Anglo-Chinese School

(Independent)

PRELIMINARY EXAMINATION 2015
YEAR 4 EXPRESS
ADDITIONAL MATHEMATICS **4047/01**
PAPER 1

Thursday **30 July 2015** **2 hours**

Additional Materials:
Answer Paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your candidate number in the spaces provided on the answer paper/answer booklet.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This question paper consists of 6 printed pages

17 **[Turn Over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

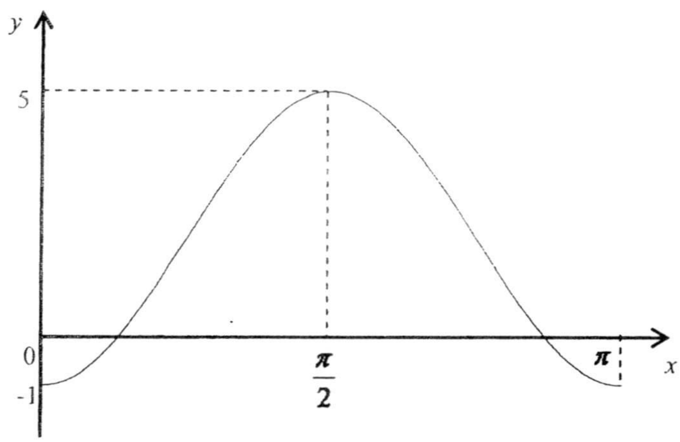
1 The sides AB and BC of a triangle are $(2\sqrt{3}+2\sqrt{6})\,cm$ and $(8\sqrt{2}-2)\,cm$ respectively and $\angle ABC$ is $60°$. Show that the area of ΔABC is $(p+q\sqrt{2})\,cm^2$ where p and q are constants to be determined. [3]

2 Find the range of values of k for which $x^2+2k(k+x)>3k+4$ for all real values of x. [3]

3 Solve the equation $|2x-3|+6x=|9-6x|+4$. [4]

4 The polynomial $f(x)$ is divisible by $(2x-3)$ and leaves a remainder of -2 when divided by $(x-1)$. Find the remainder when $f(x)$ is divided by $2x^2-5x+3$. [4]

5 The diagram below shows the graph of $y=c+a\cos bx$ where a, b and c are constants.



(i) Use the graph to determine the value of a, of b and of c. [3]

(ii) By using the values of a, b and c found in **(i)**, determine the equation of the straight line that needs to be drawn on the same diagram to solve

$$\sec bx=\frac{a\pi}{x-\pi c}.$$ [2]

6 (i) Given that $\dfrac{x^3-2x^2-x+3}{x^2-2x+1} = x+\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$, where A and B are constants.

Find the value of A and of B. [4]

(ii) Hence find $\displaystyle\int \frac{x^3-2x^2-x-4}{x^2-2x+1}\,dx$. [3]

7 The roots of the equation $2x^2-8x+3=0$ are α and β.

(i) Express $\alpha^2-\alpha\beta+\beta^2$ in terms of $\alpha+\beta$ and $\alpha\beta$. [1]

(ii) Find a quadratic equation with integer coefficients whose roots are α^3 and β^3. [6]

8 (a) Find all the values of x between $0°$ and $360°$ for which

$$\frac{1}{\sec^2 x} + 3\sin\frac{x}{2}\cos\frac{x}{2} = 0.$$ [4]

(b) Find all the exact angles between 0 and π, which satisfy the equation

$$\sin(x-\frac{\pi}{5})-\cos\frac{\pi}{10}=0.$$ [4]

9 A curve is such that $\dfrac{d^2y}{dx^2}=16\cos^2 2x-4\sin 4x-8$ and the gradient of the normal to the curve at $x=\dfrac{\pi}{4}$ is 1.

