1 Given that $9^{x-1} \times 2^{2x+2} = 6^{3x}$,

(I) find the exact value of 6^{x}, [3]

(II) hence solve the equation $9^{x-1} \times 2^{2x+2} = 6^{3x}$. [2]

2 The gradient function at any point (x, y) on a curve is given by $e^{x} - 1$. It is given that the curve passes through the point $(1, e)$.

(I) Find the equation of the curve. [3]

(II) Show that the equation of the tangent to the curve at the point where the curve cuts the y-axis is parallel to the x-axis. [2]

3 The roots of the equation $3x^2 - 6kx - 6k + 22 = 0$ are α and β. If $\alpha^3 + \beta^3 = 24$, find the values of k. [7]

4 Show that the curve $y = x^2 + k^2 + 1 - kx$ is always positive for all real values of x. [3]

5 By expressing $\sin 3x$ as $\sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. [3]

Hence

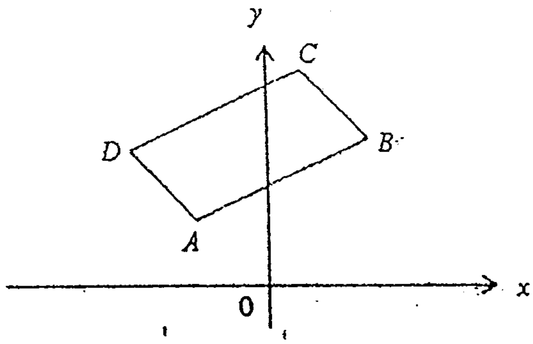
(I) solve the equation $6\sin x = 1 + 8\sin^3 x$ for $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$, [4]

(II) state the maximum value of $5 + 16\sin^3 x - 12\sin x$. [2]

6 Sketch the graph of $y = |3\sin 2x - 1|$ for $0 < x < \pi$. [2]

Hence find the range of values of c such that the equation $|3\sin 2x - 1| = c$ has 2 distinct real roots. [2]

7 The diagram shows a parallelogram $ABCD$. The coordinates of A, B and D are $(-2,1)$, $(4,3)$ and $(-5,4)$ respectively.



(i) Find the coordinates of C. [3]

(ii) Find the area of the parallelogram $ABCD$. [2]

(iii) Given further that the perpendicular bisector of DC cuts the x-axis at $(h,0)$, find the value of h. [3]

(iv) Explain why the diagonal AC and BD are not perpendicular to each other. Justify your answer with appropriate workings. [2]

8 It is given that $f'(x) = \dfrac{8}{(2x+1)^n}$, where n is a constant.

(a) Given further that $x \neq -\dfrac{1}{2}$ and $f(1) = 0$, find an expression for $f(x)$ if

(i) $n = 1$, [3]

(ii) $n = 4$. [3]

(b) Write down the range of values of n for which $f(x)$ does not have any stationary points. Support your answer with appropriate workings. [2]

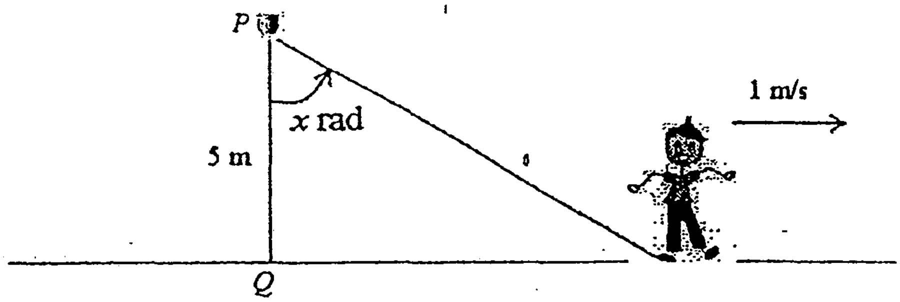
9 Two variables, x and y, are related by an equation $y = k(x-1)^h$, where k and h are constants. The table below shows their experimental values obtained.

x	2.26	3.0	4.0	4.5	5.2	7.0
y	7.94	20.0	45.8	61.3	88.2	180

(i) Express the equation $y = k(x-1)^h$ in a form suitable for drawing a straight line graph. [1]

(ii) Using graph paper, draw this straight line graph. Hence find the value of h and k. [6]

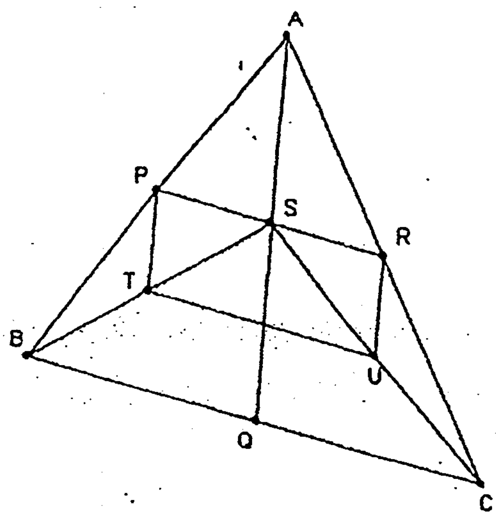
10 The diagram shows a boy walking away from the surveillance camera on a straight path. He is moving at a constant speed of 1 m/s. The surveillance camera is mounted at a point P which is 5 m vertically above a point Q along the path. The camera follows the motion of the boy rotating at P with an angle of x radian.



