TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2014
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4047/01

Paper 1

Wed 27August 2014

2 hours

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

1 (a) Find the exact value of x if $27^x = \sqrt{3\sqrt{27}}$. [3]

(b) Find the range of values of m for which $x^2 - 2mx + 2m$ is greater than -3 for all real values of x. [4]

2 The roots of the quadratic equation $2x^2 - 4x - 1 = 0$ are α and β.

(i) Show that $\alpha^3 + \beta^3 = 11$. [3]

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [3]

3 (i) Sketch the graph of $y = |3 - 2x| - 2$ for the domain $0 \le x \le 4$. [3]

(ii) Find the range of values of x for which $|3 - 2x| - 2 \le x - 2$ for the domain $0 \le x \le 4$. [4]

4 Show that $y = \ln\left(\frac{3-5x}{2+3x}\right)$ has no stationary point for $-\frac{2}{3} < x < \frac{3}{5}$. [4]

5 Given that $\int_0^4 f(x)\,dx = \int_4^7 f(x)\,dx = 5$,

(i) find $\int_7^4 2f(x)\,dx$. [2]

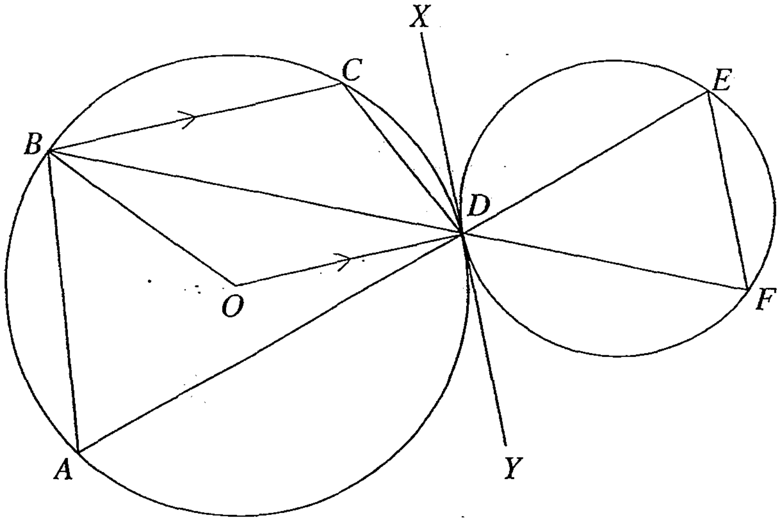
(ii) Explain the geometrical interpretation of $\int_0^7 f(x)\,dx = a$. State the value of a. [2]

(iii) Given also that $\int_0^{12} f(x)+1\,dx = 27$, evaluate $\int_7^{12} f(x)\,dx$. [3]

6 Find all the angles between $0°$ and $360°$ which satisfy the equation

$5\sec y - 3\cos y = 5\tan y$. [4]

7 The diagram shows 2 circles touching each other at point D.
ADE and BDF are straight lines and they cut the circles at points A, B, E and F. O is the centre of the bigger circle, BC is parallel to OD and XY is a tangent to the circles at D.



(i) Show that BD bisects $\angle OBC$. [3]

(ii) Prove that triangle ABD is similar to triangle EFD. [4]

8 A moving particle travelling in a straight line passes a fixed point O. Its velocity, v ms^{-1} is given by $v = 8 - 5\sin 3t$, where t is the time in seconds after passing O.

Calculate

(i) the initial acceleration, [2]

(ii) the distance of the particle from O at $t = \frac{\pi}{3}$ s, [3]

(iii) the average velocity of the particle during the first $\frac{\pi}{3}$ seconds. [2]

9 Variables x and y are related by the equation $y = \frac{11x-1}{9-x}$. Given that x and y are functions of t and y increases from an initial value of 2.9 at a constant rate of 0.005 units per second. Find, after 20 seconds,

(i) the value of y, [1]

(ii) the corresponding rate of change of x. [4]