(i) Find $\dfrac{dy}{dx}$. [3]

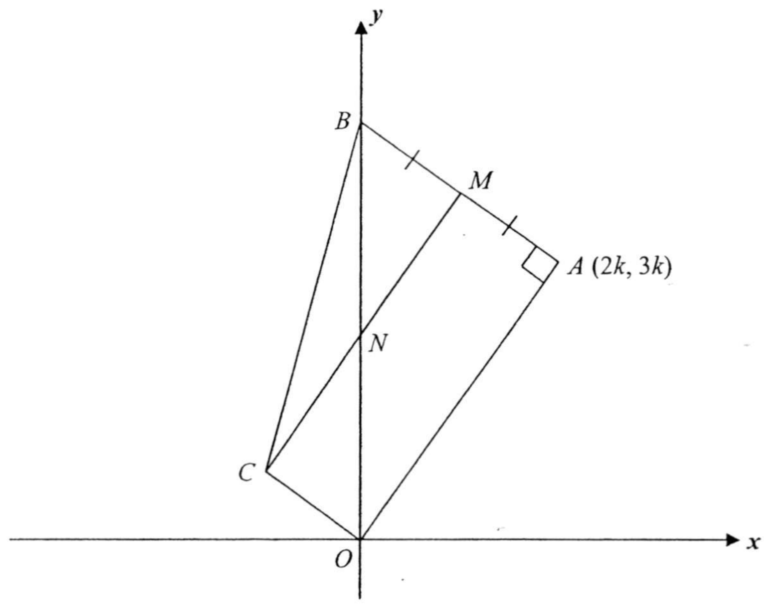
(ii) Hence solve $\dfrac{dy}{dx}=2$ for $0\le x\le 1$. [6]

10 (i) Solve $2+\ln(4-x)=0$. [2]

(ii) Sketch the graph of $y=2+\ln(4-x)$ showing clearly the asymptote and the y-intercept. [3]

(iii) Find the area of the region bounded by the curve $y=2+\ln(4-x)$, the x-axis, the y-axis and the line $x=3$. [5]

11 The diagram shows a quadrilateral $OABC$.



The coordinates of A are $(2k, 3k)$ and the length of OA is $\sqrt{52}$ units.

(i) Calculate the value of k. [2]

AB is perpendicular to OA and B lies on the y-axis.

(ii) Find the coordinates of B. [3]

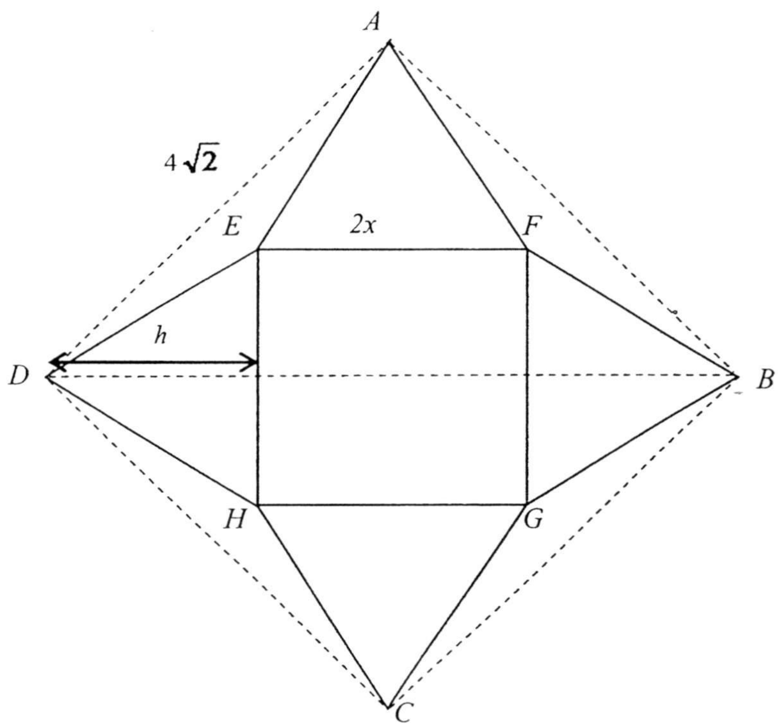
CM, the perpendicular bisector of AB, cuts the y-axis at N and OC is parallel to AB.

Find

(iii) the coordinates of C, [3]

(iv) the ratio of the area of the triangle OCN to the area of the triangle OCB. [2]

12



The diagram above shows a square piece of cardboard paper $ABCD$ of side $4\sqrt{2}$ metres.

Triangles AED, AFB, DHC and BGC are cut off leaving a figure in the shape of a square $EFGH$ of side $2x$ metres with 4 identical isosceles triangles attached to the sides. The height of each triangle is h metres. Mark wants to fold the paper to make a pyramid with $EFGH$ as the base.

