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AHS Prelim. Am 2015 Paper 1

1. A curve has the equation $y = \dfrac{\ln x}{x^2}$.

   (i) Find $\dfrac{dy}{dx}$. [2]

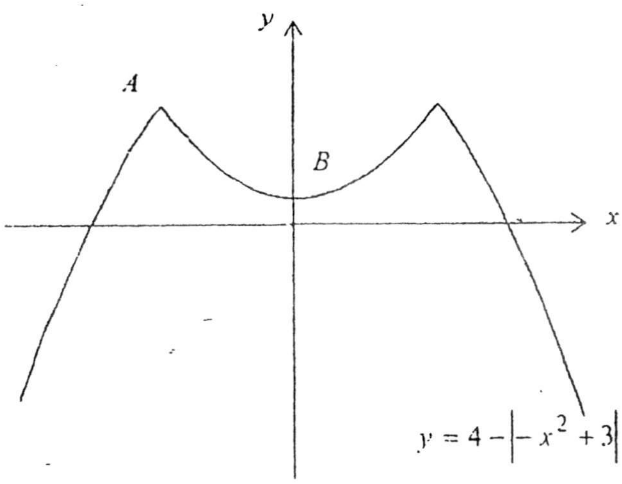
   (ii) Hence, find the range of values of x, such that y is increasing. [2]

2. The diagram below shows the graph of $y = 4 - \left|-x^2 + 3\right|$.

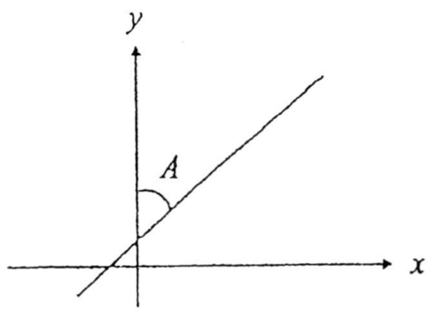
   (i) Show that the coordinates of A is $\left(-\sqrt{3}, 4\right)$. [1]

   (ii) State the coordinates of B. [2]

   (iii) Find the exact value of m, for $m < 0$ for which the equation $mx + 1 = 4 - \left|-x^2 + 3\right|$ has exactly 3 solutions. [2]



3. In the diagram shown, the line forms an angle A with the y-axis. Given that the gradient of the line is 3, without using a calculator, find the exact value of $\cos A$. [3]



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4. (i) Show that

$$\sqrt{\frac{1-\sin x}{1+\sin x}} = \sec x - \tan x \text{ , when } -90^\circ < x < 90^\circ. \qquad [5]$$

(ii) Hence, explain why x must be acute for the identity to be true. [1]

5. Given that the coefficient of $\frac{1}{x^3}$ is 512 in the expansion $\left(\frac{2}{x}+ax^2\right)^9$, where $a<0$.

(i) Find the value of a [3]

(ii) Hence, using the value of a found in (i), show that the term in $\frac{1}{x^4}$ does not exist in the expansion $\left(\frac{2}{x}+ax^2\right)^9\left(\frac{1}{8x}+\frac{x^2}{12}\right)$. [3]

6. Express $\frac{x^3+5x^2+2x-1}{2x^4+x^2}$ in partial fractions. [6]

7. The equation $6x^2+7x-3=0$ has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(i) Find the value of $(\alpha+\beta)$ and of $\alpha\beta$. [3]

(ii) Hence, or otherwise, find the exact value of $\left(\alpha^3+\beta^3\right)$. [3]

8. Solve the equation $\lg\left(4^x-10\right)-x\lg 2=\lg 3$. [5]

9. The voltage V, in volts, of an electrical signal in an electrical system is given by the formula $V = 4\sin\pi t$ where t is in seconds.

(i) Find the exact rate of change of voltage after $\frac{1}{4}$ seconds have elapsed. [2]

(ii) Find the exact times when the rate of change of voltage is $2\pi\sqrt{3}$ volts per second for $0 < t < 4$. [3]

(iii) Given that current (I in amperes) supplied to the system is governed by the equation $I = \frac{V}{5}$, find the rate of change of current when the rate of change of voltage is 2 volts per second. [2]

10. In the diagram shown below, $ABCD$ is a parallelogram with points $A(2, 8)$, $B(-1, 1)$ and $C(4, -2)$. M is the midpoint of BD and the perpendicular bisector of BD passes through the y-axis at P.

