南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2014

Subject	:	**Additional Mathematics**
Paper	:	**4047/01**
Level	:	**Secondary Four Express**
Date	:	**15 September 2014**
Duration	:	**2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correcting fluid / tape.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of 5 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Given that $\sin A = -\frac{3}{5}$ where $180° < A < 270°$ and $\sin B = \frac{5}{13}$ where $90° < B < 180°$, find $\tan(A+B)$ without using a calculator. [2]

2. A prism has a square base of side of $(2+\sqrt{3})$ m, and its volume is $(11+6\sqrt{3})$ m^3. Find, without using a calculator, the height of the prism in the form $\left(a-b\sqrt{3}\right)$ m, where a and b are integers. [4]

3. Find, in ascending powers of x, the first 4 terms of the expansion of $\left(1-\frac{x}{3}\right)^{10}$.

Hence, find the coefficient of x in the expansion of $\left(2+\frac{3}{x}\right)^2\left(1-\frac{x}{3}\right)^{10}$. [4]

4 (a) Find the value of a and k for which $\{x : -4 < x < 7\}$ is the solution set of $k - x^2 > ax$. [2]

(b) If the line $y = 4x - 1$ meets the curve $ky - 8x = 2kx^2 + 1$, find the range of values of k. [4]

5. Given the roots of $9x^2 - 13x + 36 = 0$ are α^2 and β^2, find the quadratic equation(s) whose roots are α^3 and β^3. [7]

6. Prove the identity $\dfrac{\cot^2\theta - \cos^2\theta}{\operatorname{cosec}^2\theta \tan^2\theta} = \cot^2\theta \cos^4\theta$. [3]

7 (a) Solve the equation $3\log_8 x - 4 = 4\log_x 8$ [4]

(b) Solve the simultaneous equations

$$\log_3\left(\frac{x-5y}{x-y}\right) = 2$$

$$5e^{2x-\frac{1}{2}y} = 6 - e^{x-y}$$

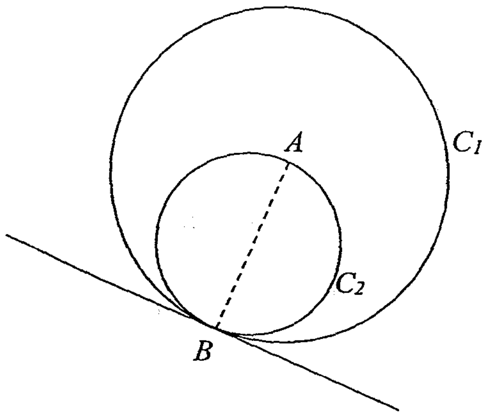
[7]

8. Water is poured into an empty inverted right circular cone at a constant ...
The cone has a base radius of 6 cm and a height of 12 cm.

(i) Find the rate of change of the height of water level when the wate... 3 cm high.

(ii) State, with reason, whether this rate will increase or decrease as ...

9.



The diagram shows two circles C_1 and C_2. The equation of circle C_1 wi... $x^2+y^2-8x+2y-63=0$. AB is the diameter of circle C_2, and the tang... at point B has a gradient of $-\frac{1}{2}$. Find

(i) the centre and radius of circle C_1,
(ii) the coordinates of point B given that its y-coordinate is negative,
(iii) the equation of circle C_2.

10 (a) Without the use of a calculator, find x such that $\cos 2x = \sin 230°$ whe... $-180° \le x \le 180°$.

(b) Evaluate $\cos\left(\cos^{-1}\left(-\frac{3}{5}\right)-\cos^{-1}\left(\frac{12}{13}\right)\right)$ without using a calculator.

11. The variables x and y are related by the equation $y=10^{-k}A^x$, where k and A are constants. Using experimental values of x and y, a graph was drawn in which $\lg y$ was plotted on the vertical axis against x on the horizontal axis. The straight line which was obtained passed through the points (20, 0.36) and (35, 1.20). Find

(i) the values of k and A, [3]

(ii) the coordinates of the point on the line at which $y=10^{-\frac{x}{20}}$ [...]

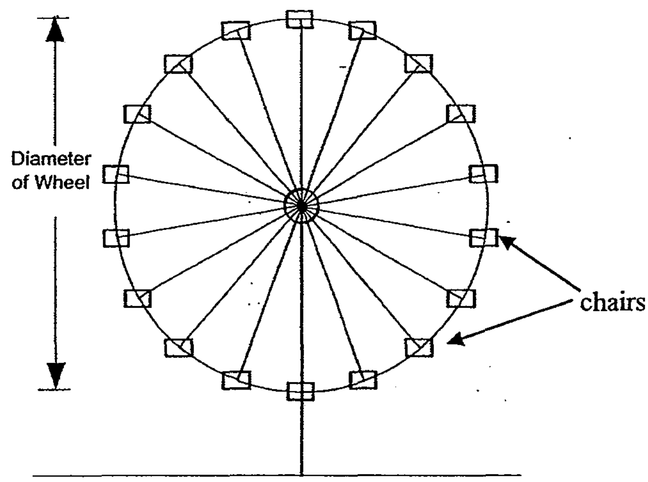
12. (a) The normal to the curve $y=\frac{2\ln x}{x^2}$,at the point where the curve crosses the x-axis, passes through the point $(4, p)$. Find the value of p. [4]

(b) Given that $\int_1^6 f(x)\,dx=9$ and $\int_1^2 f(x)\,dx=3$, find

(i) $\int_6^1\left[\frac{5}{x^2}-f(x)\right]dx$. [3]

(ii) the value of the constant m for which $\int_2^6[f(x)+mx]\,dx=-42$. [3]

13.



A Ferris wheel with a radius of 8 m is rotating at a rate of 3 revolutions per minute. The height of a chair on the Ferris wheel (measured from the ground) can be modelled by the equation, $h=a\cos kt+b$ where a, b and k are constants and t is the time in seconds after the ride starts. At the start of a ride when $t=0$, a chair on the Ferris wheel starts at the lowest point, which is 2 m above the ground.

(i) Explain why the maximum height of any chair on the Ferris wheel is 18 m above the ground. [1]

(ii) Show that the value of k is $\frac{\pi}{10}$. [2]

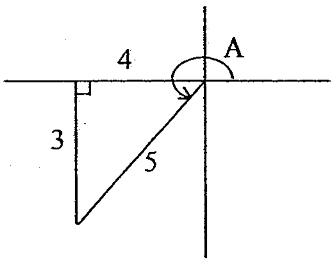
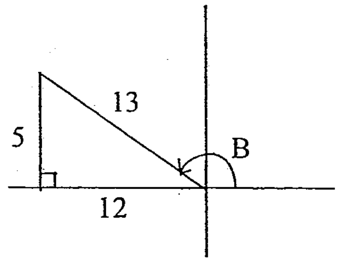
(iii) Find the values of a and b. [4]

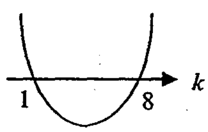
~ End of Paper ~

Answer Key

1. $\frac{16}{63}$	2. $\left(5-2\sqrt{3}\right)$
3. $1-\frac{10}{3}x+5x^2-\frac{40}{9}x^3+\ldots$; $6\frac{2}{3}$	4(a) $a=-3$, $k=28$ (b) $k\le 1$ *or* $k\ge 8$
5. $27x^2\pm 35x+216=0$	7(a) $x=\frac{1}{4}$ or 64 (b) $x=-1.61$, $y=-3.22$
8(i) 0.849 cm/s (ii) the rate will decrease	9(i) Centre: (4, -1), Radius = 8.94 units (ii) $B(0,-9)$ (iii) $(x-2)^2+(y+5)^2=20$
10(a) $-110°$, $-70°$, $70°$, $110°$ (b) $-\frac{16}{65}$	11(i) 0.76; 1.14 (ii) (7.17, - 0.358)
12(a) -3/2 (b)(i) $4\frac{5}{6}$ (ii) - 3	13(iii) - 8; 10

