______________________________ (   )   Class: __________

6



In the diagram, M is the midpoint of the line joining the points $A(3, 10)$ and $B(-5, 6)$. The perpendicular bisector of AB intersects the line l at the point P. Given that line l is parallel to the line $6y + 7x = 0$, find the coordinates of P. [6]

7



The diagram above shows part of the graph of $y = |3 - x|$. In each of the following cases, determine the number of intersections of the line $y = mx + c$ with $y = |3 - x|$, justify your answer.

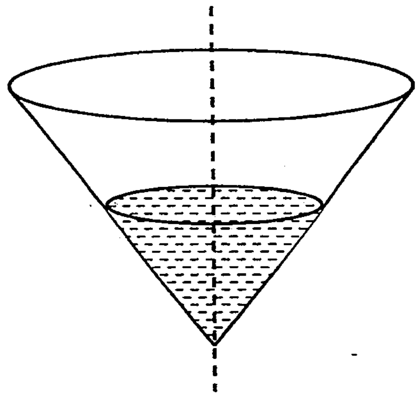
(i) $m = 1$ and $c > 1$ [2]

(ii) $m = \frac{1}{3}$ and $c = 0$ [2]

(iii) $m = -\frac{1}{3}$ and $c < 1$ [2]

Name: ______________________________ (   )   Class: _________

8



A container is in the shape of an inverted right circular cone which has a vertical axis and a base radius that is equal to its height. Water is poured into the vessel at a constant rate of 50 cm^3/s. The depth of the water is x cm.

(i) Show that the volume of water in the container is $V = \frac{1}{3}\pi\ x^3 cm^3$. [1]

Calculate, at the instant when depth of water is 20 cm, the rate of increase of

(ii) the depth of the water, in terms of π, [3]

(iii) the area of the horizontal surface of the water. [3]

9 The roots of the quadratic equation $2x^2 - 4x + 5 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha + \beta)$ and $\alpha\beta$. [1]

(ii) Find a quadratic equation whose roots are α^3 and β^3. [6]

10 (i) Express the equation $3\tan^2\theta = 5 - 2\sec\theta$ as a quadratic equation in terms of $\sec\theta$. [2]

(ii) Hence solve the equation $3\tan^2\theta = 5 - 2\sec\theta$ for $0 \le \theta \le 360°$. [4]

(iii) State the number of solutions of the equation $3\tan^2\theta = 5 - 2\sec\theta$ in the range $-540° \le \theta \le 540°$. [1]

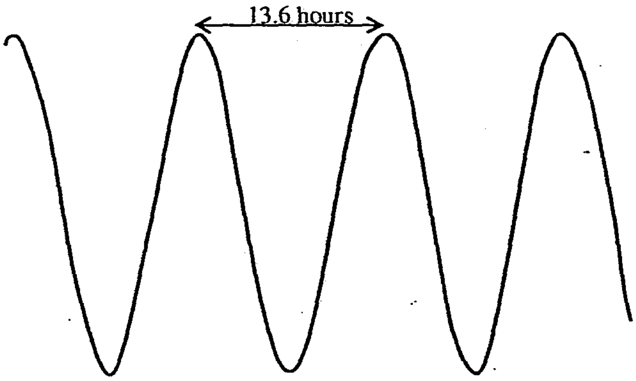
Name: ______________________________ (   )   Class: ________

11 The height of the tides in Singapore is modelled by the equation $h = 1.55 + 1.35\cos kt$, where k is a constant, and t is the time in hours after midnight. The diagram below shows the model of the height of the tides. The average time difference between high tides is 13.6 hours.

(i) Explain why this model suggest that highest tide for the day is 2.9 m. [1]

(ii) Show that the value of k is $\frac{5\pi}{34}$. [2]

(iii) Find the time for which the height of the tide first reaches 1.6 m, leaving your answer in 24-hour notation. [4]



12 A particle travels in a straight line, so that t seconds after passing through a fixed point O, its acceleration a m/s^2, is given by $a = 6t - 30$. The initial velocity of the particle is 72 m/s. Find

(i) the deceleration of the particle when $t = 3$, [1]

(ii) an expression for the velocity of the particle in terms of t, [2]

(iii) the speed of the particle when $t = 5$, [1]

(iv) an expression for the displacement of the particle in terms of t, [1]

(v) the total distance travelled by the particle for the first 6 seconds. [3]

Name: ________________________________ (   )   Class: __________

13 **(a)** The table shows experimental values of two variables x and y.

x	1	2	3	4	5
y	0.5	0.5	1.5	3.5	6

It is known that x and y are related by an equation of the form $a(x+y-b)=bx^2$, where a and b are constants.

(i) Draw a straight line graph of $(x + y)$ against x^2, using the scale of 2 cm to represent 5 units on the x^2-axis and 2 cm to represent 1 unit on the $(x + y)$ axis. [3]

(ii) Use your graph to estimate the value of a and of b. [2]

(b) In order that $y = h(1+x)^k$, where h and k are unknown constants may be represented by a straight line graph, it need to be expressed in the form $Y = mX + c$, where X and Y are functions of x and/or y, and m and c are constants.

(i) Determine an expression for Y and for X. [2]

(ii) Explain how the straight line may be used to determine the value of h. [1]

