Geylang Methodist School (Secondary)
Preliminary Examination 2014

ADDITIONAL MATHEMATICS **4047/01**

Paper 1

4 Express

Additional materials :Writing Paper **2 hours**

Setter : MrWong Han Ming / MrJohney Joseph **1Sep 2014**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all**the questions.
Write your answers on the separate Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 Without using a calculator, express $\cos 75°$ in the form

$\frac{\sqrt{a}-\sqrt{b}}{4}$, where a and b are integers. [3]

2 Find the coordinates of the point of intersection of the graphs of $y = 3x + 5$ and $y = |2-5x|+3$. [4]

3 The equation $2x^2-6x+1=0$ has roots α and β. Without solving for α and β, find an equation that has roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [6]

4 In the binomial expansion of $(2+x)^n$, where n is a positive integer, the coefficient of x and the coefficient of x^2 is in the ratio 4 : 15.
Find the value of n. [6]

5 At the beginning of year 1900, the population of a country was 580 000. Due to a disease, the population decreased such that after a period of t years, the new population was given by the equation $580\,000e^{kt}$, where k is a constant. Given that the population after 2 years was 480 000,

(i) find the value of k, [2]

(ii) calculate, to the nearest whole number, the population of the country after 5 years, [2]

(iii) find the year in which the population first fall below 100 000. [2]

6 **(a)** Given that $\lg x = a$ and $\lg y = b$, express $\lg\sqrt{\dfrac{100x^8}{y^5}}$ in terms of a and b. [3]

(b) Solve $\log_{16}[\log_2(6x-18)] = \log_9 3$. [3]

7 **(i)** Show that $\tan\theta + \cot\theta = 2\operatorname{cosec}2\theta$. [4]

(ii) Hence, solve $2\tan\theta + 2\cot\theta = -5$ for $0 \le \theta \le \pi$. [2]

8 It is given that $3^{x+1} \times 2^{2x+1} = 2^{x+2}$.

(i) Find the exact value of 6^x. [2]

(ii) Hence find the value of x correct to two decimal places. [2]

9 Given that $\dfrac{4x^3+7x^2-13x-2}{x^2+2x-3} = Ax + B + \dfrac{x+C}{x^2+2x-3}$,

(i) find the values of each of the integers A, B and C. [4]

Hence, using partial fractions and the values of A, B and C obtained in part **(i)**,

(ii) find $\displaystyle\int \frac{4x^3+7x^2-13x-2}{x^2+2x-3}\,dx$. [5]

10 **(i)** Differentiate $x^2 \ln x + x$ with respect to x. [2]

(ii) Hence evaluate $\displaystyle\int_1^3 x\ln x\,dx$, correct to one decimal place. [4]

11 Variables x and y are connected by the equation $y = ax^n$, where a and n are constants. Using experimental values of x and y, a graph was drawn in which $\lg y$ was plotted on the vertical axis against $\lg x$ on the horizontal axis.
The straight line which was obtained passes through the points (2.3, 1.5) and (4.2, 5.3).

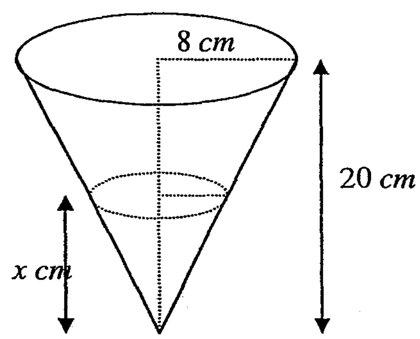
Calculate

(i) the value of n, [2]

(ii) the value of a correct to 2 significant figures, [1]

(iii) thecoordinates of the point on the line at which $y = \frac{1}{2}x$. [3]

12



[The volume of a cone of height H and radius R is given by $\frac{1}{3}\pi R^2 H$]

The diagram shows a hollow conical tank of height 20 cm and radius 8 cm. The tank is held with its circular rim horizontal. Water is then poured into the empty tank at a constant rate of 10 cm^3 s^{-1}. After t seconds, the depth of water is x cm.