(i) Show that $h = 4 - x$. [2]

(ii) Show that the volume of the pyramid , $V\ m^3$, is given by $V = \frac{8}{3}x^2\sqrt{4-2x}$. [4]

(iii) Hence find the maximum volume of the pyramid. [4]

(Proof of maximum is not required)

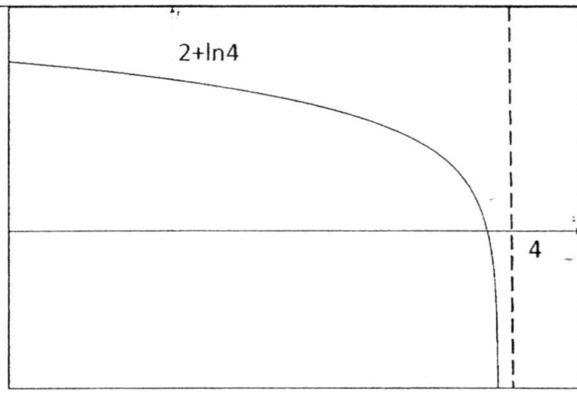
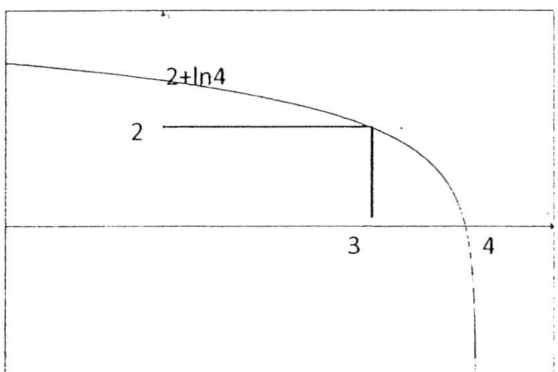
-- **END OF PAPER 1** --

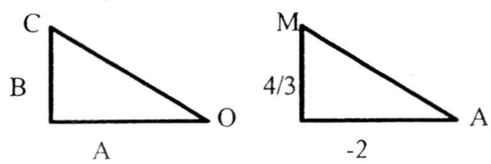
Marking Scheme for Additional Mathematics 2015 Preliminary Examination Paper 1

1		$Area = \frac{1}{2}(2\sqrt{3} + 2\sqrt{6})(8\sqrt{2} - 2)\sin 60$ $= (\sqrt{3} + \sqrt{6})(8\sqrt{2} - 2)\frac{\sqrt{3}}{2}$ $= (6\sqrt{6} - 2\sqrt{3} + 8\sqrt{12})\frac{\sqrt{3}}{2}$ $= 9\sqrt{2} + 14\sqrt{3} \times \frac{\sqrt{3}}{2}$ $= 9\sqrt{2} + 21$
2		$x^2 + 2k(k + x) > 3k + 4$ $D < 0$ $x^2 + 2k^2 + 2kx - 3k - 4 > 0$ $4k^2 - 4(2k^2 - 3k - 4) < 0$ $-4k^2 + 12k + 16 < 0$ $k^2 - 3k - 4 > 0$ $k < -1 \ or \ k > 4$
3		$\|2x - 3\| + 6x = \|9 - 6x\| + 4$ $\|2x - 3\| - \|9 - 6x\| = 4 - 6x$ $\|2x - 3\| - 3\|2x - 3\| = 4 - 6x$ $\|2x - 3\| = 3x - 2$ $2x - 3 = 3x - 2 \ or \ 2x - 3 = 2 - 3x$ $x = -1(na) \ or \ x = 1$
4		$f(x) = (2x^2 - 5x + 3)Q(x) + ax + b$ $= (2x - 3)(x - 1)Q(x) + ax + b$ $\frac{3}{2}a + b = 0$ $3a + 2b = 0 ------(1)$ $a + b = -2 ------(2)$ $Solve: a = 4, \ b = -6$ $The\ remainder\ is\ 4x - 6$
5	(i)	$y = c + a\cos bx$ $a = -3$ Period is π. Therefore $b = 2$ $c = 2$

