	Class	Register Number
Name ______________________________		

4047/01

14/S4PR1/AM/1

ADDITIONAL MATHEMATICS

PAPER 1

Friday **9 May 2014** **2 hours**

NIL SINE LABORE

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Material: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
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Omission of essential working will result in loss of marks.
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If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

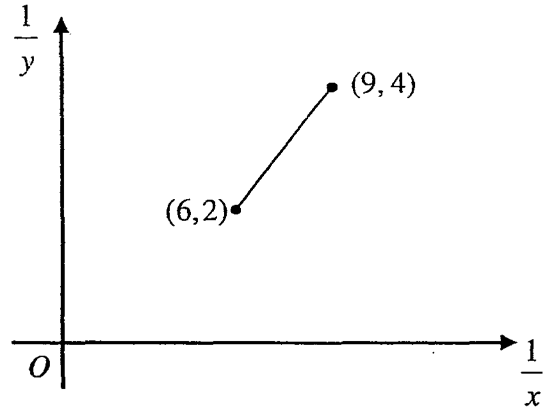
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that $p=3-2\sqrt{3}$, express $(p+1)^2-\dfrac{6}{p}$ in the form of $a+b\sqrt{3}$, where a and b are integers. [4]

2



The diagram shows part of a straight line graph $\dfrac{py+qx}{xy}=1$ where $\dfrac{1}{y}$ is plotted against $\dfrac{1}{x}$. The line passes through the points $(6, 2)$ and $(9, 4)$. Find the values of p and of q. [4]

3 Solve the simultaneous equations

$$25^x=\frac{1}{125^y},$$
$$\frac{27^y}{\sqrt{3}}\div\left(\sqrt{3}\right)^{x-1}=27.$$ [5]

4 Differentiate and simplify each of the following with respect to x.

(i) $\cos^2(2-3x)$ [2]

(ii) $\ln\left(\sqrt{(2x+1)(3x-4)}\right)$ [3]

5 Given that $\int_1^3 f(x)\,dx=\int_3^5 f(x)\,dx=55$, find the value of

(i) $\int_1^5 3f(x)\,dx+\int_5^3 f(x)\,dx$, [3]

(ii) the constant k, for which $\int_3^5\left[f(x)-kx^2\right]dx=6$. [2]

6 Given that the gradient function of a curve, $\frac{dy}{dx}=k\left(\cos x-\frac{3}{4}\sin 2x\right)$, where k is a constant, and the curve passes through the points $(0,4)$ and $\left(\frac{\pi}{2},0\right)$, find the equation of the curve. [5]

7 Given that $\sin A=q$ where A is an obtuse angle, obtain an expression in terms of q, for

(i) $\cos 2A$, [1]

(ii) $\tan^2 A$, [2]

(iii) $\sin 3A$. [3]

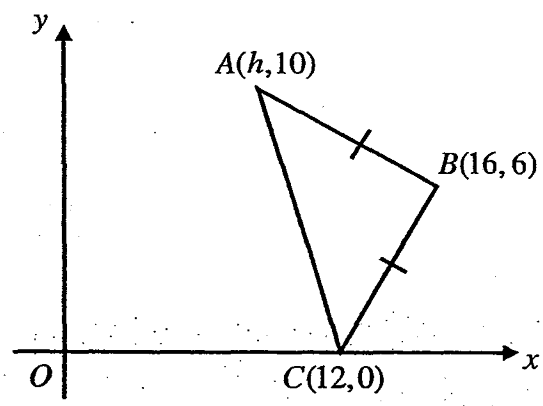
8 **(i)** Solve the equation $\left|2x^2-15\right|=-x$. [3]

(ii) Sketch on the same diagram, the graphs of $y=\left|2x^2-15\right|$ and $y=-x$. [3]

(iii) Hence, solve $\left|2x^2-15\right|<-x$. [1]

9 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a triangle ABC with vertices $A(h,10)$, $B(16,6)$ and $C(12,0)$.



Given that $AB=BC$,

(i) find the value of h. [3]

A line is drawn from point B to meet the y-axis at D such that $AD=CD$. Find the

(ii) equation of BD and the coordinates of D, [4]

(iii) area of quadrilateral $ABCD$. [2]

10 Solve the following equations.

(i) $e^{x+1}-8e+13e^{1-x}=0$ [4]

(ii) $\log_2(3-x)-\log_4\left(x^2+15\right)=-1$ [5]

11 The function $f(x)=2-3\sin 3x$ is defined for $0\le x\le\pi$.

(i) State the period and amplitude of f. [2]

(ii) Solve $f(x)=0$ for $0\le x\le\pi$. [3]

(iii) Sketch the graph of $y=f(x)$ for $0\le x\le\pi$. [3]

(iv) On the diagram drawn in part **(iii)**, draw the graph of $y=-\dfrac{2x}{\pi}+1$. [1]

(v) State the number of solutions, for $0\le x\le\pi$, of the equation $1+\dfrac{2x}{\pi}-3\sin 3x=0$. [1]

12 The expression $2x^3+mx^2+nx+3$, where m and n are constants, has a factor of $x-1$ and leaves a remainder of -9 when divided by $x+2$.

(i) Show that $m=-1$ and $n=-4$. [3]

(ii) Hence solve the equation $2x^3-x^2-4x+3=0$. [3]

(iii) Express $\dfrac{3x^2+4}{2x^3-x^2-4x+3}$ in partial fractions. [5]

End of Paper

Answer Key

1 $34-12\sqrt{3}$

2 $q=-\frac{1}{2}$ and $p=\frac{1}{3}$

3 $x=-1\frac{1}{5}$, $y=\frac{4}{5}$

4 i) $6\sin(2-3x)\cos(2-3x)$

ii) $\dfrac{12x-5}{2(2x+1)(3x-4)}$

5 i) 275

ii) $k=1\frac{1}{2}$

6 $y=-16\sin x-6\cos 2x+10$

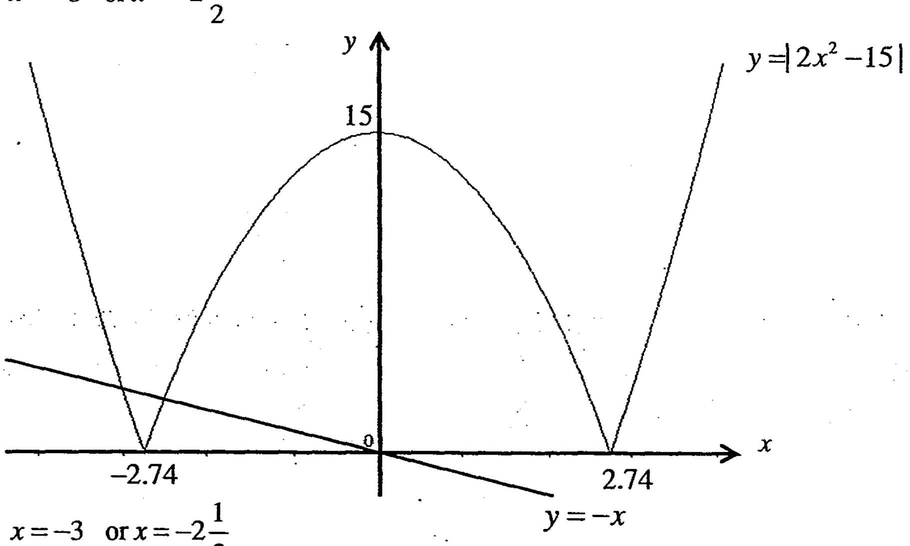
7 i) $1-2q^2$

ii) $\dfrac{q^2}{(1+q)(1-q)}$

iii) $q(3-4q^2)$

8 i) $x=-3$ or $x=-2\frac{1}{2}$

ii)



iii) $x=-3$ or $x=-2\frac{1}{2}$

9 i) $h=10$

ii)

$\therefore$ Equation of BD is $y=\frac{1}{5}x+2\frac{4}{5}$

$\therefore$ Coordinate of D is $\left(0,2\frac{4}{5}\right)$.

iii) 83.2 sq units

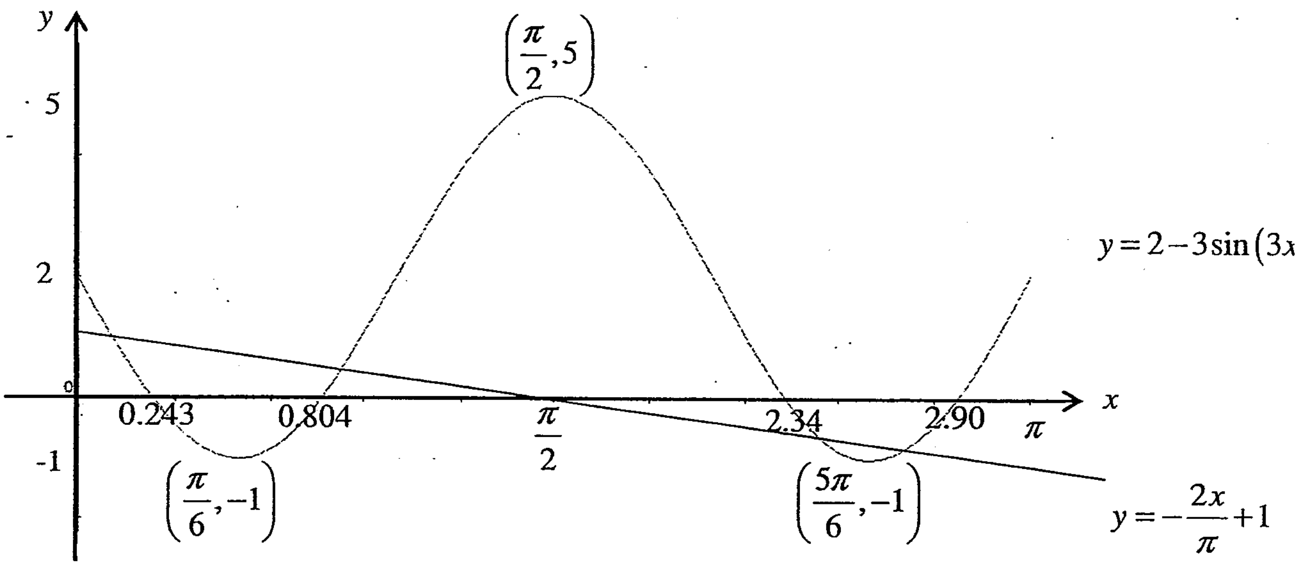
10 i) $x\approx 0.819$ or $x\approx 1.75$