(i) Express the distance, s metre, moved by the boy from Q in terms of x. [1]

(ii) Hence find the rate of change in the angle of rotation of the surveillance camera when the boy is 7 m from Q. [4]

11 In triangle ABC, P, Q and R are the midpoints of AB, BC and AC respectively. The lines AQ and PR intersect at S. T and U are the midpoints of BS and SC respectively. Prove that $PRUT$ is a parallelogram. [4]



12(a) Given that $y = x(x+k)^3$ has a stationary point at $(2, 0)$,

(i) find the value of k, [1]

(ii) hence find the other stationary point and determine the nature of all the stationary points. [3]

(b) The table shows the values of x and the sign of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for a particular function $y = f(x)$. Without finding the function $y = f(x)$, write down all the x-coordinates of the stationary points and determine the nature of each stationary point. [3]

x	0^-	0	0^+	1^-	1	1^+	2^-	2	2^+
Sign of $\frac{dy}{dx}$	+	0	–	–	–	–	–	0	+
Sign of $\frac{d^2y}{dx^2}$	–	–	–	–	0	+	+	+	+

Legend: – denotes negative and + denotes positive

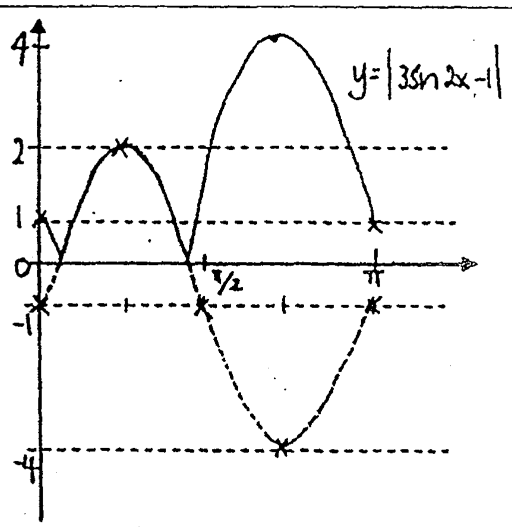
13 The blood pressure P (in mmHg) of a patient can be modelled by the equation $P = 125 - 35\cos\left(\frac{\pi}{14}t\right)$, where t is the time (in minutes) at which the blood pressure is measured.

(i) Explain why this model suggests that the blood pressure, P (in mmHg) of the patient is $90 \le P \le 160$. [2]

(ii) The patient's blood pressure is considered as 'normal' if it is less than 120 mmHg. Find the length of time for which the patient's blood pressure is not 'normal'. [4]

END OF PAPER 1

Answer Key

1. (i) $6^x = \frac{4}{9}$ (ii) -0.453 (3 s.f.)	2. (i) $y = e^x - x + 1$ (ii) Since the gradient at x = 0 is zero, the equation of tangent is parallel to the x-axis.
3. $\alpha + \beta = 2k$ $\alpha\beta = \frac{22-6k}{3}$ $k = -\frac{1}{2}, -3, 2$	4. Since $b^2 - 4ac < 0$ and coefficient of x^2 is positive, y does not cuts the x-axis. Hence, the curve is always positive.
5. (i) $x = \frac{\pi}{2}, \frac{5\pi}{2}, -\frac{7\pi}{2}, -\frac{11\pi}{2}$ (ii) 9 7. (i) C = (1,6) (ii) Area of $ABCD$ = 24 $unit^2$ (iii) $h = x = \frac{1}{3}$ (iv) Since $m_{BD} \times m_{AC} \neq -1$, hence diagonal AC and BD are not perpendicular to each other.	6  $2 < c < 4$ and c = 0
8. (ai) $\therefore f(x) = 4\ln(2x+1) - 4\ln 3$ or $\therefore f(x) = 4\ln\frac{(2x+1)}{3}$	(aii) $f(x) = \frac{4}{81} - \frac{4}{3(2x+1)^3}$
8. (b) Hence $n \geq 0$ for $f'(x)$ to have no stationary points.	
9. (i) Plotted $\lg y = \lg k + h\lg(x-1)$ (ii) $h = 2 \pm 0.2$	
10 (i) the horizontal distance from Q. $= 5\tan x$	(ii) $\frac{dx}{dt} = 0.0676$ rad/s or $\frac{5}{74} rad/s$

12.	a(i) k= -2 a(ii) $x=2, \frac{1}{2}$ $\left(\frac{1}{2}, -1\frac{11}{16}\right)$ is a minimum point. (2, 0) is a point of inflexion.
12 (b)	$x=0$ and $x=2$ $x=0$ is a maximum point as $\frac{d^2y}{dx^2}\Big\vert_{x=0}<0$ or gradient changes from + to –. $x=2$ is a minimum point as $\frac{d^2y}{dx^2}\Big\vert_{x=0}>0$ or gradient changes from – to +.
13	(i) $90 \le P \le 160$ (ii) Length of time = 21.6390 - 6.36098 = 15.3 mins