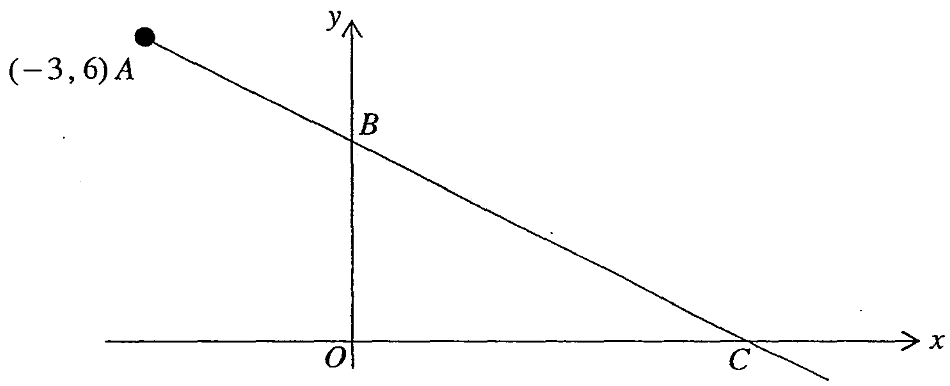
10 A line passing through $A\ (-3, 6)$ cuts the y-axis at B and x-axis at C. Given that $AB : BC = 1 : 2$, find

(i) the coordinates of B and of C, [3]

(ii) the equation of the line AC. [2]

Another line passing through M, the midpoint of AC, cuts the x-axis at N such that $\angle MNC = \angle MCN$.

(iii) Find the equation of the line MN. [3]



11 The table below shows the experimental values of x and of y which are known to be related by the equation $y = p(x+5)^{\frac{3}{2}} - q\sqrt{x+5}$, where p and q are constants.

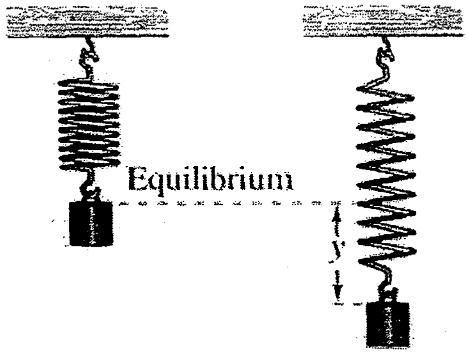
x	0.5	1	1.5	2	2.5
y	24.6	28.2	31.9	35.7	39.7

(i) On graph paper, plot $\frac{y}{\sqrt{x+5}}$ against x and draw a straight line graph. The vertical $\frac{y}{\sqrt{x+5}}$-axis should start at 9 and have a scale of 2 cm to 1. [3]

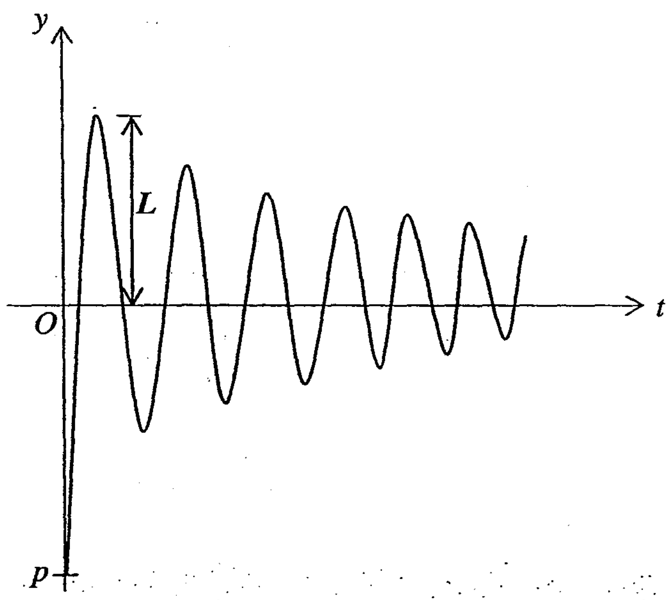
(ii) Use the graph to estimate the value of p and of q. [4]

(iii) By plotting another suitable straight line on the same axes, solve graphically the equation $p(x+5)^{\frac{3}{2}} = \sqrt{x+5}(x+10+q)$. [3]

12 The figure below shows a weight oscillating at the end of a spring. The displacement from equilibrium is given by $y = -2e^{-t}\cos 4t$, $t \geq 0$, where y is the displacement in centimetres and t is the time in seconds.