-End of Paper-

Comment [CLYA7]: Insert this at the end of the paper

FMS(S) 2014 Sec Four Express Additional Mathematics Prelim Paper 1 Answers

No.		No.	
1.	$k=0$ (NA) or $k=6$ Since 6 is multiple of 3, therefore, k is a multiple of 3 (proved).	2.	$\angle PAB = \angle BCA$ (alt segment) $\angle PBA = \angle BCA$ (alt segment) Or $\angle PBA = \angle PAB$ (PA = PB, tan fr ext. pt$P$) $\angle APB = 180° - 2\angle PAB$ ($\angle$ sum of Δ) = $180° - 2\angle BCA$ $\angle BAC = 180° - 2\angle BCA$ ($AC = AB$, isos.Δ) Hence, $\angle APB = \angle BAC$ (shown)
3.	(i) $6^x = \frac{2}{3}$ (ii) -0.23	4.	Coordinates of the stationary point is $(-\frac{1}{2}, 3)$ The stationary point is a minimum point.
5.	(i) -7 (ii) 189 (iii) $7+2q$	6.	Coordinates of P is (7, –8).
7.	(i) Line parallel to R.H. arm; since c > 1 >–3 intersects L.H. arms; $\therefore$ 1 intersection. Or Draw a line which is parallel to $y = x-3$ and c > 1. (ii) Through origin; since $m<$ 1 intersects both arms; $\therefore$ 2 intersections. Or Draw a line that passes through the origin and gradient is greater than 0 but less than 1. (iii) With $m = -\frac{1}{3}$ and $c<1$; x-coordinate of intersection with x-axis is 3; $\therefore$ 0 intersections Or Draw a line with $m = -\frac{1}{3}$; intersection with y-axis at 1 and x-axis at 3 respectively.	8.	(i) Volume of water in container, V = $\frac{1}{3}\pi(x)^2(x) = \frac{1}{3}\pi\ x^3$ (shown) (ii) $\frac{1}{8\pi}$ cm/s (ii) 400 cm^2/s
9.	(i) $(\alpha+\beta)^2 - 3\alpha\beta$ (ii)New equation: $x^2+7x+\frac{125}{8}=0$	10.	(i) $3\sec^2\theta + 2\sec\theta - 8 = 0$ (ii) $\theta = 41.40°, 318.6°$ or $\theta = 120°, 240°$. (iii) 12 solutions

<table>
<tr>
<td>11.</td>
<td>(i) Maximum height when $\cos kt = 1$, height = 2.9 m
(ii) $k = \dfrac{5\pi}{34}$ (shown)
(iii) The time is 0319.</td>
<td>12.</td>
<td>(i) Deceleration = 12 m/s^2
(ii) Velocity, $v = 3t^2 - 30t + 72$
(iii) Speed is 3 m/s
(iv) $\therefore s = t^3 - 15t^2 + 72t$
(v) Total distance travelled = 112 + (112 – 108) = 116 m
Or integration method: $\int_0^4 v\,dt + \int_4^6 v\,dt = 116$ m</td>
</tr>
<tr>
<td>13.</td>
<td>(ai)
<table>
<tr><td>x^2</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr>
<tr><td>$x+y$</td><td>1.5</td><td>2.5</td><td>4.5</td><td>7.5</td><td>11</td></tr>
</table>
$(x+y) = b + \dfrac{b}{a}x^2$
(aii) $a = 2.50 \pm 0.4$, $b = 1.0 \pm 0.2$</td>
<td>13.</td>
<td>(bi) $Y = \log y$ and $X = \log(1+x)$

(bii) Find the vertical intercept which is $\log h$
$h = 10^{\text{vertical intercept}}$ or $h = e^{\text{vertical intercept}}$.</td>
</tr>
</table>

Secondary 4 Express
Preliminary Examination 2014
Additional Mathematics
Paper 1
Marking Scheme

No	Working	Description	Marks allocated
1	Sub. $y=\frac{1}{2}x+3$ into $y^2=kx$	Eliminates y completely	M1
	$(\frac{1}{2}x+3)^2=kx$		
	$\frac{1}{4}x^2+3x+9=kx$		
	$\frac{1}{4}x^2+x(3-k)+9=0$	Correct quadratic equation	A1
	Line is tangent to curve $\Rightarrow b^2-4ac=0$ $\Rightarrow(3-k)^2-36=0$ $9-6k+k^2-9=0$ $k(k-6)\ =0$ $k=0$ (NA) or $k=6$	Use of b^2-4ac	M1
	Since 6 is multiple of 3, therefore, k is a multiple of 3.(proved).	Show that $k=6$ and conclusion	A1
2	$\angle PAB=\angle BCA$ (alt segment)	Angles in alternate segment	[B1]
	$\angle PBA=\angle BCA$ (alt segment) Or $\angle PBA=\angle PAB$ ($PA=PB$, tan from ext. pt$P$) $\angle APB=180°-2\angle PAB$ ($\angle$ sum of Δ) $=180°-2\angle BCA$	Show 2 other angles equal - alternate segment or isos, $PA=PB$	[B1] [B1] reason
	$\angle BAC=180°-2\angle BCA$ ($AC=AB$, isos..Δ) Hence, $\angle APB=\angle BAC$ (shown)	Deduce correctly.	[A1]
3(i)	$3^{x+1}\times2^{2x+1}=2^{x+2}$ $3^{x+1}=2^{x+2}\div2^{2x+1}$ $3^{x+1}=2^{-x+1}$	Apply law of indices to reduce equation to 2 terms of base 2 and 3 respectively.	B1
	$3\times3^x=\frac{2}{2^x}$	Break up indices	B1
	$3^x\times2^x=\frac{2}{3}$		
	$6^x=\frac{2}{3}$		B1

No	Working	Description	Marks allocated
3(ii)	$x\lg 6=\lg\frac{2}{3}$ $x=\frac{\lg\frac{2}{3}}{\lg 6}$ $=-0.2263$ $=-0.23$		[M1] [A1]
4	$\frac{dy}{dx}=8x+\frac{1}{x^2}$ Stationary point $\Rightarrow\frac{dy}{dx}=0$ $8x+\frac{1}{x^2}=0$ $8x^3=-1$ $x=-\frac{1}{2}$ $y=3$ $\therefore$ coordinates of the stationary point is $(-\frac{1}{2},3)$ $\frac{d^2y}{dx^2}=8-\frac{2}{x^3}$ When $x=-\frac{1}{2}$, $\frac{d^2y}{dx^2}=24>0$ $\therefore$ The stationary point is a minimum point.	Differentiate $y$wrt. x When $8x+\frac{1}{x^2}=0$ Stationary point is $(-\frac{1}{2},3)$ Any valid method Conclusion is a minimum point	B1 M1 A1 M1 A1
5(i)	$\int_3^1 px^2dx=-7$		B1
5(ii)	$\int_1^3 px^2dx=7$ $\left[\frac{px^3}{3}\right]_1^3=7$ $\frac{27p-p}{3}=7$ $p=\frac{21}{26}$ $\int_3^9 px^2dx=\left[\frac{px^3}{3}\right]_3^9$ $=\frac{729p-27p}{3}$ $=\frac{702p}{3}$ $=\frac{702}{3}\times\frac{21}{26}$ $=189$	Integrate wrt. x Obtain value of p	M1 B1 A1