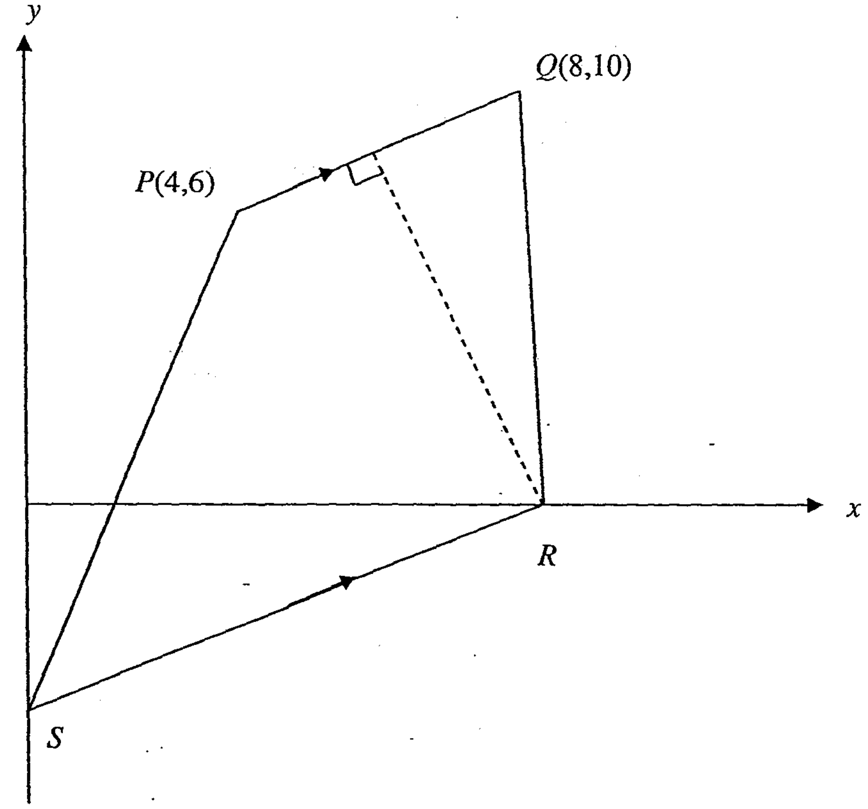
(i) Show that, the volume of the water, V cm^3, at time t is given by
$V = \frac{4}{75}\pi x^3$. [2]

(ii) Find $\frac{dV}{dx}$ and hence, find the rate of change of the depth when $x = 4$. [3]

(iii) State, with reason, whether this rate will increase or decrease as t increases. [2]

[Turn over

13 **(a)**



The diagram shows a trapezium $PQRS$ in which PQ is parallel to SR. The point P is $(4, 6)$ and the point Q is $(8, 10)$. R and S lie on the x and y axes respectively. The point R lies on the perpendicular bisector of PQ. Find

(i) the coordinates of R and of S, [4]

(ii) the area of trapezium $PQRS$. [2]

(b) Solve the equation $3^{3x} + 3^{2x+1} - 10(3^x) - 24 = 0$. [5]

- End of Paper -

A Maths P1 Solutions.

1) $\cos 75^\circ = \cos(30^\circ + 45^\circ)$

$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{1}{2} \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

2) $y = 3x + 5$

$y = |2 - 5x| + 3$

$3x + 5 = |2 - 5x| + 3$

$3x + 2 = |2 - 5x|$

$3x + 2 = 2 - 5x$ OR $3x + 2 = -(2 - 5x)$

$8x = 0$ $\qquad$ $3x + 2 = 5x - 2$

$x = 0$ $\qquad$ $4 = 2x$

$x = 2$

$\therefore y = 5$ $\qquad$ $y = 11$

$\therefore$ Coordinates are $(0, 5)$ & $(2, 11)$

3) $2x^2 - 6x + 1 = 0$

$\alpha + \beta = 3$

$\alpha\beta = \frac{1}{2}$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\frac{1}{2})^2}$$