In order for the spring to start oscillating, the weight has to be first pulled down to a displacement of p cm. The motion of the spring is commonly known as Damped Harmonic Motion and the graph of the displacement from equilibrium is shown below.



(i) State the value of p. [1]

(ii) Show that $\dfrac{dy}{dt} = 2e^{-t}(4\sin 4t + \cos 4t)$. [3]

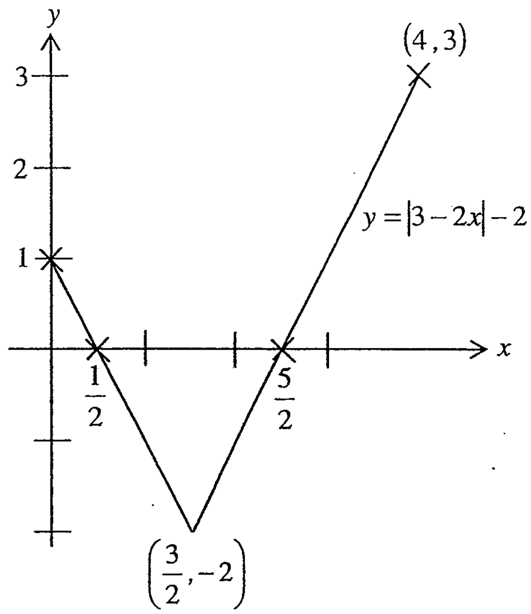
(iii) L is the displacement from equilibrium after the initial weight pull.
Find L. [4]

---------- **End of Paper** ----------

Answers

1.(a) $x=\frac{5}{12}$ (b) $-1<m<3$ 2. (ii) $x^2+88x-8=0$

3. (i)



(ii) $1\leq x\leq 3$

4. $\frac{dy}{dx}=-\frac{19}{(3-5x)(2+3x)}$ 5. (i) -10

5. (ii) The **area** bounded by $f(x)$ **and the** x**-axis** between $x=0$ **and** $x=7$ is a units^2. $a=10$ (iii) 5 6. $y=41.8°$, $138.1°$

7. (i) Let $\angle ODB=x$
$\angle OBD=x$ (base $\angle$s of isos. Δ)
$\angle DBC=x$ (alt. $\angle$s)
Since $\angle OBD=\angle DBC$,
$\Rightarrow BD$ bisects $\angle OBC$. (shown)

(ii) $\angle BDA=\angle FDE$ (vert. opp. $\angle$s)
$\angle BDX=\angle FDY$ (vert. opp. $\angle$s)
$\angle BDX=\angle BAD$ (alt. seg. thm)
$\angle FDY=\angle FED$ (alt. seg. thm)
$\Rightarrow \angle BAD=\angle FED$
ΔABD is similar to ΔEFD

8. (i) $a=-15$ m/s^2 (ii) $s=5.04$ m (iii) 4.82 m/s

9. (i) 3 (ii) 0.0025 units/sec 10. (i) $B=(0,\ 4)$, $C=(6,\ 0)$

10 (ii) $y=-\frac{2}{3}x+4$ (iii) $y=\frac{2}{3}x+2$ 11. (ii) $p=2$, $q=0.5$ (iii) $x=0.5$

12. (i) $p=-2$ (iii) $t=0.72415$, $L=0.941$ cm

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2014
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4047/02

Paper 2

Friday 29August 2014

2 hours 30 minutes

Additional Materials: Writing Paper

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You may use an HB pencil for any diagrams or graphs.
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The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
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$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 (i) Sketch the graph of $y = x^{\frac{2}{3}}$. [1]

(ii) On a separate diagram, sketch the graph of $y = \ln(-x)$. [2]

(iii) Find the value of x where the tangents of both the graphs are parallel. [4]

2 (i) Express $\dfrac{-7x^2+6x-36}{(2-x)(x^2+9)}$ in partial fractions. [6]

(ii) Hence, find $\displaystyle\int \frac{-7x^2+6x-36}{(2-x)(x^2+9)}\,dx$. [2]

3 It is studied that the population, B, of a certain species of butterfly increases exponentially. Given that $B = 800(3)^{kt}$, where k is a constant and t is the time in days after the study is conducted.