	(ii)	$\sec bx = \dfrac{a\pi}{x-\pi c}$ $\cos 2x = \dfrac{x-2\pi}{-3\pi}$. $-3\pi\cos 2x = x - 2\pi$ $2\pi - 3\pi\cos 2x = x$ $2 - 3\cos 2x = \dfrac{x}{\pi}$ $Draw\ y = \dfrac{x}{\pi}$
6	(i)	$\dfrac{x^3-2x^2-x+3}{x^2-2x+1} = x + \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$ $By\ long\ div, \dfrac{x^3-2x^2-x+3}{x^2-2x+1} = x + \dfrac{3-2x}{(x-1)^2}$ $\dfrac{3-2x}{(x-1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$ $3-2x = A(x-1)+B$ $Sub\ x=1: B=1$ $Compare\ x: A=-2$
	(ii)	$\int \dfrac{x^3-2x^2-x-4}{x^2-2x+1}dx = \int (x - \dfrac{2}{x-1} + \dfrac{1}{(x-1)^2} - \dfrac{7}{(x-1)^2})dx$ $= \dfrac{x^2}{2} - 2\ln(x-1) + \dfrac{6}{x-1} + c$
7	(i)	$2x^2-8x+3=0$ $\alpha^2 - \alpha\beta + \beta^2 = (\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta)$ $\alpha+\beta = 4$ $\alpha\beta = \dfrac{3}{2}$ $\alpha^3+\beta^3 = (4)(16-\dfrac{9}{2})$ $= 46$
	(ii)	$\alpha^3\beta^3 = \dfrac{27}{8}$ $x^2 - 46x + \dfrac{27}{8} = 0$ $8x^2 - 368x + 27 = 0$

8	(a)	$\frac{1}{\sec^2 x} + 3\sin\frac{x}{2}\cos\frac{x}{2} = 0$ $\cos^2 x + \frac{3}{2}\sin x = 0$ $2\sin^2 x - 3\sin x - 2 = 0$ $\sin x = -\frac{1}{2}$ $x = 210^\circ, 330^\circ$
	(b)	$\sin(x - \frac{\pi}{5}) = \cos\frac{\pi}{10}$ $= \sin(\frac{\pi}{2} - \frac{\pi}{10})$ $= \sin\frac{2\pi}{5}$ $x - \frac{\pi}{5} = \frac{2\pi}{5}, \frac{3\pi}{5}$ $x = \frac{3\pi}{5}, \frac{4\pi}{5}$
9	(i)	$\frac{d^2y}{dx^2} = 16\cos^2 2x - 4\sin 4x - 8$ $\frac{dy}{dx} = \int(16\cos^2 2x - 4\sin 4x - 8)\,dx$ $= \int(8\cos 4x - 4\sin 4x)\,dx$ $= 2\sin 4x + \cos 4x + c$ $grad\ of\ \tan gent = -1$ $-1 = 2\sin\pi + \cos\pi + c$ $c = 0$ $\frac{dy}{dx} = 2\sin 4x + \cos 4x$
	(ii)	$2\sin 4x + \cos 4x = R\sin(4x + \alpha)$ $R = \sqrt{5}\quad and\ \alpha = 0.4636$ $\sqrt{5}\sin(4x + 0.4636) = 2$ $\sin(4x + 0.4636) = 0.8944$ $4x + 0.4636 = 1.1071,\ 2.0345,$ $x = 0.161,\ 0.393$

10	(i)	$2 + \ln(4 - x) = 0$ $\ln(4 - x) = -2$ $4 - x = e^{-2}$ $x = 3.86$
	(ii)	
	(iii)	$y = 2 + \ln(4 - x)$ $4 - x = e^{y-2}$ $x = 4 - e^{y-2}$ $Area = 2 \times 3 + -\int_2^{2+\ln 4} (4 - e^{y-2})\ dy$ $= 6 + \left[4y - e^{y-2}\right]$ $= 6 + \left[(4(2 + \ln 4) - e^{\ln 4}) - (8 - 1)\right]$ $= 6 + [8 + 4\ln 4 - 4 - 7]$ $= 8.55 units^2$ 
11	(i)	$\sqrt{4k^2 + 9k^2} = \sqrt{52}$ $13k^2 = 52$ $k = 2$

	(ii)	Grad of OA = $\frac{3}{2}$ Grad of AB = -$\frac{2}{3}$ $y=-\frac{2}{3}x+c$ $6=-\frac{8}{3}+c$ $c=\frac{26}{3}$ $B(0,\frac{26}{3})$
	(iii)	M= $(2, \frac{22}{3})$  a = 2-4 = -2 b = 4/3 C $(-2, \frac{4}{3})$
	(iv)	M is midpt of AB, MN// AO By midpt thm, N is midpt of OB Therefore $\frac{\Delta OCN}{\Delta OCB}=\frac{1}{2}$
12	(i)	$(h+x)^2+(h+x)^2=(4\sqrt{2})^2$ $2(h+x)^2=32$ $(h+x)=4$ $h=4-x$
	(ii)	Let ht of pyramid be y

