ANDERSON SECONDARY SCHOOL
Preliminary Examination 2015
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: /

INDEX NUMBER:

ADDITIONAL MATHEMATICS

4047/01

Paper 1

28 August 2015

2 hours

0800 – 1000h

Additional Materials: Writing paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

This document consists of 4 printed pages.

Setter: Miss Leow Hwee Fen & Miss Oh Hui Ying

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$
$$\sec^2 A=1+\tan^2 A.$$
$$\operatorname{cosec}^2 A=1+\cot^2 A.$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$
$$a^2=b^2+c^2-2bc\cos A.$$
$$\Delta=\frac{1}{2}bc\sin A.$$

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Answer all questions

1 Solve the following equations.

(a) $2(3^x)+3^{-x}=3$ [3]

(b) $2\sqrt{3-2x}=x+1$ [3]

2 The line $2y-x=3$ meets the curve $x^2-xy-y^2=1$ at points A and B. Find the length of AB, giving your answer expressed in the form $a\sqrt{b}$, where a and b are integers. [5]

3 Write down and simplify the first 3 terms, in ascending powers of x, in the expansion of $\left(2-\frac{x}{3}\right)^5$. Given that the first three terms in the expansion of $\left(1+px+x^2\right)\left(2-\frac{x}{3}\right)^5$ are $32-qx+2qx^2$, find the value of p. [5]

4 A circle C_1 passes through points $P(0, 2)$, $Q(7, 3)$ and $R(8, -4)$ where $PQRS$ is a square.

(a) Find the coordinates of the centre and the radius of the circle C_1. [2]

(b) Find the equation of another circle C_2, in the form $x^2+y^2+ax+by+c=0$, that is the reflection of the circle C_1, in the line $y=x$. [2]

(c) Justify if the point $(2, 7)$ lies inside or outside the circle C_1. [2]

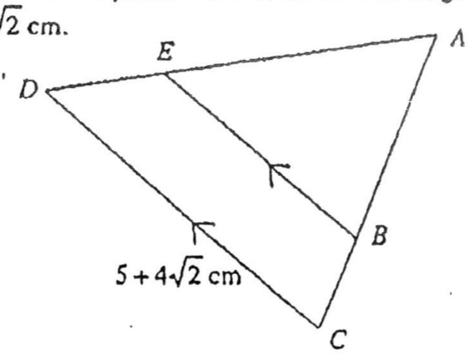
5 Prove the identity $\dfrac{\sec x+2\sin x}{2\cos x-\sec x}=\dfrac{1+\tan x}{1-\tan x}$. [5]

6 The diagram shows a triangle $ABCDE$, such that EB is parallel to DC, the ratio of lengths $AB : BC$ is $4:\sqrt{8}$ and length of DC is $5+4\sqrt{2}$ cm.

By leaving your answer in the form $a+b\sqrt{c}$, calculate

(a) the ratio $\dfrac{BE}{CD}$, [3]

(b) the length of BE. [3]



7 Given that $f(x)=-2+x^2$ and $g(x)=|x+1|-1$,

(a) Find the coordinates of the points of intersection of the graphs $y=f(x)$ and $y=g(x)$. [4]

(b) On the same axes, sketch the graphs of $y=f(x)$ and $y=g(x)$ for $-2\le x\le 2$. [3]

(c) Hence solve the inequality $x^2\le|x+1|+1$. [2]

8 (a) Sketch the graph of $y=2-e^{3x}$ for all real values of x, showing clearly all points of intersection with the axes, if any. [2]

(b) By adding a suitable straight line, explain how the number of solutions to the equation $x=\ln\sqrt[3]{4-x}$ can be obtained. [2]

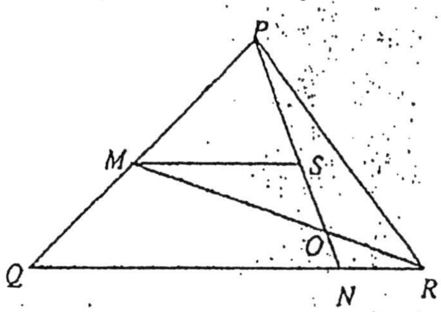
9 A and B lie in the same quadrant such that $\sin A = \frac{3}{5}$ and $\tan B = -\frac{5}{12}$. If the value of A and of B is between 0 and 2π, find, without using the calculator, the values of

(a) $\sin B$, [1]

(b) $\cot(A-B)$, [2]

(c) $\cos\frac{B}{2}$. [3]