ii) $x=1$

11 i) Period $=\frac{2\pi}{3}$, amplitude $=3$

ii) $x\approx 0.243,\quad 0.804,\quad 2.34,\quad 2.90$

iii)



iv) 4 solutions

12 ii) $x=1$ or $x=-1\frac{1}{2}$

iii) $\frac{3x^2+4}{2x^3-x^2-4x+3}=\frac{16}{25(x-1)}+\frac{7}{5(x-1)^2}+\frac{43}{25(2x+3)}$

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Solution:

$$(p+1)^2-\frac{6}{p}$$

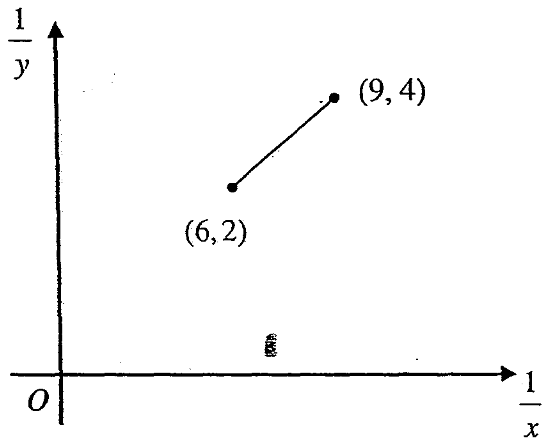
$$=\left(3-2\sqrt{3}+1\right)^2-\frac{6}{3-2\sqrt{3}}\left(\frac{3+2\sqrt{3}}{3+2\sqrt{3}}\right)$$ B1 × conjugate surd

$$=\left(4-2\sqrt{3}\right)^2-\frac{6\left(3+2\sqrt{3}\right)}{9-12}$$

$$=16-16\sqrt{3}+12+2\left(3+2\sqrt{3}\right)$$ B2 $16-16\sqrt{3}+12,\quad +2\left(3+2\sqrt{3}\right)$ oe

$$=34-12\sqrt{3}$$ A1

2



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The line passes through the points (6, 2) and (9, 4). Find the values of p and of q. [4]

Solution:

$$\frac{py+qx}{xy}=1$$

$$\frac{p}{x}+\frac{q}{y}=1$$

$$\frac{q}{y}=-p\left(\frac{1}{x}\right)+1$$

$$\frac{1}{y}=-\frac{p}{q}\left(\frac{1}{x}\right)+\frac{1}{q}$$ M1

$$Y=mX+c$$

Sub gradient $=\frac{4-2}{9-6}=\frac{2}{3}$ and $(6,2)$ into

$$Y=mX+c$$

$$2=\frac{2}{3}(6)+c$$

$$c=-2$$

$\therefore \frac{1}{q}=-2$ and $-\frac{p}{q}=\frac{2}{3}$ M2

$q=-\frac{1}{2}$ and $p=\frac{1}{3}$ A1

3 Solve the simultaneous equations

$$25^x = \frac{1}{125^y}\ ,\ \text{--------------------}(1)$$

$$\frac{27^y}{\sqrt{3}} \div \left(\sqrt{3}\right)^{x-1} = 27\ \text{--------------------}(2)$$

[5]

Solution:

From (1)

$$25^x = \frac{1}{125^y}$$

$$5^{2x} = \frac{1}{5^{3y}}$$

$$5^{2x} = 5^{-3y}$$

$$2x = -3y\text{---------}(3)$$ A1 oe

From (2)

$$\frac{27^y}{\sqrt{3}} \div \left(\sqrt{3}\right)^{x-1} = 27$$

$$\frac{3^{3y}}{3^{\frac{1}{2}}} \div 3^{\frac{1}{2}x-\frac{1}{2}} = 3^3$$

$$3^{3y-\frac{1}{2}-(\frac{1}{2}x-\frac{1}{2})} = 3^3$$

$$3y - \frac{1}{2}x = 3\text{----------}(4)$$ A1 oe

Sub $2x = -3y$ into (4) M1

$$3y - \frac{1}{2}x = 3$$

$$-2x - \frac{1}{2}x = 3$$

$$-2\frac{1}{2}x = 3$$

$$x = -1\frac{1}{5}$$

Sub $x = 2$ into (3) : $y = \frac{4}{5}$

$\therefore x = -1\frac{1}{5},\ y = \frac{4}{5}$ A2

4 Differentiate and simplify each of the following with respect to x.

(i) $\cos^2(2-3x)$ [2]

(ii) $\ln\left(\sqrt{(2x+1)(3x-4)}\right)$ [3]

Solution:

(i)

$$\frac{d}{dx}\cos^2(2-3x)$$
$$=-2\cos(2-3x)\sin(2-3x).(-3) \qquad \text{B1}$$
$$=6\sin(2-3x)\cos(2-3x) \qquad \text{A1}$$
$$=3\sin[2(2-3x)]$$
$$=3\sin(4-6x)$$

ii)

$$\frac{d}{dx}\ln\left(\sqrt{(2x+1)(3x-4)}\right)$$
$$=\frac{d}{dx}\left[\frac{1}{2}\ln(2x+1)+\frac{1}{2}\ln(3x-4)\right]$$

B1 power law, product rule

$$=\frac{1}{2x+1}+\frac{3}{2(3x-4)}$$

A2 differentiation

$$=\frac{6x-8+6x+3}{2(2x+1)(3x-4)}$$
$$=\frac{12x-5}{2(2x+1)(3x-4)}$$

Alternative

(ii)

$$\frac{d}{dx}\ln\left(\sqrt{(2x+1)(3x-4)}\right)$$
$$=\frac{d}{dx}\left[\frac{1}{2}\ln(6x^2-5x-4)\right]$$

B1 power law & expansion

$$=\frac{1}{2}\left(\frac{12x-5}{6x^2-5x-4}\right)$$

M1 differentiation

$$=\frac{12x-5}{2(2x+1)(3x-4)}$$

A1

5 Given that $\int_1^3 f(x)\,dx = \int_3^5 f(x)\,dx = 55$, find the value of

(i) $\int_1^5 3\,f(x)\,dx + \int_5^3 f(x)\,dx$ [3]

(ii) the constant k, for which $\int_3^5 \left[f(x) - kx^2 \right] dx = 6$. [2]

Solution:

(i)

$$\int_1^5 3\,f(x)\,dx + \int_5^3 f(x)\,dx$$

$$= 3\left[\int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx \right] - \int_3^5 f(x)\,dx \quad \text{B1}$$

$$= 3(110) - 55 \quad \text{B1}$$

$$= 275 \quad \text{A1}$$

(ii)

$$\int_3^5 \left[f(x) - kx^2 \right] dx = 6$$

$$\int_3^5 f(x)\,dx - k\int_3^5 x^2\,dx = 6$$

$$k\int_3^5 x^2\,dx = \int_3^5 f(x)\,dx - 6$$

$$k\left[\frac{x^3}{3} \right]_3^5 = 55 - 6 \quad \text{B1 integration of } x^2 \text{ and sub 55}$$

$$k\left(\frac{5^3}{3} - \frac{3^3}{3} \right) = 49$$

$$k = 1\frac{1}{2} \quad \text{A1}$$

6 Given that the gradient function of a curve, $\frac{dy}{dx} = k\left(\cos x - \frac{3}{4}\sin 2x\right)$, where k is a constant and the curve passes through the points $(0, 4)$ and $\left(\frac{\pi}{2}, 0\right)$, find the equation of the curve. [5]

Solution:

Let the curve be y. It passes through $(0, 4)$ and $\left(\frac{\pi}{2}, 0\right)$

$$\frac{dy}{dx} = k\left(\cos x - \frac{3}{4}\sin 2x\right)$$

$$= k\cos x - \frac{3k}{4}\sin 2x$$

$$y = \int\left(k\cos x - \frac{3k}{4}\sin 2x\right) dx$$

$$y = k\sin x + \frac{3k}{8}\cos 2x + c \text{-----------}(1) \qquad \text{A1}$$

At $(0, 4)$,

$$4 = k\sin 0 + \frac{3k}{8}\cos 0 + c \qquad \text{M1}$$

$$4 = \frac{3k}{8} + c$$

$$32 = 3k + 8c \text{-------------}(2)$$

At $\left(\frac{\pi}{2}, 0\right)$,

$$0 = k\sin\frac{\pi}{2} + \frac{3k}{8}\cos\pi + c \qquad \text{M1}$$

$$0 = k + \frac{3k}{8}(-1) + c$$

$$0 = 8k - 3k + 8c$$

$$-5k = 8c \text{-----------------}(3)$$

Sub (3) into (2): $\quad 32 = 3k - 5k$ $\qquad$ M1

$$k = -16$$

Sub $k = -16$ into (3):

$$c = \frac{-5(-16)}{8} = 10$$

$\therefore$ Equation of curve is $y = -16\sin x - 6\cos 2x + 10$ $\qquad$ A1 $\quad$ oe