(i) State the initial number of butterflies. [1]

(ii) Given that the population tripled in 18 days, show that the value of $k = \dfrac{1}{18}$. [2]

(iii) Find the number of butterflies after 30 days giving your answer to the nearest whole number. [2]

(iv) Find the number of days taken for the population to exceed 100 000. [3]

4 (a) Solve the equation $\log_3(2x+1) - \log_3(x-1) = 2$. [3]

(b) Solve the equation $\log_4 x - \log_x 16 = 1$. [4]

(c) Evaluate $\lg\dfrac{1}{2} + \lg\dfrac{2}{3} + \lg\dfrac{3}{4} + \lg\dfrac{4}{5} + ... + \lg\dfrac{99}{100}$. [2]

5 (i) Find $\frac{d}{dx} x\cos^2 x$. [3]

(ii) Using your answer to part (i), find $\int_0^{\frac{\pi}{4}} x\sin 2x \, dx$. [6]

6 (i) In the expansion of $(2+ax)^n$, where n is a positive integer, the coefficients of x and x^2 are in the ratio 1:7. Express a in terms of n. [4]

In the expansion of $(2+x-3x^2)(2+ax)^n$, the constant term is 512.

(ii) Find the value of n and hence, show that $a = 4$. [2]

(iii) Hence find the coefficient of x in the expansion of $(2+x-3x^2)(2+ax)^n$. [3]

7 It is given that $f(x) = \frac{(x+2)^2}{p-x}, x \neq p$, where p is an integer.

(i) Obtain and simplify an expression for $f'(x)$ in terms of p. [3]

(ii) The only values of x for which $f(x)$ is an increasing function of x are those values for which $-2 < x < 10$. Show that the value of $p = 4$. [4]

(iii) Hence, find the equation of the normal to the curve at the point where the curve crosses the x-axis. [3]

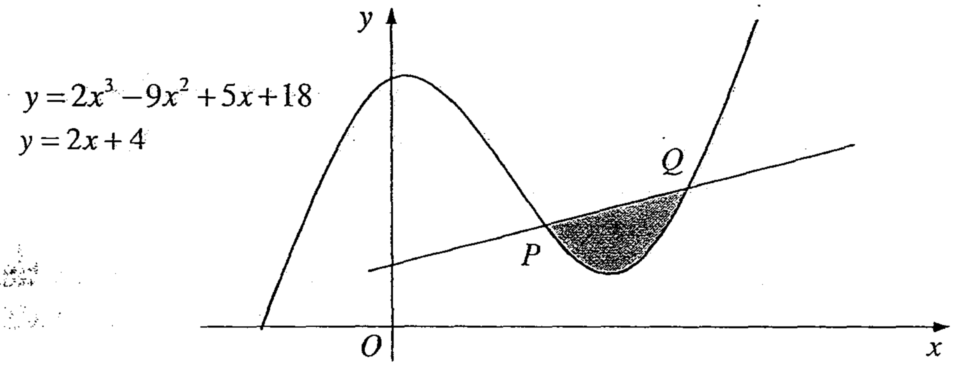
8 A circle with centre A, has a radius of 13 units. The points (–3, 7) and (–3, –17) are on the circumference of this circle. The centre of the circle lies on the right of the y-axis.

(i) Find the equation of the circle. [4]

(ii) Find the equation of the tangent to the circle at (–3, –17). [3]

(iii) Another circle is to be drawn with its centre at (24, 30), such that the two circles do not overlap. What is the maximum radius of the new circle? [3]

9



The line $y=2x+4$ cuts the curve $y=2x^3-9x^2+5x+18$ at P and Q.

