聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS **4047 / 01**

Name : ____________________ **Date** : **12 August 2014**

Register No : ____________________ **Duration** : **2 h**

Class : ____________________

Additional Materials: 7 Sheets of Writing Paper

Instruction to Candidates

Write your register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.

The use of a scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Setter: Mr Adrian Goh
Vetter: MsKohSwee Kun
MdmHayati
Mrs Nathan

2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1. Find the set of values of the constant k for which the line $y = 3x - k$ intersects the curve $y = 2x^2 + kx - 3$ at two distinct points. [5]

2. Find the x-coordinate of the stationary point of the curve $y = \frac{1}{2}e^{2x} - e^{x} - 2x + 1$. [5]

3. The roots of the quadratic equation $4x^2 - x + 16 = 0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, where $\alpha + \beta > 0$. [6]

4. It is given that $\int_a^b \frac{5x}{2} dx = 5$, where a and b are constants.

   Find the possible values of k if $\int_{ka}^{kb} \frac{5x}{2} dx = 20$. [4]

5. (a) Express $\left(\frac{4}{2+\sqrt{5}} - 3 - 2\sqrt{5}\right)^2$ in the form $a + b\sqrt{5}$ where a and b are integers. [4]

   (b) Given that $ab - 4b + a - 4$ can be expressed as $(a - 4)(b + 1)$, solve
   $$6^x - 4(3^x) + 2^x - 4 = 0.$$ [4]

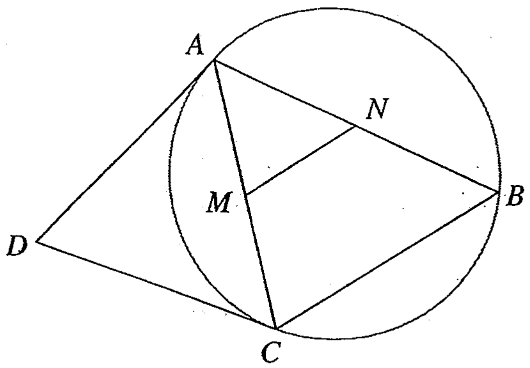
6. The variables x and y are connected by the equation $xy^2 - px - qy = 0$, where p and q are constants. Using experimental values of x and y, a graph was drawn in which y^2 was plotted on the vertical axis against $\frac{y}{x}$ on the horizontal axis. The straight line which was obtained passed through the points $(1, 4\frac{1}{3})$ and $(6, 6)$.

   Estimate

   (a) the value of p and of q, [4]

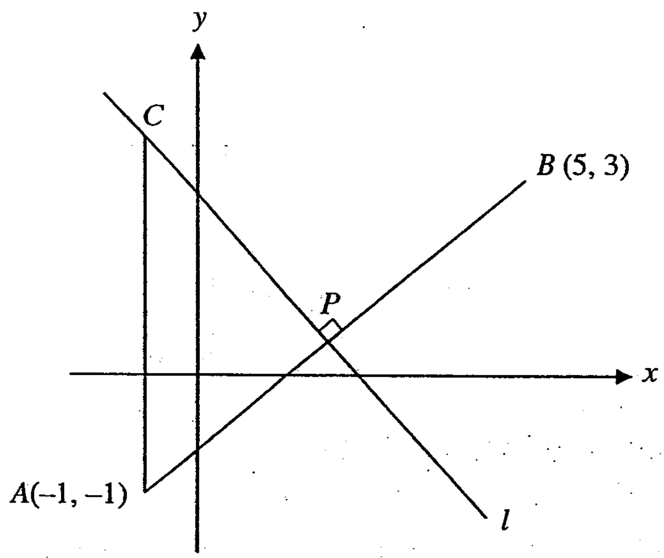
   (b) the values of x when $\frac{y}{x} = 3$. [3]

7.



The diagram shows points A, B and C lying on a circle.
The tangents to the circle at A and C intersect at D.
M and N are the mid-points of AC and AB respectively.
The line AC bisects $\angle BCD$.
Prove that ΔAMN is similar to ΔDCA. [4]

8.



In the diagram, l is the perpendicular bisector of the line joining A (–1, –1) and B (5, 3).
C is a point on l such that AC is parallel to the y-axis.

(a) Find the coordinates of C. [4]

(b) Write down the coordinates of D such that $ACPD$ is a parallelogram. [1]

(c) Find the area of the parallelogram $ACPD$. [2]

9. A particle, travelling in a straight line, passes a fixed point O on the line with a velocity of 2ms^{-1}. The acceleration, $a\text{ms}^{-2}$, of the particle, t s after passing O, is given by $a=-\dfrac{1}{(t+1)^2}$.

(a) Obtain an expression, in terms of t, for the velocity of the particle after passing O. [3]

(b) Show that the object will never come to rest. [1]

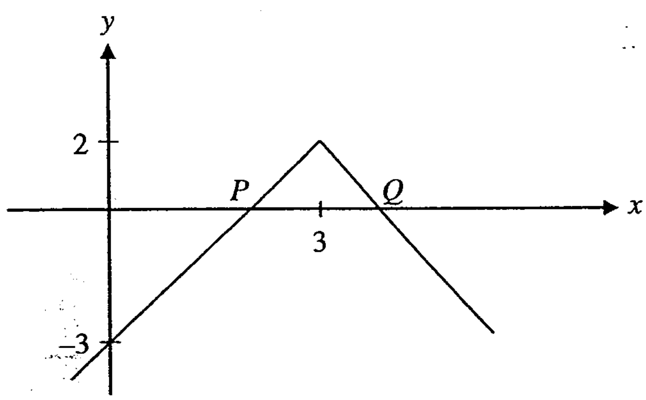
(c) Find the total distance travelled by the particle between $t=0$ and $t=5$. [3]

10. (a) Show that $\cot 2A$ can be expressed as $\dfrac{1}{2\tan A}-\dfrac{1}{2}\tan A$. [2]

(b) **Hence** solve $\tan A(3-4\cot 2A)=3$ for $0°\le A\le 360°$. [5]

(c) Using your answers in (b), state the number of solutions of the equation $\tan\frac{1}{2}A(3-4\cot A)=3$ in the range $-360°\le A\le 720°$. [1]

11.



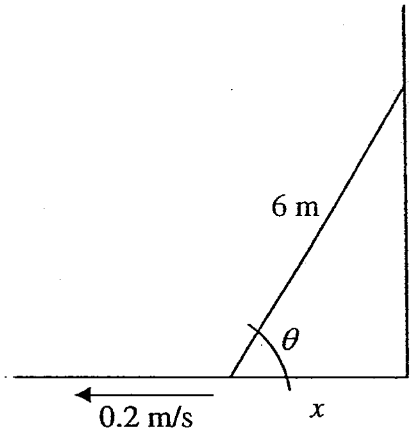
The diagram above shows part of the graph of $y=a-|bx+c|$ where a, b and c are constants.

(a) Find the values of a, b and c. [3]

(b) Find the x-coordinates of P and Q. [3]

(c) **Hence** sketch the graph of $y=|a-|bx+c||$. [2]

12.