7 Given that $\sin A = q$ where A is an obtuse angle, obtain an expression in terms of q, for

(i) $\cos 2A$, [1]

(ii) $\tan^2 A$, [2]

(iii) $\sin 3A$. [3]

Solution:

$\sin A = q$, $\cos A = -\sqrt{1-q^2}$, $\tan A = -\dfrac{q}{\sqrt{1-q^2}}$

(i) $\cos\ 2A = 1 - 2\sin^2 A$

$= 1 - 2q^2$ A1

(ii) $\tan^2 A = \left(-\dfrac{q}{\sqrt{1-q^2}}\right)^2$ B1

$= \dfrac{q^2}{(1+q)(1-q)}$ A1

(iii) $\sin 3A = \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cos^2 A + \cos 2A \sin A$ B1 use of formula

$= 2q\left(-\sqrt{1-q^2}\right)^2 + \left(1-2q^2\right)q$ M1 substitution

$= 2q\left(1-q^2\right) + q(1-2q^2)$

$= 2q - 2q^3 + q - 2q^3$

$= 3q - 4q^3$

$= q(3 - 4q^2)$ A1

Alternative

(ii) $\tan^2 A = \dfrac{\sin^2 A}{\cos^2 A}$

$= \dfrac{q^2}{\left(-\sqrt{1-q^2}\right)^2}$ B1

$= \dfrac{q^2}{(1+q)(1-q)}$ or $\dfrac{q^2}{1-q^2}$ A1

8 **(i)** Solve the equation $|2x^2 - 15| = -x$. [3]

(ii) Sketch on the same diagram, the graphs of $y = |2x^2 - 15|$ and $y = -x$. [3]

(iii) Hence, find the solution for $|2x^2 - 15| < -x$. [1]

Solution:

(i) $|2x^2 - 15| = -x$

$2x^2 - 15 = -x$ or $2x^2 - 15 = x$ M1

$2x^2 + x - 15 = 0$ $\quad$ $2x^2 - x - 15 = 0$

$(2x-5)(x+3) = 0$ $\quad$ $(2x+5)(x-3) = 0$ M1

$x = 2\frac{1}{2}$ or $x = -3$ $\quad$ $x = -2\frac{1}{2}$ or $x = 3$

(NA) $\quad$ (NA)

$\therefore x = -3$ or $x = -2\frac{1}{2}$ A1

(ii) B1 shape of $y = |2x^2 - 15|$

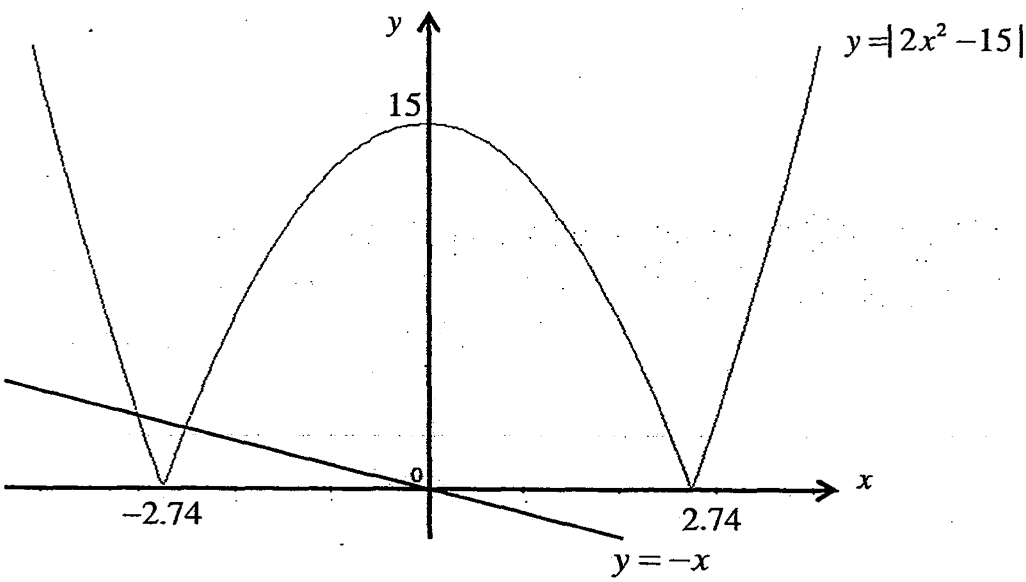
B1 line $y = -x$

B1 x and y intercepts and label

When $|2x^2 - 15| = 0$

$2x^2 - 15 = 0$

$x = \pm\sqrt{7.5}$



(iii) $-3 < x < -2\frac{1}{2}$ A1

9 Solutions to this question by accurate drawing will not be accepted.

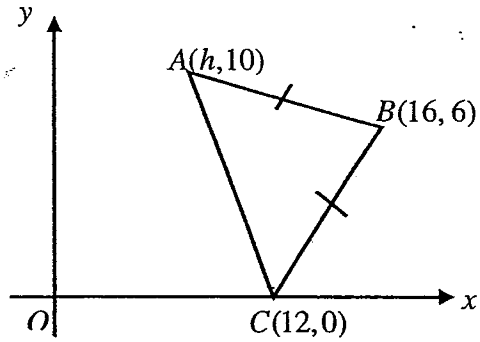
The diagram shows a triangle ABC with vertices $A(h,10)$, $B(16,6)$ and $C(12,0)$. Given that $AB = BC$,

(i) find the value of h. [3]

A line is drawn from point B to meet the y-axis at D such that $AD = CD$. Find

(ii) the equation of BD and the coordinates of D. [4]
(iii) the area of quadrilateral $ABCD$. [2]

Solution:



(i) $AB = BC$

$\sqrt{(h-16)^2+(10-6)^2} = \sqrt{(16-12)^2+(6-0)^2}$ M1

$h^2-32h+256+16 = 16+36$

$h^2-32h+220 = 0$

$(h-10)(h-22) = 0$ M1

$h = 10$ or $h = 22$ (NA) A1

Alternative

(i) $4^2+(h-16)^2 = 4^2+36$ M1

$(h-16)^2 = 36$

$h-16 = -6$ or $h-16 = 6$ M1

$h = 10$ or $h = 22$ (NA) A1

(ii) Since $AD = CD$,

BD is the perpendicular bisector of AC

Midpoint $AC = \left(\dfrac{10+12}{2}, \dfrac{10+0}{2}\right)$

$= (11,5)$

Gradient $BM = \dfrac{6-5}{16-11} = \dfrac{1}{5}$ M1

Sub gradient and (16,6) into $y = mx + c$

$6 = \frac{1}{5}(16) + c$ M1

$c = 2\frac{4}{5}$

$\therefore$ Equation of BD is $y = \frac{1}{5}x + 2\frac{4}{5}$ A1

$\therefore$ Coordinate of D is $\left(0, 2\frac{4}{5}\right)$. A1

Alternative

(ii) Gradient $AC = \dfrac{0-10}{12-10} = -5$

Gradient $BD = \dfrac{1}{5}$ M1

Sub gradient and (16,6) into $y = mx + c$

$6 = \frac{1}{5}(16) + c$ M1

$c = 2\frac{4}{5}$

$\therefore$ Equation of BD is $y = \frac{1}{5}x + 2\frac{4}{5}$ A1

(iii) Area $= \dfrac{1}{2}\begin{vmatrix} 10 & 0 & 12 & 16 & 10 \\ 10 & 2.8 & 0 & 6 & 10 \end{vmatrix}$

$= \frac{1}{2}\left[(28+0+72+160)-(0+33.6+0+60)\right]$ M1

$= \frac{1}{2}(260-93.6)$

$= 83.2$ sq units A1

10 Solve the following equations.

(i) $e^{x+1}-8e+13e^{1-x}=0,$ [4]

(ii) $\log_2(3-x)-\log_4(x^2+15)=-1$ [5]

Solution:

(i) $e^{x+1}-8e+13e^{1-x}=0$

$$e^x.e-8e+\frac{13e}{e^x}=0$$

$$e.e^{2x}-8e.e^x+13e=0$$ M1 multiply by e^x

$$e^{2x}-8e^x+13=0$$

$$e^x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(13)}}{2}$$ M1

$e^x\approx 2.268$ or $e^x\approx 5.732$

$x\approx 0.819$ or $x\approx 1.75$ A2

(ii) $\log_2(3-x)-\log_4(x^2+15)=-1$

$$\log_2(3-x)-\frac{\log_2(x^2+15)}{\log_2 4}=-1$$ M1 change of base law

$$\log_2(3-x)-\frac{1}{2}\log_2(x^2+15)=-1$$

$$\log_2\left(\frac{3-x}{\sqrt{x^2+15}}\right)=\log_2 2^{-1}$$ M1 quotient law & power law

$$\therefore \frac{3-x}{\sqrt{x^2+15}}=\frac{1}{2}$$ M1 anti log

$$\sqrt{x^2+15}=6-2x$$

$$x^2+15=36-24x+4x^2$$

$$3x^2-24x+21=0$$

$$x^2-8x+7=0$$

$(x-7)(x-1)=0$ M1

$x=7$ or $x=1$ A1

NA

Alternative

(ii) $\log_2(3-x)-\log_4(x^2+15)=-1$

$\log_2(3-x)-\dfrac{\log_2(x^2+15)}{\log_2 4}=-1$ M1 change of base law

$\log_2(3-x)-\dfrac{1}{2}\log_2(x^2+15)=-1$

$2\log_2(3-x)-\log_2(x^2+15)=-2$

$\log_2\dfrac{(3-x)^2}{x^2+15}=-2$ M1 quotient law & power law

$\therefore \dfrac{9-6x+x^2}{x^2+15}=2^{-2}$ M1 anti log

$x^2+15=36-24x+4x^2$

$3x^2-24x+21=0$

$x^2-8x+7=0$

$(x-7)(x-1)=0$ M1

$x=7$ or $x=1$ A1

NA

11 The function $f(x) = 2 - 3\sin 3x$ is defined for $0 \le x \le \pi$.

(i) State the period and amplitude of f. [2]

(ii) Solve $f(x) = 0$ for $0 \le x \le \pi$. [3]

(iii) Sketch the graph of $y = f(x)$ for $0 \le x \le \pi$. [3]

(iv) On the diagram drawn in part **(iii)**, draw the graph of $y = -\dfrac{2x}{\pi} + 1$. [1]

(v) State the number of solutions, for $0 \le x \le \pi$, of the equation $1 + \dfrac{2x}{\pi} - 3\sin 3x = 0$. [1]

Solution:

(i) Period $= \dfrac{2\pi}{3}$, amplitude $= 3$ B2

(ii) $f(x) = 0$ $\qquad 0 \le x \le \pi$

$2 - 3\sin 3x = 0$ $\qquad 0 \le 3x \le 3\pi$

$\sin 3x = \dfrac{2}{3}$

$3x$ in 1st/2nd quad

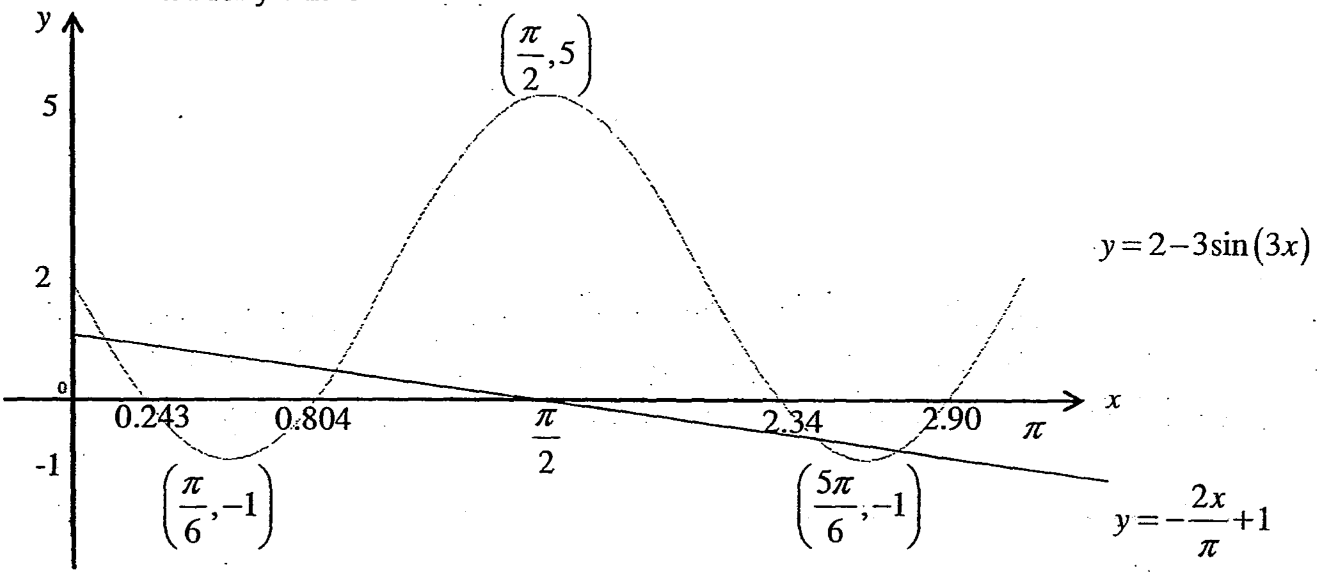
Basic □ ≈ 0.7297 A1

$3x = 0.7297, \quad \pi - 0.7297, \quad 2\pi + 0.7297, \quad 3\pi - 0.7297$

$x \approx 0.243, \quad 0.804, \quad 2.34, \quad 2.90$ A2

(iii), B1 for shape and y values
B1 for x intercepts & values
B1 for y values

(iv) B1 for straight line



(v) $1 + \dfrac{2x}{\pi} - 3\sin 3x = 0$

$2 - 3\sin 3x = -\dfrac{2x}{\pi} + 1$

$\therefore$ There are 4 solutions. A1

12 The expression $2x^3+mx^2+nx+3$, where m and n are constants, has a factor of $x-1$ and leaves a remainder of -9 when divided by $x+2$.

(i) Show that $m=-1$ and $n=-4$. [3]

(ii) Hence solve the equation $2x^3-x^2-4x+3=0$ [3]

(iii) Express $\dfrac{3x^2+4}{2x^3-x^2-4x+3}$ in partial fractions. [5]

Solution:

(i) Let $f(x)=2x^3+mx^2+nx+3$

By factor theorem,

$f(1)=0$

$2+m+n+3=0$ M1

$m=-n-5$ ------------(1)

By remainder theorem,

$f(-2)=-9$

$2(-2)^3+m(-2)^2+n(-2)+3=-9$ M1

$-16+4m-2n+3=-9$

$4m-2n=4$ ------------(2)

Sub (1) into (2):

$4(-n-5)-2n=4$

$-4n-20-2n=4$

$-6n=24$

$n=-4$

Sub $n=-4$ into (1) ,

$\therefore\ m=-1$ (shown) A1 sub and follow through

(ii) Let $2x^3-x^2-4x+3=(x-1)(ax^2+bx+c)=0$

Comparing coefficient of

x^3: $2=a$

x^0: $3=-c$ $\quad\therefore c=-3$

x^2: $-1=b-a$ $\quad\therefore b=1$

$\therefore 2x^3-x^2-4x+3=0$

$(x-1)(2x^2+x-3)=0$ A1 for $(2x^2+x-3)$

$(x-1)(x-1)(2x+3)=0$ M1

$\therefore x=1$ or $x=-1\dfrac{1}{2}$ A1

Alternative

(ii)
$$\begin{array}{r|l}
 & 2x^2+x-3 \\
x-1 & 2x^3-x^2-4x+3 \\
 & 2x^3-2x^2 \\ \hline
 & x^2-4x \\
 & x^2-x \\ \hline
 & -3x+3 \\
 & -3x+3 \\ \hline
 & 0
\end{array}$$

(iii) Let $\dfrac{3x^2+4}{2x^3-x^2-4x+3}=\dfrac{3x^2+4}{(x-1)^2(2x+3)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{2x+3}$ M1

$\therefore\ 3x^2+4=A(x-1)(2x+3)+B(2x+3)+C(x-1)^2$

When $x=1$,

$7=5B \qquad \therefore\ B=\dfrac{7}{5}$ A1

When $x=-1\dfrac{1}{2}$,

$10\dfrac{3}{4}=6\dfrac{1}{4}C \qquad \therefore\ C=\dfrac{43}{25}$ A1

Comparing coefficient of x^2:

$3=2A+C \qquad \therefore\ A=\dfrac{16}{25}$ A1

$\therefore\ \dfrac{3x^2+4}{2x^3-x^2-4x+3}=\dfrac{16}{25(x-1)}+\dfrac{7}{5(x-1)^2}+\dfrac{43}{25(2x+3)}$ A1

End of Paper

	Class	Register Number
Name ________________		

4047/02 **14/S4PR1/AM/2**

ADDITIONAL MATHEMATICS **PAPER 2**

Monday **12May 2014** **2 hours 30 minutes**

NIL SINE LABORE

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 6 printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 (i) Prove that $\dfrac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = -\cot A$. [3]

(ii) Hence, solve $\dfrac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = 4\cos^2 A$ for $0 \le A \le \pi$, giving your answers in terms of π. [4]

2 (a) Write down and simplify the first three terms in the expansion of $\left(x+\dfrac{1}{200x^2}\right)^{10}$ in descending powers of x. Hence evaluate $(1.005)^{10}$ correct to five significant figures. [4]

(b) Find the coefficient of the third term in the expansion of $\left(a-\dfrac{1}{x}\right)^4$ and the coefficient of the second term in the expansion of $\left(2+\dfrac{x}{a}\right)^6$ in terms of a.

