1 The curve $y=(4+p)x^2-4x+p$ has a maximum point and cuts the line $y=-1$ at two distinct points. Find the range of values of p. [5]

2 A curve has the equation $y=3xe^{2x+1}$.

(i) Show that the curve has only one stationary point. [4]

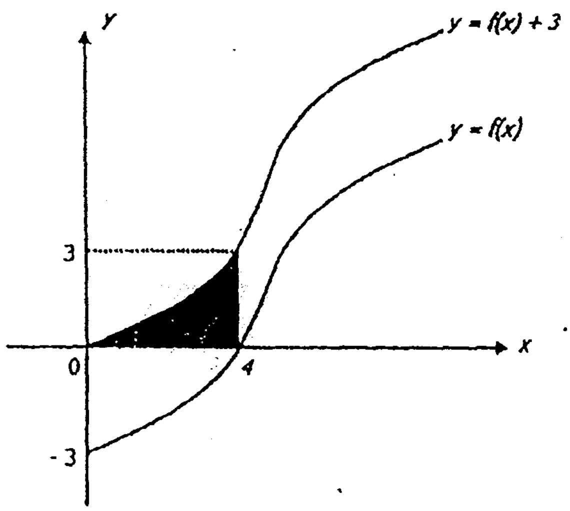
(ii) Determine the nature of this stationary point. [2]

3 The diagram below shows the graphs of $y=f(x)$ and $y=f(x)+3$.

Given that $\int_0^8 f(x)\,dx=16$ and $\int_0^4 f(x)\,dx=-7$, evaluate

(i) $\int_4^8 f(x)\,dx$, [1]

(ii) the area of the shaded region, [2]



4 Using a separate diagram for each part, represent on the number line the solution set of

(i) $|x|<3$ [1]

(ii) $(2x+1)^2>9$ [2]

Hence, state the range of values of x which satisfy both inequalities. [2]

5 Solve the following equations in the given range:

(a) $\cos\left(2x-\frac{\pi}{6}\right)+4\sin 2x=0,\quad 0\le x\le 3$ [4]

(b) $\sec x(\tan x-2)=2\operatorname{cosec} x,\quad -180^\circ<x<180^\circ$ [4]

Begin this Section on a fresh sheet of paper.

6 Given that $\alpha^2+\beta^2=14$ and that $\alpha\beta=7$.

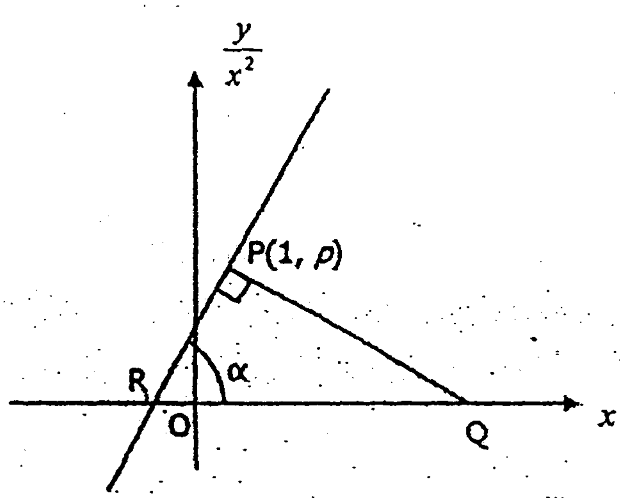
(i) find the value of $\alpha+\beta$, where $\alpha+\beta>0$, [2]

(ii) Find the quadratic equation whose roots are $\alpha-1$ and $\beta-1$. [3]

7 (i) Solve $\log_{25}[\log_2(10x+2)]=\log_9 3$. [4]

(ii) The solution of $2^{2x+3}=2^{x+1}+3$ can be expressed in the form $\log_2\frac{p}{q}$, where p and q are integers. Find the value of p and of q. [5]

8 The figure shows part of the straight line graph which is obtained by plotting $\frac{y}{x^2}$ against x for a relation between x and y given by $4y=8x^3+12x^2$. This straight line graph passes through $P(1,p)$ and cuts the x-axis at R. Q is a point on the horizontal axis such that $\angle QPR=90^\circ$ and $\angle QRP=\alpha$.