In the diagram, a ladder, 6 m, is leaning against a vertical wall. The distance between the base of the ladder and the wall is x m. A force is exerted, pulling the base of the ladder away from the wall at a constant rate of 0.2 m/s.

(a) (i) Show that $x = 6\cos\theta$. [1]

(ii) Find $\dfrac{d\theta}{dt}$ at the instant when $\theta = 0.367$ radians. [3]

(b) If it took 20 seconds for the ladder to be flat on the floor, find

(i) the initial value, in radians, of θ, [2]

(ii) the time taken for $\dfrac{d\theta}{dt}$ to reach a value of -0.0385. [5]

End of Paper

Answers for 2014 Prelim Sec 4E A Math Paper 1

1. $k < 3$ or $k > 11$

2. $x = 0.693$

3. $x^2 - \frac{1}{2}\sqrt{17}x + 2 = 0$

4. $k = 2$ or -2

5. (a) $141 - 44\sqrt{5}$
   (b) $x = 2$

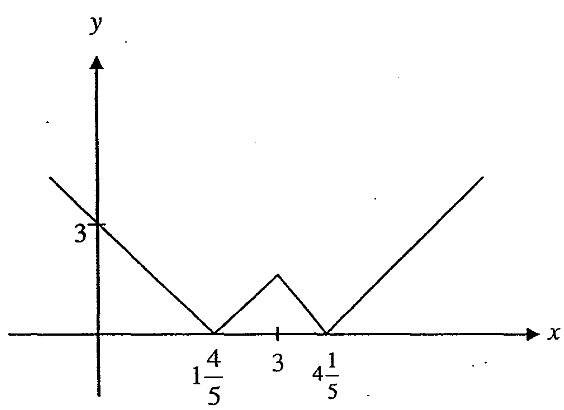
6. (a) $q = \frac{1}{3}, p = 4$
   (b) $x = 0.745, \ -0.745$

8. (a) C is $(-1, 5.5)$
   (b) D is $(2, -5\frac{1}{2})$
   (c) 19.5 sq units

9. (a) $v = \frac{1}{(t+1)} + 1$
   (b) Show $\frac{1}{(t+1)} + 1 > 0$ for all non negative values of t.
   (c) 6.79 (3sf)

10. (b) $A = 111.8°, 291.8°$
    or $45°$ or $225°$
    (c) 6

11. (a) $a = 2$
    $c = 5$
    $b = -1\frac{2}{3}$
    (b) $x = 1\frac{4}{5}$
    or $x = 4\frac{1}{5}$
    (c)



12. (aii) $\frac{d\theta}{dt} = -0.0929$ rad / s (3sf)
    (bi) 1.23 (3sf)
    (bii) 5.01 secs

Marking Scheme for 2014 Prelim A Math Paper 1

1.	$3x - k = 2x^2 + kx - 3$	[M1 for single equation]
	$0 = 2x^2 + (k-3)x + k - 3$	
	For distinct roots,	[M1 for showing knowledge that discriminant must be greater than 0]
	$b^2 - 4ac > 0$	
	$(k-3)^2 - 4(2)(k-3) > 0$	[M1 for correct substitution]
	$(k-3)(k-11) > 0$	[M1 for factorising]
	$k < 3$ or $k > 11$	[A1 for answer]

2.	$\frac{dy}{dx} = e^{2x} - e^x - 2$	[M1]
	At stat. point,	
	$e^{2x} - e^x - 2 = 0$	[M1]
	$(e^x - 2)(e^x + 1) = 0$	[M1]
	$e^x = 2$ or -1 (rej)	[A1 for rejecting]
	$x = 0.693$	[A1]

3.	Sum of roots: $\alpha^2 + \beta^2 = \frac{1}{4}$		[M1]
	Product of roots: $\alpha^2\beta^2 = 4$		[M1]
	$(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$		[M1]
	$= \frac{1}{4} + 2(2)$	or $\frac{1}{4} + 2(-2)$ (rej)	(cannot use $\alpha\beta = -2$ because $(\alpha+\beta)^2 > 0$)
	$\alpha + \beta = \sqrt{\frac{17}{4}}$		[M1]
	$\alpha\beta = 2$		[M1]
	The equation is		
	$x^2 - \frac{1}{2}\sqrt{17}x + 2 = 0$	(o.e)	[A1]

4.	$\left[\frac{5x^2}{4}\right]_a^b = 5$	[M1 for correct integration]
	$\frac{5}{4}(b^2 - a^2) = 5$	[M1 for forming equation in a and b]
	Also, $\frac{5}{4}(k^2b^2 - k^2a^2) = 20$	
	$\frac{5}{4}k^2(b^2 - a^2) = 20$	
	$k^2(5) = 20$	[M1 for removing a and b through substitution]
	$k = 2$ or -2	[A1]

5. (a) $\left(\frac{8-4\sqrt{5}}{4-5}-3-2\sqrt{5}\right)^2$ [M1 for rationalising denominator]

$= \left(2\sqrt{5}-11\right)^2$ [M1 for correct simplifying]

$= 20-44\sqrt{5}+121$

$= 141-44\sqrt{5}$ [A1 for 141]

[A1 for $-44\sqrt{5}$]

(b) Seeing 6^x as $2^x \times 3^x$ [M1]

$a=2^x, b=3^x$ [M1]

$\left(2^x-4\right)\left(3^x+1\right)=0$

$2^x=4$ or $3^x=-1$ (reject) [M1]

$x=2$ [A1]

6. (a) $xy^2 = px+qy$

$y^2 = p+q\left(\frac{y}{x}\right)$ [M1 for changing to linear form]

Gradient $= \frac{6-4\frac{1}{3}}{6-1}$ [M1]

$q=\frac{1}{3}$ [A1]

Sub (6, 6),

$6=p+\frac{1}{3}(6)$

$p=4$ [A1]

(b) When $\frac{y}{x}=3$,

$y^2=4+\frac{1}{3}(3)$ [M1]

$y=\pm\sqrt{5}$

Sub into original equation to find x:

$x(5)=4x+\frac{1}{3}\left(\sqrt{5}\right)$ or $x(5)=4x+\frac{1}{3}\left(-\sqrt{5}\right)$ [M1]

$x=0.745, -0.745$ [A1 for 2 answers]

7. Using Alt Seg Theorem $\angle DAC = \angle ABC$ [M1 for applying alt segthm]
Using Mid Point Theorem $MN // CB$ [M1 for mid pt theorem]
Therefore $\angle DCA = \angle ACB$ ("bisects")
$= \angle AMN$ (corresponding angle)
$\angle DAC = \angle ABC$ (altseg, shown above)
$= \angle ANM$ (corresponding angle)
[M1 for showing 2 angles same]

Since 2 angles are the same, the 3rd angle must be the same, therefore ΔAMN is similar to ΔDCA. [A1 for appropriate conclusion, either through showing this statement or calculating 3rd angle]

8. (a) Coordinate of P is $(2,1)$ [M1]

Gradient of AB is $\frac{2}{3}$

Gradient of l is $-\frac{3}{2}$ [M1]