No	Working	Description	Marks allocated
5(iii)	Express $\int_1^3 (px^2+q)\,dx$ in terms of the constant q. $\int_1^3 (px^2+q)\,dx$ $=\left[\frac{px^3}{3}\right]_1^3+[qx]_1^3$ $or\left[\frac{px^3}{3}+qx\right]_1^3$ $=7+(3q-q)$ $=7+2q$	Integrate $\int_1^3 (px^2+q)\,dx$	M1 A1
6	$M=(-1, 8)$ Gradient $AB=\frac{1}{2}$ Gradient of $\perp$ bisector of $AB=-2$ Equation BP: $y-6=-\frac{7}{6}(x+5)$ $y=-\frac{7}{6}x+\frac{1}{6}$ Equation MP: $y-8=-2(x+1)$ $y=-2x+6$ At point P, $-2x+6=-\frac{7}{6}x+\frac{1}{6}$ $x=7$ $y=-8$ Coordinates of P is $(7, -8)$		B1 M1 B1 B1 M1 A1
7(i)	Line parallel to R.H. arm; since $c>1>-3$ intersects L.H. arms; $\therefore$ 1 intersection Or Draw a line which is parallel to $y=x-3$ and $c>1$	Must write the word, line is parallel	B1 B1
7(ii)	Through origin; since $m<1$ intersects both arms; $\therefore$ 2 intersections Or Draw a line that passes through the origin and gradient is greater than 0 but less than 1.		B1 B1
7(iii)	With $m=-\frac{1}{3}$ and $c<1$; x-coordinate of intersection with x-axis is 3; $\therefore$ 0 intersections		B1 B1

No	Working	Description
8(i)	Volume of water in container, V = $\frac{1}{3}\pi(x)^2(x)$ $=\frac{1}{3}\pi\ x^3$	Show volume of water = $\frac{1}{3}\times\pi\times(radius)^2\times(height)$
8 (ii)	Given $\frac{dV}{dt}=50cm^3/s$ $\frac{dV}{dx}=\pi x^2$ $\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}$ $\frac{dV}{dx}=\pi x^2 \Rightarrow \frac{dx}{dV}=\frac{1}{\pi x^2}$ $\frac{dx}{dt}=\frac{1}{\pi x^2}\times 50$ When $x=20$, $\frac{dx}{dt}=\frac{1}{\pi(20)^2}\times 50$ $=\frac{1}{8\pi}$ cm/s	Differentiate $\frac{dV}{dx}$ Uses chain rule correctly
8(iii)	Surface area of water, A = $\pi(x)^2=\pi x^2$ $\frac{dA}{dx}=2\pi x$ $\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt}$ $\frac{dA}{dx}=2\pi x\times\frac{1}{2\pi}$ When $x=20$, $\frac{dA}{dt}=2\pi(20)^2\times\frac{1}{2\pi}$ $=400$ cm²/s	Differentiate $\frac{dA}{dx}$ Uses chain rule correctly Follow through from 8(ii)
9(i)	$\alpha^2-\alpha\beta+\beta^2$ $=(\alpha+\beta)^2-2\alpha\beta-\alpha\beta$ $=(\alpha+\beta)^2-3\alpha\beta$	
9(ii)	$2x^2-4x+5=0$ $\alpha+\beta=2,\ \alpha\beta=\frac{5}{2}$ $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)$ $=2(2^2-3\times\frac{5}{2})$ $=-7$ $\alpha^3\beta^3=\left(\frac{5}{2}\right)^3=\frac{125}{8}$ New equation: $x^2+7x+\frac{125}{8}=0$ or $8x^2+56x+125=0$	Sum of roots; product of roots Correct use of factors of $\alpha^3+\beta^3$ Follow through (*ft*) on $\alpha+\beta=2$, $\alpha\beta=\frac{5}{2}$

No	Working	Description	Marks allocated
10(i)	$3\tan^2\theta = 5 - 2\sec\theta$ $3(\sec^2\theta - 1) = 5 - 2\sec\theta$ $3\sec^2\theta - 3 = 5 - 2\sec\theta$ $3\sec^2\theta + 2\sec\theta - 8 = 0$	Uses $\tan^2\theta = \sec^2\theta - 1$ (correct formula) Correct equation in any form	M1 A1
10(ii)	$3\sec^2\theta + 2\sec\theta - 8 = 0$ $(3\sec\theta - 4)(\sec\theta + 2) = 0$ $\sec\theta = \frac{4}{3}$ or $\sec\theta = -2$ $\cos\theta = \frac{3}{4}$ or $\cos\theta = -\frac{1}{2}$ $\theta = 41.40°$ or $318.6°$ or $\theta = 120°$ or $240°$	Attempt as solution (solving method – factorization or general solution) Use $\sec\theta = \frac{1}{\cos\theta}$ FT for 360 - 41.40° or 180 – 60	M1 M1 A1, A1
10(iii)	4 times/cycle × 3 cycles = 12 angles		B1
11(i)	Maximum height when cos kt = 1, height = 2.9 m		B1
11(ii)	Period between high tides = 13.6 hours $\frac{2\pi}{k} = 13.6$ $\frac{2\pi}{13.6} = k$ $k = \frac{5\pi}{34}$	Period of graph	M1 A1
11(iii)	When h = 1.6 $1.6 = 1.55 + 1.35\cos kt$ $\cos kt = \frac{1}{27}$ $\frac{5\pi}{34}t = 1.5338 \rightarrow t = 3.3198$ The time is 0319.		M1 A1 M1 A1
12(i)	$a = 6t - 30$ When t = 3, a = $6(3) - 30$ = -12 m/s^2 Deceleration = 12 m/s^2		B1
12(ii)	Velocity, v = $\int 6t - 30\,dt$ $= \frac{6t^2}{2} - 30t + c$ $= 3t^2 - 30t + c$ When t = 0, v = 72, $\therefore c = 72$ Velocity, $v = 3t^2 - 30t + 72$	Integrate acceleration	M1 A1
12(iii)	When t= 5, velocity, v = $3(5)^2 - 30(5) + 72$ = -3 m/s Speed is 3 m/s	FT if velocity has a + c as constant and speed is positive value instead of negative value	B1 / FT1