If the coefficient of the second term of $\left(2+\dfrac{x}{a}\right)^6$ is four times the coefficient of the third term of $\left(a-\dfrac{1}{x}\right)^4$, find the value of a. [4]

3 The roots of the quadratic equation $2x^2+x-5=0$ are α and β.

(i) Express $\alpha^2-\alpha\beta+\beta^2$ in terms of $(\alpha+\beta)$ and $\alpha\beta$. [1]

(ii) Hence, find the exact value of $\alpha^2+\beta^2$. [3]

(iii) Find a quadratic equation whose roots are α^3 and β^3. [4]

4 (i) Differentiate $\dfrac{2x}{\sin x}$ with respect to x. [3]

(ii) Using your answer to part (i), find $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(\frac{1}{\sin x}-\frac{x\cos x}{\sin^2 x}\right)\mathrm{d}x$. [5]

5 A curve has the equation $y = f(x)$, where $f(x) = (2-x)e^{-3x}$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. Leave your answer in terms of e. [5]

(iii) Determine, with explanation, whether f is an increasing or decreasing function for $x < 2\frac{1}{3}$. [2]

6 A circle with centre C passes through points $A(-1,7)$ and $B(0,8)$.

(i) Explain why the perpendicular bisector of AB will pass through C. [1]

(ii) Given further that the line $y = 2x - 2$ passes through the centre of the circle, show that the coordinates of C is $(3,4)$. [4]

(iii) Hence, find the equation of the circle in the general form. [3]

7 (a) Find the exact values of k for which the line $y - 3x = k$ is a tangent to the curve $y^2 + 4x + 5 = x^2$. [5]

(b) Calculate the smallest negative integer k for which the equation $y = 4x^2 + (12-4k)x + 15 - 7k$ is always positive for all real values of x. [3]

(c) Solve the inequality $4x^2 - 1 > (2x+1)(x-3)$. [2]

8 **Answer the whole of this question on a sheet of graph paper.**

The population P, in millions, of a country was recorded in certain years and the results are shown in the table below:

Year	1978	1984	1988	1993
P	12.40	14.48	16.76	21.22

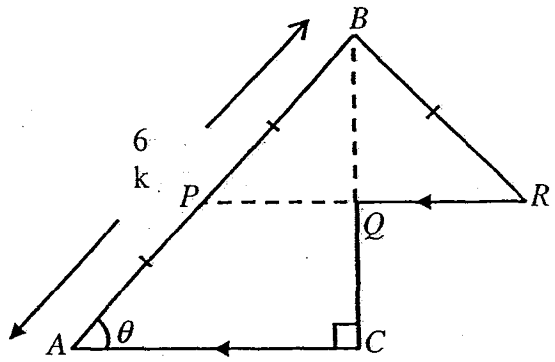
It is given that $P = 10 + ab^x$, where x is the time measured in years from January 1975 and a and b are constants. By plotting $\lg(P-10)$ against x, draw a line graph for the given data for $0 \le x \le 20$. Use a scale of 2 cm to represent 0.1 units on the vertical axis and 4 cm to represent 5 units on the horizontal axis. [4]

Use your graph to estimate,

(i) the value of a and of b. [3]

(ii) the year in which the population will reach 23.8 millions. [3]

9



In the diagram, $ABRQC$ represents a cycling course. From point A, a cyclist travels along straight tracks AB, BR, RQ and QC, returning along the track CA to finish at A. The total length of the course is L km.

It is given that $AB = 6$ km and P is the mid-point of AB. The length of each of the tracks AP, PB and BR are equal and track QC is also perpendicular to track AC. Track QR is parallel to the track AC and angle BAC is $\theta°$.

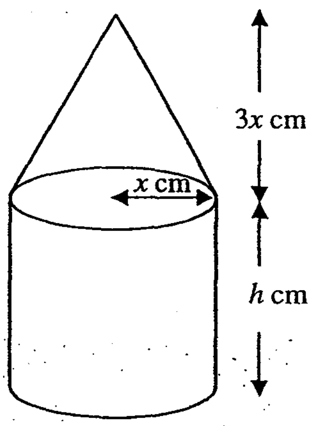
(i) Show that L can be expressed as $p + q\sin\theta + r\cos\theta$, where p, q and r are constants to be found. [3]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$ where $R > 0$, and α is acute. [5]

The total length of the course is found to be 16.5 km.

(iii) Find the value of θ. [2]

10

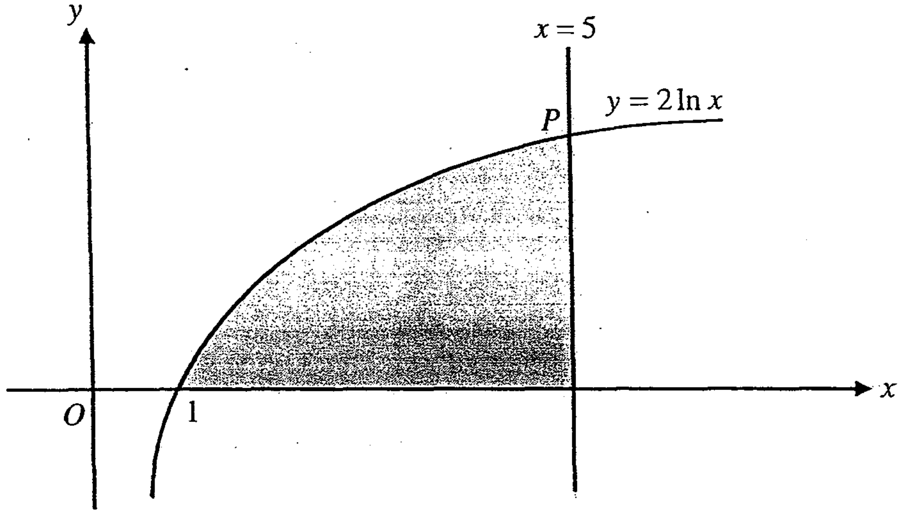


The diagram shows a solid comprising of a right-circular cone of height $3x$ cm sitting on a cylinder of base radius x cm and height h cm. The volume of the solid is 250 cm^3.

(i) Express h in terms of x and hence show that the total surface area, S cm^2, of the solid is given by $S = \pi(\sqrt{10} - 1)x^2 + \dfrac{500}{x}$. [6]

(ii) Given that x can vary, find, the value of x for which S has a stationary value. Find this value of S and determine whether it is a maximum or a minimum value. [4]

11 **(a)** The diagram below shows part of the curve $y = 2\ln x$ and the line $x = 5$.



Find the

(i) coordinates of P, [2]

(ii) area of the shaded region. [3]

(b) The concentration of caffeine, C %, in Peter's blood when he drinks a cup of coffee in the morning is represented by $C = 0.2xe^{-1.8x}$, where x is the number of hours after drinking a cup of coffee.

(i) Calculate the value of C when $x = 2$. [1]

(ii) Find the rate of change of caffeine concentration in Peter's blood, 1.5 hours after drinking a cup of coffee. [3]

(iii) What is the maximum level of caffeine concentration in Peter's blood in the morning? [3]

End of Paper

2014 A MATH PAPER 2 ANSWER KEY

1 (i) Proof (ii) $A=\frac{\pi}{2},\frac{7\pi}{12},\frac{11\pi}{12}$

2 (a) $x^{10}+\frac{1}{20}x^7+\frac{9}{8000}x^4+....$, 1.0511 **(b)** $T_3=6a^2$, $T_2=\frac{192}{a}$, $a=2$

3 (i) $\alpha^2-\alpha\beta+\beta^2=(\alpha+\beta)^2-3\alpha\beta$ **(ii)** $\alpha^2+\beta^2=5\frac{1}{4}$ **(iii)** $8x^2+31x-125=0$

4 (i) $\frac{d}{dx}\left(\frac{2x}{\sin x}\right)=\frac{2(\sin x-2x\cos x)}{\sin^2 x}$ **(ii)** -6.28

5 (i) $\therefore f'(x)=e^{-3x}(3x-7)$ **(ii)** $y=e^6x-2e^6$ **(iii)** decreasing function

6 (i)By the property of circle, the perpendicular bisector of a chord passes through the centre of a circle.

(ii) Proof (iii) $x^2+y^2-6x-8y+9=0$

7 (a) $k=-6+6\sqrt{2}$ or $k=-6-6\sqrt{2}$ **(b)** -2 **(c)** $x<-2$ or $x>-\frac{1}{2}$

8 (i) $a=1.78,\ b=1.12$ **(ii)** 1995

9 (i) $L=9+9\cos\theta+3\sin\theta$ **(ii)** $L=9+9.49\cos(\theta-18.4°)$ **(iii)** $\theta=56.2°$

10 (i) $h=\frac{250-\pi x^3}{\pi x^2}$, $S=\pi\left(\sqrt{10}-1\right)x^2+\frac{500}{x}$ **(ii)** $x=3.33$ cm , Minimum $S=225$

11 (a) (i) $P(5,2\ln 5)$ **(ii)** 8.09 units^2

(b) (i) 0.0109 **(ii)** -0.0228 % per hour **(iii)** 0.0409 %

	Class	Register Number
Name MARK SCHEME		

4047/02 **14/S4PR1/AM/2**

ADDITIONAL MATHEMATICS PAPER 2

Monday 12May 2014 2 hours 30 minutes

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Materials: Answer Paper
Graph Paper

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$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 (i) Prove that $\dfrac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = -\cot A$. [3]

$$\frac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = \frac{\sin A\cos B + \cos A\sin B + \sin B\cos A - \cos B\sin A}{\cos A\cos B - \sin A\sin B - [\cos B\cos A + \sin B\sin A]} \text{ ------- [B1]}$$

$$= \frac{2\sin B\cos A}{-2\sin B\sin A} \text{ ------- [B1]}$$

$$= -\frac{\cos A}{\sin A}$$

$$= -\cot A \quad \text{(shown)} \quad \text{-------- [A1]}$$

(ii) Hence, solve $\dfrac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = 4\cos^2 A$ for $0 \le A \le \pi$, giving your answers in terms of π. [4]

$$\frac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = 4\cos^2 A$$

$\therefore -\cot A = 4\cos^2 A$ --------- [B1]