NCHS 2014 PRELIM EXAM (2) ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

Qn No	Suggested Solutions
1(i)	$9^{x-1}\times 2^{2x+2}=6^{3x}$ $3^{2x-2}\times 2^{2x+2}=6^{3x}$ $3^{2x}\times\frac{1}{9}\times 2^{2x}\times 4=6^{3x}$ $6^{2x}\left(\frac{4}{9}\right)=6^{3x}$ $6^{x}=\frac{4}{9}$
(ii)	$9^{x-1}\times 2^{2x+2}=6^{3x}$ $6^{x}=\frac{4}{9}$ $x=\dfrac{\lg\frac{4}{9}}{\lg 6}$ $=-0.453$ (3 s.f.)
2(i)	$\frac{dy}{dx}=e^{x}-1$ $y=\int e^{x}-1\,\mathrm{dx}$ $=e^{x}-x+c$ ...... (1) Sub $(1,e)$ into (1), $e=e-1+c$ $c=1$ $y=e^{x}-x+1$ ....(2)
(ii)	When the curve cuts the $y-$axis, sub $x=0$ into (2), $y=e^{0}-0+1$ $=2$ $(0,2)$ At $(0,2)$, $\frac{dy}{dx}=e^{0}-1=0$ Since the gradient at x = 0 is zero, the equation of tangent is parallel to the x-axis.
3	$3x^{2}-6kx-6k+22=0$ $\alpha+\beta=2k$ $\alpha\beta=\frac{22-6k}{3}$

	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $24=(2k)^3-3\left(\frac{22-6k}{3}\right)(2k)$ $24=8k^3-2k(22-6k)$ $24=8k^3-44k+12k^2$ $2k^3+3k^2-11k-6=0$ Let $f(k)=2k^3+3k^2-11k-6$ $f(2)=2(2)^3+3(2)^2-11(2)-6$ $=0$ $(k-2)$ is a factor. $f(-3)=2(-3)^3+3(-3)^2-11(-3)-6 \quad =0$ $(k+3)$ is a factor. $f(-\frac{1}{2})=2(-\frac{1}{2})^3+3(-\frac{1}{2})^2-11(-\frac{1}{2})-6 \quad =0$ $(2k+1)$ is a factor. $\therefore 2k^3+3k^2-11k-6=0$ $(2k+1)(k+3)(k-2)=0$ $k=-\frac{1}{2},-3,2$
4	$x^2+k^2+1-kx=0$ b^2-4ac $=(-k)^2-4(1)(k^2+1)$ $=-3k^2-4$ $=-(3k^2+4)$ since $(3k^2+4)>0$ $\therefore -(3k^2+4)<0$ Since since $b^2-4ac<0$ and coefficient of x^2 is positive, y does not cuts the x-axis. Hence, the curve is always positive.
5	$\sin 3x$ $=\sin(2x+x)$ $=\sin 2x\cos x+\cos 2x\sin x$ $=2\sin x\cos^2 x+\sin(1-2\sin^2 x)$ $=2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x$ $=2\sin x-2\sin^3 x+\sin x-2\sin^3 x$ $=3\sin x-4\sin^3 x$ (shown)
5(i)	$6\sin x=1+8\sin^3 x$

$3\sin x - 4\sin^3 x = \frac{1}{2}$

$\sin 3x = \frac{1}{2}$

$basic\angle = \frac{\pi}{6}$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, -\left(\pi + \frac{\pi}{6}\right), -\left(2\pi - \frac{\pi}{6}\right)$

$x = \frac{\pi}{2}, \frac{5\pi}{2}, -\frac{7\pi}{2}, -\frac{11\pi}{2}$

5(ii)

$5 + 16\sin^3 x - 12\sin x$

$= 5 - 4(3\sin x - 4\sin^3 x)$

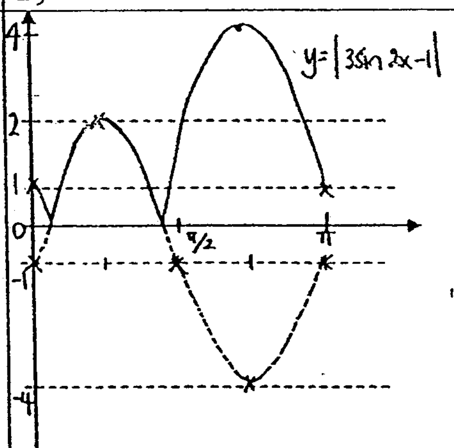
$= 5 - 4\sin 3x$

Maximum value $5 + 16\sin^3 x - 12\sin x$

$= 5 - (-4)$

$= 9$

6



$2 < c < 4$ and $c = 0$

7(i)

Midpoint BD = midpoint AC

$\left(\frac{4-5}{2}, \frac{3+4}{2}\right) = \left(\frac{x-2}{2}, \frac{y+1}{2}\right)$

$-1 = x - 2$ $\qquad$ $3 + 4 = y + 1$

$x = 1$ $\qquad$ $y = 6$

$C = (1, 6)$

(ii)

