1 Given that the line $y=x+a$ is a tangent to the curve $y^2=bx$, where a and b are positive integers, prove that $\frac{b}{a}$ is a perfect square. [4]

2 Find the coordinates of the stationary point of the curve $f(x)=1-\frac{2}{x^2+1}$, and determine the nature of the stationary point. [5]

3 It is given that $\int_{-3}^{2} f(x)\,dx=a$ and $\int_{-1}^{-3} f(x)\,dx=b$, where a and b are constants, and $f(x)>0$ for all real values of x.

Find in terms of a and b,

(i) $\int_{-1}^{2} f(x)\,dx$, [2]

(ii) $\int_{-1}^{2} [\,2f(x)-x\,]\,dx$. [3]

4 It is given that $2^{3-x}=3^{2(1-x)}$.

(i) Find the exact value of 4.5^x. [3]

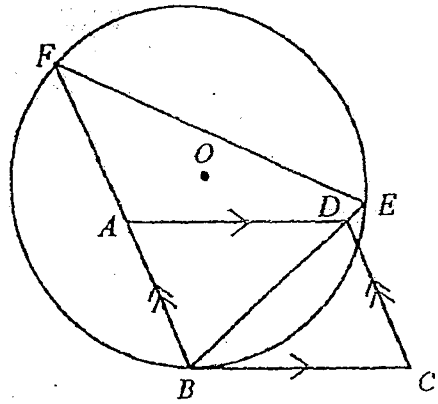
(ii) Hence find the value of x, correct to 3 significant figures. [2]

5 The roots of the equation $x^2+px+5=0$, where p is an integer, are α and β. The roots of the equation $x^2+18x+q=0$, where q is an integer, are α^3 and β^3.

(i) Express $\alpha^2-\alpha\beta+\beta^2$ in terms of $(\alpha+\beta)$ and $\alpha\beta$. [1]

(ii) Find the value of p and of q. [6]

6



In the diagram, BC is a tangent to the circle BEF, and $ABCD$ is a parallelogram.

Prove that

(i) $\angle BAD = \angle FEB$, [3]

(ii) ΔBCD is similar to ΔFEB, [2]

(iii) $BD \times BE = BA \times BF$. [2]

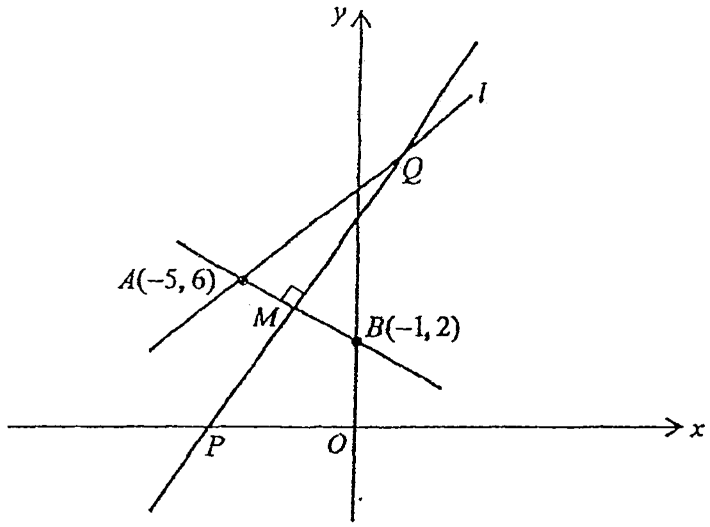
7 (i) Express the equation
$$8\cot^2 2\theta = 16\operatorname{cosec} 2\theta - 15$$
as a quadratic equation in cosec 2θ. [2]

(ii) Hence solve the equation
$$8\cot^2 2\theta = 16\operatorname{cosec} 2\theta - 15$$
for $0° \le \theta \le 180°$. [4]

(iii) State the number of solutions of the equation
$$8\cot^2 2\theta = 16\operatorname{cosec} 2\theta - 15$$
in the range $-360° \le \theta \le 360°$. [1]

PLMGS 2014 4E/5N

8 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, M is the midpoint of the line joining the points $A(-5, 6)$ and $B(-1, 2)$. The perpendicular bisector of AB intersects the x-axis at the point P and the line l at the point Q.