(i) Find the coordinates of P and of Q. [5]

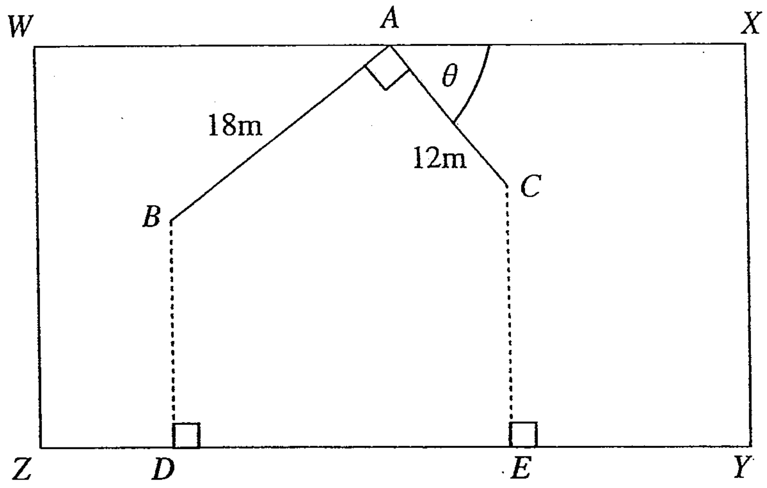
(ii) Calculate the area of the shaded region. [4]

10 **(a)** (i) Prove the identity $\dfrac{\tan\theta}{\sec\theta-1}+\dfrac{\tan\theta}{\sec\theta+1}=\dfrac{2}{\sin\theta}$. [3]

(ii) Hence, solve the equation $\dfrac{\tan\theta}{\sec\theta-1}+\dfrac{\tan\theta}{\sec\theta+1}=3$, for $0<\theta<2\pi$. [4]

(b) Given that angle A is obtuse and $\sin A=\dfrac{3}{4}$, express, without using a calculator, $\dfrac{1}{1+\tan A}$ in the form $\dfrac{a+b\sqrt{7}}{2}$, where a and b are integers. [4]

11



The diagram shows a rectangle $WXYZ$. Two lines AB and AC are to be drawn on the rectangle such that $AB = 18$ m, $AC = 12$ m and angle $BAC = 90°$.

AC makes an angle of θ with WX, where θ can vary such that $0° < \theta < 90°$.
D and E are the feet of the perpendiculars from B and C to the line YZ respectively.

(i) Show that DE can be expressed as $12\cos\theta + 18\sin\theta$. [2]

(ii) Express DE in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Find the range of values of DE. [2]

(iv) Given that $DE = 20$ m, find the values of θ. [2]

End of paper

7

Answers:

1(iii) -1.84

2(i) $A=-4, B=3, C=0$ 2(ii) $4\ln(2-x)+\frac{3}{2}\ln(x^2+9)+c$

3(i) 800 (ii) $\frac{1}{18}$ (iii) 4992 (iv) 80

4(a) $\frac{10}{7}$ 4(b) 16, 0.25 4(c) -2

5(i) $-x\sin 2x+\cos^2 x$ 5(ii) 0.25

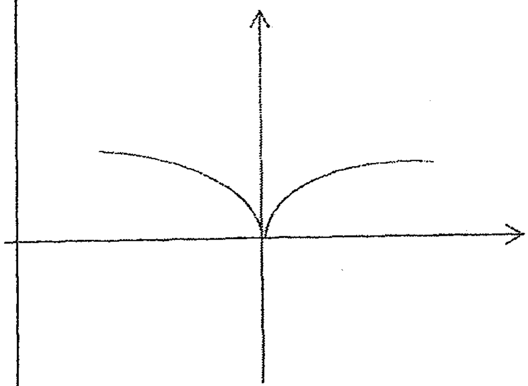
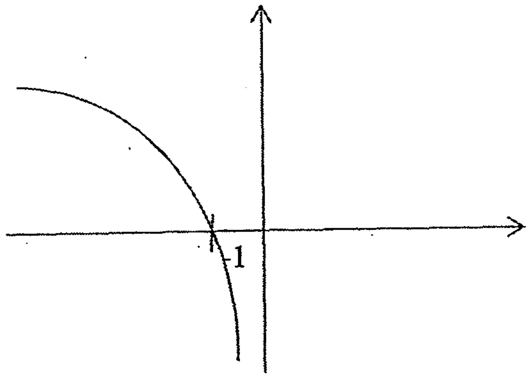
6(i) $a=\frac{28}{n-1}$ 6(ii) $n=8, a=4$ 6(iii) 8448

7(i) $\frac{-x^2+2px+4p+4}{(p-x)^2}$ 7(ii) 4 7(iii) $x=-2$

8(i) $(x-2)^2+(y+5)^2=169$ 8(ii) $y=-\frac{5}{12}x-\frac{73}{4}$ 8(iii) 28.3 9(i) $P(2,8), Q(3.5,11)$ 9(ii) 4.22

10a(ii) 0.411, 2.73 10(b) $\frac{-7-3\sqrt{7}}{2}$

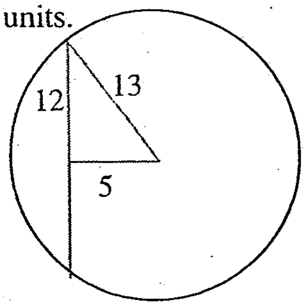
11(i) $12\cos\theta+18\sin\theta$ 11(ii) $6\sqrt{13}\cos(\theta-56.3°)$ 11(iii) $12\le DE\le 6\sqrt{13}$ 11(iv) 33.9, 78.7

No	Solution	Marks	Remarks
1(i)		B1 (graph) symmetry	
1(ii)		B1 (graph) G1 Show intercept	
1(iii)	$\frac{d}{dx}(x^{\frac{2}{3}}) = \frac{2}{3}x^{-\frac{1}{3}}$ $\frac{d}{dx}\ln(-x) = \frac{1}{x}$ $\frac{2}{3}x^{-\frac{1}{3}} = \frac{1}{x}$ $x = -1.84$	B1 B1oe M1 equate A1 (reject positive)	
			7 marks
2(i)	$\frac{-7x^2+6x-36}{(2-x)(x^2+9)} = \frac{A}{2-x} + \frac{Bx+C}{x^2+9}$ $-7x^2+6x-36 = A(x^2+9)+(Bx+C)(2-x)$ $x=2, A=-4$ $x=0, C=0$ $B=3$	B1 M1 multiply throughout by denominator M1 substitution or compare coefficient A1, A1, A1	
2(ii)	$\int \frac{-4}{2-x} + \frac{3x}{x^2+9}dx = 4\ln(2-x) + \frac{3}{2}\ln(x^2+9) + c$	√B1, for both ln (follow through only if ln) B1 +c seen	
			8 marks

No	Solution	Marks	Remarks
3(i)	Sub $t=0$, Initial population = 800	B1	
3(ii)	$\frac{800(3)^{18k}}{800}=3$ $18k=1$ $k=\frac{1}{18}$	M1 Sub t=18 M1 (3 seen) AG	
3(iii)	$B=800(3)^{\frac{30}{18}}$ $=4992$	M1 A1 (not 3sf)	
3(iv)	$100000=800(3)^{\frac{t}{18}}$ $125=3^{\frac{t}{18}}$ $\frac{\ln 125}{\ln 3}=\frac{t}{18}$ $t=79.1$ 80 days	M1 equate or inequality M1 A1	
			8 marks
4a	$\log_3(2x+1)-\log_3(x-1)=2$ $\log_3\frac{2x+1}{x-1}=2$ $\frac{2x+1}{x-1}=9$ $2x+1=9x-9$ $x=\frac{10}{7}$	M1 quotient law in log M1 changing to index form or getting rid of log A1oe	
4b	$\log_4 x-\log_x 16=1$ $\log_4 x-\frac{2}{\log_4 x}=1$ $(\log_4 x)^2-\log_4 x-2=0$ $\log_4 x=2$ or $\log_4 x=-1$ $x=16$ or $\frac{1}{4}$	M1 change base M1 $\log_4 16=2$ (power law) M1 forming quadquad A1 both answers	
4c	$\lg\left(\frac{1}{2}\times\frac{2}{3}\times....\times\frac{99}{100}\right)$ $=\lg\frac{1}{100}$ $=-2$	M1 product law A1	
			9 marks