$$= \frac{3^2 - 2(\frac{1}{2})}{1/4}$$

$$= \frac{8}{1/4}$$

$$= 32$$

$$\frac{1}{\alpha^2} \times \frac{1}{\beta^2} = \frac{1}{(\frac{1}{2})^2}$$

$$= 4$$

$\therefore$ Eqn is $x^2 - 32x + 4 = 0$

4) $(2+x)^n = 2^n + \binom{n}{1}2^{n-1}x$

$+ \binom{n}{2}2^{n-2}x^2$

$+ \cdots$

coeff of $x = n2^{n-1}$

coeff of $x^2 = \frac{n(n-1)}{2}2^{n-2}$

$$\frac{n2^{n-1}}{\frac{n(n-1)}{2}2^{n-2}} = \frac{4}{15}$$

$$15n2^{n-1} = 2n(n-1)2^{n-2}$$

$$15(2) = 2(n-1)$$

$$15 = n-1$$

$$n = 16$$

5(i) $580\,000e^{kt}$

when $t=2$

$580\,000\ e^{2k} = 480\,000$

$$e^{2k} = \frac{48}{58}$$

$$2k = \ln\frac{48}{58}$$

$$k = -0.0946209982$$

$$\approx -0.0946$$

5(ii) new population $= 580000e^{5k}$

$= 361377.495$

≈ 361377

(iii) $580000e^{kt} = 100000$

$e^{kt} = \frac{10}{58}$

$kt = \ln\frac{10}{58}$

$t = \ln\frac{10}{58} \div k$

$= 18.5778357$

$\therefore$ 18 years

The year in which the population first hit below 100 000 is 1918

6a) $\lg\sqrt{\frac{100x^8}{y^5}} = \frac{1}{2}\lg\frac{100x^8}{y^5}$

$= \frac{1}{2}\left[\lg 100 + \lg x^8 - \lg y^5\right]$

$\rightarrow = \frac{1}{2}\left[10^{2} + 8\lg x - 5\lg y\right]$

$= \frac{1}{2}\left[\cancel{10}^{1} + 8a - 5b\right]$

$= 5 + 4a - \frac{5}{2}b$

6b) $\log_{16}\left[\log_2(6x-18)\right] = \log_9 3$

$\log_{16}\left[\log_2(6x-18)\right] = \log_9 9^{\frac{1}{2}}$

$\log_{16}\left[\log_2(6x-18)\right] = \frac{1}{2}$

$\log_2(6x-18) = 16^{\frac{1}{2}}$

$\log_2(6x-18) = 4$

$6x-18 = 2^4$

$6x-18 = 16$

$6x = 34$

$x = 5\frac{2}{3}$

7(i) $\tan\theta + \cot\theta = 2\operatorname{cosec} 2\theta$

$\text{RHS} = 2\operatorname{cosec} 2\theta$

$= \dfrac{2}{\sin 2\theta}$

$= \dfrac{2}{2\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$\text{LHS} = \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta} = \text{RHS}$

7(ii) $2\tan\theta + 2\cot\theta = -5$

$4\operatorname{cosec} 2\theta = -5$

$\operatorname{cosec} 2\theta = \frac{-5}{4}$

$\sin 2\theta = -\frac{4}{5}$

basic $\measuredangle = 0.927295218$

$2\theta = 4.068887872,\ 5.355890089$

$\theta = 2.03,\ 2.68$

Paper I sol.
(JJ)

8 (i) $3^{x+1} \times 2^{2x+1} = 2^{x+2}$

$3^x \times 3 \times 2^{2x} \times 2 = 2^x \times 4$

$3^x \times 2^x = \frac{2}{3}$

$6^x = \frac{2}{3}$

(ii) $x \lg 6 = \lg \frac{2}{3}$

$x = \frac{\lg(\frac{2}{3})}{\lg 6}$

$= -0.23$

9 (i)

$$\begin{array}{r|l} & 4x \quad -1 \\ \hline x^2+2x-3 & 4x^3 + 7x^2 - 13x - 2 \\ & 4x^3 + 8x^2 - 12x \\ \hline & \qquad -x^2 - x - 2 \\ & \qquad -x^2 - 2x + 3 \\ \hline & \qquad\qquad x - 5 \end{array}$$

$$\frac{4x^3+7x^2-13x-2}{x^2+2x-3} = 4x-1 + \frac{x-5}{x^2+2x-3}$$

$\therefore A = 4, \; B = -1, \; C = -5$

(ii) $x^2+2x-3 = (x+3)(x-1)$

Let $\dfrac{x-5}{(x+3)(x-1)} = \dfrac{P}{x+3} + \dfrac{Q}{x-1}$

$x-5 = P(x-1) + Q(x+3)$

$P = 2 \;,\; Q = -1$

$\therefore \dfrac{x-5}{(x+3)(x-1)} = \dfrac{2}{x+3} - \dfrac{\cancel{1}}{x-1}$

$$\int \frac{4x^3+7x^2-13x-2}{x^2+2x-3}\,dx = \int \left(4x-1 + \frac{2}{x+3} - \frac{1}{x-1}\right)$$

$$= 2x^2 - x + 2\ln(x+3) - \ln(x-1) +$$

10 (i) $\dfrac{d}{dx}(x^2\ln x + x) = x^2 \times \dfrac{1}{x} + (\ln x)2x + 1$

$= x + 2x\ln x + 1$

$$\int_1^3 [x + 2x\ln x + 1]\,dx = \left[x^2\ln x + x\right]_1^3$$

$$\int_1^3 2x\ln x\,dx = \left[x^2\ln x + x\right]_1^3 - \left[\frac{x^2}{2} + x\right]_1^3$$

$$= \left[x^2\ln x - \frac{x^2}{2}\right]_1^3$$

$$= \left(9\ln 3 - \frac{9}{2}\right) - \left(0 - \frac{1}{2}\right)$$

$$= 9\ln 3 - 4$$

$$~~= 5.89 = 5.9~~ \;(1 d.p)$$

$$\therefore \int_1^3 x\ln x\,dx = \frac{9\ln 3 - 4}{2}$$

$$= 2.9 \;(1 d.p)$$

11. (i) $y = ax^n$

$\lg y = n \lg x + \lg a$

$n = \text{gradient}$

$= \dfrac{5.3 - 1.5}{4.2 - 2.3} = 2$

(ii) $5.3 = 2 \times 4.2 + \lg a$

$\lg a = -3.1$

$a = 10^{-3.1}$

$= 0.00079.$

(iii)