	(iii)	$y^2 + x^2 = (4-x)^2$ $y^2 = 16 - 8x$ $y = \sqrt{16-8x}$ $V = \frac{1}{3}4x^2\sqrt{16-8x}$ $= \frac{4}{x}x^2 2\sqrt{4-2x}$ $= \frac{8}{3}x^2\sqrt{4-2x}$ $\frac{dV}{dx} = \frac{8}{3}[x^2(\frac{1}{2})(-2)(4-2x)^{-\frac{1}{2}} + 2x\sqrt{4-2x}]$ $= \frac{8}{3}\frac{x[-x+2(4-2x)]}{\sqrt{4-2x}}$ $-x + 8 - 4x = 0$ $x = 1.6m$ $Max\ V = 6.11$

Anglo-Chinese School
(Independent)

THE BEST IS YET TO BE

PRELIMINARY EXAMINATION 2015
YEAR 4 EXPRESS
ADDITIONAL MATHEMATICS **4047/02**
PAPER 2

Tuesday **4 August 2015** **2 hours 30 minutes**

Additional Materials:
Answer Paper (10 sheets)

READ THESE INSTRUCTIONS FIRST

Write your candidate number in the spaces provided on the answer paper/answer booklet.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This question paper consists of 6 printed pages

23

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 Given that $\int_0^5 f(x)\,dx = 12$ and $\int_2^5 f(x)\,dx = 4$, evaluate

$\int_2^1 [2x - f(x)]\,dx + \int_0^1 f(x)\,dx$. [3]

2 The equations of two curves are $y = \frac{1}{4}x^{\frac{2}{3}}$ for $x > 0$ and $y = 4x^{-\frac{2}{3}}$ for $x > 0$.

(i) Find the coordinates of the point(s) of intersection of the graphs. [2]

(ii) Sketch these graphs on the same axes, indicating the point(s) of intersection clearly. [2]

3 Variables x and y are related by the equation $y = \frac{p - x}{x + q}$, where p and q are constants.

When the graph of $x(1 + y)$ against y is drawn, a straight line is obtained. The line has a gradient of $-1\frac{1}{3}$ and passes through the point (3, 2).

(i) Calculate the value of p and of q. [4]

(ii) Given that this line passes through (6, k), find x in terms of k. [2]

4 The equation of a curve is given by $y = \frac{\ln(x-3)^2}{2x-6}$, $x > 3$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Find the set of values of x for which y is a decreasing function. [2]

(iii) Evaluate $\int_4^5 \frac{\ln\sqrt{x-3}}{(x-3)^2}\,dx$, leaving your answer in the form $a + b\ln 2$, where a and b are constants. [3]

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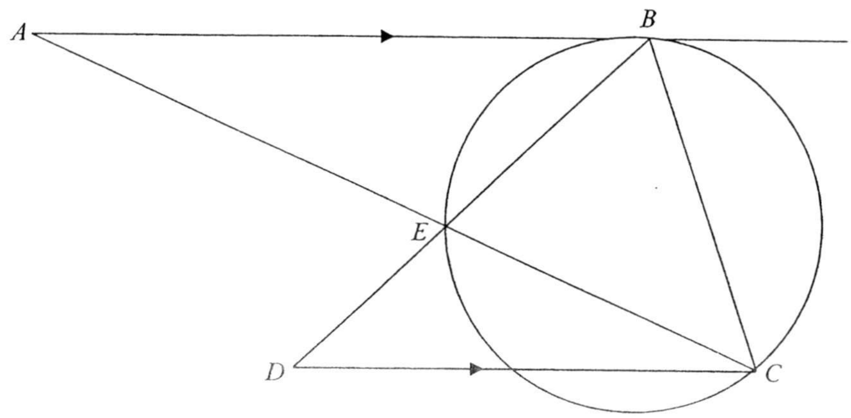
5 In the diagram, AB is a tangent to the circle at the point B, BED is a straight and $AB \parallel DC$. The points A, E and C lie on a straight line and $AE : EC = 2 : 1$.