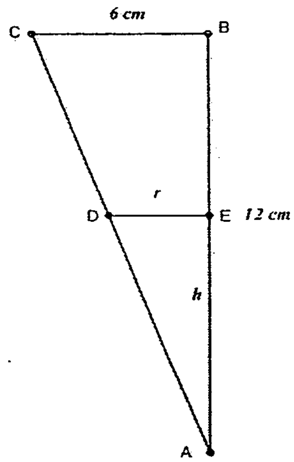
AM Prelim 2 Paper 1 Solutions

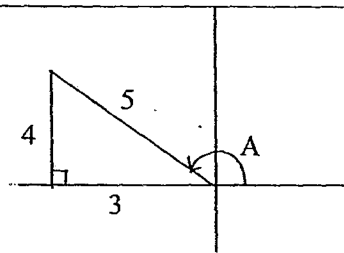
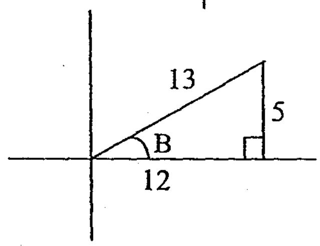
S/no	Solutions
1	 $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ $=\dfrac{\frac{3}{4}-\frac{5}{12}}{1-\frac{3}{4}\left(-\frac{5}{12}\right)}$ $=\dfrac{4}{12}\div\left(1+\dfrac{5}{16}\right)$ $=\dfrac{1}{3}\times\dfrac{16}{21}=\dfrac{16}{63}$
2	$a-b\sqrt{3}=\dfrac{11+6\sqrt{3}}{\left(2+\sqrt{3}\right)^2}$ $=\dfrac{11+6\sqrt{3}}{7+4\sqrt{3}}$ $=\dfrac{11+6\sqrt{3}}{7+4\sqrt{3}}\times\dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}$ $=\dfrac{77+42\sqrt{3}-44\sqrt{3}-72}{49-48}$ $a-b\sqrt{3}=5-2\sqrt{3}$ Hence the height of the prism is $\left(5-2\sqrt{3}\right)m$.

3	$\left(1-\frac{x}{3}\right)^{10}=1-\binom{10}{1}\left(\frac{x}{3}\right)+\binom{10}{2}\left(\frac{x}{3}\right)^2-\binom{10}{3}\left(\frac{x}{3}\right)^3+\ldots$ $=1-\frac{10}{3}x+5x^2-\frac{40}{9}x^3+\ldots$ $\left(2+\frac{3}{x}\right)^2\left(1-\frac{x}{3}\right)^{10}$ $=\left(4+\frac{12}{x}+\frac{9}{x^2}\right)\left(1-\frac{10}{3}x+5x^2-\frac{40}{9}x^3+\ldots\right)$ *Coefficient of* x: $(4)\left(-\frac{10}{3}\right)+(12)(5)+(9)\left(-\frac{40}{9}\right)$ $=-\frac{40}{3}+60-40$ $=6\frac{2}{3}$
4(a)	$k-x^2>ax$ $x^2+ax-k<0$ *if* $-4<x<7$ *is the solution, then* $(x+4)(x-7)<0$ $x^2-3x-28<0$ *By comparing,* $a=-3,\quad k=28$
4(b)	*At pt of intersection,* $k(4x-1)-8x=2kx^2+1$ $2kx^2+(8-4k)x+(1+k)=0$ *Since line meets curve, discriminant:* $(8-4k)^2-4(2k)(1+k)\geq 0$ $64-64k+16k^2-8k-8k^2\geq 0$ $8k^2-72k+64\geq 0$ $k^2-9k+8\geq 0$ $(k-8)(k-1)\geq 0$ $k\leq 1\quad or\quad k\geq 8$ 

5	$9x^2-13x+36=0$ $\alpha^2\beta^2=(\alpha\beta)^2=\frac{36}{9}=4$ $\alpha\beta=2 \quad or \quad -2$ $\alpha^2+\beta^2=\frac{13}{9}$ $(\alpha+\beta)^2-2\alpha\beta=\frac{13}{9}$ $(\alpha+\beta)^2=\frac{13}{9}+2\alpha\beta$ $When \quad \alpha\beta=-2, \quad (\alpha+\beta)^2=\frac{13}{9}-4<0 \quad (NA \sin ce\,(\alpha+\beta)^2\geq 0)$ $When \quad \alpha\beta=2, \quad (\alpha+\beta)^2=\frac{13}{9}+4=\frac{49}{9}$ $\alpha+\beta=\pm\frac{7}{3}$ $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=\pm\frac{7}{3}\left(\frac{13}{9}-2\right)$ $\left.\begin{array}{l}=\pm\frac{7}{3}\left(-\frac{5}{9}\right)=\pm\frac{35}{27}\\ \alpha^3\beta^3=(\alpha\beta)^3=2^3=8\end{array}\right\}$ Equations required: $x^2\pm\frac{35}{27}x+8=0$ $27x^2\pm 35x+216=0$
6	$\frac{\cot^2\theta-\cos^2\theta}{\operatorname{cosec}^2\theta\tan^2\theta}$ $=\frac{\frac{\cos^2\theta}{\sin^2\theta}-\cos^2\theta}{\frac{1}{\sin^2\theta}\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}$ $=\cos^2\theta\left[\cos^2\theta\left(\frac{1}{\sin^2\theta}-1\right)\right]$ $=\cos^4\theta(\operatorname{cosec}^2\theta-1)$ $=\cos^4\theta\cot^2\theta$

7(a)	$3\log_8 x - 4 = \dfrac{4}{\log_8 x}$ $3(\log_8 x)^2 - 4\log_8 x - 4 = 0$ Let $y = \log_8 x$ $\therefore 3y^2 - 4y - 4 = 0$ $(3y+2)(y-2) = 0$ $y = -\dfrac{2}{3}$ or $y = 2$ $\log_8 x = -\dfrac{2}{3}$ or $\log_8 x = 2$ $x = 8^{-\frac{2}{3}}$ or $x = 8^2$ $x = \dfrac{1}{4}$ or $x = 64$
7(b)	$\log_3\left(\dfrac{x-5y}{x-y}\right) = 2$ $\dfrac{x-5y}{x-y} = 3^2 = 9$ $x - 5y = 9x - 9y$ $4y = 8x$ $y = 2x$ $5e^{2x-\frac{1}{2}y} = 6 - e^{x-y}$ $5e^{2x-x} = 6 - e^{x-2x}$ $5e^x = 6 - e^{-x}$ Multiply by e^x $5e^{2x} - 6e^x + 1 = 0$ $(5e^x - 1)(e^x - 1) = 0$ $e^x = \dfrac{1}{5}$ or $e^x = 1$ $x = \ln\dfrac{1}{5}$ or $x = 0$ *When* $x = -1.609,\ y = -3.219$ *When* $x = 0, y = 0, \log_3\left(\dfrac{x-5y}{x-y}\right)$ *is undefined, so NA* *Ans*: $x = -1.61,\ y = -3.22$