$y = \frac{1}{2}x$

$\lg y = \lg x - \lg 2$ ——— (1)

$\lg y = 2 \lg x - 3.1$ ——— (2)

$0 = \lg x - 3.1 + \lg 2$

$\lg x = 3.1 - \lg 2$

$= 2.80$

$\lg y = 2.50$

$\therefore (2.80, 2.50).$

12(i) $\frac{x}{20} = \frac{r}{8}$

$r = \frac{2}{5}x$

$V = \frac{1}{3}\pi\left(\frac{2}{5}x\right)^2 x$

$= \frac{4\pi}{75}x^3$

(ii) $\frac{dv}{dx} = \frac{4\pi}{75}(3x^2) = \frac{4\pi}{25}x^2$

when $x = 4$, $\frac{dv}{dx} = \frac{4\pi}{25} \times 4^2 = \frac{64\pi}{25}$

$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

$10 = \frac{64\pi}{25} \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{25 \times 10}{64\pi} = 1.24$ cm/s.

(iii) $\frac{dx}{dt} = \frac{dv}{dt} \div \frac{dv}{dx}$

$= 10 \div \frac{4\pi x^2}{25}$

$= \frac{125}{2\pi x^2}$

$\therefore$ The rate decreases as t increases.

13 (a) Midpoint of PQ is $(6, 8)$

Gradient of PQ $= \dfrac{10-6}{8-4} = 1$

Equation of the perpendicular bisector is

$y - 8 = -1(x - 6)$

$y = -x + 14$

$\therefore R(14, 0)$

OR
use PR=QR

Equation of SR is

$y - 0 = 1(x - 14)$

$y = x - 14$

$\therefore S(0, -14)$

(b) $3^{3x} + 3^{2x+1} - 10(3^x) - 24 = 0$

Let $y = 3^x$

$y^3 + 3y^2 - 10y - 24 = 0$

Let $f(y) = y^3 + 3y^2 - 10y - 24$

$f(-2) = -8 + 12 + 20 - 24 = 0$

$\therefore y+2$ is a factor.

Geylang Methodist School (Secondary)
Preliminary Examination 2014

ADDITIONAL MATHEMATICS 4047/02

Paper 2

4 Express

Additional materials :Writing Paper **2 hours 30 minutes**

Setter : MrJohney Joseph **29Aug 2014**

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The total number of marks for this paper is 100.

This document consists of **6**printed pagesincluding the cover page

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Mathematical Formulae

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Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A curve is given by the equation $y = 2 + \frac{c}{x}$ where c is a positive constant.

(i) Explain with working why the line $y = 2x - 2$ intersects the curve at two distinct points for all values of c. [3]

Given that the gradient of the curve $y = 2 + \frac{c}{x}$ at the point A $(k, 6)$ is -4.

(ii) Show that c is a multiple of k^2. [2]

(iii) Find the value of c. [2]

2 (i) Sketch the graph of $y = 9x^{\frac{1}{3}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = 4x^{\frac{-1}{3}}$ for $x > 0$. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(iv) Determine, with explanation, whether there is a common tangent for the two curves. [4]

3 It is given that $\int_1^5 px\,dx = 10$, where p is a constant.

(i) Find the value of the constant p. [2]

(ii) Express $\int_1^5 (px + q)\,dx$ in terms of the constant q. [2]

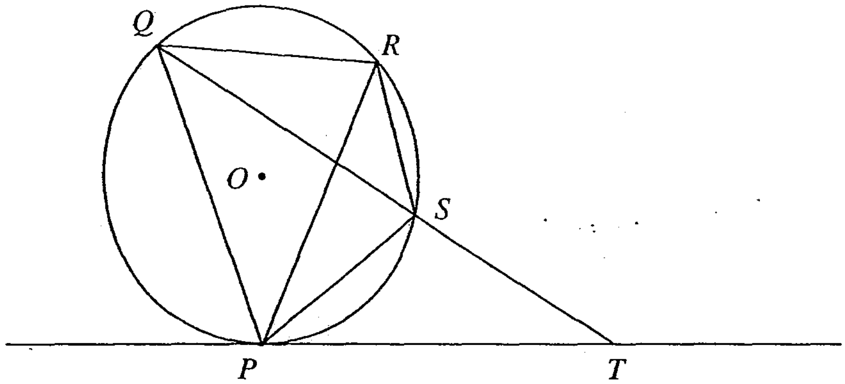
4 The height h metres of water at a jetty on a particular day is given by $h = 1.5\,(2 + \sin\frac{\pi t}{6})$ where t is the number of hours after midnight.