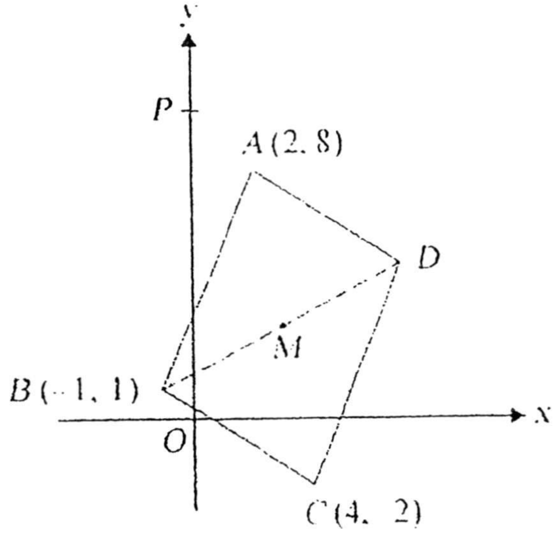
Find

(i) the coordinates of M, [1]

(ii) the coordinates of D, [2]

(iii) the equation of the perpendicular bisector of BD, [2]

(iv) and the area of quadrilateral $ACBP$. [2]



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11. Given that $y = x^3 + ax^2 + bx + 3$ has a stationary point $(1, 0)$,

(i) find the values of a and of b, [3]

(ii) find the coordinates of the other stationary point, [3]

(iii) and determine the nature of these stationary points. [3]

12. (i) Differentiate the following with respect to x.

(a) $\dfrac{(e^x)^4 e^{-x}}{e^{x+1}}$ [2]

(b) $\ln(\cos^2 x)$ [2]

(ii) Hence, or otherwise, find $\int \dfrac{5}{2x-3} - 4e^{2x-1} - 2\tan x \, dx$ [4]

13. The table shows experimental values of two variables, x and y, which are connected by the equation $y = ae^{bx-1}$.

x	1	2	3	4	5
y	1.89	2.30	2.82	3.44	4.20

(a) Plot $\ln y$ against x and draw a straight line graph. [3]

(b) Use your graph to estimate the value of a and of b. [3]

(c) By drawing a suitable line on your graph, solve the equation $2.46 = ae^{bx-1}$. [2]

END OF PAPER

AHS Prelim AM P1

1. (i) $\frac{1-2\ln x}{x^3}$ (ii) $0 < x < e^{\frac{1}{2}}$

2. (ii) B (0,1) (iii) $-\sqrt{3}$

3. $\frac{3}{\sqrt{10}}$

4. (ii) $\cos x - \sqrt{1-\sin^2 x}$. Since $\cos x$ must be positive, x is in the 1st or 4th quadrant. $\therefore$ x must be acute.

5. (i) $-\frac{1}{3}$ (ii) Term with $\frac{1}{x^4} = \left(-\frac{768}{x^6}\right)\left(\frac{x^2}{12}\right) + \left(\frac{512}{x^3}\right)\left(\frac{1}{8x}\right) = 0$
   $\therefore$ it does not exist.

6. $\frac{2}{x} - \frac{1}{x^2} + \frac{7-3x}{2x^2+1}$

7. (i) $\frac{7}{3}$, -2 (ii) $\frac{721}{27}$

8. $x = 2.32$

9. (i) $\frac{dv}{dt} = \frac{4\pi}{\sqrt{2}}$ v/s (ii) t = $\frac{1}{6}, \frac{11}{6}, 2\frac{1}{6}, 3\frac{5}{6}$ sec (iii) $\frac{dI}{dt} = \frac{2}{5}$ Amperes/sec

10. (i) m (3,3) (ii) D (7,5) (iii) y = $-2x + 9$ (iv) $30\frac{1}{2}$ units^2

11. (i) a = 1, b = -5 (ii) $\left(-\frac{5}{3}, \frac{256}{27}\right)$ (iii) max point

12. (i)(a) $2e^{2x-1}$ (b) $-2\tan x$ (ii) $\frac{5}{2}\ln(2x-3) - 2e^{-2x-1} + \ln\cos^2 x + c$

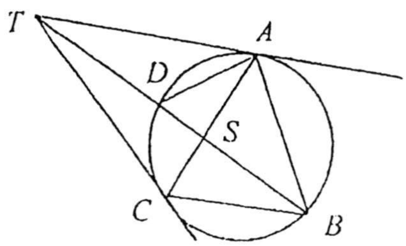
13. (b) a = 4.22, b = 0.2 (c) $x = 2.3$

AHS Prelim. 2015 Am Paper 2

1. The curve $\frac{(x-2)^2}{4}+(y-3)^2=4$ and the line $2y+x=12$ intersect at the points P and Q. Find the exact distance between P and Q. [5]

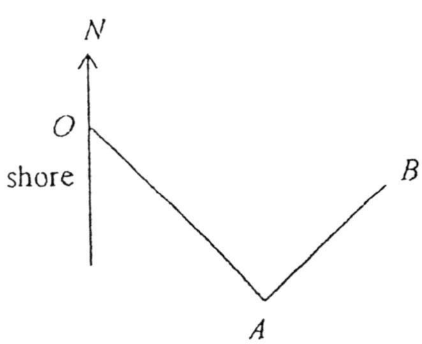
2. Find the values of a and b for which the function $f(x)=2x^4-7x^3+ax^2+bx-21$ is exactly divisible by x^2-2x-3.
Hence determine, showing all necessary working, the number of real roots of the equation $f(x)=0$. [8]

3.