8(i)	*By similar* Δ, $\frac{r}{6}=\frac{h}{12}$ $r=\frac{h}{2}$ $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $=\frac{\pi}{12}h^3$ $\frac{dV}{dh}=\frac{\pi}{4}h^2, \quad \therefore \frac{dh}{dV}=\frac{4}{\pi h^2}$ $\frac{dh}{dt}=\frac{dh}{dV}\mid_{h=3}\times\frac{dV}{dt}$ $=\frac{4}{\pi 3^2}\times 6$ $=\frac{8}{3\pi}$ or $0.849 cm/s$  The water level is increasing at a rate of $\frac{8}{3\pi}$ or $0.849\ cm/s$
8(ii)	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}=\frac{4}{\pi h^2}\times 6=\frac{24}{\pi h^2}$ As t increases, h increases, therefore the rate will decrease.
9(i) 9(ii)	$x^2+y^2-8x+2y-63=0$ $(x-4)^2-16+(y+1)^2-1-63=0$ $(x-4)^2+(y+1)^2=80$ Centre: (4, -1), Radius = $\sqrt{80}=8.94$ units *eq of AB*: $\frac{y+1}{x-4}=2$ $y=2x-9$ *At B*, $(x-4)^2+(2x-8)^2=80$ $(x-4)^2+4(x-4)^2=80$ $5(x-4)^2=80$ $(x-4)^2=16$ $x-4=4$ *or* -4 $x=8$ *or* 0 $y=7$ *or* -9 $\therefore B(0,-9)$ *since the y-coord is negative*

9(iii)	centre of C_2: $\left(\frac{4+0}{2}, \frac{-1-9}{2}\right) = (2, -5)$ radius of C_2: $\sqrt{(2-0)^2 + (-5+9)^2} = \sqrt{20}$ eq of C_2: $(x-2)^2 + (y+5)^2 = 20$
10(a)	$\cos 2x = \sin 230°$ $= -\sin 50°$ $= -\cos 40°$ basic $\angle = 40°$ since $-180° \le x \le 180°$ $-360° \le 2x \le 360°$ $\therefore\ 2x = 140°,\ 220°,\ -140°,\ -220°$ $x = -110°,\ -70°,\ 70°,\ 110°$
10(b)	Let $A = \cos^{-1}\left(-\frac{3}{5}\right)$ $\cos A = -\frac{3}{5}$  Let $B = \cos^{-1}\left(\frac{12}{13}\right)$ $\cos B = \frac{12}{13}$  $\cos\left[\cos^{-1}\left(-\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right)\right]$ $= \cos(A-B)$ $= \cos A \cos B + \sin A \sin B$ $= \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$ $= \frac{-36+20}{65}$ $= -\frac{16}{65}$
11(i)	$y = 10^{-k} A^x$ $\lg y = -k + x \lg A$ $\lg A = \frac{1.20 - 0.36}{35 - 20} = 0.056$ $A = 1.14$ (to 3 sf) Using $(35, 1.20)$: $1.20 = -k + 35(0.056)$ $k = 0.76$

11(ii)	$y=10^{-\frac{x}{20}}$ $\lg y=-\dfrac{x}{20}=-k+x\lg A$ $=-0.76+0.056x$ $\left(0.056+\dfrac{1}{20}\right)x=0.76$ $x=7.16981=7.17$ $\lg y=-\dfrac{x}{20}=-0.358$ $The\ po\text{int}\ is\ (7.17,-0.358)$
12(a)	when $y=0,\dfrac{\ln x^2}{x^2}=0$ $x=1$ $\dfrac{dy}{dx}=\dfrac{x^2\left(\dfrac{2}{x}\right)-2\ln x\times(2x)}{x^4}$ $=\dfrac{2x-4x\ln x}{x^4}=\dfrac{2-4\ln x}{x^3}$ gradient of tangent at this point $=\dfrac{2-4\ln(1)}{(1)^3}=2$ gradient of normal at this point $=-\dfrac{1}{2}$ Equation of normal: $y-0=-\dfrac{1}{2}(x-1)$ $y=-\dfrac{1}{2}x+\dfrac{1}{2}$ when $x=4,\ p=4\left(-\dfrac{1}{2}\right)+\dfrac{1}{2}=-\dfrac{3}{2}$
12(b)(i)	$\int_6^1\left[\dfrac{5}{x^2}-f(x)\right]dx$ $=\int_1^6\left[f(x)-\dfrac{5}{x^2}\right]dx$ $=\int_1^6 f(x)dx+\left[\dfrac{5}{x}\right]_1^6$ $=(9)+\dfrac{5}{6}-5=4\dfrac{5}{6}$

(ii)	$\int_2^6 [f(x)+mx]\,dx = -42$ $\int_2^6 f(x)\,dx + \left[\frac{mx^2}{2}\right]_2^6 = -42$ $(9-3)+\frac{m}{2}(36-4) = -42$ $16m = -48$ $m = -3$
13(i)	$\max h = 8+8+2 = 18\,m$ $\therefore \max$ *height of a chair is* 18 *m above the ground*
13(ii)	$h = a\cos kt + b$ $3\ rev/60\ s$ $1\ rev/20\ s$ $\frac{1}{2} rev/10\ s$ $period = \frac{2\pi}{k} = 20$ $k = \frac{\pi}{10}$
13(iii)	$h = a\cos kt + b$ $When\ t = 0, \quad a\cos 0 + b = 2$ $a + b = 2 .... eq1$ $When\ t = 10, \quad h\ is\ \max:$ $a\cos\left(\frac{\pi}{10}\right)(10) + b = 18$ $-a + b = 18 ..... eq2$ $eq1 + eq2: \quad 2b = 20$ $b = 10$ $a = -8$

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Answer **all** the questions.
Write your answers on the separate writing paper provided.
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The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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The total number of marks for this paper is 100.

This paper consists of 8 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1. (a) The term containing the highest power of x in the polynomial $f(x)$ is $2x^4$ and the roots of $f(x)=0$ are 2 and -7. $f(x)$ has a remainder of -72 when divided by $(x+1)$, and a remainder of -80 when divided by $(x-1)$.

(i) Find the expression for $f(x)$ in descending power of x. [4]

(ii) Explain why the equation $f(x)=0$ has exactly 2 real roots. [2]

(b) Express $\dfrac{6x^2-3x-2}{4x^3-12x^2+7x-21}$ in partial fractions. [6]

2. (a) (i) Sketch the graph of $y=5-|2x^2-x-15|$ for $-4\le x\le 5$. [4]

(ii) Hence find the value(s) of k such that the equation $|2x^2-x-15|=5-k$ has exactly 2 solutions. [2]

(b) Solve the equation $|-3x+18|=6x+|x-6|$. [4]

3. (a) (i) Solve the equation $6\sin^2\dfrac{x}{2}-2\cos^2 x=1$ for $0°<x<360°$. [4]

(ii) State the number of solutions of the equation $6\sin^2\dfrac{x}{2}-2\cos^2 x=1$ in the range $-1080°<x<720°$. [1]

(b) Solve the equation $\sin 2y=\cos^2 y$ for $0\le y\le 9$. [5]

4. The function f is defined by $f(x)=a\tan\left(\dfrac{x}{2}\right)+b$, where a and b are positive integers and $-\pi\le x\le\pi$. The graph of $y=f(x)$ meets the y-axis at the point where $y=3$, and $f\left(\dfrac{\pi}{2}\right)=7$.