Area of $ABCD$

$= \frac{1}{2}\begin{vmatrix} 1 & -5 & -2 & 4 & 1 \\ 6 & 4 & 1 & 3 & 6 \end{vmatrix}$

$= \frac{1}{2}\left[4 - 5 - 6 + 24 - (-30 - 8 + 4 + 3)\right] = 24 \text{ unit}^2$

NCHS 2014 PRELIM EXAM (2) ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

(iii)	$m_{DC}=\frac{6-4}{1-(-5)}=\frac{1}{3}$ $m_{\perp}=-3$ Midpoint of DC = $\left(\frac{-5+1}{2},\frac{4+6}{2}\right)$ $=(-2,5)$ Equation of perpendicular bisector: $y-5=-3(x+2)$ $y=-3x-1$ Sub $y=0$, $0=-3x-1$ $h=x=-\frac{1}{3}$
(iv)	$m_{DB}=\frac{4-3}{-5-4}=-\frac{1}{9}$ $m_{AC}=\frac{6-1}{1-(-2)}=\frac{5}{3}$ Since $m_{BD}\times m_{AC}\neq -1$, hence diagonal AC and BD are not perpendicular to each other.
8(ai)	*If* $n=1$ $f'(x)=\frac{8}{(2x+1)}$ $f(x)$ $=\int \frac{8}{(2x+1)}dx$ $=4\ln(2x+1)+c$ Sub $f(1)=0$, $0=4\ln 3+c$ $c=-4\ln 3$ $\therefore f(x)=4\ln(2x+1)-4\ln 3$ or $\therefore f(x)=4\ln\frac{(2x+1)}{3}$
8(aii)	*If* $n=4$ $f'(x)=\frac{8}{(2x+1)^4}$ $f(x)$ $=\int 8(2x+1)^{-4}dx$ $=-\frac{4}{3}(2x+1)^{-3}+c$ Sub $f(1)=0$,

	$0=-\frac{4}{3}(3)^{-3}+c$ $c=\frac{4}{81}$ $\therefore f(x)=\frac{4}{81}-\frac{4}{3(2x+1)^3}$
8(b)	At stationary point, $f'(x)=0$ For $\frac{8}{(2x+1)^n}=0$ to be defined, $n<0$. Hence $n\geq 0$ for $f'(x)$ to have no stationary points.
9	(i) Plotted $\lg y=\lg k+h\lg(x-1)$ (ii) G1 – All points plotted correctly; G1 – Join all points with a best straight line and appropriate choice of scale T1- Table $k=5.01\pm 1.4$ $h=2\pm 0.2$
10(i)	Let s be the horizontal distance from Q. $\tan x=\frac{-s}{5}$ $s=5\tan x$
10(ii)	When $s=7$, $7=5\tan x$ $x=0.95055$ $\frac{ds}{dx}=5\sec^2 x$ $\frac{dx}{dt}=\frac{dx}{ds}\times\frac{ds}{dt}$ $=\frac{1}{5\sec^2 x}\times 1$ $=\frac{\cos^2 x}{5}$ Sub $x=0.95055$, $\frac{dx}{dt}=\frac{\cos^2 0.95055}{5}$ $=0.0676$ rad/s or $\frac{5}{74}rad/s$
11	In ΔABC: $AP=PB$ (given) $AR=RC$ (given) $PR=\frac{1}{2}BC$ and $PR//BC$ (Midpoint Theorem) In ΔSBC:

$ST = BT$ (given)

$SU = UC$ (given)

$TU = \frac{1}{2}BC$ and $TU // BC$ (Midpoint Theorem)

$\therefore PR = TU$ and $PR // TU$

Hence $PRUT$ is a parallelogram. (shown)

12(ai)

Sub (2, 0) into $y = x(x+k)^3$,

$0 = 2(2+k)^3$

$k = -2$

12(aii)

$y = x(x-2)^3$

$\frac{dy}{dx} = x(3)(x-2)^2 + (x-2)^3$

$= (x-2)^2(4x-2)$

At stationary point, $\frac{dy}{dx} = 0$.

$(x-2)^2(4x-2) = 0$

$x - 2 = 0$ or $(4x-2) = 0$

$x = 2, \frac{1}{2}$

Sub $x = 0.5$, $y = 0.5(0.5-2)^3 = -1\frac{11}{16}$

Ans: $\left(\frac{1}{2}, -1\frac{11}{16}\right)$

x	0.4	0.5	0.6
$\frac{dy}{dx}$	-	0	+

$\left(\frac{1}{2}, -1\frac{11}{16}\right)$ is a minimum point.

x	1.9	2	2.1
$\frac{dy}{dx}$	+	0	+

(2, 0) is a point of inflexion.

12(b)

$x = 0$ and $x = 2$

$x = 0$ is a maximum point as $\frac{d^2y}{dx^2}\Big|_{x=0} < 0$ or gradient changes from + to −.

$x = 2$ is a minimum point as $\frac{d^2y}{dx^2}\Big|_{x=0} > 0$ or gradient changes from − to +.