No	Solution	Marks	Remarks
5(i)	$\frac{d}{dx}(\cos^2 x) = -2\sin x\cos x$ $\frac{d}{dx}(x\cos^2 x)$ $= x(-2\sin x\cos x) + \cos^2 x$ (accept this) $= -x\sin 2x + \cos^2 x$	B1 diff cos$x$ = -sin$x$ B1 chain rule 2cos$x$( ) M1 product rule	
4(ii)	$\int -x\sin 2x + \cos^2 x dx = x\cos^2 x + c$ $\int \cos^2 x dx$ $= \int \frac{\cos 2x + 1}{2} dx$ $= \frac{1}{4}\sin 2x + \frac{1}{2}x + c$ $-\int x\sin 2x dx = x\cos^2 x - \frac{1}{4}\sin 2x - \frac{1}{2}x + c$ $\int_0^{\frac{\pi}{4}}$ $= -\left[(\frac{\pi}{8} - \frac{1}{4} - \frac{\pi}{8}) - (0 - 0 - 0)\right]$ $= \frac{1}{4}$	M1 double angle B1 working backwards statement M1 reducing to power 1 B1 integration of cos $2x$ M1 substitution A1	
			9 marks
6(i)	$(2+ax)^n = 2^n + n2^{n-1}ax + \binom{n}{2}2^{n-2}a^2x^2 + \ldots$ Coeff of $x = n2^{n-1}a$ Coeff of $x^2 = \frac{n(n-1)}{2}2^{n-2}a^2$ $\frac{\frac{n(n-1)}{2}2^{n-2}a^2}{n2^{n-1}a} = 7$ $a = \frac{28}{n-1}$	B1 B1 M1 equate A1	
6(ii)	$(2+x-3x^2)(2^n + n2^{n-1}ax + \binom{n}{2}2^{n-2}a^2x^2 + \ldots)$ $2^{n+1} = 512$ $n = 8$ $a = 4$	A1 A1 (provided eqn is correct)	

No	Solution	Marks	Remarks
6(iii)	Coeff of x $=2\binom{8}{1}2^7(4)^1+1(2)^8$ $=8448$	B1, B1 for each term A1	
			9 marks
7(i)	$f'(x)$ $=\dfrac{(p-x)(2(x+2))+(-1)(x+2)^2}{(p-x)^2}$ $=\dfrac{-x^2+2px+4p+4}{(p-x)^2}$	M1 quotient rule (all correct) M1 chain rule 2(x+2) A1 ISW	
7(ii)	$\dfrac{-x^2+2px+4p+4}{(p-x)^2}>0$ $-x^2+2px+4p+4>0$ Given $-2<x<10$, $(x+2)(x-10)<0$ $-x^2+8x+20>0$ Comparing the two inequalities, $2p=8$ $p=4$	B1 $f'(x)>0$ M1 M1 comparing both inequalities A1	
7(iii)	$f'(x)=\dfrac{-x^2+8x+20}{(p-x)^2}$ At $y=0, x=-2$. Gradient = 0 Equation: $x=-2$	M1 finding x coord M1 finding gradient B1 oe	
			10 marks