In the diagram, A, B, C and D are points on the circle. $TDSB$ and ASC are straight lines. TA and TC are tangents. Prove that
(a) $\angle ACB=\angle ATD+\angle ABD$, and [3]
(b) $\angle ATC=180°-2\angle ABC$. [3]

4. The diagram shows the route of a fishing boat. The boat leaves the point O from the shore and sails in a straight line for 5 km to a point A, at a bearing of $(090°+\theta)$. At A, the boat makes a right-angled turn and sails for 2 km to the point B to continue fishing. The angle $OAB=90°$ and the shortest distance from B to the shore is L km.



(a) Show that $L=5\cos\theta+2\sin\theta$. [2]
(b) Express L in the form $R\cos(\theta-\alpha)$, where $R>0$ and $0°<\alpha<90°$. [3]
(c) State the maximum value of L and find the corresponding value of θ. [3]

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5. (a) Express $\tan^2\alpha - \cos^2\beta$ in the form $A\sec^2 B\alpha + C\cos 2\beta + D$ where $A, B, C,$ and D are constants. [3]

(b) Hence, or otherwise, evaluate $\int_1^3 \left(\tan^2 3x - \cos^2 \frac{x}{2}\right) dx$. [5]

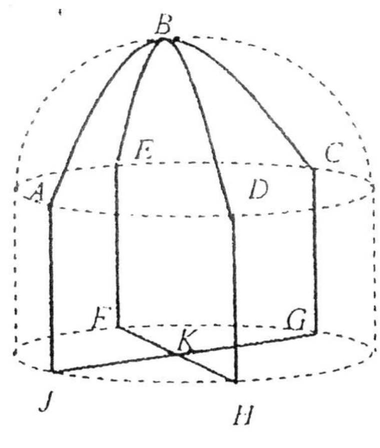
6. The curve $y = P\cos Qx + R$ has a period of $720°$, a maximum value of 8 and a minimum value of -4.

(a) Given that P is a negative constant and Q and R are positive constants, find the value of P, of Q and of R. [4]

(b) Solve the equation $y = 3$ where $0° < x < 360°$. [2]

(c) Sketch the graph of y for $0° \le x \le 360°$. [3]

7. A container in the shape of a cylinder with a hemisphere on top is to be decorated by gold wires.



The wires AC and DE go across the hemisphere and intersect at B, the highest point of the hemisphere. The wires AJ, EF, CG and DH run down the sides of the cylinder. The wires GJ and FH cross at right angles at K where K is the centre of the base. The total length of wire is 30 cm. The height of the cylinder is h cm and the radius of the hemisphere is r cm. The volume of the container is V cm^3.

(a) Express h in terms of r. [2]

(b) Show that $V = \frac{\pi r^2}{6}(45 - 2r - 3\pi r)$. [3]

(c) Find the stationary value of V and determine its nature. [5]

8. (a) Show that the equation $2^{2x} = \frac{1}{2}[3(2^x) + 2]$ is satisfied by only one value of x. [3]

(b) Given that $m = a^s$, $n = a^t$ and $m^t n^s = a^{\frac{2}{u}}$, where $a > 0$ and $a \neq 1$, show that $st = \frac{1}{u}$. [4]

(c) Without using calculators, find the value of k, in the form $\frac{x + y\sqrt{5}}{2}$, such that $k\sqrt{3} - k\sqrt{15} = -2\sqrt{3}$. [4]

(d) Differentiate $e^{-x}\sqrt{1+3x}$ with respect to x. [4]

9. In the Chingay Parade procession held at the heartlands early this year, the Pioneer Generation Float was travelling on a straight road with a velocity, v ms^{-1}, given by the equation $v = 5t - \frac{1}{2}t^2 + 4$, where t is the time after passing a fixed point A.

(a) Show that the maximum velocity is reached 5 s later. [3]

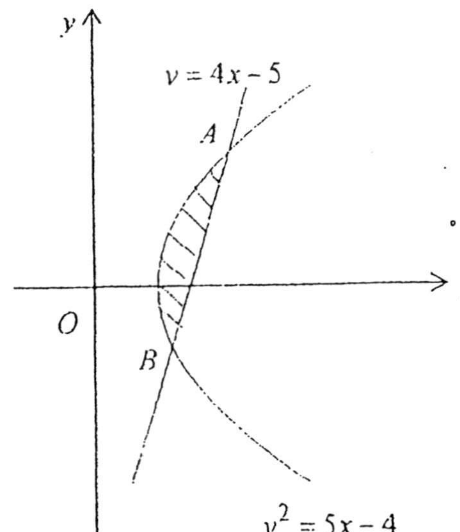
(b) Sketch the velocity-time graph for the first 5 s. [3]

Upon reaching its maximum velocity, the float started to decelerate uniformly at 1.5 ms^{-2}, before coming to a rest at point B to allow residents to take photographs.