NCHS 2014 PRELIM EXAM (2) ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

13(i)	$P=125-35\cos\left(\frac{\pi}{14}t\right)$ $P_{max}=125-35(-1)=160$ $P_{min}=125-35(1)=90$ Hence $90\leq P\leq 160$
13(ii)	$125-35\cos\left(\frac{\pi}{14}t\right)=120$ $35\cos\left(\frac{\pi}{14}t\right)=5$ $\cos\left(\frac{\pi}{14}t\right)=\frac{1}{7}$ Basic $\angle=1.4274$ $\left(\frac{\pi}{14}t\right)=1.4274, 2\pi-1.4274$ $t=6.36098, 21.6390$ Length of time = 21.6390 - 6.36098 = 15.3 mins

Answer ALL Questions

1. The initial mass of a radioactive substance is $5\,\mu g$. Given that the mass of a radioactive substance, m μg, at time t minutes, is given by $m = Ae^{kt}$, where A and k are constants.

   (i) Find the value of A. [1]

   (ii) Hence, given further that the mass of the radioactive substance decays to one third of its initial mass when $t = 60$ min, find the value of k. [2]

   (iii) With the values found in (i) and (ii), find the rate at which m is decreasing when $t = 10$ min. [3]

2. (i) Show $\frac{d}{dx}\left(\tan^3 2x\right) = 6\left(\sec^4 2x - \sec^2 2x\right)$. [3]

   (ii) Hence find $\int_{\frac{\pi}{3}} \sec^4 2x \, dx$. [4]

3. (i) Prove the identity $\frac{2\cos A}{2\sin A - \operatorname{cosec} A} = -\tan 2A$. [3]

   (ii) Hence solve the equation $\frac{6\cos 2x}{2\sin 2x - \operatorname{cosec} 2x} + \sqrt{3} = 0$, for $0 < x < \pi$. Express your answers in term of π. [5]

4. (i) Given that $\log_4(2a-1) + \log_2 2b = \log_2 b$ where $b \neq 0$, find the value of a. [4]

   (ii) Given that $3^k = h$, find $\log_3\left(\frac{27}{h}\right)$ and $\log_h 9$ in terms of k. [5]

5. (i) Sketch the graph of $y = -\frac{8}{27}x^3$. [1]

   (ii) Sketch the graph of $y = -\frac{27}{8}x^{-1}$ on the same diagram as (i). [2]

   (iii) Point A and B are the two intersections of your graphs. Calculate the coordinates of points of intersection of your graphs. Hence write down two facts about OA and OB, where point O is at the origin. [6]

6. (i) Express $\frac{2x^4 + 3x^3 - x^2 + 4}{(x^2 - 4)(x - 2)}$ as partial fraction. [7]

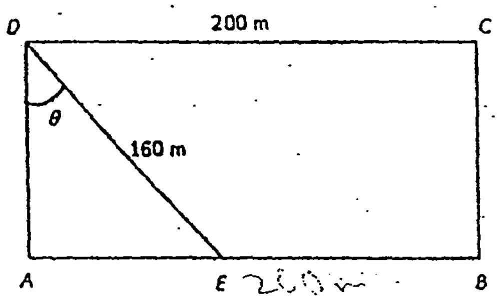
   (ii) Hence find $\int \frac{2x^4 + 3x^3 - x^2 + 4}{(x^2 - 4)(x - 2)} dx$ for $x > 2$. [4]

7. Sketch the graphs of $y = x^2 - 2x + 2$ and $y = 2x + 2$ on the same diagram showing the coordinates of the intersections. Hence find the area enclosed by the two graphs. [9]

8. (i) Given that the coefficient of x^6 and x^{10} in the expansion of $\left(x^3 - \frac{2}{x}\right)^{10}$ are a and b respectively, find the value of $\frac{a}{b}$. [5]

(ii) Given the first three terms in the expansion, in ascending powers of x, of $(2 + px)^n$ are $2048 + 16x + qx^2$ where n, p and q are constants. Find the value of n, p and q. [6]

9. The diagram shows a rectangular field $ABCD$ with length 200 m. Students ran along the straight tracks BC, CD, DE and EB to complete the running course. Given that $DE = 160$m and $\angle ADE = \theta$ where $0 < \theta < \frac{\pi}{2}$.



(i) Show that the total distance, P m, of the running course is given by $P = r + h\sin\theta + k\cos\theta$, where r, h and k are constants to be found. [3]

(ii) Express P in the form $r + R\cos(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Find the value of θ which $P = 0.65$ km. [4]

10. Given that the y-axis is a tangent to the circle C_1 at point A which the y-coordinate is 4 and the circle passes through point $B(4, 5)$. Find

(i) the coordinates of the centre and radius of the circle, [5]

(ii) the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants, [2]

(iii) the equation of another circle C_2 which is the image of the circle C_1 being reflected along the x-axis. Express the equation of circle C_2 in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are constants. [2]