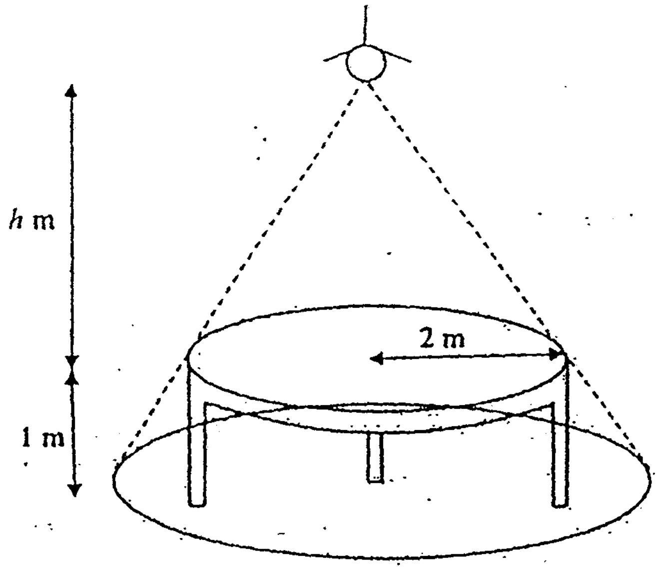


Find the value of p, of α and the x-coordinate of Q. [7]

9 A round stool has radius 2 m and height 1 m.

A small lamp O is placed h metres vertically above the centre of the stool, where $0 < h < 4$.

The lamp casts a shadow of the stool onto the ground.



(i) Show that the area of the shadow, A m^2, is given by $A = \dfrac{4\pi(h+1)^2}{h^2}$. [2]

(ii) The lamp is lowered vertically at a constant rate of $\dfrac{1}{8}$ m/s, find the rate of change of A when $h = 3$. [4]

Begin this Section on a fresh sheet of paper.

10 The function f is defined by $f(x) = 2 - 5x - x^2$.

(i) Express the function in the form $f(x) = a - (x+b)^2$, stating the maximum value of $f(x)$. [3]

(ii) Sketch the graph of $y = |f(x)|$ for $-6 \le x \le 1$, indicating the coordinates of the maximum point and the points of intersection with the coordinate axes. [3]

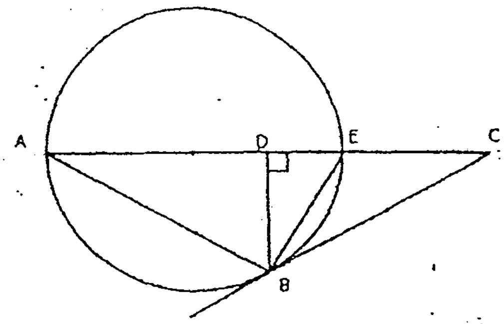
(iii) Hence, state the range of values of z in the range $-6 \le x \le 1$ for which the equation $|f(x)| = z$ has 4 solutions. [1]

11 (a) In the figure, the tangent to the circle at B meets the diameter AE produced at C. Given that BD is perpendicular to AC, prove that

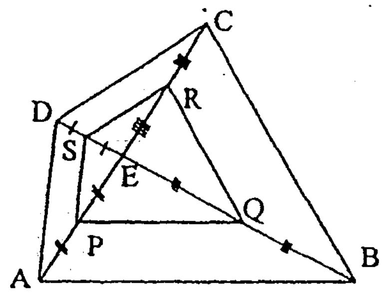
(i) EB bisects $\angle CBD$, [3]

(ii) ΔABC is similar to ΔBEC, [2]

(iii) $AB \times BC = AC \times BE$. [2]



(b) In the diagram, E is the point of intersection of the diagonals AC and BD of the quadrilateral $ABCD$. The points P, Q, R and S are the midpoints of AE, BE, CE and DE respectively.
Prove that the perimeter of $ABCD$ is twice the perimeter of $PQRS$. [3]



12 A particle moves in a straight line such that t seconds after passing through point O, its displacement, s metres, is given as $s = k\sin(\pi t)$, where k is a constant.

(i) Express the velocity v ms^{-1}, of the particle in terms of π, k and t. [1]

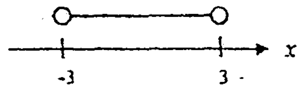
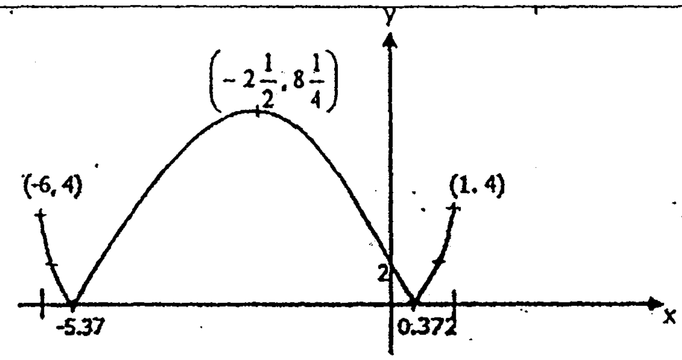
(ii) The initial velocity of the particle is 4π ms^{-1}. Find the value of k. [2]