No	Solution	Marks	Remarks
8(i)	Midpoint of chord joining (–3, 7) and (–3, –17) is (–3, –5). Centre lies on $y = -5$. Radius = 13, length of chord 24 units. By Pythagoras,  Therefore, x-coordinate = –3+5=2 Centre (2, –5) Equation: $(x-2)^2+(y+5)^2=169$	B1 y coord SOI M1 using half the length of chord and pythagoras thm (students can use diagram to do this) Or substituting into equation of circle to find x coord A1 B1 o.e (13^2 not accepted)	
8(ii)	Gradient of centre to (–3, –17) $=\frac{12}{5}$ Gradient of tangent $=-\frac{5}{12}$ $y+17=-\frac{5}{12}(x+3)$ $y=-\frac{5}{12}x-\frac{73}{4}$	M1 tan perp. Radius M1 forming equation B1 oe	SOI
8(iii)	$\sqrt{(24-2)^2+(30-(-5))^2}$ $=\sqrt{1709}$ $\sqrt{1709}-13=28.3$	M1 find distance between centres M1, A1	
			10 marks
9(i)	$2x^3-9x^2+5x+18=2x+4$ $2x^3-9x^2+3x+14=0$ By trial and error, first factor : $(x-2)$ $2x^3-9x^2+3x+14=(x-2)(2x^2+bx-7)$ Comparing coefficients of x, $3=-2b-7$ $b=-5$ $(x-2)(2x^2-5x-7)=(x-2)(2x-7)(x+1)$ $x=2, x=3.5, x=-1$ $P(2,8)$ $Q(3.5,11)$	M1 equating B1 first factor M1 comparing coefficient or long division A1 coordinates of P A1 coordinates of Q	

No	Solution	Marks	Remarks
9(ii)	$\int_2^{3.5} 2x+4-(2x^3-9x^2+5x+18)dx$ $=\left[-\frac{x^4}{2}+3x^3-\frac{3x^2}{2}-14x\right]_2^{3.5}$ $=(-\frac{441}{32}-(-18))$ $=\frac{135}{32}$ $=4.21875$	M1 equation on top-equation below M2 integration A1	-1 mark for each error
			9 marks
10ai	$\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$ $=\frac{\tan\theta(\sec\theta+1+\sec\theta-1)}{\sec^2\theta-1}$ $=\frac{\tan\theta(2\sec\theta)}{\tan^2\theta}$ $=\frac{2\sec\theta}{\tan\theta}$ $=\frac{\frac{2}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$ $=\frac{2}{\sin\theta}$	M1 combine fractions M1 identity M1 reciprocal functions or any other method (arrives at answer) AG	
10aii	$\frac{2\tan\theta}{\sec\theta-1}+\frac{2\tan\theta}{\sec\theta+1}=10$ $\frac{4}{\sin\theta}=10$ $\sin\theta=\frac{2}{5}$ $\theta=0.411,2.73$	M1 using identity proven in (i) M1 A1, A1	

No	Solution	Marks	Remarks
10b	$\tan A = -\frac{3}{\sqrt{7}}$ $\frac{1}{1+\tan A}$ $=\frac{1}{1-\frac{3}{\sqrt{7}}}$ $=\frac{\sqrt{7}}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}$ $=\frac{7+3\sqrt{7}}{-2}$ $=\frac{-7-3\sqrt{7}}{2}$	B1 ratio for tangent S.O.I M1 attempt to simplify fraction within fraction M1 rationalising by mutliplying conjugate surd A1	
			11 marks
11(i)	$AD'=18\sin\theta$ $AC'=12\cos\theta$ $DE=12\cos\theta+18\sin\theta$	B1 B1 Must have clear presentation	
11(ii)	$R=\sqrt{18^2+12^2}$ $=6\sqrt{13}$ or $\sqrt{468}$ $\alpha=\tan^{-1}(\frac{18}{12})$ $=56.3099°$ $R=6\sqrt{13}\cos(\theta-56.3°)$	M1 M1 A1 B1	
11(iii)	$12\le DE\le 6\sqrt{13}$ or $\sqrt{468}$ or 21.63	B1 upper limit B1 lower limit	
11(iv)	$6\sqrt{13}\cos(\theta-56.3°)=20$ $\theta=78.72°$	M1 A1	
			10 marks