Equating gradient of PC to l OR Finding equation of l $2y = -3x + 8$

[M1]

When $x = -1$, $y = 5.5$.
C is $(-1, 5.5)$ [A1]

(b) D is $(2, -5\frac{1}{2})$ [B1]

(c) Area $= 2\left(\frac{1}{2}\times 3\times 6.5\right)$ (area of 2 triangles) [M1, or for using "shoelace" method]

$= 19.5$sq units. [A1]

9 (a) $v = \int -\frac{1}{(t+1)^2} dt$

$= \frac{1}{(t+1)} + c$ [M1]

When $t = 0$, $v = 2$. [M1]
$2 = 1 + c$
$c = 1$

$v = \frac{1}{(t+1)} + 1$ [A1]

(b) since $t > 0$,

So, $t + 1 > 0$

So, $\frac{1}{(t+1)} > 0$

So, $\frac{1}{(t+1)} + 1 > 0$ for all non negative values of t. [B1]

(c) $s = \int_0^5 \left(\frac{1}{t+1} + 1 \right) dt$

$= [\ln(t+1) + t]_0^5$ [M1]

$= \ln 6 + 5 - 0$ [M1]

$= 6.79$ (3sf) [A1]

OR

$s = \int \left(\frac{1}{t+1} + 1 \right) dt$

$= \ln(t+1) + t + c$ [M1]

$t = 0, s = 0,$

so $c = 0$

Therefore, $s = \ln(t+1) + t$ [M1]

Since never come to rest,

Dist $= \ln(5+1) + 5$

$= 6.79$ [A1]

10. (a) $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$ [M1 for knowing to use double angle for tan 2*A*]

Cot $2A = \frac{1 - \tan^2 A}{2\tan A}$

$= \frac{1}{2\tan A} - \frac{1}{2}\tan A$ (shown) [A1]

(b) $\tan A\left(3 - 4\left(\frac{1}{2\tan A} - \frac{1}{2}\tan A\right)\right) = 3$ [M1 for applying earlier part]

$3\tan A - 2 + 2\tan^2 A = 3$ [M1]

$(2\tan A + 5)(\tan A - 1) = 0$ [M1]

$A = 111.8°, 291.8°$ or $45°$ or $225°$ [A1 for 2-3 answers]

[A2 for 4 answers]

(c) 6 [B1]

11. (a) $a = 2$ [B1]

$c = 5$ [B1]

$b = -1\frac{2}{3}$ [B1]

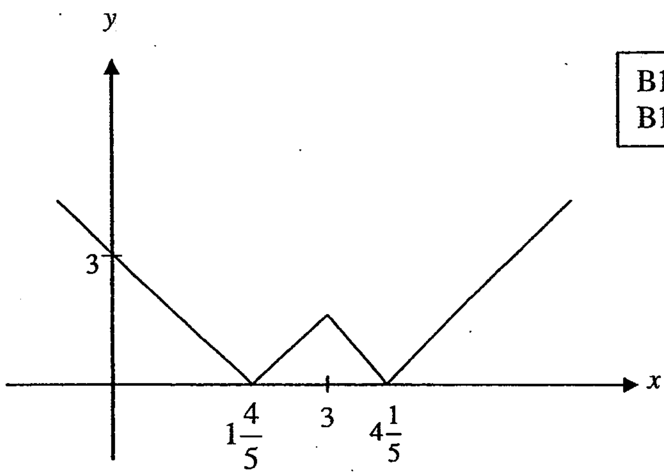
(b) When $y = 0$,

$\left|-\frac{5}{3}x + 5\right| = 2$ [M1]

$-\frac{5}{3}x + 5 = 2$ or $-\frac{5}{3}x + 5 = -2$ [M1]

$x = 1\frac{4}{5}$ or $x = 4\frac{1}{5}$ [A1]

(c)



B1 for "flipping lines"
B1 for correct y intercept value

12. (ai) $\cos\theta = \frac{x}{6}$

$x = 6\cos\theta$ [B1, must show previous line]

(aii) $\frac{dx}{d\theta} = -6\sin\theta$ [M1]

$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$

$0.2 = -6\sin 0.367 \times \frac{d\theta}{dt}$ [M1]

$\frac{d\theta}{dt} = -0.0929$ rad / s (3sf) [A1]

(bi) distance moved in 20s = 4 m

Therefore, initial value of $x = 6 - 4 = 2$ m. [M1]

Initial value of $\theta = \cos^{-1}\frac{2}{6}$

$= 1.23$ (3sf) [A1]

(bii) When $\frac{d\theta}{dt} = -0.0385$,

$0.2 = \frac{dx}{d\theta} \times (-0.0385)$ [M1]

$\frac{dx}{d\theta} = \frac{0.2}{-0.0385}$

$-6\sin\theta = \frac{0.2}{-0.0385}$ [M1]

$\sin\theta = \frac{0.2}{6 \times 0.0385}$

$\theta = 1.04675$ [M1]

When $\theta = 1.04675$,

$x = 6 \cos 1.04675$ [M1]

$= 3.00233$

Distance moved = 1.00233

Time taken = 1.00233 / 0.2

= 5.01 secs [A1]

聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS
PAPER 2 **4047/02**

Name : ______________________ **Date** : **6 Aug 2014**

Register No : ______________________ **Duration :2h30 min**

Class : ______________________

Additional Materials needed
Writing Paper (8 sheets)

Instructions to Candidates

Write your register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: MsKohSwee Kun
Vetters: Mdm Noor Hayati
Mrs Nathan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

$$a^2=b^2+c^2-2bc\cos A.$$

$$\Delta=\frac{1}{2}bc\sin A.$$

Answer all the questions.

1 **(i)** Sketch the graph of $y = 5x^{\frac{1}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = \frac{1}{5}x^{-\frac{3}{2}}$ for $x > 0$. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

2 **(a)** **(i)** Find the first four terms of the expansion, in ascending powers of x, of $(3+x)^5$. [2]

Hence obtain

(ii) the coefficient of x^3 in the expansion of $(3+x)^5(2x^2+x-1)$, [2]

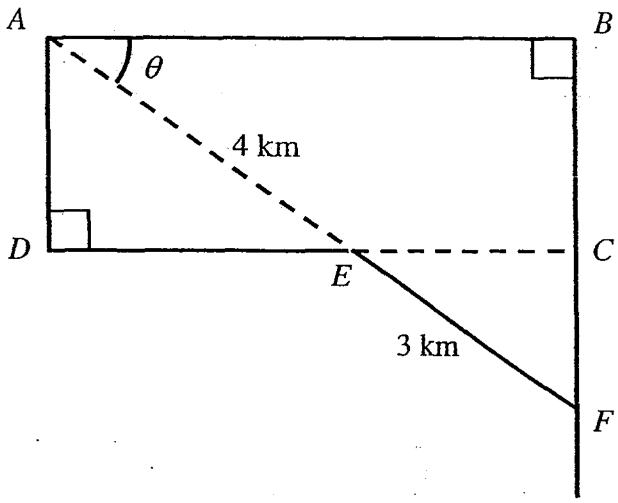
(iii) the coefficient of y^3 in the expansion of $(3-y+y^2)^5$. [2]

(b) Determine if the term independent of x exists in the expansion of $(x^2-2x)^{100}$. [2]

3 **(i)** Differentiate $3x\cos x$ with respect to x. [2]

(ii) Using your answer to part **(i)**, find $\int 2x\sin x \, dx$ and **hence** evaluate $\int_0^{\frac{\pi}{2}} 2x\sin x \, dx$. [5]

4



The diagram shows a Long Distance Race Route *FBADE*.
ABCD is a rectangular plot of land. A line through *A*, at an angle θ to *AB* intersects *DC* at *E* and *BC* produced at *F*.
During Holy Innocents' High School Long Distance Race, runners will start from point *F* and run along the straight paths *FB*, *BA*, *AD*, *DE* and return along the path *EF* to finish at *F*.
$AE = 4$ km and $EF = 3$ km.
The total length of the race is L km.