(i) State the period of f. [1]

(ii) Find the values of a and b. [2]

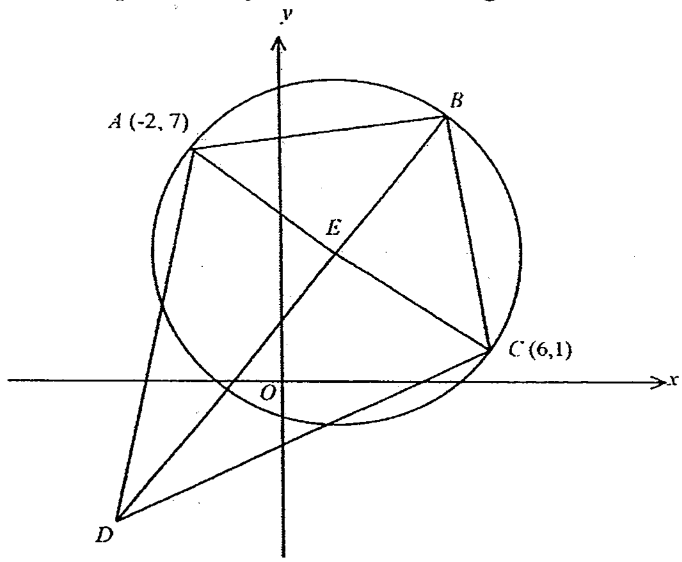
(iii) Sketch the graph of $y=f(x)$. [2]

5. Differentiate $\tan^3 6\theta$ with respect to θ. Hence find [1]

(i) $\int\tan^2 6\theta\sec^2 6\theta\, d\theta$, [1]

(ii) $\int\sec^4 6\theta\, d\theta$. [3]

6. **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a kite $ABCD$ in which A, B and C are points on a circle C_1. The coordinates of A and C are $(-2, 7)$ and $(6, 1)$ respectively.

The lines AC and BD intersect at point E, and point B lies on the curve $4y = x^2 + 7$.

(i) Find the coordinates of B, if the coordinates are integer values. [5]

(ii) Given that $BE : ED = 2 : 3$, find the coordinates of D. [3]

(iii) Show that point E is the centre of the circle C_1. [3]

(iv) Calculate the area of the kite $ABCD$. [2]

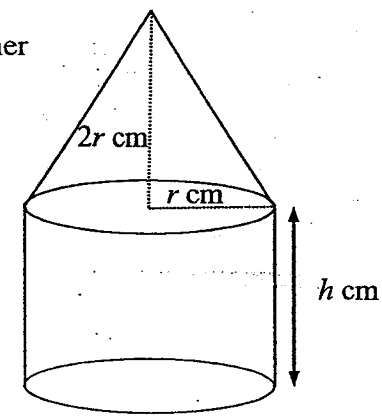
7. A container consists of a cone (of radius r cm) fixed to the top of a right circular cylinder of radius r cm and height h cm. Given that the height of the cone is $2r$ cm and that the volume of the container is 75π cm^3,

(i) show that $h = \dfrac{75}{r^2} - \dfrac{2}{3}r$, [2]

(ii) determine, with explanation, whether h is an increasing or decreasing function, [2]

(iii) show that the total surface area, A cm^2, of the container is given by

$$A = \left(\sqrt{5} - \frac{1}{3}\right)\pi r^2 + \frac{150\pi}{r}.$$ [2]



If r varies,

(iv) showing full working, determine whether the stationary value of A is a minimum or maximum. [4]

8. A particle moves in a straight line so that its displacement, s metres, from a fixed point O is given by $s = 4e^{-5t} + 10t - 7$ for $0 \le t \le T$, where t is the time in seconds after passing O, and T is the time in seconds when the particle first comes to rest.

(i) Find the initial position of the particle. [1]

(ii) Find the initial velocity of the particle. [2]

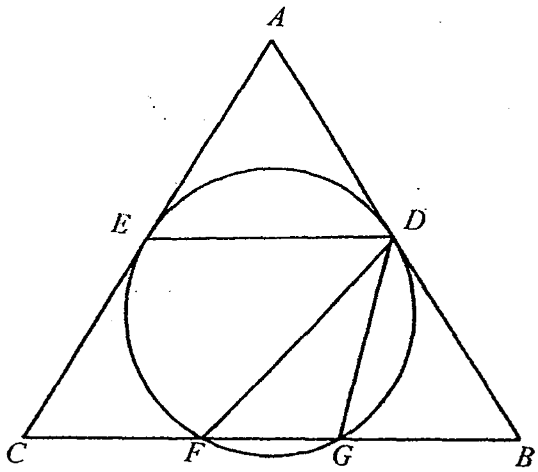
(iii) Show that $T = \dfrac{\ln 2}{5}$. [2]

The particle then travels at a constant acceleration of 10 m/s^2 for the next 2 seconds.

(iv) Sketch the velocity-time graph for the particle for $0 \le t \le (T+2)$. [2]

(v) Find the total distance travelled by the particle in the first $(T+2)$ seconds. [3]

9. In the diagram, AB and AC are tangents to the circle at point D and E respectively. BC intersects the circle at points F and G.

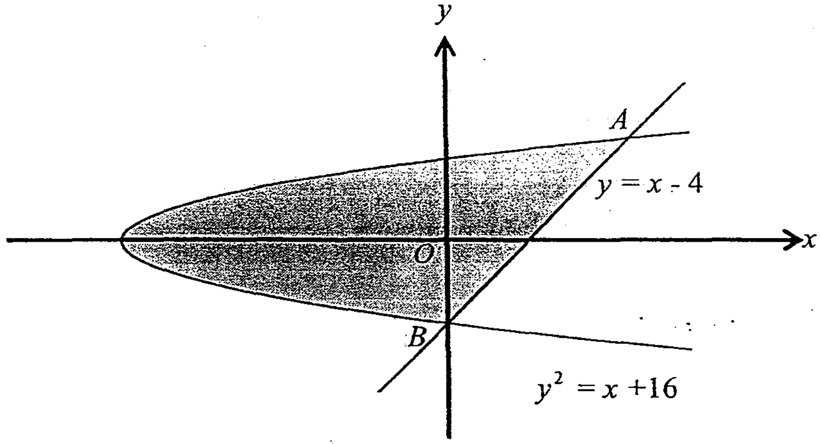


Given $AE = EC$ and $AD = DB$, prove that

(i) $AE \times BC = AC \times ED$, [3]

(ii) $BD^2 = BG \times BF$. [3]

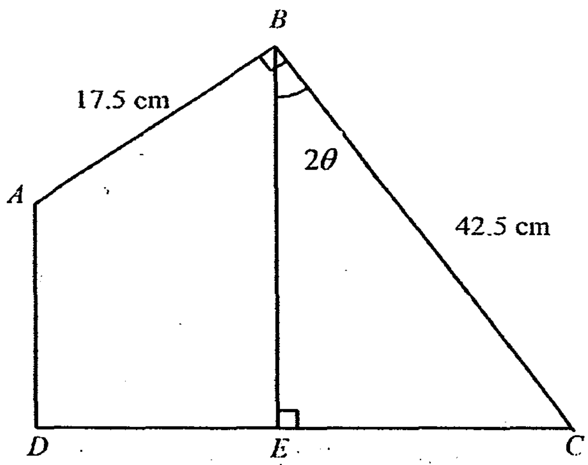
10. The diagram shows part of the curve $y^2 = x + 16$. The line $y = x - 4$ meets the curve at points A and B.



(i) Find the coordinates of A and B. [3]

(ii) Calculate the area of shaded region bounded by the curve and the straight line AB. [4]

11. The diagram shows a glass window, $ABCD$, consisting of a trapezium $ABED$ in which $AD // BE$ and a right-angled triangle BCE. It is given that $AB = 17.5$ cm, $BC = 42.5$cm, and AB and BC intersect each other perpendicularly at B, and $\angle EBC = 2\theta$, where θ varies.