10 In ΔPQR, M is the mid-point of PQ. PN and MR intersect at O.



Given that $OR : OM = PS : PN = 1 : 2$, prove that

(a) MS is parallel to QN, [2]

(b) ΔMSO is similar to ΔRNO, [2]

(c) $OP = 5\,NO$. [2]

11 The table shows some experimental values of two variables x and y which are known to be related by the equation $y = ax(x+b)$.

x	1.5	2.5	3.5	4.5	5.5
y	10.1	20.6	34.2	50.7	70.1

Using a suitable scale, plot the graph of $\frac{y}{x}$ against x to represent the above data and use it to estimate

(a) the value of a and of b, [4]

(b) the value of x when $y = 9x$. [1]

12 A function is given by $y = \frac{9x-3b}{4x-1}$ where $x \neq a$ and $x > 0$.

(a) State the value of a. [1]

(b) Determine the range of values of b if y is an increasing function. [3]

(c) Given that $b = 3$ and that x and y vary with time t, find the value(s) of x if $\frac{dy}{dt} = 12\frac{dx}{dt}$. [3]

13 An electronic gadget was programmed to travel in a straight line. It started through a fixed point O with a velocity of 3 m/s. Its acceleration, a m/s^2, is given by $a = 2 - 2t$, where t seconds is the time after passing O. Find

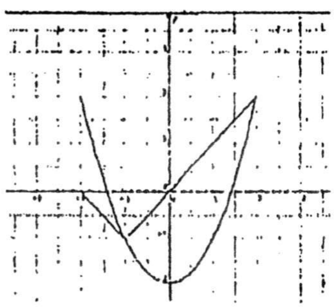
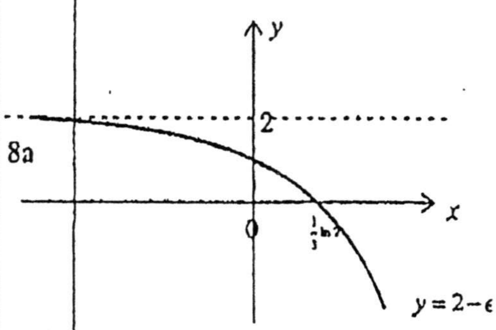
(a) its maximum velocity, [3]

(b) its deceleration when it changes its direction of motion, [3]

(c) the total distance travelled during the first four seconds of motion. [4]

 [End of Paper]

Prelim AM Paper 1 Answer Key

1(a)	$x = -0.631$ or $x = 0$	9a	$\frac{5}{13}$
1(b)	$x = 1$	9b	$-3\frac{15}{16}$
2	$7\sqrt{5}$ units	9c	$\frac{\sqrt{26}}{26}$
3	$32 - \frac{80}{3}x + \frac{80}{9}x^2 + \ldots$; $p = \frac{1}{3}$	10	Proof
4a	$(4, -1)$; 5 units	11a	$a = 1.5$; $b = 3$
4b	$x^2 + y^2 + 2x - 8y - 8 = 0$	11b	$x = 3$
4c	(2, 8) lies outside the circle C_1.	12a	$a = \frac{1}{4}$
5	Proof	12b	$b > \frac{3}{4}$
6a	$2 - \sqrt{2}$	12c	$x = \frac{5}{8}$
6b	$(2 + 3\sqrt{2})$ cm	13a	4 m/s
7a	$(-1, -1)$ and $(2, 2)$	13b	4 m/s^2
7b		13c	$11\frac{1}{3}$m
		9a	$\frac{5}{13}$
		9b	$-3\frac{15}{16}$
		9c	$\frac{\sqrt{26}}{26}$
		11a	$a = 1.5$; $b = 3$
7c	$-1 \leq x \leq 2$	11b	$x = 3$
8a		12a	$a = \frac{1}{4}$
		12b	$b > \frac{3}{4}$
		12c	$x = \frac{5}{8}$
		13a	4 m/s
		13b	4 m/s^2
8b	Add the line $y = x - 2$. The number of intersection points of $y = 2 - e^{3x}$ and $y = x - 2$ gives the number of solutions for $x = \ln\sqrt[3]{4 - x}$.	13c	$11\frac{1}{3}$m

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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2015
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: /

INDEX NUMBER:

ADDITIONAL MATHEMATICS 4047/02

Paper 2

27 August 2015

2 hours 30 minutes

0800 – 1030h

Additional Materials: Writing paper

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where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

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Answer all questions

1 When $(1-2p)^n$ is expanded in ascending powers of p, the sum of the constant term, the coefficients of p and p^2 is 161. If n is a positive integer, find the value of n. [4]

2 (a) Find the smallest positive integer, p such that $4(px-5)=x^2$ has real roots. [4]

(b) Find the range of values of m for which the graph of $y=mx^2-4x+m$ lies entirely below the line $y=3$. [4]

(c) Given that the line $y=4x+k$ is a tangent to the curve $y^2=mx$, where k and m are constants, prove that $\frac{k}{m}=\frac{1}{16}$. [4]

3 Marcus believes that the depth of water, d metres, at the end of a jetty, t hours after low tide, can be modelled by the equation $d=a+b\cos kt$ where a, b and k are constants.