(i) Explain why the maximum height of water at the jetty for the particular day is 4.5 m. [1]

(ii) Find the height of water at 9 am. [2]

(iii) Find the first two values of t for which the height of water is 2 m. [4]

5 In the figure below, $RS = PS$ and the line PT is tangent to the circle.
Line QT bisects angle PQR and cuts the circle at point S.



Prove that

(i) $\angle TPS = \angle SPR$, [2]

(ii) ΔSPT is similar to ΔPQT, [2]

(iii) $PT \cdot PQ = QT \cdot RS$. [4]

6 A curve has an equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x^2+5}{x-2}$ for $x \neq 2$.

(i) Obtain an expression for $\mathrm{f}'(x)$. [2]

(ii) Find the range of values of x for which f is an increasing function. [3]

(iii) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

7 A particle travels in a straight line, so that t seconds after passing through a fixed point O, the velocity v ms^{-1}, is given by $v = \dfrac{2t-4}{t+2}$. Find

(i) the initial velocity of the particle, [1]

(ii) the acceleration of the particle when it comes to instantaneous rest, [3]

(iii) an expression for the displacement in terms of t. [4]

8 **(i)** Prove that $\sin^6 x + \cos^6 x - \sin^2 x \cos^2 x = \cos^2 2x$. [4]

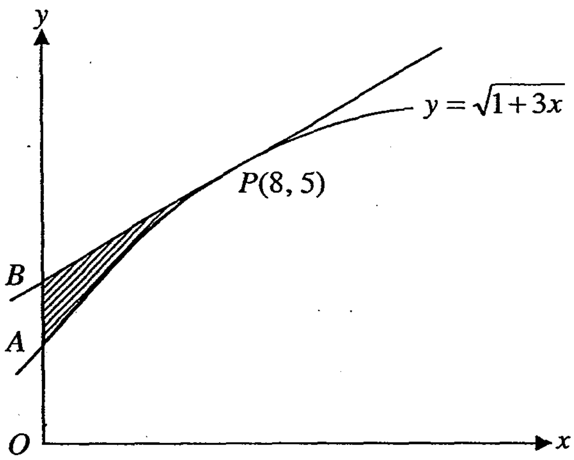
(ii) Solve the equation $\sin^6 x + \cos^6 x - \sin^2 x \cos^2 x - 1 = 0$, for $0 \leqslant x \leqslant \pi$, giving your answers in terms of π. [4]

(iii) Given that $\sin^6 A + \cos^6 A - \sin^2 A \cos^2 A = 3\sin^2 2A$, where $45° \leqslant A \leqslant 90°$, **without using a calculator,**

(a)deduce that $\tan 2A = -\dfrac{1}{\sqrt{3}}$, [2]

(b)find the value of $\tan A$. [3]

9

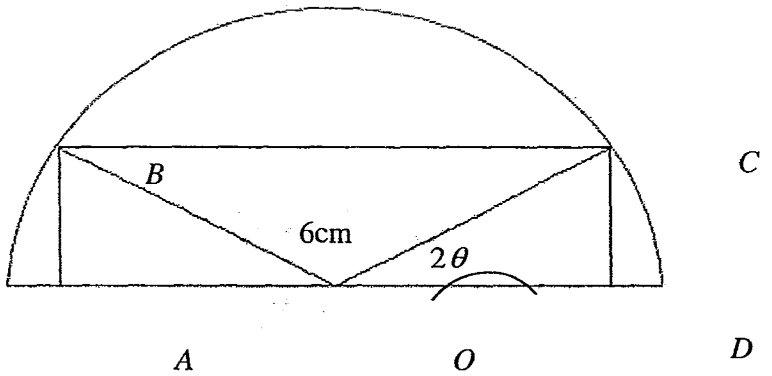


The diagram shows part of the curve $y = \sqrt{1+3x}$, intersecting the y-axis at A. The tangent to the curve at the point $P(8, 5)$ intersects the y-axis at B. Find

(i) the equation of BP, [4]

(ii) the area of the shaded region ABP. [4]

10 The diagram shows a rectangle $ABCD$ inscribed in a semicircle, centre O and radius6 cm, such that $\angle BOC = 2\theta$ radians.