<table>
<tr><th>No</th><th>Working</th><th>Description</th><th>Marks allocated</th></tr>
<tr><td>12(iv)</td><td>Displacement, s = $\int 3t^2 - 30t + 72dt$
$= t^3 - 15t^2 + 72t + c$
When t=0, s = 0, $\therefore c = 0$
$\therefore s = t^3 - 15t^2 + 72t$</td><td>Exact answer for displacement</td><td>B1</td></tr>
<tr><td>12(v)</td><td>When v = 0, $3t^2 - 30t + 72 = 0$
$t^2 - 10t + 24 = 0$
$(t-6)(t-4) = 0$
t = 4 or t = 6
When $t = 4$, s = 112
When $t = 6$, s = 108
Total distance travelled = 112 + (112 – 108)
= 116 m
Or integration method: $\int_0^4 vdt + \int_4^6 vdt = 116$ m</td><td>Find the time when v = 0

Find the distance travelled when t= 4 or $t = 6$ (does not matter if the integration in (iv)) is incorrect

Answer (follow through from (iv))</td><td>M1

M1

A1 / FT1</td></tr>
<tr><td>13(a) (i)</td><td><table><tr><td>x^2</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr><tr><td>$x+y$</td><td>1.5</td><td>2.5</td><td>4.5</td><td>7.5</td><td>11</td></tr></table>
$a(x+y-b) = bx^2$
a(x+y) – ab = bx^2
$(x+y) = b + \frac{b}{a}x^2$

Refer to attached graph</td><td>Table of values

Plot the points and best fit line</td><td>B1

P1, S1</td></tr>
<tr><td>13(a)(ii)</td><td>Vertical intercept, $b = 1.0 \pm 0.2$
Gradient of line , $\frac{b}{a} = 0.40 \pm 0.2$
$a = 2.5 \pm 0.4$</td><td>Value of b = vertical intercept

Value for a</td><td>B1

B1</td></tr>
<tr><td>13(b)(i)</td><td>$y = h(1+x)^k$
Apply either natural logarithm or common logarithm
log y = log h + log $(1 + x)^k$
log y = log h + k log (1 + x)
Y = log y and X = log(1 + x)</td><td></td><td>B1, B1</td></tr>
<tr><td>13(b)(ii)</td><td>Find the vertical intercept which is log h
$h = 10^{\text{vertical intercept}}$ or $h = e^{\text{vertical intercept}}$</td><td>Value of h</td><td>B1</td></tr>
</table>

NAME: ______________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMARY EXAMINATION 2014
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS **4047/02**

Paper 2

Date: 27August 2014 **Duration: 2 hours 30 minutes**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
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Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Total	/ 100

Setters: Miss Thio Lay Hong and Mr James Quek

This question paper consists of 7 printed pagesincluding the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Over a period of 10 years from 1 January 2001 to 1 January 2011, the population, P, of a city increased exponentially from 50000 to 250000.

Given that $P = Ae^{kt}$, where A and k are constants and t is the time in years from 1 January 2001, find

(i) the value of A and of k. [3]

If the population continues to increase at the same rate,

(ii) in which year will it first exceed 1 million? [3]

2 **(a)** Given that $\log_{13} x + \log_{13} y = \log_{13}(x - y)$, express x in terms of y. [3]

(b) Given that $u = \log_3 z$, find, in terms of u,

(i) $\log_3 \frac{3}{z}$, [1]

(ii) $\log_3 27z$, [1]

(iii) $\log_z 9$. [2]

3 **(i)** Differentiate $x\cos 2x$ with respect to x. [3]

(ii) Using your answer to part **(i)**,find $\int x\sin 2x \, dx$ and hence show that $\int_{\pi}^{2\pi} x\sin 2x \, dx = -\frac{\pi}{2}$. [5]

4 (i) Write down the first three terms in the expansion, in ascendi
$(1-px)^{12}$, where p is a constant.

(ii) Find, in terms of p, the coefficient of x^2 in the
$(1+3x-5x^2)(1-px)^{12}$.

In the expansion of $(1+3x-5x^2)(1-px)^{12}$, the sum of the coefficients of
x^2 is $-4\frac{2}{3}$.

(iii) Find the values of p in its exact form.

5 (i) Sketch the graph of $y=3e^{2x+1}$.

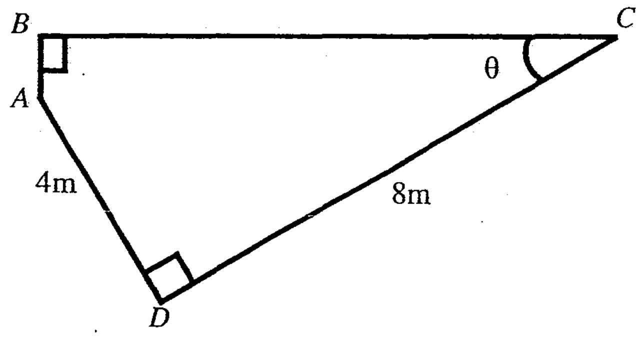
(ii) On the same diagram, sketch the graph of $y=3e^{-2x-1}$.

(iii) Calculate the coordinates of the point of intersection of your graph

(iv) Determine, with explanation, whether the tangents to the graphs
intersection are perpendicular.

Name: ______________________________ (   )   Class: __________

6 The diagram shows a model of a playground $ABCD$. The angles ABC and ADC are right angles. AD is 4m and CD is 8m. The perimeter of the playground is P m.