(i) Show that L can be expressed as $a\sin\theta + b\cos\theta + c$, where a, b and c are constants to be found. [3]

(ii) Express L in the form $R\cos(\theta - \alpha) + c$, where $R > 0$ and α is an acute angle. [3]

The total length of the race is found to be 18 km.

(iii) Find the value of θ. [2]

5 A curve has the equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x-2}{2x+1}$ for $x > 0$.

(i) Obtain an expression for $\mathrm{f}'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

(iii) Determine, with explanation, whether f is an increasing or decreasing function. [1]

(iv) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

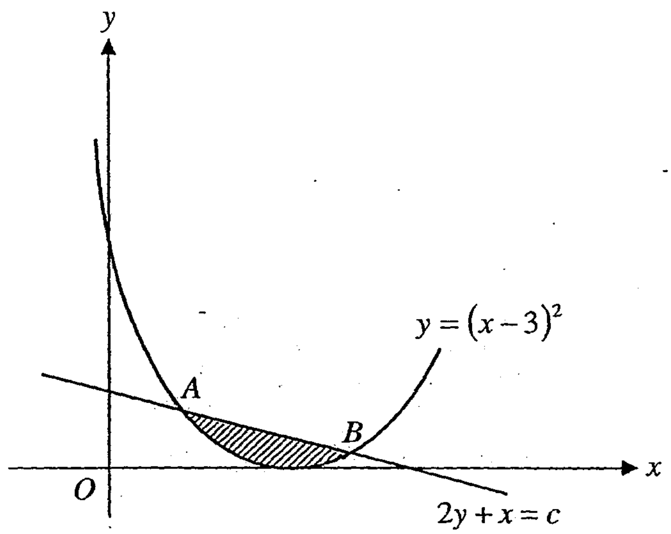
6 Given that $\dfrac{ax^3+x^2-14x-12}{x^2-9}=2x+b+\dfrac{4x+c}{x^2-9}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part (i), find

(ii) $\displaystyle\int\frac{ax^3+x^2-14x-12}{x^2-9}\,dx$. [6]

7



The diagram shows part of the curve $y=(x-3)^2$ intersecting the line $2y + x = c$, where c is a constant, at points A and B.

The tangent at B is perpendicular to the line $2y + x = c$.

(i) Find the x-coordinate of B. [3]

(ii) **Hence** show that $c = 6$. [2]

(iii) Using your answer to part **(ii)**, find the area of the shaded region. [7]

8 (a) (i) Prove the identity $\tan 2A(2\cos A - \sec A) = 2\sin A$. [4]

(ii) **Hence** solve the equation $\tan 2A(2\cos A - \sec A) = 1$, for $0 < x \leq \pi$, giving your answers in terms of π. [3]

(b) (i) Given that P is acute and $\tan P = \frac{4}{3}$, without using a calculator, find the value of $\sin P$ and of $\cos P$. [2]

(ii) Using the values of $\sin P$ and $\cos P$ found in part **(i)**, show that $3\cos(Q+P) + 4\sin(Q+P) = 5\cos Q$. [3]

9 A circle, centre C, passes through the points $P(1, -2)$, $Q(9, -2)$ and $R(5, 6)$.

(i) Show that the coordinates of C is (5, 1). [6]

(ii) Find the radius of the circle. [2]

(iii) Find the equation of the circle in the form $x^2 + y^2 + fx + gy + h = 0$, where f, g and h are integers. [2]

(iv) Does the point (10, 4) lie outside, inside or on the circle? Justify your answer. [2]

10 (a) Given that $\log_x(p^2q^3) = m$ and $\log_x(pq^2) = n$, find $\log_x \sqrt{pq}$ in terms of m and n. [3

(b) Solve the equations

(i) $3^{x+1} = 5^x$, [2

(ii) $\log_3(4x) + \log_3(x-1) = 1$. [4

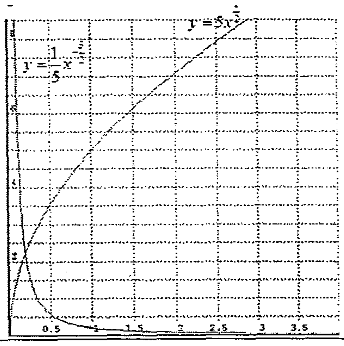
(c) The population of a village at the beginning of the year 2004 was 350. The populatio increased so that, after a period of n years, the new population is given by $350(1.08)^n$.

Find

(i) the population at the beginning of the year 2014, [

(ii) the year in which the population first reached 3680. [

End of Paper

Question	Answer		Question	Answer
1(i) & (ii)			5(iv)	Since $(2x+1)^3>0$ for $x>0$, then $f''(x)=\dfrac{-20}{(2x+1)^3}<0\Rightarrow$ Gradient is an decreasing function.
1(iii)	(0.2, 2.24)		6(i)	$a=2$, $b=1$ and $c=-3$
1(iv)	Since product of gradients $=5.59\times-16.77$ $\neq-1$, therefore the tangents at the point of intersection are not perpendicular.		6(ii)	$x^2+x+\frac{5}{2}\ln(x+3)$ $+\frac{3}{2}\ln(x-3)+c$
2(a)(i)	$243+405x+270x^2+90x^3$		7(i)	4
2(a)(ii)	990		7(ii)	Sub B(4, 1) into the line equation $2y+x=c$ to show $c=6$.
2(a)(iii)	–630		7(iii)	$2\frac{29}{48}$ or 2.60
2(b)	The term independent of x does not exist as there should only be 101 terms in the expansion of $(x^2-2x)^{100}$ and so it is not possible to obtain any term when $r=200$ is substituted.		8(a)(i)	Refer to next page
3(i)	$3\cos x-3x\sin x$		8(a)(ii)	$A=\frac{\pi}{6},\ \frac{5\pi}{6}$
3(ii)	$\int 2x\sin x\,dx=2\sin x-2x\cos x+c$; 2		8(b)(i)	$\sin P=\frac{4}{5}$, $\cos P=\frac{3}{5}$
4(i)	L $=FB+BA+AD+DE+EF$ $=11\sin\theta+11\cos\theta+3$		8(b)(ii)	Refer to next page
4(ii)	$L=15.6\cos(\theta-45°)+3$		9(i)	Refer to next page
4(iii)	$\theta=60.4°$		9(ii)	5 units
5(i)	$\dfrac{5}{(2x+1)^2}$		9(iii)	$x^2-10x+y^2-2y+1$ $=0$
5(ii)	$y=10-5x$		9(iv)	Outside the circle
5(iii)	Since $(2x+1)^2>0$ for $x>0$, then $f'(x)=\dfrac{5}{(2x+1)^2}>0\Rightarrow f(x)$ is an increasing function.		10(a)	$\log_x\sqrt{pq}=\dfrac{m-n}{2}$