(iii) Find the value of t at which the particle first comes to instantaneous rest. [3]

(iv) Find the total distance travelled by the particle in the first second. [3]

End of Paper

Answer key

1.		$-5<p<-4$	2.		The stationary point is a minimum point at $x=-\frac{1}{2}$
3.	(i)	(i) $\int_{4}^{5} f(x)\,dx = 16-(-7)=23$	4.	(i)	$x<3$ and $-x<3$ $x>-3$ 
	(ii)	$5 units^2$		(ii)	Solution set: $\{x : x \in \Re,\ -3<x<-2 \ \text{or}\ 1<x<3\}$
5.	(a)	$x=1.48 rad\ (3sf)$	6	(i)	$\alpha+\beta=\sqrt{28}$ or $2\sqrt{7}$
	(b)	$x=-110.1°, -36.2°, 69.9°, 143.8°$		(ii)	Equation: $x^2-(2\sqrt{7}-2)x+(8-2\sqrt{7})=0$
7.	(i)	$x=3$	8.		x- coordinate of Q is 11.
	(ii)	$p=3, q=4$	11.	(a)	(ii) ΔABC is similar to ΔBEC. (AA similarity)
9.	(ii)	$\frac{4\pi}{27} m^2/s$			
10.	(i)	Maximum value of $f(x)=\frac{33}{4}$ or $8\frac{1}{4}$			
	(ii)				
	(iii)	$0<z\leq 4$			
12.	(i)	$v=k\pi\cos(\pi t)$			
	(ii)	$k=4$			
	(iii)	t = 0.5s			
	(iv)	Total distance travelled by the particle in the first second = $2(4)=8m$			

Answer all questions.

Section A.

1. A cup of hot drink, initially at 95° C was left to stand in a room.

   The temperature, $\theta° C$, at time t minutes is given by the equation $\theta = Ae^{-kt} + 28$,

   where A and k are constants. At 6 minutes, the temperature dropped to 50° C.

   (i) Find the value of A and of k. [3]

   (ii) Find the time T minutes, where T is a <u>whole number</u>, after which the temperature is below 35° C. [2]

   (iii) Calculate the rate at which the temperature decreases when $t = 10$. [2]

   (iv) State the temperature the drink will reach when it is left to stand for a long time. [1]

2. Given that the roots of $3x^2 - 13x + 2 = 0$ are α and β,

   (i) State the value of (a) $\alpha + \beta$, [1]

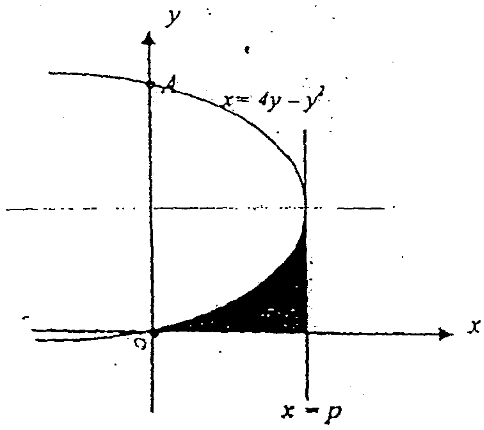
   (b) $\alpha\beta$. [1]

   (ii) Given further that $\alpha = \ln p$ and $\beta = \ln q$, find the value of each of the following:

   (a) $\ln p^2 \times \ln q^2$, [1]

   (b) $\log_q p + \log_p q$. [3]

3.



The above diagram shows a graph of the curve $x = 4y - y^2$. The curve meets the y-axis at the origin and point A. The line $x = p$ is a tangent to the curve.

(i) Find the coordinates of point A, and hence the equation of the line of symmetry. [2]

(ii) Show that $p = 4$. [2]

(iii) Calculate the area of the shaded region. [3]

4. (a) Expand $(1 + 2x)^5$ and $(1 - 2x)^5$ in ascending powers of x.