(i) Show that $P = 4\cos\theta + 12\sin\theta + 12$. [4]

(ii) Express P in the form $R\cos(\theta - \alpha) + 12$ where $R>0$ and α is acute. [4]

(iii) Find the value of θ for which P is a maximum. [2]

7 A curve has the equation $y = f(x)$, where $f(x) = \dfrac{2x-3}{4+3x}$ for $x > 0$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

(iii) Determine, with explanation, whether $f(x)$ is an increasing or decreasing function. [1]

(iv) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

8 Given that $\dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = ax + b + \dfrac{2x + c}{x^2 - 9}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part (i), find

(ii) $\displaystyle\int \frac{4x^3 - x^2 - 34x + 12}{x^2 - 9}\,dx$. [6]

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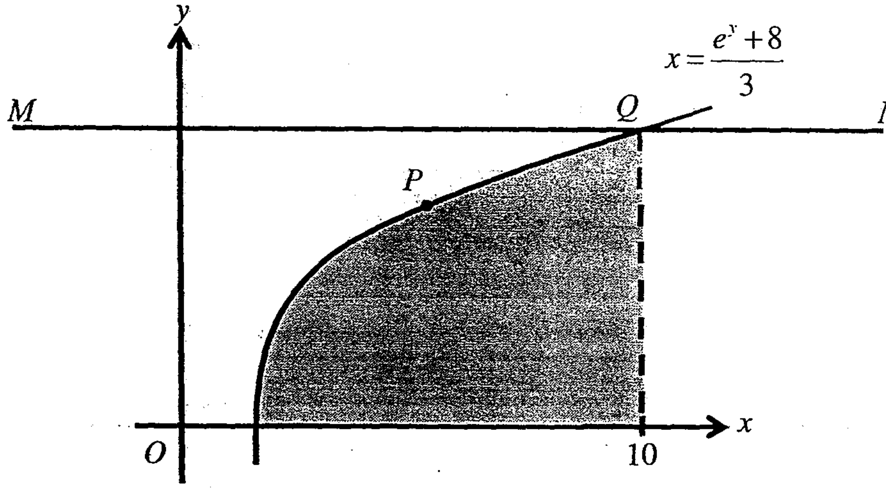
9



The diagram shows part of the curve $x=\dfrac{e^y+8}{3}$ intersecting the horizontal line MN at point Q with x-coordinate = 10. The point P lies on the curve and the tangent at P is parallel to the line $5y=x+6$.

(i) Find the x-coordinate of P. [4]

(ii) Find the area of the shaded region. [6]

10 **(i)** Prove the identity $\dfrac{2\tan x}{1+\tan^2 x}-\sin 2x=0$. [3]

(ii) Solve the equation $\dfrac{8\tan x}{2+2\tan^2 x}=-1$, for $0<x<\pi$, giving your answers in terms of π. [3]

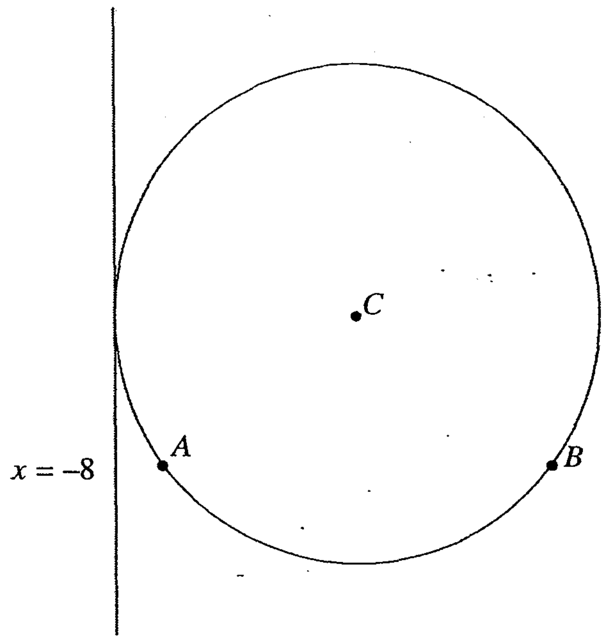
(iii) Given that $\dfrac{2+2\tan^2 x}{16\tan x}=-\dfrac{1}{3}\sec 2x$ and **without the use of calculator,**

(a) deduce that $\tan 2x=-\dfrac{3}{4}$, [2]

(b) find the possible values of $\tan x$. [3]

Name: ______________________________ (   )   Class: __________

11 The diagram shows two points $A(-6, 2)$ and $B\ (10, 2)$ on the circumference of a circle whose centre, C, lies above the x-axis. The line $x = -8$ is a tangent to the circle.



(i) Show that the radius of the circle is 10 units. [4]

(ii) Find the coordinates of C. [4]

(iii) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$. [2]

(iv) Find the equations of the tangents to the circle parallel to the x-axis. [2]

~End of Paper~

FMS(S)Secondary 4 Express 2014
Additional MathematicsPreliminary ExaminationPaper 2Answers

1(i) $A = 50\,000,\ k = 0.161$ 1(ii)2019

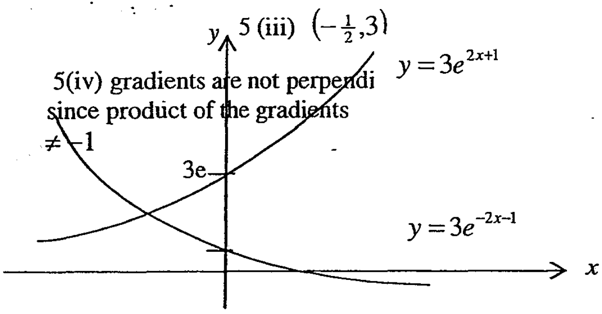
2(a) $x = \dfrac{-y}{y-1}$ or $\dfrac{y}{1-y}$ 2(b) (i) $1-u$ 2(b) (ii) $3+u$ 2(b) (iii) $\dfrac{2}{u}$

3(i) $-2x\sin 2x + \cos 2x$ 3(ii) $\frac{1}{4}\sin 2x - \frac{1}{2}x\cos 2x + C$

4(i) $1 - 12px + 66p^2x^2$ 4(ii) $66p^2 - 36p - 5$ 4(iii) $\dfrac{2}{3}$ *or* $\dfrac{2}{33}$

5(i)
5(ii)

5 (iii) $\left(-\frac{1}{2}, 3\right)$



5(iv) gradients are not perpendi since product of the gradients $\neq -1$

6(ii) $P = 4\sqrt{10}\cos(\theta - 71.6^0) + 12$ 6(iii) $\theta = 71.6^0$

7(i) $\dfrac{17}{(4+3x)^2}$ 7(ii) $y = -\frac{17x}{4} + \frac{51}{8}$ or $8y = -34x + 102$ 7(iii) Increasing function 7(iv) decreasing function