Given that the line l is parallel to the line PB, find

(i) the coordinates of Q, [4]

(ii) the area of the quadrilateral $APBQ$. [2]

9 A particle travels in a straight line, so that t seconds after passing through a fixed point O, its velocity, $v\ \mathrm{ms}^{-1}$, is given by $v = 3(e^{kt} + 2)$, where k is a constant. The initial acceleration of the particle is $\frac{3}{5}\mathrm{ms}^{-2}$.

(i) Show that the value of k is $\frac{1}{5}$. [2]

(ii) Explain why the particle would never return to O. [2]

(iii) Find the total distance travelled by the particle in the first 5 seconds. [3]

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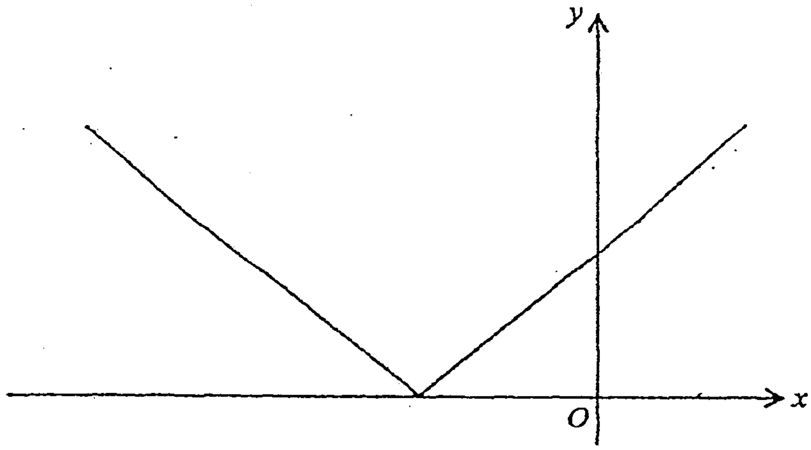


The diagram shows part of the graph of $y=|x+2|$. In each of the following cases, determine the number of intersections of the line $y=mx+c$ with $y=|x+2|$, justifying your answer.

(i) $m=1$ and $c>2$ [2]

(ii) $m=\frac{1}{2}$ and $c<1$ [2]

(iii) $m=-\frac{1}{2}$ and $c=0$ [2]

11 The function f is defined, for all values of x by $f(x)=2-a\sin bx$ where a and b are positive integers. The minimum value of $f(x)$ is -1. Two complete cycles of the graph of $y=f(x)$ can be sketched in the interval $-\pi \leq x \leq \pi$.

(i) Find the value of a and of b. [2]

Using the values of a and of b found in part (i),

(ii) find, in radians, the largest negative value of x for which $f(x)=2$, [1]

(iii) sketch the graph of $y=f(x)$ for $-\pi \leq x \leq \pi$, [2]

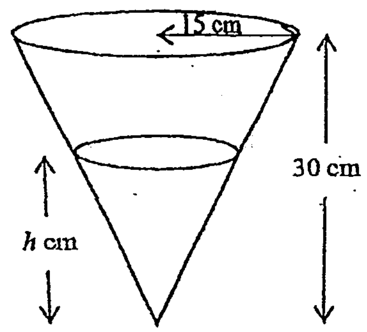
(iv) identify a possible pair of values for c and d, if the equation $a\sin bx + c\cos dx = 2$, where c and d are positive integers, has exactly 5 solutions in the interval $-\pi \leq x \leq \pi$. [2]

12 Using experimental values of variables x and y, a graph was drawn in which $\ln y$ was plotted on the vertical axis against x on the horizontal axis. The straight line that was obtained passed through the points $(2, 3)$ and $(10, 7)$.