(i) Prove that $\angle BCE = \angle BDC$. [2]

(ii) Hence, show that ΔBCE is similar to ΔBDC. [2]

(iii) Prove that $3AE \times CD = AB \times AC$. [3]



6 The equation of a circle, C_1, is $x^2 + y^2 + kx - (k+2)y + 7 = 0$, where k is a constant.

(i) Find the coordinates of the centre in terms of k. [2]

Given that the centre of the circle lies on the line $2x + 5y - 11 = 0$,

(ii) show that $k = 4$. [2]

(iii) Find the equation of the circle, C_2, which is a reflection of C_1 in the line $x = 1$. [3]

(iv) Explain why the two circles do not intersect each other. [1]

7 The height of the tides at a certain place can be modelled by the equation $h = 2(3.25 - \sin kt)$, where k is a constant, and t is the time in hours after midnight. The average time difference between high tides is 14.5 hours.

(i) Explain why this model suggests that the lowest tide for the day is 4.5 m. [1]

(ii) Show that the value of k is $\frac{4\pi}{29}$. [2]

(iii) Find the height of the tide at 2 am. [1]

(iv) Find the time for which the height of the tide first reaches 7.0 m, leaving your answer in 24 hour notation. [4]

8 **(a)** Given that $\frac{d}{dx}[F(x)] = \frac{9}{2}\sqrt{3x-1} - \frac{3}{\sqrt{3x-1}}$, evaluate $F(3) - F(1)$, giving your answer in the form $k\sqrt{2}$. [4]

(b) The equation of a curve is given by $y = \operatorname{cosec}^2\left(\frac{x}{2} - \frac{\pi}{6}\right)$, where $0 < x < \frac{\pi}{2}$.

Given that x is increasing at 0.3 radian per second, find the rate of change of y with respect to time when $x = \frac{5\pi}{6}$. [4]

9 In the expansion of $\left(x^3 - \frac{k}{x}\right)^9$, where k is a constant, the coefficient of $\frac{1}{x}$ is -4608.

(i) Show that $k = 2$. [3]

(ii) Explain why there is no term independent of x in the expansion of $\left(x^3 - \frac{k}{x}\right)^9$. [1]

(iii) Find the coefficient of x^2 in the expansion of $\left(2x^3 + \frac{1}{x}\right)\left(x^3 - \frac{k}{x}\right)^9$. [4]

10 The gradient of a curve is $\frac{e^{2x}+1}{e^{2x}}$ and $P(0, -1)$ is a point on the curve.

(i) Show that the curve has no stationary point. [2]

(ii) Find the equation of the curve. [2]

The tangent and normal to the curve at P intersect the x-axis at Q and R respectively.

(iii) Find the area of the triangle PQR. [5]

11 **(a)** Given that $2^{2x-3} = \frac{1}{4^{x-1}}$, evaluate 16^{x}. [3]

(b) Solve the equation $\log_x 5 - \frac{2}{\log_{\sqrt{x}} 2} = \log_{x^2}\left(\frac{25}{4}\right)$. [6]

12 A particle P travelling in a straight line passes a fixed point O. Its velocity, v ms^{-1}, is given by the equation $v = t^2 - 6t + 8$, where t is the time in seconds after passing O.

(i) Find the times when P is instantaneously at rest. [2]

(ii) Find the total distance travelled by P when its velocity reaches 8 ms^{-1} again. [5]

(iii) Will P return to O in the course of its motion? Explain your answer clearly. [2]

The particle P is at point A when its velocity reaches 8 ms^{-1} again. It continues its motion at this velocity for 1 second and then decelerates uniformly until it comes to a complete rest at point B in another 2 seconds.

(iv) Find the distance AB. [2]

13 **(i)** Prove that $\tan\frac{\theta}{2} + \cot\frac{\theta}{2} = 2\operatorname{cosec}\theta$. [3]

(ii) Hence, solve $\tan\theta + \cot\theta = (\operatorname{cosec}4\theta)(\sin 2\theta + \cos 2\theta)$ for $0^{o} \leq \theta \leq 180^{o}$. [5]

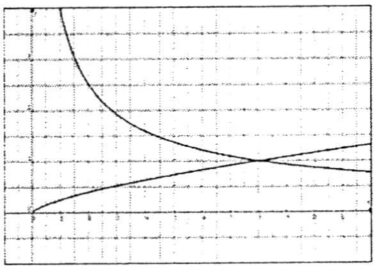
Given that $2\tan A + 2\cot A = 5$ and $0 < A < \frac{\pi}{4}$,