15.7 sq units

8(i) $a = 4,\ b = \square 1,\ c = 3$

8 (ii) $\dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = 4x - 1 + \dfrac{1}{2(x+3)} + \dfrac{3}{2(x-3)}$

$$\frac{(4x-1)^2}{8} + \frac{1}{2}\ln(x+3) + \frac{3}{2}\ln(x-3) + C \text{ or } 2x^2 - x + \frac{1}{2}\ln(x+3) + \frac{3}{2}\ln(x-3) + C$$

9(i) $x = 7\dfrac{2}{3}$ or 7.67 9(ii) $\left(\dfrac{22}{3}\ln 22 - 7\right)$ units^2 or 15.7 sq units

10(i) $LHS = \dfrac{2\tan x}{\sec^2 x} - \sin 2x = \dfrac{2\sin x}{\cos x} \div \dfrac{1}{\cos^2 x} - \sin 2x = \dfrac{2\sin x}{\cos x} \times \cos^2 x - \sin 2x$
$= 2\sin x\cos x - \sin 2x = \sin 2x - \sin 2x = 0$

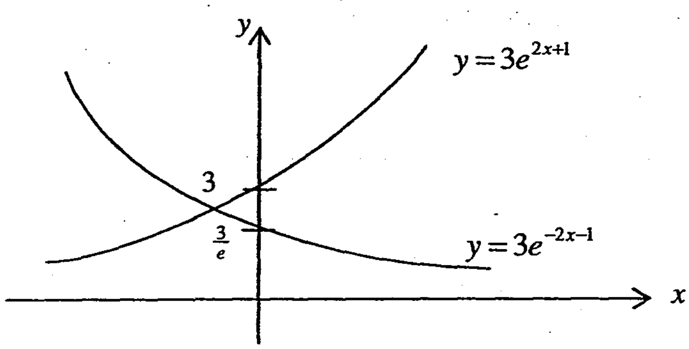
10(ii) $x = \dfrac{7\pi}{12}, \dfrac{11\pi}{12}$ 10(iii)(b) $-\dfrac{1}{3}$ or 3

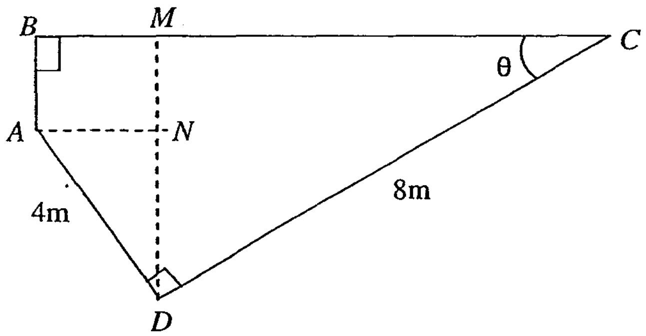
11(i) mid pt of chord formed by (-6,2) and (10,2) = (2,2) ; centre lies on $x = 2$;
distance between tangent at -8 and centre =10; hence radius = 10 units

11 (ii) are (2,8) 11(ii) $(x-2)^2 + (y-8)^2 = 100$ or $x^2 + y^2 - 4x - 16y - 32 = 0$ 11(iii) $y = 18$ or $y = -2$

Fairfield Methodist School (Secondary)
Secondary 4 Express 2014
Additional MathematicsPreliminary ExaminationPaper 2
Marking Scheme

No	Working / Description	Allocation of Marks
1i	A = 50 000 $P = Ae^{kt}$ $250000 = 50000e^{10k}$ $5 = e^{10k}$ $\ln 5 = 10k$ $k = 0.161$	B1 M1 A1
1ii	$1000000 = 50000e^{0.1609t}$ $20 = e^{0.1609t}$ $\ln 20 = 0.1609t$ $t = 18.6$ The year is 2019	M1 A1 FT1
2a	$\log_{13} x + \log_{13} y = \log_{13}(x-y)$ $\log_{13} xy = \log_{13}(x-y)$ $xy = x-y$ $xy - x = -y$ $x = \frac{-y}{y-1}$ or $\frac{y}{1-y}$	M1 for product law M1 removing log A1
2bi	$\log_3 \frac{3}{z}$ $= 1-u$	B1
2bii	$\log_3 27z$ $= \log_3 27 + \log_3 z$ $= 3+u$	B1
2biii	$\log_z 9$ $= 2\log_z 3$ $= \frac{2}{\log_3 z}$ $= \frac{2}{u}$	M1 Change of base A1
3(i)	$\frac{d}{dx}(\cos 2x) = -2\sin 2x$ $\frac{d}{dx}(x\cos 2x) = x(-2\sin 2x) + \cos 2x(1)$ $= -2x\sin 2x + \cos 2x$	B1 M1(product rule) FT1

No	Working / Description	Allocation of Marks
3(ii)	From (i), $\frac{d}{dx}(x\cos 2x) = -2x\sin 2x + \cos 2x$ $\int \frac{d}{dx}(x\cos 2x)dx = \int -2x\sin 2x dx + \int \cos 2x dx$ $x\cos 2x + C_1 = \int -2x\sin 2x dx + \int \cos 2x dx$ $\int 2x\sin 2x dx = \int \cos 2x dx - x\cos 2x - C_1$ $= \frac{1}{2}\sin 2x + C_2 - x\cos 2x - C_1$ $\int x\sin 2x dx = \frac{1}{4}\sin 2x - \frac{1}{2}x\cos 2x + C$ $\int_{\frac{\pi}{4}}^{\pi} x\sin 2x dx =$ $\left[\frac{1}{4}\sin 2(2\pi) - \frac{1}{2}(2\pi)\cos 2(2\pi)\right] - \left[\frac{1}{4}\sin 2(\pi) - \frac{1}{2}(\pi)\cos 2(\pi)\right]$ $= [0-\pi] - \left[0 - \frac{\pi}{2}\right] =$ (shown)	M1 B1 M1, −1m for not writing constant for final answer M1 B1
4(i)	$(1-px)^{12} = 1 - 12px + \binom{12}{2}(-px)^2 + ...$ $= 1 - 12px + 66p^2x^2 + ...$	B1 B1
4(ii)	$(1+3x-5x^2)(1-px)^{12} = (1+3x-5x^2)(1-12px+66p^2x^2+....)$ Coefficient of $x^2 = 66p^2 - 36p - 5$	M1 A1
4(iii)	$(1+3x-5x^2)(1-12px+66p^2x^2+....)$ Coefficient of $x = -12p+3$ $66p^2 - 36p - 5 + (-12p+3) = -4\frac{2}{3}$ $66p^2 - 48p + 2\frac{2}{3} = 0$ $198p^2 - 144p + 8 = 0$ $99p^2 - 72p + 4 = 0$ $(3p-2)(33p-2) = 0$ $p = \frac{2}{3}$ or $p = \frac{2}{33}$	B1 M1 M1 A1
5(i) &(ii)		B1 B1