(i) Show that the perimeter, P cm, of the glass window $ABCD$ is $P = 60 + 60\cos 2\theta + 25\sin 2\theta$. [3]

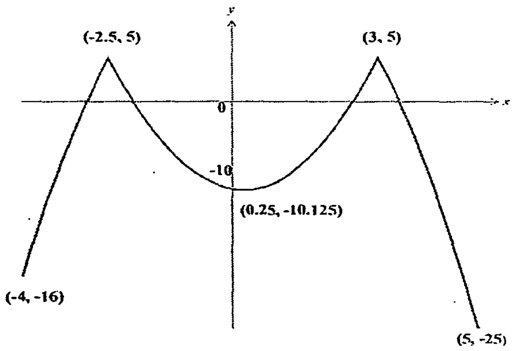
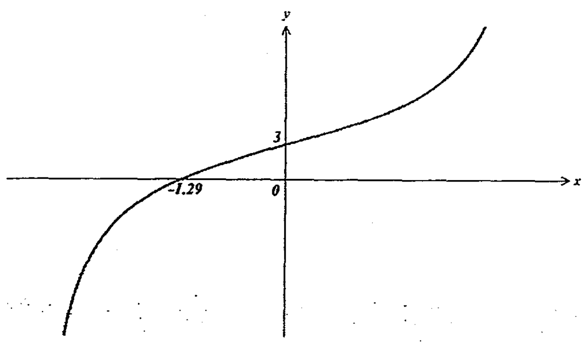
(ii) Express P in the form $a + b\sin(2\theta + \alpha)$ where $a, b > 0$ and $0° < \alpha < 90°$. [4]

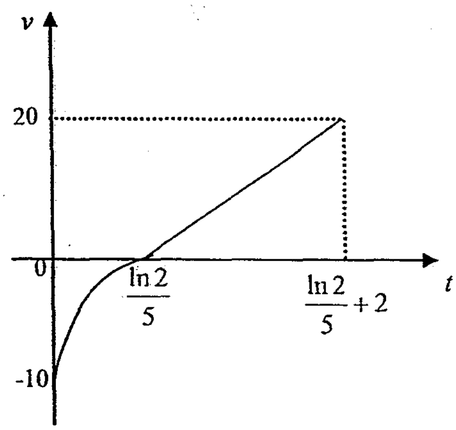
(iii) Find the value of θ for which P =100 cm. [3]

(iv) Find the maximum value of P and the corresponding value of θ. [2]

– End of Paper –

Answer Key

1.	(ai) $f(x)=2x^4+13x^3-8x^2-17x-70$ (b) $\dfrac{6x^2-3x-2}{4x^3-12x^2+7x-21}=\dfrac{1}{(x-3)}+\dfrac{2x+3}{(4x^2+7)}$
2.	(ai) $y=5-\lvert 2x^2-x-15\rvert$  (aii) Value of $k=5$ or $-16\le k<-10\dfrac{1}{8}$ (b) $x=1\dfrac{1}{2}$
3.	(ai) $x=60°,\ 300°$ (aii) 10 (b) $y=0.464,\dfrac{\pi}{2},3.61,\dfrac{3\pi}{2},6.75,\dfrac{5\pi}{2}$
4.	(i) 2π (ii) a=4, b=3 (iii) 
5.	(i) $18\tan^2 6\theta\sec^2 6\theta$ (ii) $\dfrac{1}{18}\tan^3 6\theta+C$ (iii) $\dfrac{1}{6}\tan 6\theta+\dfrac{1}{18}\tan^3 6\theta+C$
6.	(i) $B(5,8)$ (ii) $D(-2.5,-2)$ (iv) 62.5 units^2
7.	(i) $h=\dfrac{75}{r^2}-\dfrac{2}{3}r$ (iv) A is a minimum value
8.	(i) -3m from O (ii) $-10m/s$ (v) 20.6 m

	(iii) 
10.	(i) $A\ (9,5)$ and $B\ (0,-4)$ (ii) 121.5 units ^{2}
11.	(ii) $P = 60 + 65\sin(2\theta + 67.4°)$ (iii) 37.3° (iv) Corresponding value of $\theta = 11.3°$ Maximum value of $P = 125\text{m}$

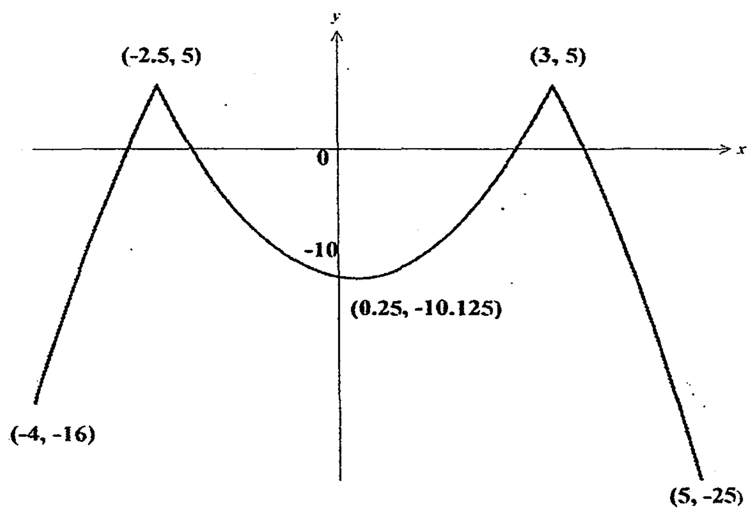
NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
1(a) (i)	$f(x) = (x-2)(x+7)(ax^2+bx+c)$ comparing coefficent of x^4 : $2x^4 = ax^4$ $a = 2$ $f(1) = -80$ $(1-2)(1+7)(2\times1^2+b\times1+c) = -80$ $b+c = 8$ ----------- (1) $f(-1) = -72$ $(-1-2)(-1+7)(2\times(-1)^2+b\times-1+c) = -72$ $9b-9c = -18$ -------- (2) Solving Equation (1) and (2) $b = 3$ $c = 5$ $\therefore f(x) = (x-2)(x+7)(2x^2+3x+5)$ $= 2x^4+13x^3-8x^2-17x-70$
1(a)(ii)	$f(x) = 0$ $(x-2)(x+7)(2x^2+3x+5) = 0$ $(x-2) = 0$ or $(x+7) = 0$ or $2x^2+3x+5 = 0$ $x = 2$ or $x = -7$ The quadratic equation $2x^2+3x+5 = 0$ has discriminant $b^2-4ac = 3^2-4\times2\times5 = -31 < 0$ It has no real roots. Hence, $f(x) = 0$ has exactly 2 real roots.
1(b)	Let $f(x) = 4x^3-12x^2+7x-21$ $f(3) = 4\times3^3-12\times3^2+7\times3-21 = 0$ $\therefore (x-3)$ is a factor of $f(x)$. By long division , $f(x) = (x-3)(4x^2+7)$ $\dfrac{6x^2-3x-2}{4x^3-12x^2+7x-21} = \dfrac{A}{(x-3)} + \dfrac{Bx+C}{(4x^2+7)}$ $6x^2-3x-2 = A(4x^2+7)+(Bx+C)(x-3)$ Sub $x = 3$, $6\times3^2-3\times3-2 = A(4\times3^2+7)$ $A = 1$