(i) Show that the equation connecting x and y can be expressed in the form $y = ae^{bx}$ where a and b are constants. [4]

(ii) Find the coordinates of the point on the line at which $y = e^{2x+1}$ [3]

13



The diagram shows a hollow conical container of height 30 cm and radius 15 cm. The container that is initially completely filled with water is held fixed with its circular rim horizontal. Water is leaking from a hole at the vertex of the container at a constant rate of $10\ \text{cm}^3\text{s}^{-1}$. After t seconds, the depth of water is h cm.

(i) Show that the volume of water in the container, $V\ \text{cm}^3$, at time t, is given by $V = \dfrac{\pi h^3}{12}$. [2]

(ii) Find the rate of change of the depth when $h = 2$. [4]

(iii) State, with a reason, whether this rate would increase or decrease as t increases. [1]

End of Paper

- 3

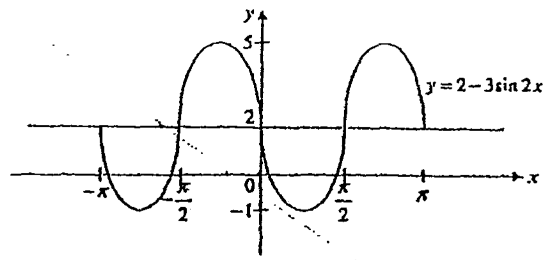
Paya Lebar Methodist Girls' School(Secondary)

Preliminary Examination 2 (2014)

Additional Mathematics (4047/1)

Answer Key

Qn	Answer	Qn	Answer
1	$\frac{b}{a}=4$, a perfect square	7	(i) $8\operatorname{cosec}^2 2\theta-16\operatorname{cosec}2\theta+7=0$ (ii) $\operatorname{cosec}2\theta=\frac{4\pm\sqrt{2}}{4}$, $\theta=23.8°,\ 66.2°$ (iii) 8 solutions
2	Stationary point $=(0,-1)$ It is a minimum point	8	(i) $Q(1,8)$ (ii) 32 square units
3	(i) $\int_{-1}^{2}\mathrm{f}(x)\,\mathrm{d}x=a+b$ (ii) $\int_{-1}^{2}[2\mathrm{f}(x)-x]\,\mathrm{d}x=2(a+b)-\frac{3}{2}$	9	(i) $k=\frac{1}{5}$ (ii) Velocity of the particle is always positive $\Rightarrow$ the particle is always moving away from O and to the right. (iii) 55.8 m (3 s.f.)
4	(i) $4.5^x=\frac{9}{8}$ (ii) $x=0.0783$ (3 s.f.)	10	(i) For $m=1$ and $c>2$, the line $y=mx+c$ is above the right arm and parallel to it. $\therefore$ The line $y=mx+c$ intersects the left arm at one point. Number of intersections = 1 (ii) For $m=\frac{1}{2}$ and $c=1$, the line $y=mx+c$ passes through the points $(-2,0)$ and $(0,1)$. For $m=\frac{1}{2}$ and $c<1$, the line $y=mx+c$ lies entirely below the graph of $y=\lvert x+2\rvert$. Number of intersection = 0 (iii) For $m=-\frac{1}{2}$ and $c=0$, the line $y=mx+c$ passes through the origin, and the points $(-2,1)$ and $(-4,2)$. The line $y=mx+c$ intersects the graph of $y=\lvert x+2\rvert$ twice. Number of intersections = 2.