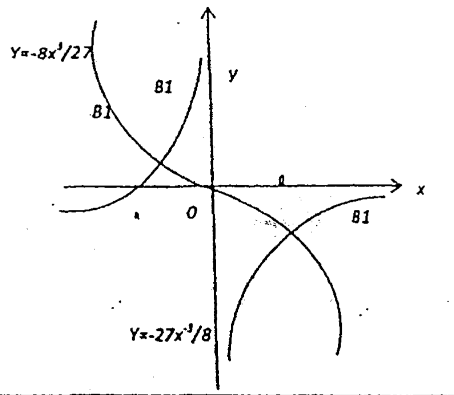
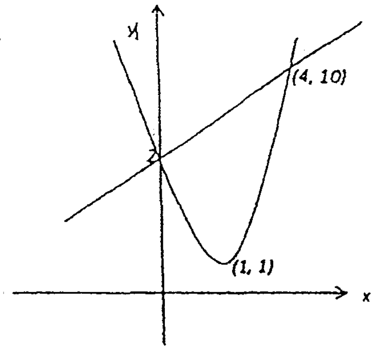
11. A curve has an equation $y = \ln\left(\frac{\sqrt{5-2x}}{3x+1}\right)$ for $1 < x < 2\frac{1}{2}$. Find

(i) $\frac{dy}{dx}$, [3]

(ii) the equation of the normal to the curve at the point A which the tangent to the curve at this point is parallel to the line $7y + 10x = 1$. Express the equation of the normal in the form of $y = mx + \ln a + b$ which m, a and b are constants. [8]

END OF PAPER 1

Answer Key

1.	(i)	$A = 5$	2.	(i)	$6(\sec^4 2x - \sec^2 2x)$
	(ii)	$k = -0.0183$		(ii)	$\sqrt{3}$
	(iii)	$rate\ decreasing = 0.0762\ \mu g/\min$	4.	(i)	$a = \frac{5}{8}$
3	(ii)	$x = \frac{\pi}{24}, \frac{7\pi}{24}, \frac{13\pi}{24}, \frac{19\pi}{24}$		(ii)	$\frac{2}{k}$
5	(i) (ii)	 	7.		
	(iii)	A(-1.5,1), B(1.5,-1) Point A, O and B are collinear. O is the mid-point of AB.			Parabolic curve with y-intercept of 2 (B1) Min pt (B1) Str. Line with y-intercepts of 2 and +ve gradient (B1) Showing values or coordinates of two intersection (B1)
6.	(i)	$\frac{2x^4+3x^3-x^2+4}{(x^2-4)(x-2)} = 2x+7+\frac{41}{2(x-2)}+\frac{14}{(x-2)^2}+\frac{1}{2(x+2)}$			$10\frac{2}{3}$
	(ii)	$x^2+7x+\frac{41}{2}\ln(x-2)-\frac{14}{(x-2)}+\frac{1}{2}\ln(x+2)+C$			
8	(i)	$\frac{a}{b} = -\frac{5}{3}$	9	(i)	$560+160\cos\theta-160\sin\theta$
	(ii)	$p = \frac{1}{704},\ q = \frac{5}{88}$		(ii)	$P = 560+226\cos(\theta+\frac{\pi}{4})$
				(iii)	$\theta = 0.376$
10.	(i)	$(2\frac{1}{8}, 4)$, $radius = 2\frac{1}{8}$	11.	(i)	$-\frac{1}{(5-2x)}-\frac{3}{3x+1}$
	(ii)	$x^2+y^2-\frac{17}{4}x+8y+16=0$		(ii)	$y = \frac{7}{10}x+\ln\frac{1}{7}-\frac{7}{5}$

Additional Mathematics – Secondary 4 Express

Nan Chiau High School

Prelim Examination (2) 2014

Marking Scheme

Qn No	Suggested Solutions	
1i	$m = Ae^0$ $A = 5$ —— B1	
1ii	$\frac{1}{3}(5) = 5e^{k(60)}$ —— M1 $k = -0.018310$ $k = -0.0183$ —— A1	
1iii	$\frac{dm}{dt} = Ake^{kt}$ —— M1 $= 5(-0.018310)e^{-0.01831(10)}$ $= -0.0762\ \mu g/\min$ —— M1 $rate\ decreasing = 0.0762\ \mu g/\min$ —— A1	
2i	$\frac{d}{dx}(\tan^3 2x) = 3\tan^2 2x(\sec^2 2x)(2)$ —— M1 $= 6\tan^2 2x(\sec^2 2x)$ $= 6(\sec^2 2x - 1)(\sec^2 2x)$ —— M1 $= 6(\sec^4 2x - \sec^2 2x)$ —— A1	
2ii	$\int_{\frac{\pi}{3}}^{\pi} \sec^4 2x\,dx = \frac{1}{6}[\tan^3 2x]_{\frac{\pi}{3}}^{\pi} + \int_{\frac{\pi}{3}}^{\pi} \sec^2 2x\,dx$ —— M2 $= \frac{1}{6}[\tan^3 2x]_{\frac{\pi}{3}}^{\pi} + \left[\frac{\tan 2x}{2}\right]_{\frac{\pi}{3}}^{\pi}$ —— M1 $= \sqrt{3}$ —— A1	