(iii) show that $\cos 2A = \frac{3}{5}$. [2]

(iv) Hence, find the **exact** value of $\cos(2A + \frac{\pi}{6})$. [2]

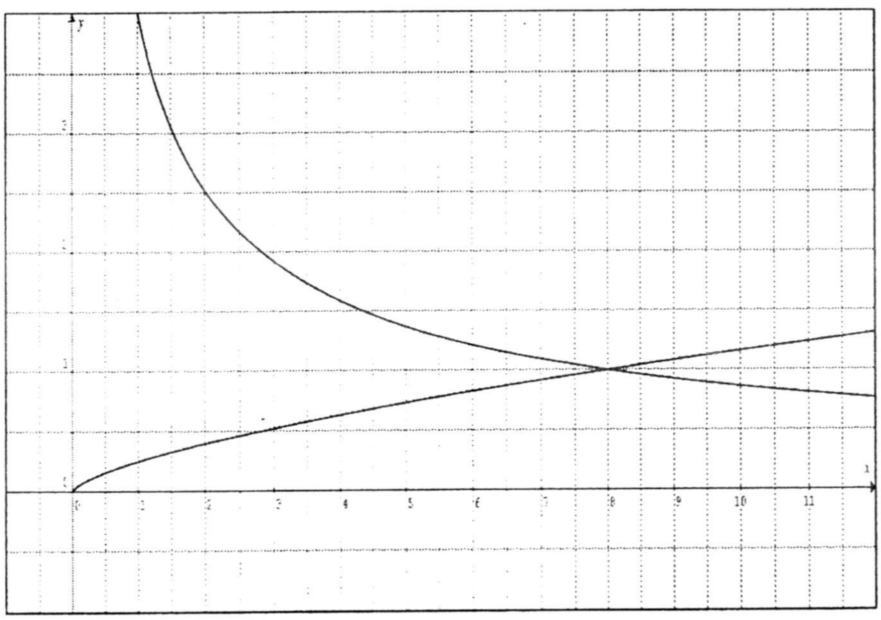
END OF PAPER 2

Answers

1	5
2(i)	(8, 1)
2(ii)	
3(i)	$p = 6, \quad q = 1\frac{1}{3}$
3(ii)	$x = \frac{k}{7}$
4(i)	$\frac{dy}{dx} = \frac{1-\ln(x-3)}{(x-3)^2}$
4(ii)	$x > e + 3 = 5.72$
4(iii)	$\frac{1}{2}\left(\frac{1}{2} - \frac{\ln 2}{2}\right) = \frac{1-\ln 2}{4}$
5	Proof
6(i)	$(-\frac{k}{2}, \frac{k+2}{2})$
6(ii)	Proof
6(iii)	$(x-4)^2 + (y-3)^2 = 6$
6(iv)	$Let\ d = distace\ from\ P_1\ to\ P_2 = 6$ $Let\ R = radius\ of\ C_1 + radius\ of\ C_2 = \sqrt{6} + \sqrt{6} = 4.89$ $Since\ R < d,\ the\ two\ circles\ do\ not\ intersect\ each\ other.$
7(i)	Lowest tide occurs when $\sin kt = 1$, lowest tide = 4.5 m
7(ii)	Proof
7(iii)	$4.98m$
7(iv)	0750
8(a)	$12\sqrt{2}$

8(b)	$-0.6\ radian/\sec$
9(i)	Proof
9(ii)	$27-4r \neq 0$ *as r must be an integer* $\Rightarrow$ *no independent term.*
9(iii)	-3840
10(i)	Proof
10(ii)	$y = x - \frac{1}{2}e^{-2x} - \frac{1}{2}$
10(iii)	$1\frac{1}{4}\ units^2$
11(a)	32
11(b)	$x = 2,\ \frac{1}{2}$
12(i)	$t = 2$ *or* $t = 4$
12(ii)	$14\frac{2}{3}\ m$
12(iii)	Proof
12(iv)	$16m$
13(i)	Proof
13(ii)	$\theta = 35.8°,\ 125.8°$
13(iii)	Proof
13(iv)	$\frac{3\sqrt{3}-4}{10}$

Marking Scheme (Additional Mathematics Paper 2/ Prelim Examination 2015)