Question	Answer
10(b)(i)	2.15
10(b)(ii)	$x=-\frac{1}{2}$ (rej) or $1\frac{1}{2}$
10(c)(i)	756
10(c)(ii)	Year 2034

8(a)(i)

$$\tan 2A(2\cos A-\sec A)$$
$$=\tan 2A\left(2\cos A-\frac{1}{\cos A}\right)$$
$$=\tan 2A\left(\frac{2\cos^2 A-1}{\cos A}\right)$$
$$=\tan 2A\left(\frac{\cos 2A}{\cos A}\right)$$
$$=\frac{\sin 2A}{\cos 2A}\left(\frac{\cos 2A}{\cos A}\right)$$
$$=\frac{\sin 2A}{\cos A}$$
$$=\frac{2\sin A\cos A}{\cos A}$$
$$=2\sin A \text{ (proven)}$$

8(b)(ii)

$$3\cos(Q+P)+4\sin(Q+P)$$
$$=3\cos Q\cos P-3\sin Q\sin P+4\sin Q\cos P+4\cos Q\sin P$$
$$=3\bullet\frac{3}{5}\cos Q-3\bullet\frac{4}{5}\sin Q+4\bullet\frac{3}{5}\sin Q+4\bullet\frac{4}{5}\cos Q$$
$$=\frac{9}{5}\cos Q-\frac{12}{5}\sin Q+\frac{12}{5}\sin Q+\frac{16}{5}\cos Q$$
$$=\frac{25}{5}\cos Q$$
$$=5\cos Q \text{ (shown)}$$

9(i)

Method to solve:

Method 1 – Using concept of perpendicular bisectors of the sides of triangle inscribed in the circle and their intersection

Method 2 – Using concept of radius, solve simultaneous equations of $PC=QC=RC$.

聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS
MARKING SCHEME

ADDITIONAL MATHEMATICS **4047/02**
PAPER 2

Name : ____________________ **Date** : **06 Aug 2014**

Register No : ____________________ **Duration :2h30 min**

Class : ____________________

Additional Materials needed
Writing Paper (10 sheets)

Instructions to Candidates

Write your register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: MsKohSwee Kun
Vetter: Mdm Noor Hayati

This paper consists of 19 printed pages, inclusive of this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

wheren is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

$$a^2=b^2+c^2-2bc\cos A.$$

$$\Delta=\frac{1}{2}bc\sin A.$$

Answer all the questions.

1 **(i)** Sketch the graph of $y = 5x^{\frac{1}{2}}$ for $x > 0$. [1]

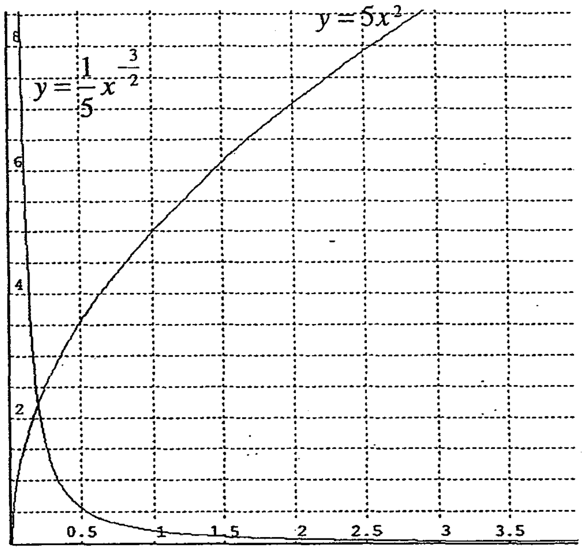
(ii) On the same diagram, sketch the graph of $y = \frac{1}{5}x^{-\frac{3}{2}}$ for $x > 0$. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

Solution:

(i) & (ii)



(iii)

$$5x^{\frac{1}{2}} = \frac{1}{5}x^{-\frac{3}{2}}$$

$25x^2 = 1$ --- M1

$x = 0.2$

Point of intersection (0.2, 2.24) --- A1 (o.e)

(iv)

$$\frac{d\left(5x^{\frac{1}{2}}\right)}{dx} = \frac{5}{2}x^{-\frac{1}{2}} = \frac{5}{2\sqrt{x}}$$ --- M1 (Differentiation)

At $x = 0.2$, gradient of tangent $= \dfrac{5}{2\sqrt{0.2}} = 5.59$

$$\frac{d\left(\frac{1}{5}x^{-\frac{3}{2}}\right)}{dx} = -\frac{3}{10}x^{-\frac{5}{2}} = -\frac{3}{10x^{\frac{5}{2}}} = -\frac{3}{10(0.2)^{\frac{5}{2}}} = -16.77$$ --- M1 (Differentiation)

Since product of gradients $= 5.59 \times -16.77 \neq -1$, therefore the tangents at the point of intersection are not perpendicular. --- M1, A1 (M1 for value of gradients)

2 (a) (i) Find the first four terms of the expansion, in ascending powers of x, of $(3+x)^5$. [2]

Hence obtain

(ii) the coefficient of x^3 in the expansion of $(3+x)^5(2x^2+x-1)$, [2]

(iii) the coefficient of y^3 in the expansion of $(3-y+y^2)^5$. [2]

(b) Determine if the term independent of x exists in the expansion of $(x^2-2x)^{100}$. [2]

Solution:

(a)(i)

$(3+x)^5$

$=3^5+\binom{5}{1}(3^4)x+\binom{5}{2}(3^3)x^2+\binom{5}{3}(3^2)x^3$

$=243+405x+270x^2+90x^3$ --- B2 (B1 for any 2 correct terms)

(a)(ii)

$(3+x)^5(2x^2+x-1)$

$=(243+405x+270x^2+90x^3+...)(2x^2+x-1)$

Terms in x^3

$=405\times 2x^3+270x^3-90x^3$ --- M1 (Expanding the terms with x^3)

$=990x^3$

Coefficient of $x^3=990$ --- A1

(a)(iii)

$(3-y+y^2)^5$

$=(3+y^2-y)^5$

$=243+405(y^2-y)+270(y^2-y)^2+90(y^2-y)^3$

$=243+405y^2-405y+270(y^4-2y^3+y^2)+90(y^2-y)(y^4-2y^3+y^2)$

$=243+405y^2-405y+270y^4-540y^3+270y^2-90y^3+....$

--- M1 (Replacing x with y^2-y and expanding to find terms that include y^3)

Terms in y^3

$=-540y^3-90y^3$

$=-630y^3$

The coefficient of y^3 is -630. --- A1

(b)

General term of the expansion of $\left(x^2-2x\right)^{100}$, $T_{r+1}=\binom{100}{r}\left(x^2\right)^{100-r}\left(-2x\right)^r$.