(c) Find the time when the float reached B. [2]

(d) Find the total distance travelled from A to B. [3]

10.



The graph above shows part of curve $y^2 = 5x - 4$ and the line $y = 4x - 5$. Find

(a) the coordinates of A and of B, and [3]

(b) the area of the shaded region. [6]

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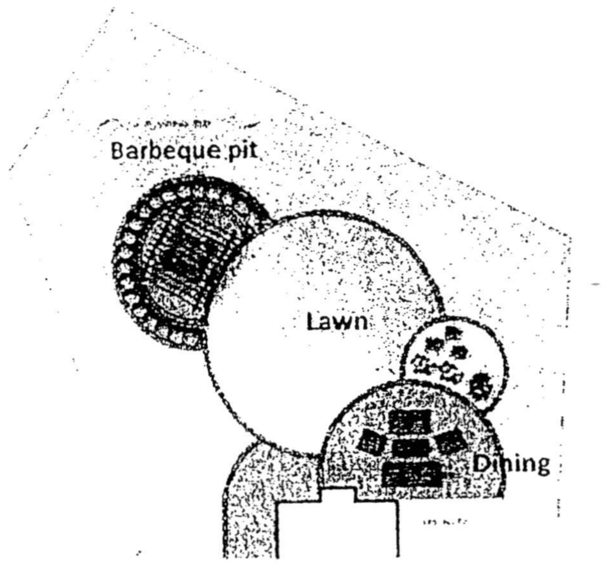
11. A landscaping company has been tasked to design the backyard for a client. The design is made up of overlapping circles as shown below. The circular lawn in the centre will be the focus point of the design and a barbeque pit will be constructed on one side of the lawn.



On the Cartesian plane, the circular lawn can be modelled by the equation of a circle, $x^2 + y^2 + 2x - 6y - 15 = 0$.

(a) Show why this model suggests that the radius of the lawn is 5 m. [2]

(b) A lamp post is positioned at a point $P(-5, 8)$ in the pit area. Determine, with working, if P lies inside or outside the lawn. [3]

(c) Two dustbins, at Q and R, will be placed on the circumference of the lawn such that Q is $(-4, -1)$ and QR is the diameter of the lawn. Find the equation of the tangent to the lawn at R. [6]

END OF PAPER

P2

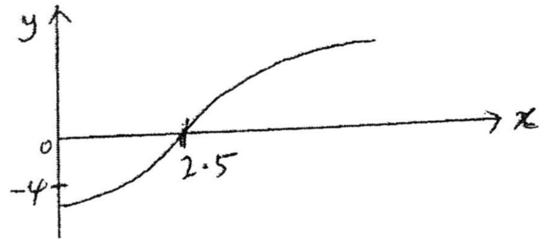
1. P (2 , 5) , Q (6 , 3) , PQ = $2\sqrt{5}$

2. a = 7, b = -5, no real roots

4. (b) $\sqrt{29}\cos(\theta - 21.8°)$ (c) 21.8°

5. (a) $\sec^2\alpha - \frac{1}{2}\cos 2\beta - \frac{3}{2}$
   (b) -2.75

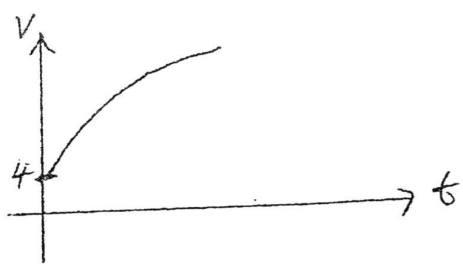
6. (a) P = -6 , Q = $\frac{1}{2}$, R = 2 (b) x = 199.2°
   (c)



7. (a) h = $\frac{15-2r-\pi r}{2}$
   (c) v = 54.2 max. value

8. (c) k = $\frac{1+\sqrt{5}}{2}$ (d) $\frac{1-6x}{2e^x\sqrt{1+3x}}$

9. (b)



   (c) t = 16
   (d) $152\frac{5}{12}$m

10. (a) A (1.8125 , 2.25) , B (1 , -1)
    (b) 1.14 units²

11. (a) $(x+1)^2 + (y-3)^2 = 5^2$
    (b) Distance = 6.40 m > radius, the lamp post lies outside the lawn
    (c) y = $-\frac{3}{4}x + \frac{17}{2}$