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution

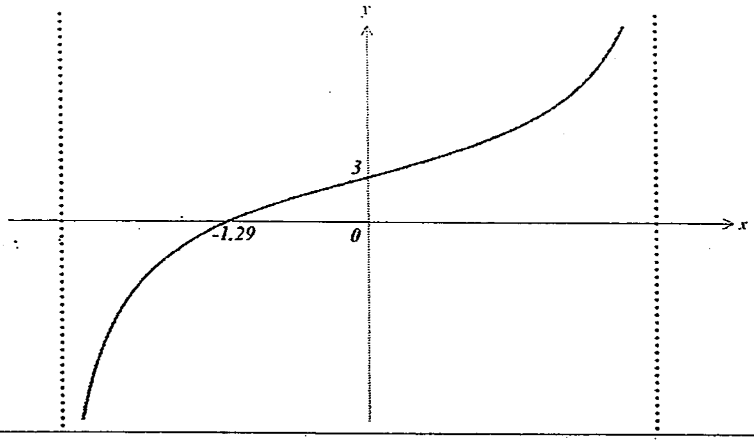
	Sub $x=0$, $6\times0^2-3\times0-2=1\times(4\times0^2+7)+C(0-3)$
	$C=3$
	Sub $x=1$, $6\times1^2-3\times1-2=1\times(4\times1^2+7)+(B+3)(1-3)$
	$B=2$
	$\dfrac{6x^2-3x-2}{4x^3-12x^2+7x-21}=\dfrac{1}{(x-3)}+\dfrac{2x+3}{(4x^2+7)}$

2(a)(i)	$y=5-\lvert 2x^2-x-15\rvert$ 
2(a)(ii)	Value of $k=5$ or $-16\le k<-10\frac{1}{8}$.
2(b)	$\lvert -3x+18\rvert=6x+\lvert x-6\rvert$ $\lvert -3(x-6)\rvert=6x+\lvert x-6\rvert$ $\lvert -3\rvert\lvert x-6\rvert-\lvert x-6\rvert=6x$ $\lvert x-6\rvert=3x$ $x-6=3x$ or $x-6=-3x$ $x=-3$ or $x=1\frac{1}{2}$ Check: $x=-3$ LHS $=\lvert -3(-3)+18\rvert=27$ RHS $=6(-3)+\lvert -3-6\rvert=-9\ne LHS$ $\therefore x=-3$ is rejected. $x=1\frac{1}{2}$ LHS $=\left\lvert -3(1\frac{1}{2})+18\right\rvert=13.5$ RHS $=6(1\frac{1}{2})+\left\lvert 1\frac{1}{2}-6\right\rvert=13.5=$ LHS

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

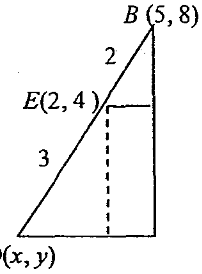
Question	Solution
	$\therefore\ x = 1\frac{1}{2}$
3(a)(i)	$6\sin^2\frac{x}{2} - 2\cos^2 x = 1$ $3(1-\cos x) - 2\cos^2 x - 1 = 0$ $3 - 3\cos x - 2\cos^2 x - 1 = 0$ $2\cos^2 x + 3\cos x - 2 = 0$ $(2\cos x - 1)(\cos x + 2) = 0$ $\cos x = -2\ (rej\ \sin ce -1 \le \cos x \le 1)$ or $\cos x = \frac{1}{2}$ $x = 60°,\ 300°$
3(a)(ii)	There are 5 complete revolutions in $-1080° < x < 720°$. No. of solutions = 5 x 2=10
3(b)	$\sin 2y = \cos^2 y$ $2\sin y\cos y = \cos^2 y$ $\cos y(2\sin y - \cos y) = 0$ $\cos y = 0$ $y = \frac{\pi}{2}\ or\ \frac{3\pi}{2}\ or\ \frac{5\pi}{2}$ $or\quad \tan y = \frac{1}{2}$ Basic angle $= 0.463648$ $y = 0.464, 3.61, 6.75 \qquad Ans: y = 0.464, \frac{\pi}{2}, 3.61, \frac{3\pi}{2}, 6.75, \frac{5\pi}{2}$
4 (i)	The period of f is 2π.
4(ii)	$a\tan\left(\frac{0}{2}\right) + b = 3$ $b = 3$ $a\tan\left(\frac{\pi/2}{2}\right) + 3 = 7$ $a\tan\left(\frac{\pi}{4}\right) = 4$ $a = 4$

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

<table>
<tr><th>Question</th><th>Solution</th></tr>
<tr><td>4(iii)</td><td>$y = 4\tan\left(\frac{x}{2}\right)+3$

</td></tr>
<tr><td>5</td><td>$\frac{d}{d\theta}\left(\tan^3 6\theta\right) = 3\tan^2 6\theta \sec^2 6\theta . 6$
$= 18\tan^2 6\theta \sec^2 6\theta$</td></tr>
<tr><td>5(i)</td><td>$\int 18\tan^2 6\theta \sec^2 6\theta \, d\theta = \tan^3 6\theta + C_1$
$\int \tan^2 6\theta \sec^2 6\theta \, d\theta = \frac{1}{18}\tan^3 6\theta + C$</td></tr>
<tr><td>5(ii)</td><td>$\int \left(\sec^2 6\theta - 1\right)\sec^2 6\theta \, d\theta = \frac{1}{18}\tan^3 6\theta + C$
$\int \sec^4 6\theta \, d\theta - \int \sec^2 6\theta \, d\theta = \frac{1}{18}\tan^3 6\theta + C$
$\int \sec^4 6\theta \, d\theta - \frac{1}{6}\tan 6\theta = \frac{1}{18}\tan^3 6\theta + C$
$\int \sec^4 6\theta \, d\theta = \frac{1}{6}\tan 6\theta + \frac{1}{18}\tan^3 6\theta + C$</td></tr>
</table>

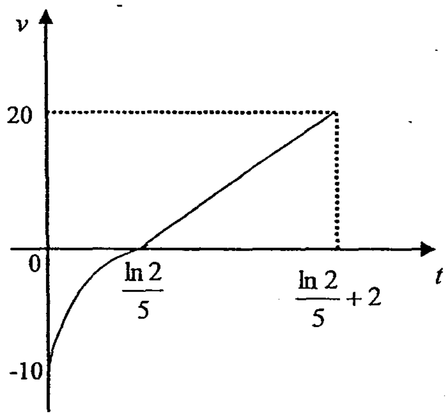
NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
6(i)	The mid - point of AC: $E=\left(\frac{6-2}{2},\frac{1+7}{2}\right)$ $=(2,4)$ gradient of $AC=\frac{7-1}{-2-6}=-\frac{3}{4}$ gradient of $BD=\frac{4}{3}$ Equation of BD: $y-4=\frac{4}{3}(x-2)$ $y=\frac{4}{3}x+\frac{4}{3}$ -------------(1) substitute (1) into equation $4y=x^2+7$ $4\left(\frac{4}{3}x+\frac{4}{3}\right)=x^2+7$ $3x^2-16x+5=0$ $(3x-1)(x-5)=0$ $x=\frac{1}{3}$ (rej $\because$ x - coordinate of B should be an integer) $\quad \therefore B(5,8)$ $\therefore x=5$ $y=8$
6(ii)	$BE=\sqrt{(5-2)^2+(8-4)^2}=5$ units $ED=5\times\frac{3}{2}=\frac{15}{2}$ units Let point D (x, y) $\sqrt{(x-2)^2+(y-4)^2}=\frac{15}{2}$ $\qquad (x-2)^2+\left(\frac{4}{3}\right)^2(x-2)^2=\frac{225}{4}$ $(x-2)^2+(y-4)^2=\frac{225}{4}$ -----------(2) Substitute $y=\frac{4}{3}(x+1)$ into eq (2) $(x-2)^2+(\frac{4}{3}(x+1)-4)^2=\frac{225}{4}$ $(x-2)^2=\frac{225}{4}\times\frac{9}{25}$ $x-2=4.5$ *or* -4.5 $x=6.5$ (rej since x - coordinate of D is negative) *or* -2.5