Hence write $(1 + 2x)^5 - (1 - 2x)^5$ in its simplest form. [3]

Use your expansion to find the exact value of

$(1+2\sqrt{2})^5 - (1-2\sqrt{2})^5$. [3]

(b) Given that a is positive, write down in descending powers of x, the first four terms in the expansion of $(x + \frac{a}{x})^8$, simplifying the coefficients. [3]

Given that the ratio of the coefficient of the fourth term to the coefficient of the second term is 7 : 9, find the value of a. [2]

Section B. (Start on a fresh sheet of paper.)

5. The lines $y = 8$ and $y = -2$ are tangents to a circle C_1.
The line $3y = x + 11$ is an equation of the diameter of the circle.
Find (i) the radius of the circle. [1]
(ii) the coordinates of the centre of circle C_1. [3]

Circle C_2 has the same centre as that of Circle C_1 but its area is twice that of the area of Circle C_1.

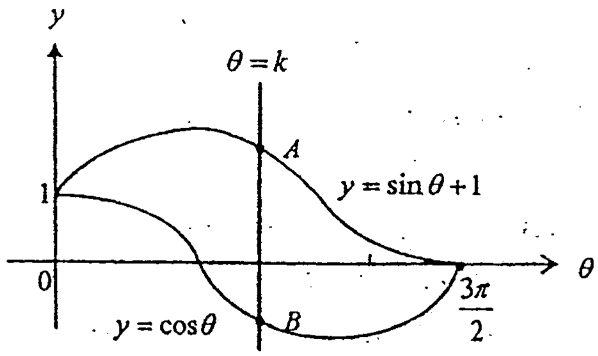
(iii) Find the equation of circle C_2 in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [3]

6. Given that $(x^4 + x^3 + 4x^2 - 3x + 1) \div (x^3 - x^2) = a + x + \dfrac{6x^2 + bx + c}{x^2(x-1)}$, find the value of each of the integer a, b, and c. [3]

Using partial fractions and the values of integer a, b, and c found,

evaluate $\displaystyle\int_3^5 \frac{x^4 + x^3 + 4x^2 - 3x + 1}{x^3 - x^2}\,dx$ [6]

7. (a)



The above are sketches of the curves $y = \sin\theta + 1$ and $y = \cos\theta$ for $0 \le \theta \le \frac{3}{2}\pi$.

The line $\theta = k$ meets the curves at points A and B respectively, where $0 \le k \le \frac{3}{2}\pi$.

Find the length of AB when $\theta = \frac{3\pi}{4}$.

Show that at this value of θ, AB is the maximum length. [4]

(b) Given $y = e^{\cos 2x}$, find $\frac{dy}{dx}$. Hence find the exact value of $\int_0^{\frac{\pi}{6}} 4e^{\cos 2x} \sin 2x dx$.

[4]

8. (i) Show that $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = \frac{2 - \sin 2\theta}{2}$. [3]

(ii) Hence, or otherwise, given that $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = \frac{7}{10}$, show that $\sin 2\theta = \frac{3}{5}$ [2]

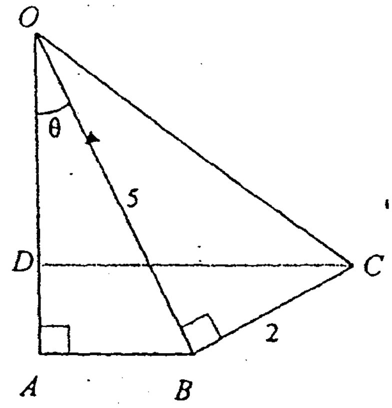
(iii) Given further that 2θ is an acute angle, without using calculator,

find the value of (a) $\cos\theta$ [2]

(b) $\tan 3\theta$. [3]

Section C (Start on a fresh sheet of paper.)

9.



The diagram shows two triangles OAB and OBC with $\angle OAB = \angle OBC = 90°$, and $\angle AOB = \theta°$.

The line CD is drawn parallel to BA.

Given that $OB = 5$ cm and $BC = 2$ cm,

(i) Show that $CD = 2\cos\theta + 5\sin\theta$. [4]

(ii) Express CD in the form $R\sin(\theta + \alpha)$. [4]

(iii) Find the maximum value of CD and the value of θ at which it occurs [3]

10. (i) Express the function $f(x) = 9x^3 + 3x^2 - 8x - 4$ as a product of three linear factors, and show that the equation $f(x) = 0$ has 2 real distinct roots. [3]

(ii) Given $y = f(x)$, find the coordinates of the turning points of the curve $y = 9x^3 + 3x^2 - 8x - 4$.

Sketch the curve $y = 9x^3 + 3x^2 - 8x - 4$.