Term independent of $x \Rightarrow$ power of $x = 0$

Therefore,
$x^{200-2r+r}=x^0$
$200-r=0$
$r=200$

--- M1 (Show that $r = 200$ if term independent of x exists)

The term independent of x does not exist as there should only be 101 terms in the expansion of $\left(x^2-2x\right)^{100}$ and so it is not possible to obtain any term when $r = 200$ is substituted.

--- A1 (Explain that it is not possible for term independent of x to exist by quoting that there are only 101 terms in the expansion)

3 **(i)** Differentiate $3x\cos x$ with respect to x. [2]

(ii) Using your answer to part **(i)**, find $\int 2x\sin x \; dx$ and **hence** evaluate $\int_0^{\frac{\pi}{2}} 2x\sin x \; dx$. [5]

Solution:

(i)

$$\frac{d(3x\cos x)}{dx}$$
$$= 3\cos x - 3x\sin x$$

--- B1 for 3 cos x and B1 for $-3x\sin x$

(ii)

$\int(3\cos x - 3x\sin x)\,dx = 3x\cos x + c_1$ --- M1 (Integration as reverse of differentiation)

$\int 3\cos x\,dx - \int 3x\sin x\,dx = 3x\cos x + c_1$

$3\int\cos x\,dx - \frac{3}{2}\int 2x\sin x\,dx = 3x\cos x + c_1$

$3\sin x - \frac{3}{2}\int 2x\sin x\,dx = 3x\cos x + c_2$ --- M1 (Differentiating 3 cos x)

$\frac{3}{2}\int 2x\sin x\,dx = 3\sin x - 3x\cos x + c_2$

$\int 2x\sin x\,dx = \frac{2(3\sin x - 3x\cos x)}{3} + c$

$\int 2x\sin x\,dx = 2\sin x - 2x\cos x + c$ --- A1 (no marks if no constant c is seen)

$\int_0^{\frac{\pi}{2}} 2x\sin x \; dx$

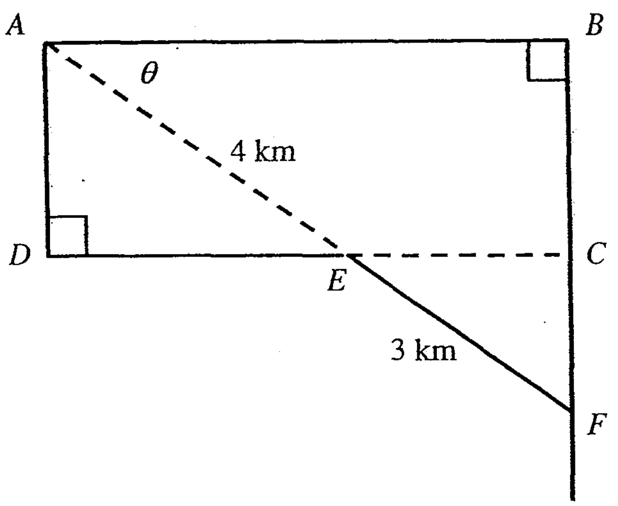
$= [2\sin x - 2x\cos x + c]_0^{\frac{\pi}{2}}$

$= \left[2\sin\frac{\pi}{2} - 2\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2} + c\right] - [2\sin 0 - 2(0)\cos 0 + c]$ --- M1

$= 2(1) - 0 + c - (0 + c)$

$= 2$ --- A1

4



The diagram shows a Long Distance Race Route $FBADE$.
$ABCD$ is a rectangular plot of land. A line through A, at an angle θ to AB intersects DC at E and BC produced at F.
During Holy Innocents' High School Long Distance Race, runners will start from point F and run along the straight paths FB, BA, AD, DE and return along the path EF to finish at F.
$AE = 4$ km and $EF = 3$ km.
The total length of the race is L km.

(i) Show that L can be expressed as $a\sin\theta + b\cos\theta + c$, where a, b and c are constants to be found. [3]

(ii) Express L in the form $R\cos(\theta - \alpha) + c$, where $R > 0$ and α is an acute angle. [3]

The total length of the race is found to be 18 km.

(iii) Find the value of θ. [2]

Solution:
(i)
$AB = 7\cos\theta$
$DE = 4\cos\theta$
$AD = BC = 4\sin\theta$
Since $ABCD$ is a rectangle, AB is parallel to DC. Therefore, angle $CEF = \theta$ (corresponding angle)
$CF = 3\sin\theta$ --- M1

L
$= FB + BA + AD + DE + EF$
$= 3\sin\theta + 4\sin\theta + 7\cos\theta + 4\sin\theta + 4\cos\theta + 3$ --- M1
$= 11\sin\theta + 11\cos\theta + 3$ --- A1

(ii)
$L = 11\sin\theta + 11\cos\theta + 3$
$L = 11\cos\theta + 11\sin\theta + 3$

$R = \sqrt{11^2 + 11^2} = \sqrt{242} = 11\sqrt{2} = 15.556$ --- M1

$\alpha = \tan^{-1}\left(\frac{11}{11}\right) = 45°$ --- M1

Therefore, $L = 15.6\cos(\theta - 45°) + 3$ --- A1

(iii)

$15.556\cos(\theta - 45°) + 3 = 18$

$\cos(\theta - 45°) = 0.96425$

$(\theta - 45°) = \cos^{-1} 0.96425$ --- M1

$\theta = 15.366 + 45$

$\theta = 60.4°$ --- A1

5 A curve has the equation $y = f(x)$, where $f(x) = \dfrac{x-2}{2x+1}$ for $x > 0$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

(iii) Determine, with explanation, whether f is an increasing or decreasing function. [1]

(iv) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

Solution:

(i)

$$f'(x) = \frac{2x+1-2(x-2)}{(2x+1)^2}$$ --- M1 (Applying Quotient Rule)

$$= \frac{5}{(2x+1)^2}$$ --- A1

(ii)

Curve crosses x-axis, y = 0.

Therefore, $\dfrac{x-2}{2x+1} = 0$

$x = 2$

At $x = 2$, gradient of tangent $= \dfrac{5}{(2(2)+1)^2} = \dfrac{1}{5}$ --- M1 (gradient of tangent)

Therefore, gradient of normal $= -5$ --- M1 (gradient of normal)

Equation of normal:

$y - 0 = -5(x-2)$ --- M1 (Form line equation)

$y = 10 - 5x$ --- A1 (o.e.)

(iii)

$$f'(x) = \frac{5}{(2x+1)^2}$$

Since $(2x+1)^2 > 0$ for $x > 0$, then $f'(x) = \dfrac{5}{(2x+1)^2} > 0 \Rightarrow$ f(x) is an increasing function.