(i) Show that the perimeter, P cm, of the rectangle is given by
$P = 12 \cos \theta + 24 \sin \theta$. [3]

(ii) Find the values of θ for which P is 25cm. [6]

11 Find the coordinates of the stationary points on the curve $y = 15 - \frac{8}{x} - 2x$ and determine the nature of each stationary point. [8]

12 A circle with centreC, pass through the points (5, – 8) and (5, 0).
The line $y = 1$ is a tangent to the circle and the circle intersects the y-axis at two points. Find

(i) the radius of the circle, [4]

(ii) the coordinates of C, [4]

(iii) the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers, [2]

(iv) the equations of the tangents to the circle parallel to the y- axis. [2]

- End of Paper -

A Maths Paper II Solution.

(JJ)

1(i) $2 + \frac{c}{x} = 2x - 2$

$2x^2 - 4x - c = 0$

$b^2 - 4ac = (-4)^2 - 4(2)(-c)$

$= 16 + 8c$

$> 0 \quad (c > 0)$

$\therefore y = 2x - 2$ intersects $y = 2 + \frac{c}{x}$ at two distint points.

(ii) $\frac{dy}{dx} = -\frac{c}{x^2}$

$-4 = -\frac{c}{k^2}$

$c = 4k^2$

(iii) $6 = 2 + \frac{c}{k}$

$k = \frac{c}{4}$

$c = 4\left(\frac{c}{4}\right)^2$

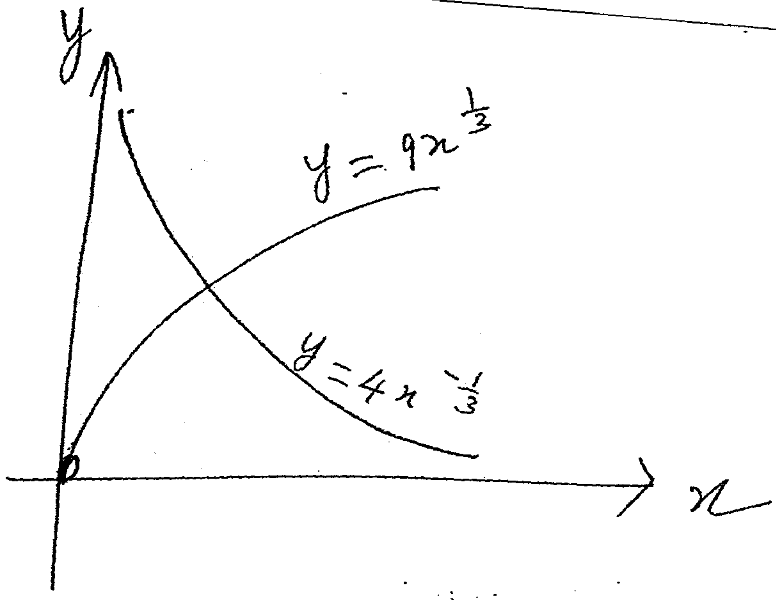
$c^2 - 4c = 0$

$c(c-4) = 0$

$c = 0$ or $c = 4$
(NA)

$\therefore \underline{\underline{c = 4}}$

2(i), (ii)



(iii) $9x^{\frac{1}{3}} = 4x^{-\frac{1}{3}}$

$x^{\frac{2}{3}} = \frac{4}{9}$

$x = \left(\frac{4}{9}\right)^{\frac{3}{2}}$

$= \frac{8}{27}$

$y = 9 \times \left(\frac{8}{27}\right)^{\frac{1}{3}} = 6$

$\therefore \left(\frac{8}{27}, 6\right)$

(iv) $y = 9x^{\frac{1}{3}}$

$\frac{dy}{dx} = 9 \times \frac{1}{3} x^{-\frac{2}{3}} = 3x^{-\frac{2}{3}}$

$y = 4x^{-\frac{1}{3}}$

$\frac{dy}{dx} = 4 \times -\frac{1}{3} x^{-\frac{4}{3}} = -\frac{4}{3} x^{-\frac{4}{3}}$

when gradients are equal

$3x^{-\frac{2}{3}} = -\frac{4}{3} x^{-\frac{4}{3}}$

$x^{\frac{2}{3}} = -\frac{4}{9}$ which is not possible as x;