$-\cos A = 4\cos^2 A\sin A$

$4\cos^2 A\sin A + \cos A = 0$

$\cos A(4\sin A\cos A + 1) = 0$ --------- [M1]

$\Rightarrow \cos A = 0$ or $4\sin A\cos A + 1 = 0$

$A = \dfrac{\pi}{2}$ --------- [A1]

$2\sin 2A = -1$

$\sin 2A = -\dfrac{1}{2}$

$\therefore$ basic $\angle, \alpha = \dfrac{\pi}{6}$

$2A = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$A = \dfrac{7\pi}{12}, \dfrac{11\pi}{12}$ --------- [A1]

2 **(a)** Write down and simplify the first three terms in the expansion of $\left(x+\frac{1}{200x^2}\right)^{10}$ in descending powers of x. Hence evaluate $(1.005)^{10}$ correct to five significant figures. [4]

$$\left(x+\frac{1}{200x^2}\right)^{10} = x^{10}+\binom{10}{1}x^9\left(\frac{1}{200x^2}\right)+\binom{10}{2}x^8\left(\frac{1}{200x^2}\right)^2+\ldots \quad \text{------ [B1]}$$

$$= x^{10}+\frac{1}{20}x^7+\frac{9}{8000}x^4+\ldots \quad \text{------ [A1]}$$

$$(1.005)^{10} = \left(x+\frac{1}{200}\right)^{10}, \quad \text{where } x=1.$$

$$= (1)^{10}+\frac{1}{20}(1)^7+\frac{9}{8000}(1)^4+\ldots \quad \text{------ [M1]}$$

$$\approx 1.0511 \quad \text{------ [A1]}$$

(b) Find the coefficient of the third term in the expansion of $\left(a-\frac{1}{x}\right)^4$ and the coefficient of the second term in the expansion of $\left(2+\frac{x}{a}\right)^6$ in terms of a.

If the coefficient of the second term of $\left(2+\frac{x}{a}\right)^6$ is four times the coefficient of the third term of $\left(a-\frac{1}{x}\right)^4$, find the value of a. [4]

For, $\left(a-\frac{1}{x}\right)^4$,

$$T_3 = \binom{4}{2}a^2\left(-\frac{1}{x}\right)^2 = \frac{6a^2}{x^2}$$

Coeff. of $T_3 = 6a^2$ --------- [A1]

For, $\left(2+\frac{x}{a}\right)^6$,

$$T_2 = \binom{6}{1}2^5\left(\frac{x}{a}\right) = \frac{192}{a}x$$

Coeff. of $T_2 = \frac{192}{a}$ --------- [A1]

Since, $\frac{192}{a} = 4\left(6a^2\right)$ --------- [M1]

$$\therefore 48 = 6a^3$$

$$a^3 = 8$$

$$a = 2 \quad \text{--------- [A1]}$$

3 The roots of the quadratic equation $2x^2 + x - 5 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha + \beta)$ and $\alpha\beta$. [1]

$\alpha^2 - \alpha\beta + \beta^2 = (\alpha + \beta)^2 - 3\alpha\beta$ -------- [B1]

(ii) Hence, find the exact value of $\alpha^2 + \beta^2$. [3]

$\alpha + \beta = -\frac{1}{2}$, $\alpha\beta = -\frac{5}{2}$ ---------- [B1 - for both]

$\therefore \alpha^2 - \alpha\beta + \beta^2 = (\alpha + \beta)^2 - 3\alpha\beta$ -------- [M1]

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{5}{2}\right)$

$= 5\frac{1}{4}$ -------- [A1]

(iii) Find a quadratic equation whose roots are α^3 and β^3. [4]

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ ------- [B1]

$= (\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]$

$= \left(-\frac{1}{2}\right)\left[\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{5}{2}\right)\right]$ ------- [M1]

$= -\frac{31}{8}$

$\alpha^3\beta^3 = (\alpha\beta)^3$

$= \left(-\frac{5}{2}\right)^3$

$= -\frac{125}{8}$ ------- [A1]

$\therefore$ Equation with new roots;

$x^2 - \left(-\frac{31}{8}\right)x + \left(-\frac{125}{8}\right) = 0$

$x^2 + \frac{31}{8}x - 15\frac{5}{8} = 0$ ------- [A1 o.e]

$8x^2 + 31x - 125 = 0$

4 **(i)** Differentiate $\dfrac{2x}{\sin x}$ with respect to x. [3]

$$\frac{d}{dx}\left(\frac{2x}{\sin x}\right)=\frac{2\sin x-2x\cos x}{\sin^2 x} \text{ ------- [B2]}$$

$$=\frac{2(\sin x-x\cos x)}{\sin^2 x} \text{ ------- [A1]}$$

(ii) Using your answer to part **(i)**, find $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(\dfrac{1}{\sin x}-\dfrac{x\cos x}{\sin^2 x}\right)\mathrm{d}x$. [5]

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(\frac{1}{\sin x}-\frac{x\cos x}{\sin^2 x}\right)\mathrm{d}x=\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(\frac{\sin x-x\cos x}{\sin^2 x}\right)\mathrm{d}x \text{ ------- [B1]}$$

$$=\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left[\frac{2(\sin x-x\cos x)}{\sin^2 x}\right]\mathrm{d}x \text{ ------- [M1]}$$

$$=\frac{1}{2}\left[\frac{2x}{\sin x}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \text{ ------- [A1]}$$

$$=\frac{1}{2}\left[\frac{3\pi}{-1}-\frac{\pi}{1}\right] \text{ ------- [M1]}$$

$$=-2\pi$$

$$=-6.28\ \ (3\text{ SF}) \text{ ------- [A1]}$$

5 A curve has the equation $y = f(x)$, where $f(x) = (2-x)e^{-3x}$.

(i) Obtain an expression for $f'(x)$. [2]

$\therefore f'(x) = -3(2-x)e^{-3x} + (-1)e^{-3x}$ --------- [B1]

$= e^{-3x}(-6+3x-1)$

$= e^{-3x}(3x-7)$ --------- [A1 o.e]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. Leave your answer in terms of e. [5]

At $f(x) = 0$,

$(2-x)e^{-3x} = 0$ --------- [B1]

$\therefore 2-x=0$ or $e^{-3x} = 0$ (NA)

$\Rightarrow x = 2$ --------- [B1]

The curve passes thru' the x-axis at $(2,0)$.

At $x = 2$,

$\therefore f'(2) = e^{-6}(6-7)$

$= -e^{-6}$

$\Rightarrow$ Gradient of normal at $x = 2$ is e^{6} --------- [M1]

$\therefore$ Equation of normal: $y = e^{6}(x-2)$ --------- [M1]

$y = e^{6}x - 2e^{6}$ --------- [A1 o.e]

(iii) Determine, with explanation, whether f is an increasing or decreasing function for $x < 2\frac{1}{3}$. [2]

$f'(x) = e^{-3x}(3x-7)$

$\Rightarrow$ For $x < 2\frac{1}{3}$, $e^{-3x} > 0$ and $(3x-7) < 0$ --------- [B1]

$\therefore e^{-3x}(3x-7) < 0$

$\Rightarrow f'(x) < 0$

$\therefore$ For $x < 2\frac{1}{3}$, f is a decreasing function. --------- [A1]

6 A circle with centre C passes through points $A(-1,7)$ and $B(0,8)$.

(i) Explain why the perpendicular bisector of AB will pass through C. [1]

By the property of circle, the perpendicular bisector of a chord passes through the centre of a circle. ------ [B1]

(ii) Given further that the line $y=2x-2$ passes through the centre of the circle, show that the coordinates of C is $(3,4)$. [4]

Mid-point of $AB = \left(\frac{-1+0}{2}, \frac{7+8}{2}\right)$

$= \left(-\frac{1}{2}, \frac{15}{2}\right)$ -------- [B1]

Gradient of $AB = \frac{8-7}{0-(-1)} = 1$

$\therefore$ Gradient of $\perp$ bisector $AB = -1$ -------- [M1]

$\therefore$ Equation of Gradient of $\perp$ bisector AB,

$y - \frac{15}{2} = -1\left[x - \left(-\frac{1}{2}\right)\right]$

$y = -x+7$ ------- (1)

Given: $y = 2x-2$ ------- (2)

Solving (1) & (2),

$2x-2 = -x+7$ -------- [M1]

$3x = 9$

$x = 3$

Subst. $x=3$ into (1), $\Rightarrow y=4$.

$\therefore$ Coordinates of C are $(3,4)$ (shown). -------- [A1]

(iii) Hence, find the equation of the circle in the general form. [3]

Coordinates of Centre is $(3,4)$.