3i	$\dfrac{2\cos A}{2\sin A-\operatorname{cosec}A}=-\tan 2A$	
	$LHS=\dfrac{2\cos A}{2\sin A-\dfrac{1}{\sin A}}$	
	$=\dfrac{2\cos A}{\dfrac{2\sin^2 A-1}{\sin A}}$	M1
	$=\dfrac{2\sin A\cos A}{2\sin^2 A-1}$	
	$=\dfrac{\sin 2A}{-\cos 2A}$	M1
	$=-\tan 2A$	A1
	$=RHS$	
3ii	$\dfrac{6\cos 2x}{2\sin 2x-\operatorname{cosec}2x}+\sqrt{3}=0$	
	$-3\tan 4x+\sqrt{3}=0$	
	$\tan 4x=\dfrac{\sqrt{3}}{3}$	M1
	$\alpha=\dfrac{\pi}{6}$	M1
	$4x=\dfrac{\pi}{6},\dfrac{7\pi}{6},\dfrac{13\pi}{6},\dfrac{19\pi}{6}$	M2
	$x=\dfrac{\pi}{24},\dfrac{7\pi}{24},\dfrac{13\pi}{24},\dfrac{19\pi}{24}$	A1
4i	$\log_4(2a-1)+\log_2 2b=\log_2 b$	
	$\dfrac{1}{2}\log_2(2a-1)+\log_2 2b=\log_2 b$	M1
	$\log_2(2a-1)4b^2=\log_2 b^2$	M1
	$(2a-1)4b^2=b^2$	
	$b^2(8a-5)=0$	M1
	$(8a-5)=0$	
	$a=\dfrac{5}{8}$	A1
	$3^k=h$	M1

4ii

$\log_3 h = k$

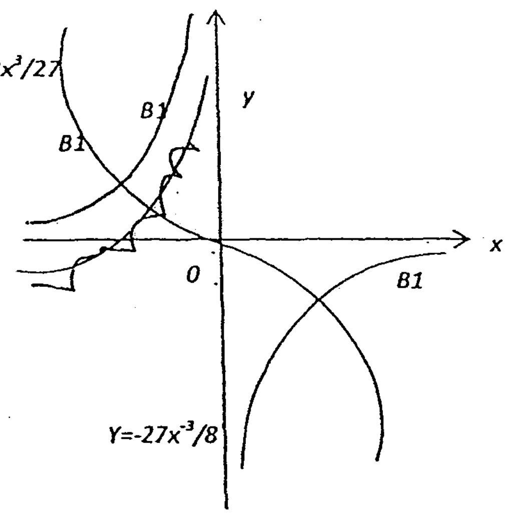
$\log_3\left(\frac{27}{h}\right) = \log_3 27 - \log_3 h$ —— M1

$= 3 - k$ —— A1

$\log_h 9 = 2\log_h 3$ —— M1

$= \frac{2}{k}$ —— A1

5i,ii



5iii

$-\frac{8}{27}x^3 = -\frac{27}{8}x^{-3}$ —— M1

$x^6 = \frac{729}{64}$

$x = 1.5$ or -1.5 —— M1

A(-1.5,1), B(1.5,-1) —— A2

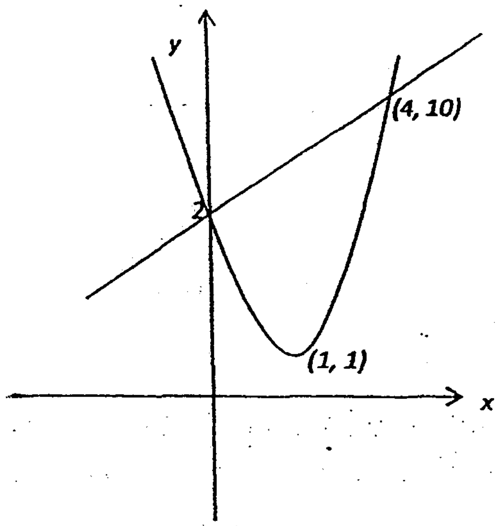
Point A, O and B are collinear. O is the mid-point of AB. (must prove). —— A2
lie on ē same pt.

6i

$\frac{2x^4+3x^3-x^2+4}{(x^2-4)(x-2)} = 2x+7+\frac{21x^2+12x-52}{(x^2-4)(x-2)}$ —— M2

$\frac{21x^2+12x-52}{(x-2)^2(x+2)} = \frac{A}{(x-2)}+\frac{B}{(x-2)^2}+\frac{C}{(x+2)}$ —— M1

$21x^2+12x-52 = A(x-2)(x+2)+B(x+2)+C(x-2)^2$

	Let $x=2$ $4B=56$ $B=14$ —— M1 Let $x=-2$ $16C=8$ $C=\frac{1}{2}$ —— M1 Let $x=0$ $-4A+2B+4C=-52$ $A=\frac{41}{2}$ —— M1 $\frac{2x^4+3x^3-x^2+4}{(x^2-4)(x-2)}=2x+7+\frac{41}{2(x-2)}+\frac{14}{(x-2)^2}+\frac{1}{2(x+2)}$ —— A1	
6ii	$\int\frac{2x^4+3x^3-x^2+4}{(x^2-4)(x-2)}dx=\int 2x+7+\frac{41}{2(x-2)}+\frac{14}{(x-2)^2}+\frac{1}{2(x+2)}dx$ —— M1 $=x^2+7x+\frac{41}{2}\ln(x-2)-\frac{14}{(x-2)}+\frac{1}{2}\ln(x+2)+C$ —— A3	
7	 Parabolic curve with y-intercept of 2 (B1) Min pt (B1) Str. Line with y –intercepts of 2 and +ve gradient (B1) Showing values or coordinates of two intersection (B1)	3