(a) He measures the depth of water at low tide to be 2 metres.
Assuming that low tides occur every 12 hours, show that $k=\frac{\pi}{6}$. [1]

(b) Marcus also measures the depth of water at high tide to be 6 metres.
Calculate the value of a and of b. [2]

(c) Sketch the graph of the equation $d=a+b\cos kt$ for $0<t<2\pi$. [3]

(d) Marcus requires the depth of water at the end of the jetty to be at least 3 metres to sail his boat. Given that the low tide on a particular day was at 0830, find the earliest time after 0830 when Marcus could sail his boat that day. [2]

(e) Marcus measured the depth of water and found that it is 5m. He then claimed that the depth of water at the end of the jetty will reach 5 m again after every 4 hours. Justify if Marcus is right or wrong. [2]

4 (a) (i) Factorise $h(x)=x^3-7x^2+2x+40$ completely. [3]

(ii) Hence, solve the equation $2y^3-7y^2+y+10=0$. [3]

(b) Find the value of n for which the division of $2x^n+3x^2-4x-10$ by $x-2$ gives a remainder of 26. [3]

5 Solve the following equations.

(a) $\log_2\sqrt{5x+1}=\log_4(x-2)+\log_2 4$ [5]

(b) $4\tan^2 x=1-8\sec x$ for $-\pi<x<2\pi$ [5]

6 (a) Consider the equation $kx^2 - k^2x - 3x + 4 = k$.
If the roots of the equation are reciprocal of each other, and β is one of roots,
(i) find the value of k, [2]
(ii) show that $\frac{7}{\beta^2+1} = \frac{2}{\beta}$. [2]

(b) The roots of the quadratic equation $2x^2 - 4x + 5 = 0$ are λ and μ.
Find the quadratic equation whose roots are $\frac{\mu}{\lambda}$ and $\frac{\lambda}{\mu}$. [4]

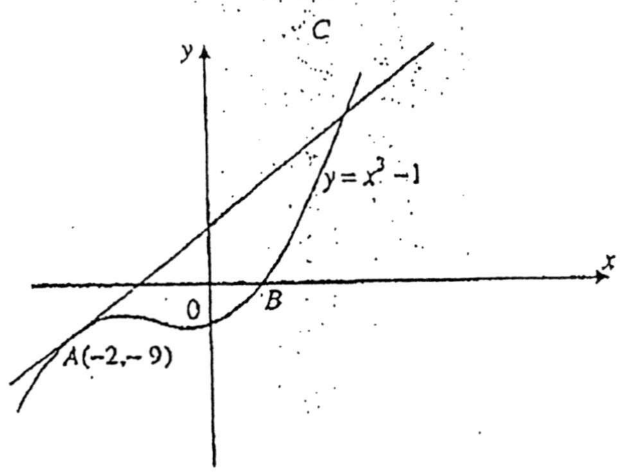
7 The term containing the highest power of x in the polynomial $f(x)$ is $3x^4$.
$x^2 - 2x + k$ is a quadratic factor of $f(x)$. $x = -1$ and $x = 2$ are roots of the equation $f(x) = 0$. $f(x)$ leaves a remainder of -36 when it is divided by x.
(a) Show that $k = 6$. [2]
(b) Determine the number of real roots of the equation $f(x) = 0$. [2]

8 A curve is defined by $y = (1-2x)^2 e^{2x}$. Find
(a) $\frac{dy}{dx}$, [2]
(b) the equation, in terms of e, of the tangent at the point where $x = 1$, [4]
(c) the x-coordinate(s) of the stationary point(s) on the curve and determine the nature of the point(s). [4]

9 (a) Express $\frac{x^2 - 3x + 5}{(x^2+x)(2x-1)}$ in partial fractions. [3]

(b) Hence, evaluate $\int_1^2 \frac{x^2 - 3x + 5}{(x^2+x)(1-2x)}\,dx$. [3]

10 The diagram shows part of the curve $y = x^3 - 1$. The tangent at A $(-2, -9)$ meets the curve again at C. Find the area of the region bounded by the two graphs. [8]