--- B1

(iv)

$f''(x) = 5 \times -2 \times 2 \times (2x+1)^{-3}$ --- M1 (Applying Quotient Rule)

$$= -\frac{20}{(2x+1)^3}$$ --- M1

Since $(2x+1)^3 > 0$ for $x > 0$, then $f''(x) = \dfrac{-20}{(2x+1)^3} < 0 \Rightarrow$ Gradient is an decreasing function. --- B1

6 Given that $\frac{ax^3+x^2-14x-12}{x^2-9}=2x+b+\frac{4x+c}{x^2-9}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part (i), find

(ii) $\int\frac{ax^3+x^2-14x-12}{x^2-9}\,dx$. [6]

Solution:

(i)

$$\frac{ax^3+x^2-14x-12}{x^2-9}=2x+b+\frac{4x+c}{x^2-9}$$

$ax^3+x^2-14x-12=(2x+b)(x^2-9)+4x+c$ --- M1

By comparing coefficient of x^3,

$a=2$ --- A1

By comparing coefficient of x^2,

$b=1$ --- A1

By comparing constant term,

$-12=-9b+c$

$-12=-9(1)+c$

$c=-3$ --- A1

(ii)

$$\frac{2x^3+x^2-14x-12}{x^2-9}=2x+1+\frac{4x-3}{x^2-9}$$

$$\frac{4x-3}{x^2-9}=\frac{4x-3}{(x+3)(x-3)}$$

$\frac{4x-3}{(x+3)(x-3)}=\frac{A}{x+3}+\frac{B}{x-3}$ --- M1 (Break into partial fractions)

$4x-3=A(x-3)+B(x+3)$

Comparing coefficient of x,

$4=A+B$ --- (1)

Comparing constant terms,

$-3=-3A+3B$

$1=A-B$ --- (2)

(1) + (2):

$5=2A$ --- M1 (Solving simultaneous equations)

$A = \frac{5}{2}$ --- A1

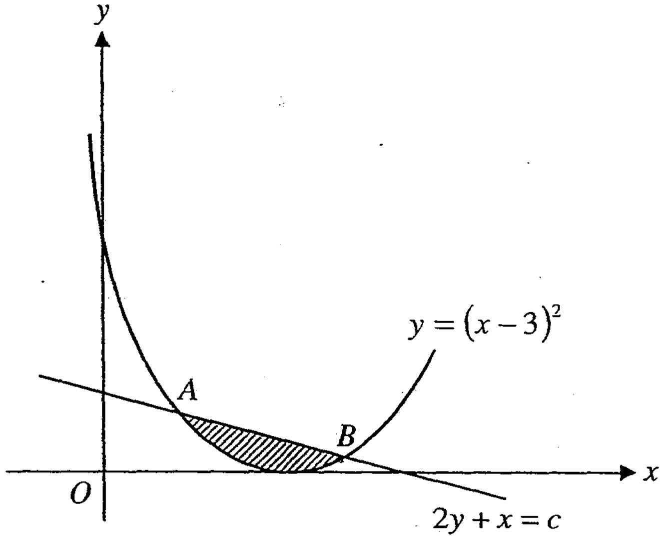
Substitute $A = \frac{5}{2}$ into (1),

$B = 4 - \frac{5}{2} = \frac{3}{2}$ --- A1

$$\int \frac{ax^3 + x^2 - 14x - 12}{x^2 - 9}\,\mathrm{d}x$$
$$= \int \frac{2x^3 + x^2 - 14x - 12}{x^2 - 9}\,\mathrm{d}x$$
$$= \int 2x + 1 + \frac{5}{2(x+3)} + \frac{3}{2(x-3)}\,\mathrm{d}x$$
$$= x^2 + x + \frac{5}{2}\ln(x+3) + \frac{3}{2}\ln(x-3) + c$$

--- B2 (minus 1 mark for each error)

7



The diagram shows part of the curve $y=(x-3)^2$ intersecting the line $2y +$ ... constant, at points A and B.

The tangent at B is perpendicular to the line $2y + x = c$.

(i) Find the x-coordinate of B.

(ii) **Hence** show that $c = 6$.

(iii) Using your answer to part **(ii)**, find the area of the shaded region.

(iii) Using **(ii)**, find the area of the shaded region.

Solution:

(i)

$$\frac{d(x-3)^2}{dx}=2(x-3)$$ --- M1

$2y+x=c$

$y=-\frac{1}{2}x+\frac{1}{2}c$

Therefore, gradient of tangent at $B = 2$ --- M1

$2(x-3)=2$

$x-3=1$

$x=4$

The x-coordinate of B is 4. --- A1

(ii)

Sub $x=4$ into $y=(x-3)^2$:

$y=(4-3)^2=1$ --- M1

Since the point B also lies on the line $2y+x=c$, it satisfies the line equation.

Sub $B(4, 1)$ into the line equation $2y+x=c$:

$2(1)+4=c$

$c=6$ (shown)

--- A1 (Obtain $c=6$ by substituting the coordinates of B into the line equation)

(iii)

$y=(x-3)^2$ --- (1)

$2y+x=6$ --- (2)

Sub (1) into (2):

$2(x-3)^2+x=6$ --- M1 (Solving simultaneous equations)

$2(x^2-6x+9)+x-6=0$

$2x^2-11x+12=0$

$(2x-3)(x-4)=0$

$x=1\frac{1}{2}$ or 4 --- M1

When $x=1\frac{1}{2}$,

$2y+1\frac{1}{2}=6$

$2y=4\frac{1}{2}$

$y=2\frac{1}{4}$ --- M1

Area of shaded region

= Area of trapezium – Area under the curve

$=\left[\frac{1}{2}\times\left(1+2\frac{1}{4}\right)\times\left(4-1\frac{1}{2}\right)\right]-\int_{1\frac{1}{2}}^{4}(x-3)^2\ dx$ --- M1 (Area of trapezium)

$=\frac{65}{16}-\left[\frac{(x-3)^3}{3}\right]_{1\frac{1}{2}}^{4}$ --- M1 (Integrating equation of curve)

$=\frac{65}{16}-\left[\frac{1}{3}-\left(-\frac{9}{8}\right)\right]$ --- M1 (Finding area using definite integral)

$= 2\frac{29}{48}$ units^2 or 2.60 units^2 --- A1

8 (a) (i) Prove the identity $\tan 2A(2\cos A - \sec A) = 2\sin A$.

(ii) **Hence** solve the equation $\tan 2A(2\cos A - \sec A) = 1$, for $0 < x \le \pi$, giving your answers in terms of π.

(b) (i) Given that P is acute and $\tan P = \frac{4}{3}$, without using a calculator, find $\sin P$ and of $\cos P$.

(ii) Using the values of $\sin P$ and $\cos P$ found in part (i), show that $3\cos(Q+P) + 4\sin(Q+P) = 5\cos Q$.