$\Longrightarrow$ There is no common tang

3 (i) $\int_1^5 px\,dx = 10$

$\left[p\frac{x^2}{2}\right]_1^5 = 10$

$p\left(\frac{25}{2} - \frac{1}{2}\right) = 10$

$12p = 10$

$p = \frac{5}{6}$

(ii) $\int_1^5 (px+q)\,dx = \int_1^5 px\,dx + \int_1^5 q\,dx$

$= 10 + q[x]_1^5$

$= 10 + q(5-1)$

$= 10 + 4q$

4 (i) $h = 1.5\left(2 + \sin\frac{\pi x}{6}\right)$

Max value of $h = 1.5(2+1)$

$= 4.5\text{ m}$ $\left(\because -1 \le \sin\frac{\pi x}{6} \le 1\right)$

(ii) When $x = 9$

$h = 1.5\left(2 + \sin\frac{9\pi}{6}\right)$

$= 1.5(2-1)$

$= 1.5\text{ m}.$

(iii) $1.5\left(2 + \sin\frac{\pi x}{6}\right) = 2$

$3 + 1.5\sin\frac{\pi x}{6} = 2$

$$\sin\frac{\pi t}{6} = -\frac{2}{3}$$

Basic Angle = 0.7297

$$\frac{\pi t}{6} = \pi + 0.7297,\ 2\pi - 0.7297$$

$$t = 7.39,\ 10.6$$

5 (i) $\angle TPS = \angle SRP$ (Alternate Segment Theorem)

$\angle SRP = \angle SPR$ ($RS = PS$)

$\therefore \angle TPS = \angle SPR$

(ii) $\angle SPT = \angle PQT$ (Alternate Segment Theorem)

$\angle T$ is common.

$\therefore \triangle SPT$ is similar to $\triangle PQT$

(iii) $\dfrac{SP}{PQ} = \dfrac{PT}{QT} = \dfrac{ST}{PT}$ ($\triangle SPT$ Similar to $\triangle PQ$

$$\frac{SP}{PQ} = \frac{PT}{QT}$$

$$PT \cdot PQ = QT \cdot SP$$

$SP = SR$ (Given)

$$\therefore PT \cdot PQ = QT \cdot SR$$

b(i) $f(x) = \dfrac{x^2+5}{x-2}$

$$f'(x) = \frac{(x-2)(2x) - (x^2+5)(1)}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2 - 5}{(x-2)^2}$$

$$= \frac{x^2 - 4x - 5}{(x-2)^2}$$

(ii) $\dfrac{x^2-4x-5}{(x-2)^2} > 0$

$x^2 - 4x - 5 > 0$

$(x-5)(x+1) > 0$

$x < -1$ or $x > 5$

But $x > 2$.

$\therefore f(x)$ is increasing when $x > 5$.

(iii) $$f''(x) = \frac{(x-2)^2(2x-4) - (x^2-4x-5)\,2(x-2)}{(x-2)^4}$$

$$= \frac{(x-2)(2x-4) - 2(x^2-4x-5)}{(x-2)^3}$$

$$= \frac{2x^2 - 4x - 4x + 8 - 2x^2 + 8x + 10}{(x-2)^3}$$

$$= \frac{18}{(x-2)^3} > 0$$

$\therefore f(x)$ is an increasing function.

7(i) $V = \dfrac{2t-4}{t+2}$

When $t = 0$

$V = -2$ m/s

(ii) When $V = 0$

$\dfrac{2t-4}{t+2} = 0$

$t = 2$

$a = \dfrac{(t+2)(2)-(2t-4)}{(t+2)^2}$

$= \dfrac{8}{(t+2)^2}$

When $t = 2$

$a = \dfrac{8}{(2+2)^2} = \dfrac{1}{2}$ m/s^2.

(iii) $S = \int v\,dt$

$= \int \dfrac{2t-4}{t+2}\,dt$

$$\begin{array}{r|l} & \;\;2 \\ \hline t+2 & 2t-4 \\ & 2t+4 \\ \hline & \;\;-8 \end{array}$$

$= \int \left(2 - \dfrac{8}{t+2}\right) dt$

$S = 2t - 8\ln(t+2) + c$

$0 = -8\ln 2 + c \Rightarrow c = 8\ln 2$

$\therefore$ $S = 2t - 8\ln(t+2) + 8\ln 2$

8 (i) $\sin^6 x + \cos^6 x - \sin^2 x \cos^2 x$

$= (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) - \sin^2 x \cos^2 x$

$= 1\ (\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) - \sin^2 x \cos^2 x$

$= \sin^4 x - 2\sin^2 x \cos^2 x + \cos^4 x$

$= (\sin^2 x - \cos^2 x)^2$

$= (-\cos 2x)^2$

$= \cos^2 2x.$

(ii) $\cos^2 2x - 1 = 0$

$\cos^2 2x = 1$

$\cos 2x = \pm 1$

$2x = 0, \pi, 2\pi$

$x = 0, \frac{\pi}{2}, \pi$
(NA) (NA)