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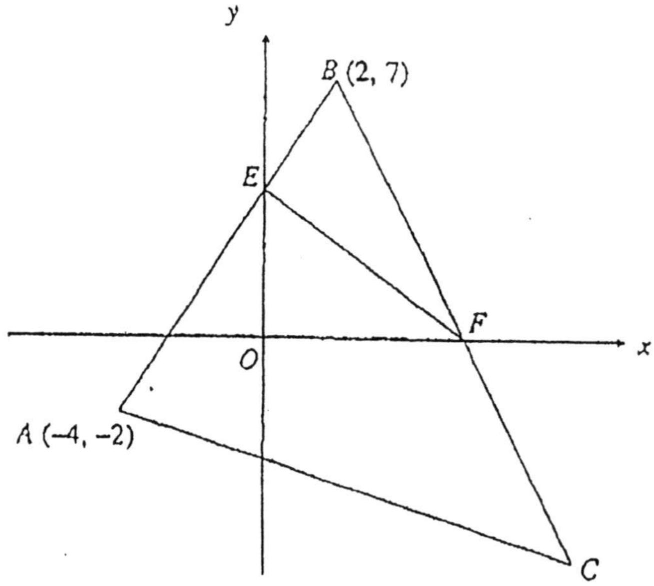
11 Solution to this question by accurate drawing will not be accepted.



The diagram shows a triangle ABC where A is $(-4, -2)$, B is $(2, 7)$ and BC is parallel to the line $2y = -4x + 1$. BC cuts the x-axis at F and AB cuts the y-axis at E.

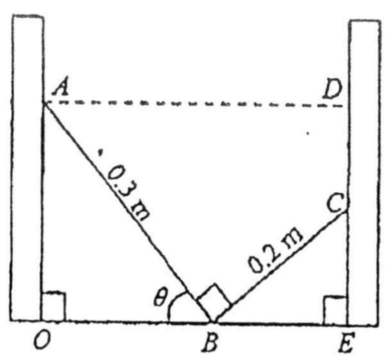
(a) Find the equation of the line BC. [2]

(b) Determine whether if EF is perpendicular to AB. [3]

(c) Given that C is equidistant from A and E, find the coordinates of C. [3]

(d) Find the length of AE, and hence, find the area of ΔAEC. [3]

12



A L-shaped ladder, ABC is wedged in between two pillars AO and DE as shown in the diagram. A and C are the points of contact between the ladder and the pillars while B is the point of contact between the ladder and the ground.

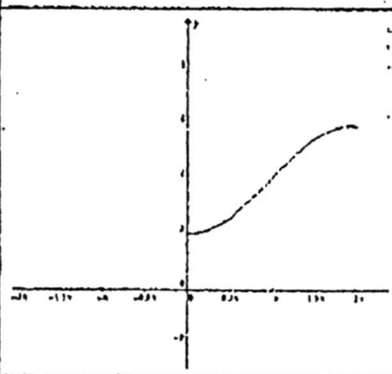
Given that $\angle OBA = \theta$, where $0° < \theta < 90°$, $AB = 0.3$ m, $BC = 0.2$ m and $CD = x$ m,

(a) show that $x = 0.3\sin\theta - 0.2\cos\theta$, [2]

(b) express x in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, [4]

(c) hence, explain if the length of CD can be 0.45 m. [2]

Prelim AM Paper 2 Answer Key

1	$n=10$	7b	2 real roots
2a	$p=3$	8a	$-2(1-2x)(1+2x)e^{2x}$
2b	$m<-1$	8b	$y=6e^2x-5e^2$
3a	$k=\frac{\pi}{6}$	8c	$x=-\frac{1}{2}$(max) & $x=\frac{1}{2}$(min)
3b	$a=4$	9a	$-\frac{5}{x}+\frac{3}{x+1}+\frac{5}{2x-1}$
3c		9b	-0.497
		10	108 sq units
		11a	$y=-2x+11$
		11b	EF is not perpendicular to AB.
		11c	$\left(\frac{17}{2},-6\right)$
		11d	$45\frac{1}{2}units^2$
		12b	$0.361\sin(\theta-33.7°)$
		12c	Length of CD cannot be 0.45 m.
3d	1030h		
3e	Wrong		
4ai	$(x-4)(x+2)(x-5)$		
4aii	$y=2$ or $y=-1$ or $y=\frac{5}{2}$		
4b	$n=4$		
5a	$x=3$		
5b	$x=-1.98$ or 1.98 or 4.30		
6ai	$k=2$		
6b	$5x^2+2x+5=0$		

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