Solution:

(a)(i)

$\tan 2A(2\cos A - \sec A)$

$= \tan 2A\left(2\cos A - \frac{1}{\cos A}\right)$ --- M1 ($\sec A = \frac{1}{\cos A}$)

$= \tan 2A\left(\frac{2\cos^2 A - 1}{\cos A}\right)$

$= \tan 2A\left(\frac{\cos 2A}{\cos A}\right)$ --- M1 ($2\cos^2 A - 1 = \cos 2A$)

$= \frac{\sin 2A}{\cos 2A}\left(\frac{\cos 2A}{\cos A}\right)$

$= \frac{\sin 2A}{\cos A}$

$= \frac{2\sin A\cos A}{\cos A}$ --- M1 ($\sin 2A = 2\sin A\cos A$)

$= 2\sin A$ (proven) --- A1

(a)(ii)

$\tan 2A(2\cos A - \sec A) = 1$

$2\sin A = 1$

$\sin A = \frac{1}{2}$ --- M1

$A = \frac{\pi}{6}, \frac{5\pi}{6}$ --- A2 (A1 for each correct answer)

(b)(i)

By Pythagoras' Theorem, hypotenuse = 5.

$\sin P = \frac{4}{5}$ --- B1

$\cos P = \frac{3}{5}$ --- B1

(b)(ii)

$3\cos(Q+P)+4\sin(Q+P)$

$= 3\cos Q\cos P - 3\sin Q\sin P + 4\sin Q\cos P + 4\cos Q\sin P$ --- M2 (Expansion)

$= 3\bullet\frac{3}{5}\cos Q - 3\bullet\frac{4}{5}\sin Q + 4\bullet\frac{3}{5}\sin Q + 4\bullet\frac{4}{5}\cos Q$

$= \frac{9}{5}\cos Q - \frac{12}{5}\sin Q + \frac{12}{5}\sin Q + \frac{16}{5}\cos Q$

$= \frac{25}{5}\cos Q$

$= 5\cos Q$ (shown) --- A1

9 A circle, centre C, passes through the points $P(1, -2)$, $Q(9, -2)$ and $R(5, 6)$.

(i) Show that the coordinates of C is $(5, 1)$. [6]

(ii) Find the radius of the circle. [2]

(iii) Find the equation of the circle in the form $x^2 + y^2 + fx + gy + h = 0$, where f, g and h are integers. [2]

(iv) Does the point $(10, 4)$ lie outside, inside or on the circle? Justify your answer. [2]

Solution:

(i)

As the circle passes through all 3 points, the intersection of the perpendicular bisectors of PR and PQ is the centre as P, Q and R to the intersection of the perpendicular bisectors have equal length which is the radius (Concept of perpendicular bisector)

Midpoint of PQ

$= \left(\frac{1+9}{2}, -2\right)$

$= (5, -2)$

Equation of the perpendicular bisector of PQ : $x = 5$

--- M1 (or able to see R is above midpoint of PQ and conclude that x-coordinate of centre is 5)

Gradient of perpendicular bisector of $PR = -\frac{1}{2}$ --- M1

Midpoint of PR

$= \left(\frac{1+5}{2}, \frac{-2+6}{2}\right)$ --- M1

$= (3, 2)$

$\frac{y-2}{x-3} = -\frac{1}{2}$ --- M1 (Forming line equation)

$2(y-2) = 3 - x$

$2y - 4 = 3 - x$

$2y = 7 - x$

Equation of the perpendicular bisector of PR : $2y = 7 - x$ ---M1

Sub $x = 5$ into $2y = 7 - x$,

$2y = 7 - 5$

$y = 1$ --- A1

Therefore, centre of the circle is $(5, 1)$

(**Alternative method:** Using concept on radius. Form equations where $PC = RC = QC$ and solve simultaneous equations)

(ii)

Radius (From P to centre of circle)

$= \sqrt{(1-5)^2 + (-2-1)^2}$ --- M1

$= 5$ units --- A1

(**Alternative method:** Use of diagram, since R is directly on top of centre, therefore $RC = 5$ units)

(iii)

Equation of circle, C:

$(x-5)^2 + (y-1)^2 = 25$ --- M1 (or for finding f, g and h)

$x^2 - 10x + y^2 - 2y + 1 = 0$ --- A1

(iv)

Length from the point (10, 4) to centre of circle

$= \sqrt{(10-5)^2 + (4-1)^2}$

--- M1 (Find length from point to centre)

$= \sqrt{25+9}$

$= \sqrt{34}$

$= 5.83$ units

Since 5.83 > radius of the circle, this implies that the point (10, 4) lies outside the circle.

--- A1

(*Do not accept sketching of diagram*)

10 **(a)** Given that $\log_x(p^2q^3) = m$ and $\log_x(pq^2) = n$, find $\log_x\sqrt{pq}$ in terms of m and n. [3]

(b) Solve the equations

(i) $3^{x+1} = 5^x$, [2]

(ii) $\log_3(4x) + \log_3(x-1) = 1$. [4]

(c) The population of a village at the beginning of the year 2004 was 350. The population increased so that, after a period of n years, the new population is given by $350(1.08)^n$.

Find

(i) the population at the beginning of the year 2014, [2]

(ii) the year in which the population first reached 3680. [2]

Solution:

(a)

$\log_x\sqrt{pq} = \log_x(pq)^{\frac{1}{2}} = \frac{1}{2}\log_x(pq)$

$\log_x(p^2q^3) - \log_x(pq^2) = m - n$ --- M1

$\log_x\frac{(p^2q^3)}{(pq^2)} = m - n$

$\log_x(pq) = m - n$ --- M1

$\frac{1}{2}\log_x(pq) = \frac{m-n}{2}$

$\log_x\sqrt{pq} = \frac{m-n}{2}$ --- A1

(b)(i)

$3^{x+1} = 5^x$

$\lg 3^{x+1} = \lg 5^x$

$(x+1)\lg 3 = x\lg 5$ --- M1

$x\lg 3 + \lg 3 = x\lg 5$

$x(\lg 5 - \lg 3) = \lg 3$

$x = \frac{\lg 3}{\lg 5 - \lg 3} = 2.15$ --- A1

Alternative Method:

$3^{x+1} = 5^x$

$3^x \bullet 3 = 5^x$

$3 = \left(\frac{5}{3}\right)^x$ --- M1

$x \lg\left(\frac{5}{3}\right) = \lg 3$

$x = 2.15$ --- A1

(b)(ii)

$\log_3(4x) + \log_3(x-1) = 1$

$\log_3[4x(x-1)] = 1$ --- M1 (Combining terms using law of log)

$4x(x-1) = 3$

$4x^2 - 4x - 3 = 0$ --- M1 (Obtaining quadratic equation)

$(2x+1)(2x-3) = 0$

$x = -\frac{1}{2}$ (rej) or $1\frac{1}{2}$ --- A1 for rejecting negative value, A1 for $1\frac{1}{2}$

(c)(i)

Population at the beginning of 2014

$= 350(1.08)^{10}$ --- M1 (Sub $n = 10$)

$= 755.62$

$= 756$ --- A1

(c)(ii)

$350(1.08)^n = 3680$

$(1.08)^n = \frac{3680}{350}$

$n = \lg\left(\frac{3680}{350}\right) \div \lg 1.08$

$n = 30.6$ --- M1 (Finding n)

Therefore, the population first reached 3680 in the year 2034. --- A1