$x = \frac{\pi}{2}$

(iii) $3\sin^2 2A = \cos^2 2A$

$\tan^2 2A = \frac{1}{3}$

$\tan 2A = -\frac{1}{\sqrt{3}}$ ($\cancel{2A}$ $90^\circ < 2A < 180^\circ$)

$$\frac{2\tan A}{1-\tan^2 A} = -\frac{1}{\sqrt{3}}$$

$$2\sqrt{3}\tan A = -1 + \tan^2 A$$

$$\tan^2 A - 2\sqrt{3}\tan A - 1 = 0$$

$$\tan A = \frac{2\sqrt{3} \pm \sqrt{(-2\sqrt{3})^2 - 4(1)(-1)}}{2}$$

$$= \frac{2\sqrt{3} \pm \sqrt{16}}{2}$$

$$= \frac{2\sqrt{3} \pm 4}{2}$$

$$= \sqrt{3} \pm 2$$

$$= \sqrt{3} + 2 \quad (\because \tan A > 0)$$

9(i) $y = \sqrt{1+3x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+3x}}(3)$$

When $x = 8$

$$\frac{dy}{dx} = \frac{3}{10}$$

Equation of BP is

$$y - 5 = \frac{3}{10}(x - 8)$$

$$10y - 50 = 3x - 24$$

$$10y = 3x + 26$$

(ii) $B(0, 2.6)$

Area of shaded region

$$= \frac{1}{2}(2.6+5)\times 8 - \int_0^8 \sqrt{1+3x}\, dx$$

$$= 30.4 - \left[\frac{(1+3x)^{\frac{3}{2}}}{3\times\frac{3}{2}}\right]_0^8$$

$$= 30.4 - \frac{2}{9}\left[25^{\frac{3}{2}} - 1^{\frac{3}{2}}\right]$$

$$= 30.4 - \frac{2}{9}(125-1)$$

$$= \underline{\underline{2.84 \text{ units}^2}}$$

10 (i) $\sin\theta = \frac{OA}{6} \Rightarrow OA = 6\sin\theta \Rightarrow AD = 12\sin\theta$

$\cos\theta = \frac{AB}{6} \Rightarrow AB = 6\cos\theta$

Perimeter $= 2(6\cos\theta + 12\sin\theta)$

$= 12\cos\theta + 24\sin\theta$

(ii) Let $12\cos\theta + 24\sin\theta = R\cos(\theta - \alpha)$

$12\cos\theta + 24\sin\theta = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha$

$R\cos\alpha = 12$

$R\sin\alpha = 24$

$\tan\alpha = 2$

$\alpha = 1.107$

$R = \sqrt{12^2 + 24^2} = \sqrt{720} = 26.83$

$26.83\cos(\theta - 1.107) = 25$

$\cos(\theta - 1.107) = \frac{25}{26.83}$

$\theta - 1.107 = 0.3715, -0.371$

$\theta = 1.48, 0.736$

11. $y = 15 - \frac{8}{x} - 2x$

$\frac{dy}{dx} = \frac{8}{x^2} - 2$

When $\frac{dy}{dx} = 0$

$\frac{8}{x^2} - 2 = 0$

$\frac{8}{x^2} = 2$

$x^2 = 4$

$x = \pm 2$

When $x = 2$

$y = 15 - \frac{8}{2} - 2 \times 2 = 7$

When $x = -2$

$y = 15 + \frac{8}{2} + 4 = 23$

$\therefore$ The stationary points are $(2,7)$ and $(-2,23)$

$\frac{d^2y}{dx^2} = -\frac{16}{x^3}$

When $x = 2$

$\frac{d^2y}{dx^2} = -\frac{16}{2^3} < 0 \quad \therefore (2,7)$ Max

When $x = -2$ and $(-2,23)$ Min.

$\frac{d^2y}{dx^2} = \frac{-16}{(-2)^3} > 0$

12 (i) Midpoint of chord joining (5, -

is $(5, -4)$

$\therefore$ The centre lies

on the line $y = -4$

$y = 1$ is tangent to the cir

$\therefore$ Radius $= 4 + 1 = 5$ uni

(ii) Let the centre be $(a,$

$$(a-5)^2 + (-4-0)^2 = 5^2$$

$$(a-5)^2 = 9$$

$$a - 5 = \pm 3$$

$$a = 2 \quad \text{OR} \quad a = 8$$

(NA

$\therefore$ Centre is $(2, -4)$

(iii) $(x-2)^2 + (y+4)^2 = 5^2$

$$x^2 - 4x + 4 + y^2 + 8y + 16 = 2$$

$$x^2 + y^2 - 4x + 8y - 5 = 0$$

(iv) For tangents parall to x-axis

$$x = 2 + 5 \quad \text{OR} \quad x = 2 - 5$$

$\therefore x = 7$ and $x = -3$ ar

tangen