5	(i) $\alpha^2-\alpha\beta+\beta^2=(\alpha+\beta)^2-3\alpha\beta$ (ii) $p=-3$, $q=125$	11	(i) $a=3$, $b=2$ (ii) Largest negative value of $x=-\frac{\pi}{2}$ (iii)  (iv) one possible pair $c=d=2$.
6	(i) Show that $\angle ADB=\angle BFE$ (ii) Show that $\angle BCD=\angle FEB$, - Show that $\angle BDC=\angle FBE$ (iii) Using part (ii), $\frac{BD}{FB}=\frac{CD}{EB}$	12	(i) $y=ae^{bx}$ where $a=e^2$ $b=\frac{1}{2}$ (ii) $\left(\frac{2}{3},\frac{7}{3}\right)$
		13	(i) $V=\frac{\pi h^3}{12}$ (ii) $-3.18\ \text{cms}^{-1}$ (iii) Since $\frac{dh}{dt}=-\frac{40}{\pi h^2}$, the rate of change of the depth will increase as t increases.

1 (i) Differentiate $x\cos\frac{x}{2}$ with respect to x. [2]

(ii) Using the answer to part (i), find $\int 2x\sin\frac{x}{2}\,dx$,

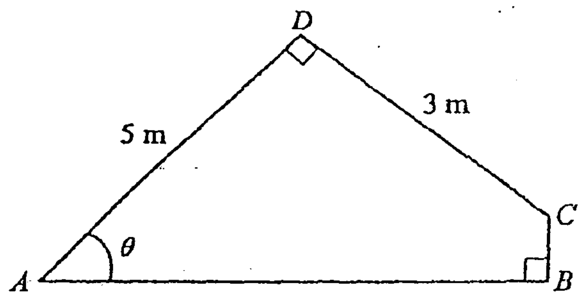
and hence show that $\int_0^{\frac{\pi}{3}} 2x\sin\frac{x}{2}\,dx = 2\left(2-\frac{\sqrt{3}\pi}{3}\right)$. [5]

2 (i) On the same diagram, sketch the graphs of $y=-\frac{1}{16}x^{\frac{4}{3}}$ and $y=-16x^{-\frac{4}{3}}$ for $x>0$. [2]

(ii) Find the coordinates of the point of intersection of the graphs. [2]

(iii) Determine, with explanation, whether the normal to the graph of $y=-\frac{1}{16}x^{\frac{4}{3}}$ and the tangent to the graph of $y=-16x^{-\frac{4}{3}}$, at the point of intersection, are perpendicular. [4]

3



The diagram shows an enclosure where $AD = 5$ m, $CD = 3$m and $\angle ADC = 90°$.
The side AD makes an acute angle θ with the side AB.
The sides AB and BC are perpendicular to each other.
The total length of the enclosure is L m.

(i) Show that L can be expressed as $a + b\cos\theta + c\sin\theta$, where a, b and c are constants to be found. [3]

(ii) Express L in the form $a + R\cos(\theta - \alpha)$ where $R > 0$ and α is an acute angle. [4]

The total length of the enclosure is found to be 11.8 m.

(iii) Find the value of θ. [2]

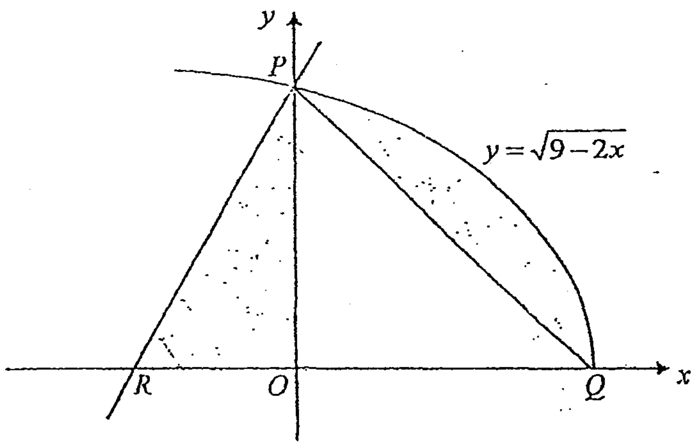
4 Given that $\dfrac{2x^3-3x^2-11x+20}{x^2-9} = 2x+a+\dfrac{bx+c}{x^2-9}$,

(i) find the value of each of the integers a, b and c. [4]

Hence using partial fractions and the values of a, b and c obtained in part (i), find

(ii) $\displaystyle\int \frac{2x^3-3x^2-11x+20}{x^2-9}\,dx$. [6]

5



The diagram shows part of the curve $y=\sqrt{9-2x}$ that crosses the axes at the points P and Q. The lines PQ and PR are perpendicular to each other and intersect the x-axis at the points R and Q.