Radius $= \sqrt{(3-0)^2 + ((8-4))^2}$ -------- [M1]

$= 5$ units

Eqn.: $(x-3)^2 + (y-4)^2 = 5^2$ -------- [M1]

In general form;

$x^2 - 6x + 9 + y^2 - 8y + 16 = 25$

$x^2 + y^2 - 6x - 8y + 9 = 0$ -------- [A1]

7 (a) Find the exact values of k for which the line $y-3x=k$ is a tangent to the curve $y^2+4x+5=x^2$. [5]

$y-3x=k \Rightarrow y=3x+k$ ------- (1) & $y^2+4x+5=x^2$ ------- (2)

Subst. (1) into (2);

$(3x+k)^2+4x+5=x^2$ ------- [M1]

$9x^2+6kx+k^2-x^2+4x+5=0$

$8x^2+(6k+4)x+(k^2+5)=0$

Since $y-3x=k$ is a tangent to the curve,

$\therefore (6k+4)^2-4(8)(k^2+5)=0$ ------- [M1 B1]

$(3k+2)^2-8(k^2+5)=0$

$9k^2+12k+4-8k^2-40=0$

$k^2+12k-36=0$

$k=\dfrac{-12\pm\sqrt{(-12)^2-4(1)(-36)}}{2(1)}$ ------- [M1]

$\therefore k=-6+6\sqrt{2}$ or $k=-6-6\sqrt{2}$ ------- [A1 o.e]

(b) Calculate the smallest negative integer k for which the equation $y=4x^2+(12-4k)x+15-7k$ is always positive for all real values of x. [3]

Since $y>0, \Rightarrow 4x^2+(12-4k)x+15-7k>0$.

$\therefore b^2-4ac<0$

$(12-4k)^2-4(4)(15-7k)<0$ --------- [B1 B1]

$(3-k)^2-(15-7k)<0$

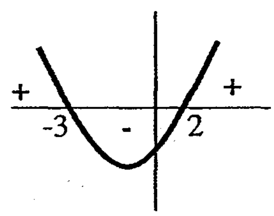
$9-6k+k^2-15+7k<0$

$k^2-k-6<0$

$(k+3)(k-2)<0$

$\Rightarrow -3<k<2$

$\therefore$ Smallest negative integer of k is -2. ------ [A1]



(c) Solve the inequality $4x^2-1>(2x+1)(x-3)$. [2]

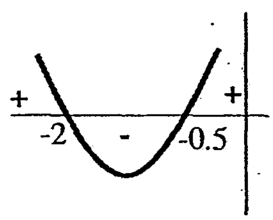
$4x^2-1>(2x+1)(x-3)$

$4x^2-1>2x^2-5x-3$

$2x^2+5x+2>0$

$(2x+1)(x+2)>0$ ------ [M1]

$\therefore x<-2$ or $x>-\dfrac{1}{2}$ ------ [A1]



8 Answer the whole of this question on a sheet of graph paper.

The population P, in millions, of a country was recorded in certain years and the results are shown in the table below:

Year	1978	1984	1988	1993
P	12.40	14.48	16.76	21.22

It is given that $P = 10 + ab^x$, where x is the time measured in years from January 1975 and a and b are constants. By plotting $\lg(P-10)$ against x, draw a line graph for the given data for $0 \le x \le 20$. Use a scale of 2 cm to represent 0.1 units on the vertical axis and 4 cm to represent 5 units on the horizontal axis. [4]

x	3	9	13	18
$\lg(P-10)$	0.380	0.651	0.830	1.05

.... [B1 with at least 3 SF]

[M1 --- Plot Points correctly]
[M1 --- Best-Fit Line]
[M1 --- Label axes, origin

Use your graph to estimate,

(i) the value of a and of b. [3]

$P = 10 + ab^x$

$P - 10 = ab^x$

$\lg(P-10) = (\lg b)x + \lg a$ ------ [A1]

$\lg a = 0.25$

$\Rightarrow a = 1.78$ ------ [A1]

$$\text{Gradient} = \lg b = \frac{1.03 - 0.55}{17.5 - 7.8}$$ ------ [M1] [with gradient Δ or coordinates]

$= 0.049485$

$\Rightarrow b = 1.12$

(ii) the year in which the population will reach 23.8 millions. [3]

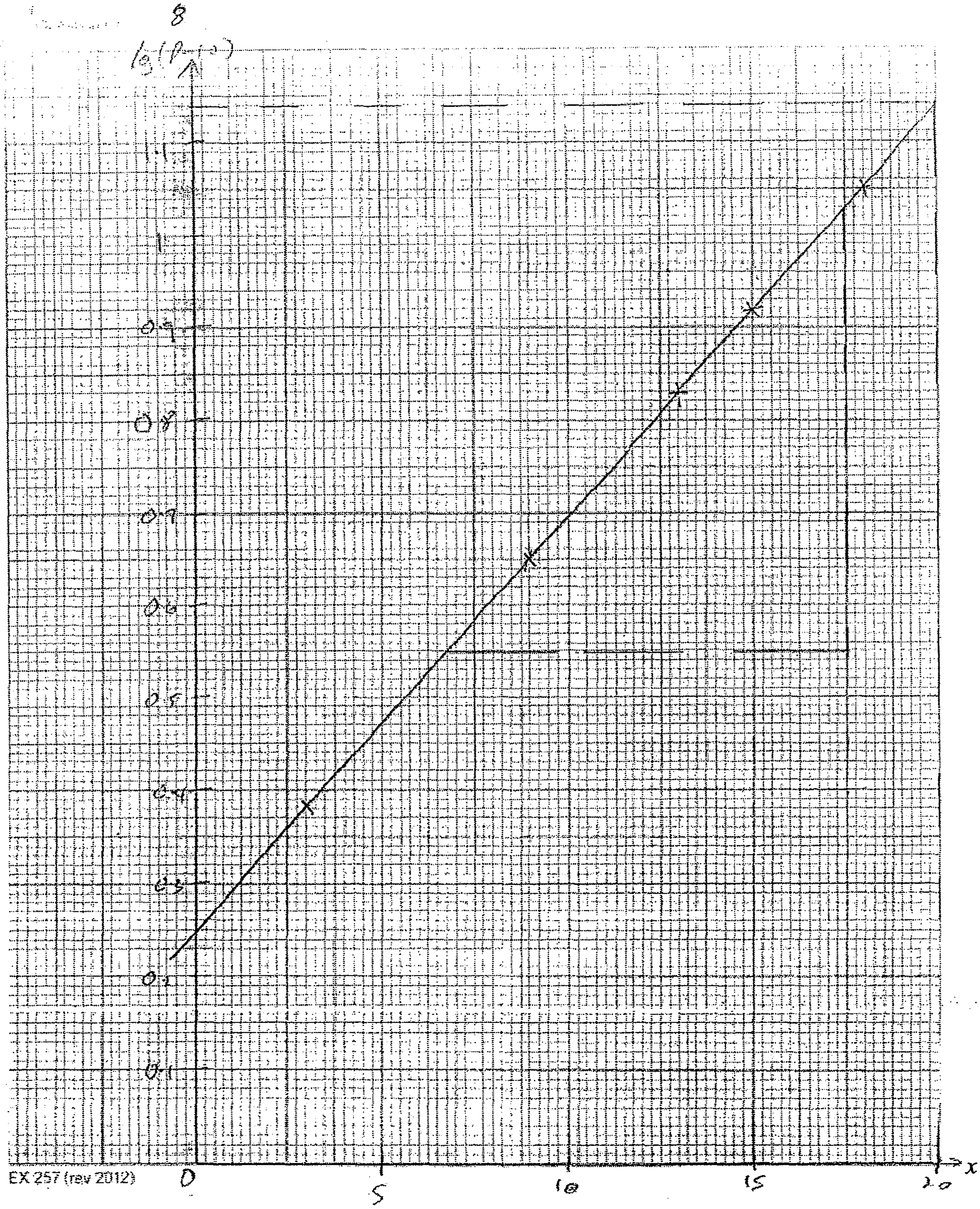
At $p = 23.8$,

$\therefore \lg(P-10) = \lg 13.8$

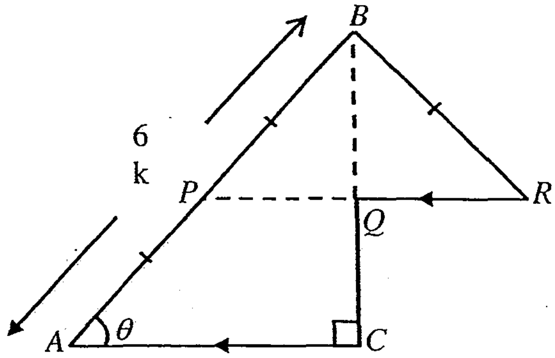
$= 1.14$ -------- [B1]

From the graph, corresponding value of x is 20. -------- [A1]

$\therefore$ The year will be in 1995. -------- [A1]


8
lg(p-p)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
0
5
10
15
20
x
EX 257 (rev 2012)

9 In the diagram, $ABRQC$ represents a cycling course. From point A, a cyclist travels along straight tracks AB, BR, RQ and QC, returning along the track CA to finish at A. The total Length of the course is L km.



It is given that $AB = 6$ km and P is the mid-point of AB. The length of each of the tracks AP, PB and BR are equal and track QC is also perpendicular to track AC. Track QR is parallel to the track AC and angle BAC is $\theta°$.