No	Working / Description	Allocation of Marks
5(iii)	$y=3e^{2x+1}$ ----------(1) $y=3e^{-2x-1}$ --------(2) $3e^{2x+1}=3e^{-2x-1}$ $e^{4x+2}=1$ $4x+2=0$ $x=-\frac{1}{2}$ From (1), $y=3e^{-2(\frac{1}{2})+1}=3$ Point of intersection = $\left(-\frac{1}{2},3\right)$	M1 A1
5(iv)	If the tangents to the graphs are perpendicular, then $m_1m_2=-1$ $y=3e^{2x+1}$ $\frac{dy}{dx}=6e^{2x+1}$ $x=-\frac{1}{2},\ \frac{dy}{dx}=6e^{2(-0.5)+1}=6$ $y=3e^{-2x-1}$ $\frac{dy}{dx}=-6e^{-2x-1}$ $x=-\frac{1}{2},\ \frac{dy}{dx}=-6e^{-2(-0.5)-1}=-6$ Since $6(-6)\neq-1$, the 2 tangents are **not** perpendicular. Note: Not necessary to substitute in $x=-\frac{1}{2}$	B1 B1 M1 A1
6(i)	 $\sin\theta=\frac{DM}{8}\Rightarrow DM=8\sin\theta$ $\cos\theta=\frac{CM}{8}\Rightarrow CM=8\cos\theta$ $\sin\theta=\frac{AN}{4}\Rightarrow AN=4\sin\theta$ $\cos\theta=\frac{DN}{4}\Rightarrow DN=4\cos\theta$ $AB=DM-DN=8\sin\theta-4\cos\theta$ $P=AB+BM+MC+CD+DA$ $=8\sin\theta-4\cos\theta+4\sin\theta+8\cos\theta+8+4$ $=4\cos\theta+12\sin\theta+12$ (shown)	B1 B1 B1 B1

No	Working / Description	Allocation of Marks
6(ii)	$4\cos\theta + 12\sin\theta = R\cos(\theta - \alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $R\cos\alpha = 4$ ----------(1) $R\sin\alpha = 12$ ---------(2) (2) $\div$ (1) : $\tan\alpha = 3 \Rightarrow \alpha = \tan^{-1}3 = 71.57^0 = 71.6^0$ $(1)^2 + (2)^2$: $R = \sqrt{4^2 + 12^2}$ $= \sqrt{160}$ $= \sqrt{16} \times \sqrt{10}$ $= 4\sqrt{10}$ or 12.649 $P = 4\sqrt{10}\cos(\theta - 71.6^0) + 12$ or $P = 12.6\cos(\theta - 71.6^0) + 12$	M1 M1 B1 A1
6(iii)	For P to be a maximum, $\cos(\theta - 71.6^0) = 1$ $\theta - 71.6^0 = 0$ $\theta = 71.6^0$	M1 A1
7(i)	$f(x) = \dfrac{2x-3}{4+3x}$ $f'(x) = \dfrac{(4+3x)(2) - (2x-3)(3)}{(4+3x)^2}$ $= \dfrac{8+6x-6x+9}{(4+3x)^2}$ $= \dfrac{17}{(4+3x)^2}$	M1 A1
7(ii)	Let $y = 0$ $2x - 3 = 0$ $x = 1.5$ (1.5, 0) $x = 1.5$ $f'(x) = \dfrac{17}{[4+3(1.5)]^2}$ $= 4/17$ Gradient of normal = $-\dfrac{17}{4}$ Equation of normal is $y - 0 = -\dfrac{17}{4}(x - 1.5)$ $y = -\dfrac{17}{4}x + \dfrac{51}{8}$ or $8y = -34x + 51$	B1 A1 M1 FT1
7(iii)	Since $(4+3x)^2 > 0$, $f'(x) = \dfrac{17}{(4+3x)^2} > 0$ f (x) is a increasing function.	B1
7(iv)	$f'(x) = \dfrac{17}{(4+3x)^2}$ $f''(x) = 17(\square 2)(4+3x)^{\square 3}(3)$ $= \dfrac{-102}{(4+3x)^3}$	M1 A1

No	Working / Description	Allocation of Marks
	Since $x > 0$, $(4+3x)^3 > 0$ $f''(x) < 0$ Hence, gradient of the curve is an decreasing function.	B1
8(i)	$\frac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = ax + b + \frac{2x + c}{x^2 - 9}$ $a = 4$ $4x - 1$ $x^2 - 9 \overline{)4x^3 - x^2 - 34x + 12}$ $-(4x^3 - 36x)$ $-x^2 + 2x + 12$ $-(-x^2 + 9)$ $2x + 3$ $\therefore \frac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = 4x - 1 + \frac{2x + 3}{x^2 - 9}$ $b = \square 1$ $c = 3$	B1 M1 A1 A1
8(ii)	$\frac{2x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$ $2x + 3 = A(x-3) + B(x+3)$ By comparing coefficients, $2 = A + B$ ---------(1) $3 = -3A + 3B$ ---------(2) (1) $\times 3$ +(2) : $9 = 6B \Rightarrow B = \frac{3}{2}$ (1): $A = \frac{1}{2}$ $\frac{2x+3}{x^2-9} = \frac{1}{2(x+3)} + \frac{3}{2(x-3)}$ $\frac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = 4x - 1 + \frac{2x+3}{x^2-9}$ $\frac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = 4x - 1 + \frac{1}{2(x+3)} + \frac{3}{2(x-3)}$ $\int \frac{4x^3 - x^2 - 34x + 12}{x^2 - 9} dx = \int (4x-1) dx + \frac{1}{2} \int \frac{1}{x+3} dx + \frac{3}{2} \int \frac{1}{x-3} dx$ $= \frac{4x^2}{2} - x + \frac{1}{2} \ln(x+3) + \frac{3}{2} \ln(x-3) + C$ $= 2x^2 - x + \frac{1}{2} \ln(x+3) + \frac{3}{2} \ln(x-3) + C$ Or $\int \frac{4x^3 - x^2 - 34x + 12}{x^2 - 9} dx = \int (4x-1) dx + \frac{1}{2} \int \frac{1}{x+3} dx + \frac{3}{2} \int \frac{1}{x-3} dx$	M1 M1 A1 A1 B2, □1 for each error