(i) Find the coordinates of P, Q and R. [4]

(ii) Find the total area of the shaded regions. [6]

6 A curve has the equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{\ln(1-x)}{x-1}$ for $x < 1$.

(i) Obtain an expression for $\mathrm{f}'(x)$. [2]

(ii) The tangent to the curve at the point where $x = -1$ intersects the y-axis at the point A. Find the exact coordinates of the point A. [4]

(iii) Showing all necessary working, find the range of values of x for which f is a decreasing function. [3]

(iv) Hence deduce the range of values of x for which f is an increasing function. [1]

7 (i) Write down the first three terms in the expansion, in ascending powers of x, of $\left(1-\dfrac{x}{3}\right)^n$, where n is a positive integer greater than 2. [2]

(ii) Find, in terms of n and p, the first three terms in the expansion, in ascending powers of x, of $\left(2+px+\dfrac{5}{2}x^2\right)\left(1-\dfrac{x}{3}\right)^n$, where p is a constant. [2]

In the expansion of $\left(2+px+\dfrac{5}{2}x^2\right)\left(1-\dfrac{x}{3}\right)^n$, in ascending powers of x, the first three terms are $2+\dfrac{31p}{3}x+\dfrac{25}{3}x^2$.

(iii) Find the value of n and of p. [illegible]

(iv) Hence find the coefficient of x^3 in the expansion of $\left(2+px+\dfrac{5}{2}x^2\right)\left(1 [illegible]\right.$ [illegible]

[Turn over

8 (i) Prove the identity $\cos 4A = 8\sin^4 A - 8\sin^2 A + 1$. [4]

(ii) Solve the equation $16\sin^4 A - 16\sin^2 A = -4$ for $0 < A < \pi$, giving your answers in terms of π. [3]

(iii) Given that $8\sin^4 \frac{x}{2} - 8\sin^2 \frac{x}{2} + 1 = 0$ and $\cos x \neq 0$,

(a) deduce that $\tan x = \pm 1$, [3]

(b) find the possible values of $\sec x$. [2]

9 The points $(-15, -5)$ and $(1, -5)$ are on the circumference of a circle whose centre lies below the x-axis. The line $y = -21$ is a tangent to the circle.

Find

(i) the centre and the radius of the circle, [8]

(ii) the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers, [2]

(iii) the equations of the tangents to the circle that are parallel to the y-axis. [2]

-2

10 (a) Given that $u = \log_7 x$, find, in terms of u,

(i) $\log_7\left(7\sqrt{x}\right)$, [1]

(ii) $\log_x\left(\frac{1}{343}\right)$. [2]

(b) Given that $\ln\left(x^2y\right) = p$ and $\ln\left(xy^2\right) = q$, express $\frac{x}{y}$ in terms of p and q. [3]

(c) In an experimental environment, the population of a certain bacteria can be modelled by the equation $P = 500 + Ae^{kt}$, where A and k are constants and t is the time in days. During the first 3 days of the experiment, the population of the bacteria decreased exponentially from 5500 to 5000, find

(i) the value of A and of k. [3]

If the population of the bacteria continues to decrease at the same rate,

(ii) after how many days would it first decrease to below 1000? [3]