(i) Show that L can be expressed as $p + q\sin\theta + r\cos\theta$, where p, q and r are constants to be found. [3]

$AP = PB = BR = 3$
$\angle BRQ = \angle BPR = \theta$
$\therefore QR = 3\cos\theta, AC = 6\cos\theta \ \& \ BC = 6\sin\theta$
$\therefore QC = \dfrac{1}{2}BC = 3\sin\theta$
$\Rightarrow L = AB + BR + QC + AC + RQ$
$\quad = 6 + 3 + 3\sin\theta + 6\cos\theta + 3\cos\theta$ -------- [M2]
$\quad L = 9 + 3\sin\theta + 9\cos\theta$ -------- [A1]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$ where $R > 0$, and α is acute. [5]

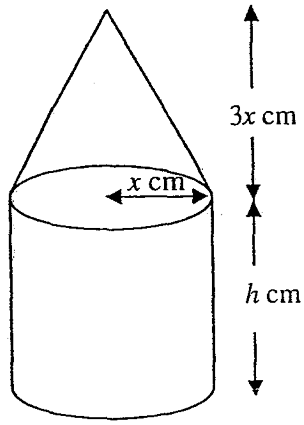
$L = 9 + 9\cos\theta + 3\sin\theta = 9 + R\cos(\theta - \alpha)$
$= 9 + R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ -------- [B1]
Comparing both sides,
$\Rightarrow R\cos\alpha = 9$ ----- (1), $\quad R\sin\alpha = 3$ ----- (2)
$(1)^2 + (2)^2 : R^2\left(\cos^2\alpha + \sin^2\alpha\right) = 9^2 + 3^2$ --------- [M3]
$\Rightarrow R = \sqrt{90} = 9.49$ (3 SF)
$(2) \div (1): \tan\alpha = \dfrac{1}{3} \quad \therefore \alpha = 18.43°$
$\therefore L = 9 + 9.49\cos(\theta - 18.4°)$ -------- [A1]

The total length of the course is found to be 16.5 km.

(iii) Find the value of θ. [2]

$9 + \sqrt{90}\cos(\theta - 18.43°) = 16.5$ -------- [M1]
$\sqrt{90}\cos(\theta - 18.43°) = 7.5$
$\theta - 18.43° = 37.76°$
$\theta = 56.2°$ (1 DP) -------- [A1]

10 The diagram shows a solid comprising of a right-circular cone of height $3x$ cm sitting ona cylinder of base radius x cm and height h cm. The volume of the solid is 250 cm^3.



(i) Express h in terms of x and hence show that the total surface area, S cm^2, of the solid is given by

$$S=\pi(\sqrt{10}-1)x^2+\frac{500}{x}.$$ [6]

Vol. of cone $=\frac{1}{3}\pi x^2(3x)=\pi x^3$

Vol. of cylinder $=\pi x^2(h)=\pi x^2h$

$\therefore \pi x^3+\pi x^2h=250$ --------- [B1]

$\pi x^2h=250-\pi x^3$

$h=\frac{250-\pi x^3}{\pi x^2}$ --------- [A1]

Slant height of cone, $l=\sqrt{(3x)^2+(x)^2}$

$=\sqrt{10x^2}$

$l=\sqrt{10}x$

Curved surface area of cone $=\pi(x)(\sqrt{10}x)$ --------- [M1]

$=\pi\sqrt{10}x^2$

Surface area of cylinder $=2\pi xh+\pi x^2$

$=2\pi x\left(\frac{250-\pi x^3}{\pi x^2}\right)+\pi x^2$ --------- [M1]

$=\frac{500-2\pi x^3}{x}+\pi x^2$

$=\frac{500}{x}-2\pi x^2+\pi x^2$

$=\frac{500}{x}-\pi x^2$

$\therefore$ Total Surface Area of solid, $S=\pi\sqrt{10}x^2+\frac{500}{x}-\pi x^2$ --------- [M1]

$\Rightarrow S=\pi\left(\sqrt{10}-1\right)x^2+\frac{500}{x}$ --------- [A1]

(ii) Given that x can vary, find, the value of x for which S has a stationary value. Find this value of S and determine whether it is a maximum or a minimum value. [4]

$$S = \pi\left(\sqrt{10}-1\right)x^2 + \frac{500}{x}$$

$$\frac{dS}{dx} = 2\pi x\left(\sqrt{10}-1\right) - \frac{500}{x^2}$$

At stationary point, $2\pi x\left(\sqrt{10}-1\right) - \frac{500}{x^2} = 0$ --------- [M1]

$$x^3 = \frac{500}{2\pi\left(\sqrt{10}-1\right)}$$

$x = 3.326 \Rightarrow x = 3.33$ cm (3 SF) --------- [A1]

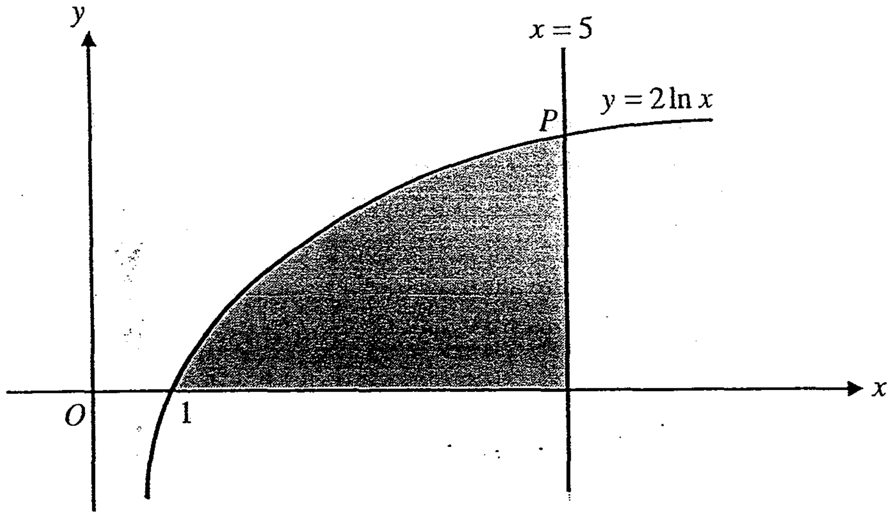
$$\frac{d^2S}{dx^2} = 2\pi\left(\sqrt{10}-1\right) + \frac{1000}{x^3}$$

At $x = 3.326$, $\frac{d^2S}{dx^2} > 0 \Rightarrow$ Minimum --------- [A1]

$\therefore$ Minimum $S = \pi\left(\sqrt{10}-1\right)(3.326)^2 + \frac{500}{3.326}$

$S = 225$ (3 SF) -------- [A1]

11 **(a)** The diagram below shows part of the curve $y = 2\ln x$ and the line $x = 5$.



Find the

(i) coordinates of P, [2]

For $y = 2\ln x$
At $x = 5$, $y = 2\ln 5$ -----[M1]
$\therefore P(5, 2\ln 5)$ -----[A1]

(ii) area of the shaded region. [3]

Area of rect. $= 5 \times 2\ln 5 = 10\ln 5$
Since $y = 2\ln x$
$\ln x = \frac{1}{2}y \Rightarrow x = e^{\frac{1}{2}y}$
Area enclosed by $x = e^{\frac{1}{2}y}$ and y – axis
$= \int_0^{2\ln 5} e^{\frac{1}{2}y}\, dy$ -----[M1]
$= \left[2e^{\frac{1}{2}y} \right]_0^{2\ln 5}$
$= 2\left[e^{\ln 5} - 1\right]$
$= 2[5 - 1]$
$= 8$
$\therefore$ Shaded area $= 10\ln 5 - 8$ -----[M1]
$= 8.09$ units^2 -----[A1]

Alternative solution
Since $y = 2\ln x$
$\ln x = \frac{1}{2}y \Rightarrow x = e^{\frac{1}{2}y}$
Area enclosed by $x = e^{\frac{1}{2}y}$ and y – axis
$= \int_0^{2\ln 5} \left(5 - e^{\frac{1}{2}y}\right) dy$ -----[M2]
$= \left[5y - 2e^{\frac{1}{2}y} \right]_0^{2\ln 5}$
$= \left[10\ln 5 - 2e^{\ln 5} + 2\right]$
$= [10\ln 5 - 10 + 2]$
$= 8.09$ units^2 -----[A1]

(b) The concentration of caffeine, C %, in Peter's blood when he drinks a cup of coffee in the morning is represented by $C = 0.2xe^{-1.8x}$, where x is the number of hours after drinking a cup of coffee.

(i) Calculate the value of C when $x = 2$. [1]

$C = 0.2xe^{-1.8x}$
At $x = 2$,
$C = 0.2(2)e^{-3.6}$
$= 0.0109 \quad (3\text{ SF})$ ------ [B1]

(ii) Find the rate of change of caffeine concentration in Peter's blood, 1.5 hours after drinking a cup of coffee. [3]

$C = 0.2xe^{-1.8x}$
$\frac{dC}{dx} = 0.2e^{-1.8x} + 0.2x\left(-1.8e^{-1.8x}\right)$ ------[B1]
$= 0.2e^{-1.8x}(1-1.8x)$
At $x = 1.5$,
$\frac{dC}{dx} = 0.2e^{-1.8(1.5)}\left[1-1.8(1.5)\right]$ ------[M1]
$= -0.0228$ % per hour. (3 SF) ------ [A1]

(iii) What is the maximum level of caffeine concentration in Peter's blood in the morning? [3]

At stationary point, $0.2e^{-1.8x}(1-1.8x) = 0$ ------[B1]
Since $e^{-1.8x} = 0$ (NA),
$\therefore 1-1.8x = 0 \Rightarrow x = 0.55556$
$\therefore \frac{d^2C}{dx^2} = -0.36e^{-1.8x}(2-1.8x)$
At $x = 0.55556$, $\frac{d^2C}{dx^2} < 0 \Rightarrow$ Maximum ------ [A1]
$\therefore$ Maximum $C = 0.2(0.55556)e^{-1.8(0.55556)}$
$= 0.0409$ % (3 SF) ------ [A1]

End of Paper