Question	Solution
	$y=\left(\frac{4}{3}\right)(-2.5+1)=-2$ $\therefore D(-2.5,-2)$ **<u>Alternative Method:</u>** Considering similar triangles / gradients  $\frac{8-4}{4-y}=\frac{2}{3}, \quad \frac{5-2}{5-x}=\frac{2}{5}$ $y=-2, \quad x=-2.5$ D(-2.5, - 2)
6(iii)	gradient of $AB=\frac{8-7}{5+2}=\frac{1}{7}$ gradient of $BC=\frac{8-1}{5-6}=-7$ gradient of $AB\times$ gradient of $BC=-1$ $\therefore AB \perp BC, \angle ABC=90°$ Since A, B and C are points on the circle, AC is the diameter of the circle. (rt$\angle$ in semi - circle) E is the mid - point of diameter AC hence it is the centre of the circle
6(iv)	Area of $ABCD$ $=\frac{1}{2}\begin{vmatrix} -2 & -2.5 & 6 & 5 & -2 \\ 7 & -2 & 1 & 8 & 7 \end{vmatrix}$ $=\frac{1}{2}(4-2.5+48+35+17.5+12-5+16)$ $=62.5$ units^2 **Alternatively,** Area of $ABCD$ $=\frac{1}{2}AC\times BD$ $=\frac{1}{2}\times 10\times 12.5$ $=62.5$ units^2
7(i)	$Volume=\frac{1}{3}\pi r^2(2r)+\pi r^2 h=75\pi$ $\frac{2}{3}r^3+r^2h=75$ $h=\frac{75}{r^2}-\frac{2}{3}r$

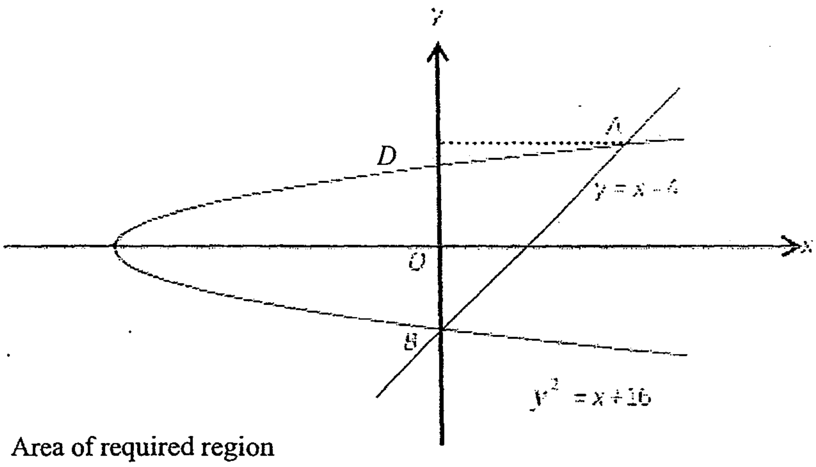
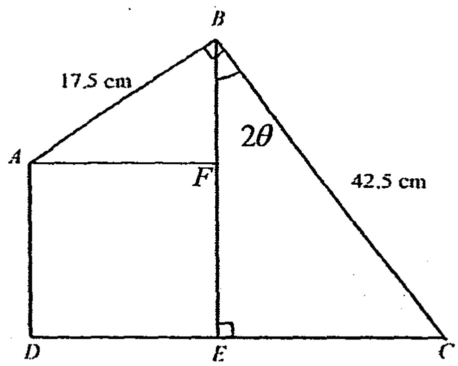
NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
7(ii)	$h=\frac{75}{r^2}-\frac{2}{3}r$ $\frac{dh}{dr}=\frac{-150}{r^3}-\frac{2}{3}$ $=-\left(\frac{150}{r^3}+\frac{2}{3}\right)<0 \text{ for all real values of } r>0$ Therefore, *h is a decreasing funct4ion.*
7(iii)	$A=\pi rl+2\pi rh+\pi r^2$ $=\pi r\sqrt{4r^2+r^2}+2\pi r\left(\frac{75}{r^2}-\frac{2}{3}r\right)+\pi r^2$ $=\pi r^2\sqrt{5}+\frac{150\pi}{r}-\frac{4\pi r^2}{3}+\pi r^2$ $=\pi r^2\sqrt{5}+\frac{150\pi}{r}-\frac{\pi r^2}{3}$ $=\left(\sqrt{5}-\frac{1}{3}\right)\pi r^2+\frac{150\pi}{r}$
7(iv)	$\frac{dA}{dr}=\left(\sqrt{5}-\frac{1}{3}\right)2\pi r-\frac{150\pi}{r^2}$ $\frac{d^2A}{dr^2}=\left(\sqrt{5}-\frac{1}{3}\right)2\pi+\frac{300\pi}{r^3}$ $\frac{d^2A}{dr^2}>0 \text{ for all real values of } r>0$ Therefore, the stationary value of A is a minimum.
8(i)	$s=4e^{-5t}+10t-7$ Initial position $=4e^{0(-5)}+10(0)-7$ $=4-7$ $=-3\text{m}$
8(ii)	$v=-20e^{-5t}+10$ Initial velocity $=-20e^{-5(o)}+10$ $=-10m/s$

Question	Solution
8(iii)	$v = -20e^{-5t} + 10$ At Instantaneous rest, v = 0 $-20e^{-5t} + 10 = 0$ $e^{-5t} = \frac{1}{2}$ $t = -\frac{1}{5}\ln\left(\frac{1}{2}\right)$ $\therefore T = -\frac{1}{5}(-\ln 2)$ $= \frac{\ln 2}{5}\ (shown)$
8(vi)	
8(v)	at t = 0, $s = -3$ m at $t = \frac{\ln 2}{5}$, $s = 4e^{-5\left(\frac{\ln 2}{5}\right)} + 10\left(\frac{\ln 2}{5}\right) - 7$ $= (2\ln 2 - 5)$m or -3.6137 m Since in the next 2 seconds, the particles travels at a constant acceleration, its velocity at $t = \frac{\ln 2}{5} + 2$ is $(0 + 2\times 10) = 20$m/s hence the distance travelled $= \frac{2\times 20}{2} = 20$m $\therefore$ the total distance travelled in the first $(T+2)$ seconds $= -3 - (2\ln 2 - 5) + 20$ $= 22 - 2\ln 2 = 20.6$ m