	$x^2-2x+2=2x+2$ — M1 $x^2-4x=0$ $x=0 \text{ or } 4$ $y=2 \text{ or } 10$ — M1 $A=\int_0^4 2x+2-(x^2-2x+2)dx$ $=\int_0^4 -x^2+4x\ dx$ — M1 $=\left[-\frac{x^3}{3}+\frac{4x^2}{2}\right]_0^4$ — M1 $=-\frac{64}{3}+32$ $=10\frac{2}{3}$ — A1	
8i	$\left(x^3-\frac{2}{x}\right)^{10}$ ${}^{10}C_4\left(x^3\right)^4\left(-\frac{2}{x}\right)^6$ — M1 $=13440x^6$ — M1 ${}^{10}C_5\left(x^3\right)^5\left(-\frac{2}{x}\right)^5$ — M1 $=-8064x^{10}$ — M1 $a=13440, b=-8064$ $\frac{a}{b}=-\frac{5}{3}$ — A1	
8ii	$(2+px)^n=2048+16x+qx^2+\ldots$ $=2^n+{}^nC_1(2)^{n-1}(px)+{}^nC_2(2)^{n-2}(px)^2+\ldots$ $2^n=2048$ — M1 $n=11$ — A1 ${}^{11}C_1(2)^{10}(px)=16x$ — M1 $p=\frac{1}{704}$ — A1	

	${}^{11}C_2(2)^9(px)^2 = qx^2$	M1
	${}^{11}C_2(2)^9\left(\frac{1}{704}x\right)^2 = qx^2$	
	$q = \frac{5}{88}$	A1
9i	$P = BC + CD + DE + EB$	
	$= 160\cos\theta + 200 + 160 + (200 - 160\sin\theta)$	M2
	$= 560 + 160\cos\theta - 160\sin\theta$	A1
9ii	$P = 560 + 160\cos\theta - 160\sin\theta$	
	$R = \sqrt{51200}$	M1
	$\tan\alpha = 1$	
	$\alpha = \frac{\pi}{4}$	M1
	$P = 560 + 226\cos(\theta + \frac{\pi}{4})$	A1
9iii	$560 + \sqrt{51200}\cos(\theta + \frac{\pi}{4}) = 650$	M1
	$\cos(\theta + \frac{\pi}{4}) = \frac{90}{\sqrt{51200}}$	
	$\alpha 1 = 1.16174$	M1
	$\theta + \frac{\pi}{4} = 1.16174$	M1
	$\theta = 0.376$	A1

10i	$mid-pt\ of\ AB=(2,4.5)$ —— M1 $m_{AB}=\frac{1}{4}$ $m_{normal}=-4$ —— M1 $4.5=-4(2)+C$ $C=12.5$ $y=-4x+12.5$ —— M1 $y-coordinate=4$ $4=-4x+12.5$ $x=\frac{17}{8}$ $(2\frac{1}{8},4),\quad radius=2\frac{1}{8}$ —— A2 OR Let C = (x; 4) —— M1 $(x-0)^2+(4-4)^2=(x-4)^2+(4-5)^2$ —— M1 $x^2=x^2-8x+16+1$ $x=\frac{17}{8}$ —— M1 $(2\frac{1}{8},4),\quad radius=2\frac{1}{8}$ —— A2	
10ii	$\left(x-\frac{17}{8}\right)^2+(y-4)^2=\left(\frac{17}{8}\right)^2$ —— M1 $x^2+y^2-\frac{17}{4}x-8y+16=0$ —— A1	
10iii	$centre\ of\ C2\ (2\frac{1}{8},-4)$ —— M1 $x^2+y^2-\frac{17}{4}x+8y+16=0$ —— A1	
11i	$y=\ln\frac{\sqrt{5-2x}}{3x+1}$ $=\frac{1}{2}\ln(5-2x)-\ln(3x+1)$ —— M1 $\frac{dy}{dx}=\frac{-2}{2(5-2x)}-\frac{3}{3x+1}$ —— M1 $=-\frac{1}{(5-2x)}-\frac{3}{3x+1}$ —— A1	

11iii	$7y+10x=1$ $m1=-\frac{10}{7}$ —— M1 $-\frac{1}{(5-2x)}-\frac{3}{3x+1}=-\frac{10}{7}$ —— M1 $60x^2-151x+62=0$ —— M1 $x=2$ or $\frac{31}{60}(rejected)$ —— M1 $y=\ln\frac{\sqrt{5-2(2)}}{3(2)+1}$ $=\ln\frac{1}{7}$ —— M1 $m2=\frac{7}{10}$ —— M1 $y=\frac{7}{10}x+C$ $\ln\frac{1}{7}=\frac{7}{10}(2)+c$ $c=\ln\frac{1}{7}-\frac{7}{5}$ —— M1 $y=\frac{7}{10}x+\ln\frac{1}{7}-\frac{7}{5}$ —— A1 *** End of Paper ***	