(Proof of nature of turning point is not required.) [5]

(iii) From your sketch,

(a) state the range of values of x for which y is a decreasing function. [1]

(b) explain why the function $g(x) = 9x^3 + 3x^2 - 8x - 5$ has only one linear factor. [1]

11. The line $y = x$ intersects the curve $y = x^{\frac{1}{3}}$ at three points A, B and O, where O is the origin.

(i) Find the coordinates of A and B. [3]

(ii) Sketch the curve $y = x^{\frac{1}{3}}$ and the line $y = x$ on the same axes. [3]

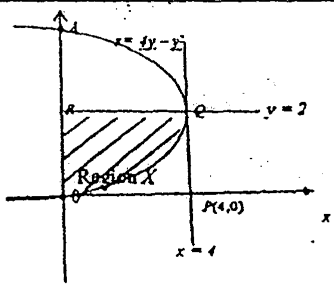
(iii) Given that the point $P\left(\frac{1}{8}, \frac{1}{2}\right)$ lies on the curve, find the perpendicular distance from the point P to the line $y = x$. [4]

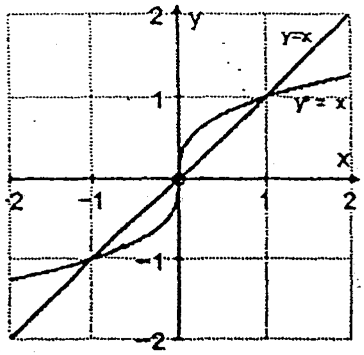
(iv) Find the gradient of the tangent to the curve at the point P.

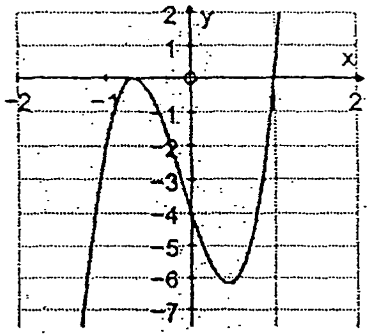
State whether the tangent drawn at point P will meet the line $y = x$, giving a reason for your answer. [3]

End of paper

Answer Key

1.	(i)	k=0.186 (3 s.f.)	2.	(i)	(a) $\alpha+\beta=\frac{13}{3}$ (b) $\alpha\beta=\frac{2}{3}$
	(ii)	T = 13.		(ii)	(a) $2\frac{2}{3}$ (b) $26\frac{1}{6}$
	(iii)	1.94^0 C/min.	4.	(a)	$596\sqrt{2}$
	(iv)	28^0C		(b)	Since $a>0, a=\frac{1}{3}$
3.	(i)	A (0,4) Equation of line of symmetry is y=2	5.	(i)	5 units.
	(iii)	 		(ii)	Center of circle (-2,3)
				(iii)	$x^2+y^2+4x-6y-37=0$.
			6	(i)	$\frac{x^4+x^3+4x^2-3x+1}{x^3-x^2}$ $=2+x+\frac{2}{x}-\frac{1}{x^2}+\frac{4}{(x-1)}$
		Required shaded area $=8-5\frac{1}{3}=2\frac{2}{3}$ units^2		(ii)	15.7 (3 sig fig.)
7	(a)	$1+\sqrt{2}$ units $\therefore$ the value of h at $\theta=\frac{3}{4}\pi$ is maximum.	8	(iii)a.	$\cos\theta=\frac{3\sqrt{10}}{10}$
	(b)	$\int_0^{\frac{\pi}{4}} 4e^{\cos 2x}\sin 2x\,dx=2(e-\sqrt{e})$		(b)	$1\frac{4}{9}$

9	(ii)	$CD = \sqrt{29}\sin(\theta + 21.80°)$
	(iii)	$\theta = 68.20°$ $= 68.2°$ (to 1 dec.place.)
11.	(i) (ii)	A and B are points (1, 1) and (−1, −1) 
	(iii)	Perpendicular distance from $(\frac{1}{8}, \frac{1}{2})$ to $(\frac{5}{16}, \frac{5}{16})$ $= \frac{3\sqrt{2}}{16}$ units
	(iv)	**since gradient of tangent and gradient of line y=x are different, the lines will meet.**

10.	(i)	$x = 1$ or $-\frac{2}{3}$
		Stationary points are $(-\frac{2}{3}, 0)$ and $(\frac{4}{9}, -6\frac{14}{81})$ Draw the line $y = 1$ on the graph of $y = f(x)$. The line meets the curve at only one point. Thus $9x^3 + 3x^2 - 8x - 5$ has only one linear factor.