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

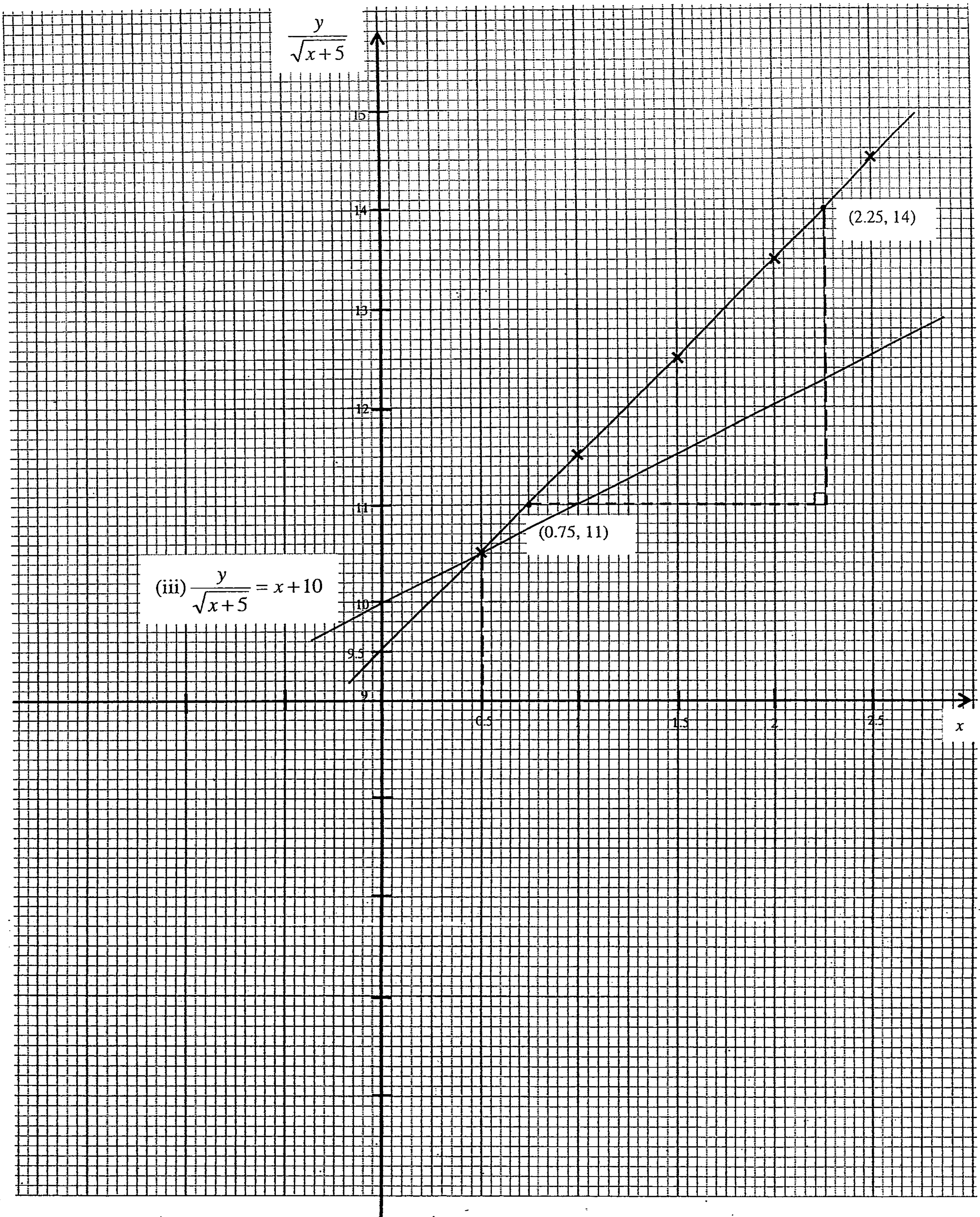
Question	Solution
9(i)	$\angle EAD = \angle CAB$ (common$\angle$) $AE = EC, \quad AD = DB$ (given) $\therefore \frac{AE}{AC} = \frac{AD}{AB} = \frac{1}{2}$ $\therefore \Delta EAD$ is similar to ΔCAB (SAS similarity) $\frac{AE}{AC} = \frac{ED}{BC}$ (corr. sides of similar Δs) $AE \times BC = AC \times ED$ (shown) **Alternatively,** $AE = EC, \quad AD = DB$ (given) $\frac{AE}{AC} = \frac{AD}{AB} = \frac{1}{2}$ $\frac{ED}{BC} = \frac{1}{2}$ (mid-point theo rem) $\frac{AE}{AC} = \frac{ED}{BC}$ $AE \times BC = AC \times ED$ (shown)
9(ii)	$\angle GBD = \angle DBF$ (common$\angle$) $\angle GDB = \angle DFB$ (tangent - chord theorem) $\therefore \Delta GBD$ is similar to ΔDBF (AA similarity) $\frac{BD}{BF} = \frac{GB}{DB}$ (corr. sides of similar Δs) $BD^2 = BG \times BF$ (shown)
10(i)	$(x-4)^2 = x+16$ $x^2 - 8x + 16 - 16 - x = 0$ $x^2 - 9x = 0$ $x(x-9) = 0$ $x = 0 \quad or \quad 9$ $y = -4 \quad or \quad 5$ $\therefore A\ (9, 5)$ and $B\ (0, -4)$

Question	Solution
10(ii)	 Area of required region $= \text{Area of } \Delta ADB - \int_4^5 (y^2-16)dy + 2\times\left\|\int_0^4 (y^2-16)dy\right\|$ $= \dfrac{9\times(5+4)}{2} - \left(\dfrac{y^3}{3}-16y\right)_4^5 + 2\left\|\left(\dfrac{y^3}{3}-16y\right)_0^4\right\|$ $= 40.5 - 4\dfrac{1}{3} + 85\dfrac{1}{3}$ $= 121.5$ units ^{2}
11(i)	$EC = 42.5\sin(2\theta)$ $BE = 42.5\cos(2\theta)$  $\angle ABF + \angle BAF = 90°$ $\angle ABF + \angle EBC = 90°$ $\angle BAF = \angle EBC = 2\theta$ $DE = AF = 17.5\cos(2\theta)$ $BF = 17.5\sin(2\theta)$ $AD = BE - BF = 42.5\cos(2\theta) - 17.5\sin(2\theta)$ $P = AB + BC + EC + DE + AD$ $= 17.5 + 42.5 + 42.5\sin(2\theta) + 17.5\cos(2\theta) +$ $42.5\cos(2\theta) - 17.5\sin(2\theta)$ $= 60 + (42.5 - 17.5)\sin(2\theta) + (17.5 + 42.5)\cos(2\theta)$ $= 60 + 60\cos(2\theta) + 25\sin(2\theta)$ (shown)

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
11(ii)	Let $60\cos(2\theta)+25\sin(2\theta) = b\sin(2\theta+\alpha)$ $= b\sin(2\theta)\cos\alpha + b\cos(2\theta)\sin\alpha$ $b\cos\alpha = 25 ------(1)$ $b\sin\alpha = 60 ------(2)$ $(1)^2+(2)^2$: $b^2(\cos^2\alpha+\sin^2\alpha)=25^2+60^2$ $b^2 = 65^2$ $b = 65$ *or* -65 $(rej \because b>0)$ $\frac{(2)}{(1)}$: $\frac{b\sin\alpha}{b\cos\alpha}=\frac{60}{25}$ $\tan\alpha = 2.4$ $\alpha = 67.380°$ $\therefore P = 60+65\sin(2\theta+67.4°)$
11(iii)	$100 = 60+65\sin(2\theta+67.380°)$ $40 = 65\sin(2\theta+67.380°)$ $\sin(2\theta+67.380°)=\frac{40}{65}$ Basic $\angle = 37.97987°$ $0° < 2\theta < 90°$ $67.380° < 2\theta+67.380° < 157.380°$ $\therefore 2\theta+67.380° = 180°-37.97987° = 142.02°$ $\theta = 37.3°$
11(iv)	Maximum value of P occurs when $\sin(2\theta+67.380°)=1$ $(2\theta+67.380°)=90°$ $\therefore$Corresponding value of $\theta = 11.3°$ Maximum value of $P = 60+65\times1 = 125$ cm

*Q*11



$\frac{y}{\sqrt{x+5}}$
15
14
13
12
11
10
9.5
9
0.5
1.5
2
2.5
x
(2.25, 14)
(0.75, 11)
(iii) $\frac{y}{\sqrt{x+5}} = x + 10$