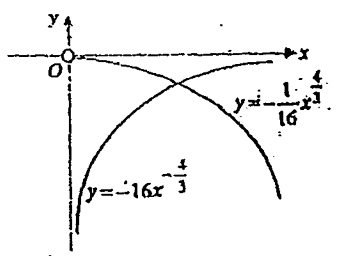
End of Paper

Paya Lebar Methodist Girls'School(Secondary)

Preliminary Examination 2 (2014)

Additional Mathematics (4047/2)

Answer Key

Qn	Answer	Qn	Answer
1	(i) $\frac{d}{dx}\left(x\cos\frac{x}{2}\right)=-\frac{x}{2}\sin\frac{x}{2}+\cos\frac{x}{2}$ (ii) $\int 2x\sin\frac{x}{2}\,dx=8\sin\frac{x}{2}-4x\cos\frac{x}{2}+C$ $\int_0^{\frac{\pi}{3}} 2x\sin\frac{x}{2}\,dx=2\left(2-\frac{\sqrt{3}\pi}{3}\right)$	7	(i) $\left(1-\frac{x}{3}\right)^n=1-\frac{n}{3}x+\frac{n(n-1)}{18}x^2+...$ (ii) $1+\left(p-\frac{2n}{3}\right)x+\left[\frac{n(n-1)}{9}-\frac{np}{3}+\frac{5}{2}\right]x^2+...$ (iii) $n=7$ and $p=-\frac{1}{2}$ (iv) $=-\frac{239}{27}$
2	(i)  (ii) Point of intersection $=(8,-1)$ (iii) The normal to the graph of $y=-\frac{1}{16}x^{\frac{4}{3}}$ is not perpendicular to the tangent to the graph of $y=-16x^{-\frac{4}{3}}$ as (gradient of normal)$\times$(gradient of tangent) $=6\times\frac{1}{6}\neq-1$	8	(i) Proof (ii) $x=\frac{\pi}{4},\frac{3\pi}{4}$ (iii) (a) $\tan x=\pm1$ (b) $\sec x=\pm\sqrt{2}$

3	(i) $L=8+2\cos\theta+8\sin\theta$ where $a=8$, $b=2$ and $c=8$ (ii) $L=8+2\sqrt{17}\cos(\theta-76.0^\circ)$ (iii) $\theta=13.4^\circ$	9	(i) Centre of circle $=(-7,-11)$ Radius of circle $=10$ units (ii) $a=14$, $b=22$ and $c=70$ (iii) $x=-17$ and $x=3$
4	(i) $\frac{2x^3-3x^2-11x+20}{x^2-9}=2x-3+\frac{7x-7}{x^2-9}$ (ii) $\int\frac{2x^3-3x^2-11x+20}{x^2-9}\,dx$ $=\int\left[2x-3+\frac{14}{3(x+3)}+\frac{7}{3(x-3)}\right]dx$ $=x^2-3x+\frac{14}{3}\ln(x+3)+\frac{7}{3}\ln(x-3)+C$	10	(a) (i) $\log_7\left(7\sqrt{x}\right)=1+\frac{u}{2}$ (ii) $\log_x\left(\frac{1}{343}\right)=-\frac{3}{u}$ (b) $\frac{x}{y}=e^{p-q}$ (c) (i) $A=5000$ $k\approx-0.0351$ (to 3 s.f.) (ii) 66 days
5	(i) $P=(0,3)$, $Q=(4\frac{1}{2},0)$, $R=(-2,0)$ (ii) Total area of shaded regions $=\frac{1}{2}(2)(3)+\int_0^{\frac{9}{2}}\sqrt{9-2x}\,dx+\frac{1}{2}\left(\frac{9}{2}\right)(3)$ $=5\frac{1}{4}$ sq. units		
6	(i) $f'(x)=\frac{1-\ln(1-x)}{(x-1)^2}$ (ii) $A=\left(0,\frac{1-3\ln 2}{4}\right)$ (iii) f is a decreasing function for $x<1-e$ (iv) f is a increasing function for $1-e<x<1$		