No	Working / Description	Allocation of Marks
	$=\frac{(4x-1)^2}{2(4)}+\frac{1}{2}\ln(x+3)+\frac{3}{2}\ln(x-3)+C$ $=\frac{(4x-1)^2}{8}+\frac{1}{2}\ln(x+3)+\frac{3}{2}\ln(x-3)+C$	
9i	Gradient of tangent at P $=\frac{1}{5}$ $y=\ln(3x-8)$ $\frac{dy}{dx}=\frac{3}{3x-8}$ $\frac{3}{3x-8}=\frac{1}{5}$ $15=3x-8$ $x=\frac{23}{3}$ or $7\frac{2}{3}$ or 7.67 (3sf)	B1 M1 for differentiation M1 A1
9ii	$x=\frac{e^y+8}{3}$ $y=\ln(30-8)$ $=\ln 22$ Area bounded curve, MN, y-axis and x-axis $\int_0^{\ln 22}\frac{e^y+8}{3}dy$ $=\frac{1}{3}\left[e^y+8y\right]_0^{\ln 22}$ $=\frac{1}{3}\left[(22+8\ln 22)-(1)\right]$ $=7+\frac{8}{3}\ln 22$ Area of rectangle = 10ln22 Shaded Area $=10\ln 22-(7+\frac{8}{3}\ln 22)$ $=\frac{22}{3}\ln 22-7$ $=15.7$ sq units	B1 (y-coordinate of MN) M1 for integration A1 B1 M1 for rect-unshaded A1

No	Working / Description	Allocation of Marks
10i	$LHS=\dfrac{2\tan x}{1+\tan^2 x}-\sin 2x$	
	$=\dfrac{\frac{2\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}}-\sin 2x$	M1 changing tangent
	$=\dfrac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x+\sin^2 x}{\cos^2 x}}-\sin 2x$	
	$=\dfrac{2\sin x}{\cos x}\times\dfrac{\cos^2 x}{\cos^2 x+\sin^2 x}-\sin 2x$	
	$=\dfrac{2\sin x}{\cos x}\times\cos^2 x-\sin 2x$	M1 Apply $\sin^2x+\cos^2x=1$
	$=2\sin x\cos x-\sin 2x$	
	$=\sin 2x-\sin 2x$	AG1
	$=0$	
10i	$LHS=\dfrac{2\tan x}{1+\tan^2 x}-\sin 2x$	Alternate Method
	$=\dfrac{2\tan x}{\sec^2 x}-\sin 2x$	M1 changing secant form
	$=\dfrac{2\sin x}{\cos x}\div\dfrac{1}{\cos^2 x}-\sin 2x$	
	$=\dfrac{2\sin x}{\cos x}\times\cos^2 x-\sin 2x$	M1 changing to cosine form
	$=2\sin x\cos x-\sin 2x$	
	$=\sin 2x-\sin 2x$	AG1
	$=0$	
10ii	$\dfrac{8\tan x}{2+2\tan^2 x}=-1$	
	$2\left(\dfrac{2\tan x}{1+\tan^2 x}\right)=-1$	
	$2\sin 2x=-1$	B1
	$\sin 2x=-\dfrac{1}{2}$	
	ref angle $=\sin^{-1}\dfrac{1}{2}=\dfrac{\pi}{6}$	M1 find ref angle
	$2x=\dfrac{7\pi}{6},\dfrac{11\pi}{6}$	
	$x=\dfrac{7\pi}{12},\dfrac{11\pi}{12}$	A1 for both answer

No	Working / Description	Allocation of Marks
10iiia	$\frac{2+2\tan^2 x}{16\tan x}=-\frac{1}{3}\sec 2x$ $\frac{16\tan x}{2+2\tan^2 x}=-3\cos 2x$ $4\left(\frac{2\tan x}{1+\tan^2 x}\right)=-3\cos 2x$ $4\sin 2x=-3\cos 2x$ $\frac{\sin 2x}{\cos 2x}=-\frac{3}{4}$ $\tan 2x=-\frac{3}{4}$	M1 A1 (must show sin2x/cos2x)
10iiib	$\tan 2x=-\frac{3}{4}$ $\frac{2\tan x}{1-\tan^2 x}=-\frac{3}{4}$ $8\tan x=-3+3\tan^2 x$ $3\tan^2 x-8\tan x-3=0$ $(3\tan x+1)(\tan x-3)=0$ $\tan x=-\frac{1}{3}$ or $\tan x=3$	M1 for quad equation M1 factorise or quad formula A1 for both answers correct
11i	mid pt of chord formed by (-6,2) and (10,2) = (2,2) centre lies on x= 2, dist b/w tangent at -8 and centre =10 Radius = 10 units	M1 mid pt formula, A1 M1 A1
11ii	$\sqrt{(2+6)^2+(y-2)^2}=100$ $64+(y-2)^2=100$ $(y-2)=\pm 6$ $y=8$ or -4 na coordinates are (2,8)	M1 form equation M1 solving A1 A1
11iii	$(x-2)^2+(y-8)^2=100$ $x^2+y^2-4x-16y-32=0$	M1 A1 or B2
11iv	y = 18 y = −2	B1 B1