Qn No.	Solution
1	$\int_2^1 [2x - f(x)]dx + \int_0^1 f(x)\,dx$ $= \left[x^2\right]_1^2 + \int_0^2 f(x)\,dx$ $= -3 + \int_0^5 f(x)]dx - \int_2^5 f(x)\,dx$ $= -3 + 12 - 4 = 5$
2(i)	$\frac{1}{4}x^{\frac{2}{3}} = 4x^{-\frac{2}{3}}$ $x^{\frac{4}{3}} = 16$ $x = 8$ $y = \frac{1}{4}(8)^{\frac{2}{3}} = 1$ $Coord\ (8, 1)$
2(ii)	
3(i)	$y = \frac{p - x}{x + q}$ $x(1 + y) = -qy + p$ -------- (1) $-q = -1\frac{1}{3} \Rightarrow q = 1\frac{1}{3}$ $2 = -\frac{4}{3}(3) + p$ $\Rightarrow p = 6$

3(ii)	$x(1+y)=k$ $7x=k$ $x=\dfrac{k}{7}$
4(i)	$y=\dfrac{\ln(x-3)^2}{2x-6}=\dfrac{\ln(x-3)}{x-3}$ $\dfrac{dy}{dx}=\dfrac{(x-3)\left(\dfrac{1}{x-3}\right)-\ln(x-3)}{(x-3)^2}$ $=\dfrac{1-\ln(x-3)}{(x-3)^2}$
4(ii)	$For\ \dfrac{dy}{dx}<0,\quad 1-\ln(x-3)<0$ $\Rightarrow \ln(x-3)>1$ $\Rightarrow x>e+3=5.72$
4(iii)	$\int_4^5 \dfrac{1-\ln(x-3)}{(x-3)^2}dx=\left[\dfrac{\ln(x-3)}{x-3}\right]_4^5$ $\int_4^5 \dfrac{1}{(x-3)^2}dx-\int_4^5 \dfrac{\ln(x-3)}{(x-3)^2}dx=\left[\dfrac{\ln(x-3)}{x-3}\right]_4^5$ $\int_4^5 \dfrac{\ln(x-3)}{(x-3)^2}dx=\int_4^5 \dfrac{1}{(x-3)^2}dx-\left[\dfrac{\ln(x-3)}{x-3}\right]_4^5$ $=\left[-\dfrac{1}{x-3}\right]_4^5-\left[\dfrac{\ln(x-3)}{x-3}\right]_4^5=\dfrac{1}{2}-\dfrac{\ln 2}{2}$ $\therefore\ \int_4^5 \dfrac{\ln\sqrt{(x-3)}}{(x-3)^2}dx=\dfrac{1}{2}\int_4^5 \dfrac{\ln(x-3)}{(x-3)^2}=\dfrac{1}{2}\left(\dfrac{1}{2}-\dfrac{\ln 2}{2}\right)=\dfrac{1-\ln 2}{4}$
5(i)	$\angle BDC=\angle ABD$ (*Alternate angles, AB // DC*) $\angle ABD=\angle BCE$ (*Alternate Segment Theorem*) $\therefore\ \angle BCE=\angle BDC$
5(ii)	*In* ΔBCE *and* ΔBDC, $\angle BCE=\angle BDC$ (*From (i)*) $\angle CBE=\angle DBC$ (*Common angles*) $\therefore\ \Delta BCE$ *and* ΔBDC *are similar triangles* (*AAA property*)

5(iii)	ΔAEB and ΔCED are similar triangles (AAA property) $\frac{AE}{CE} = \frac{AB}{CD}$ $\frac{AE}{\frac{1}{3}AC} = \frac{AB}{CD}$ (Given AE:EC = 2:1) $3AE \times CD = AB \times AC$
6(i)	$a = -\frac{k}{2}$ $b = \frac{k+2}{2}$ $Centre = (-\frac{k}{2}, \frac{k+2}{2})$
6(ii)	$2\left(-\frac{k}{2}\right) + 5\frac{(k+2)}{2} - 11 = 0$ $k = 4$
6(iii)	$C_1 = (-2, 3)$, $r = \sqrt{(-2)^2 + 3^2 - 7} = \sqrt{6}$ $Let\ P_2 = Centre\ of\ C_2$ $C_2 = (4, 3)$ $Equation\ of\ C_2$: $(x-4)^2 + (y-3)^2 = 6$
6(iv)	$Let\ d = distace\ from\ P_1\ to\ P_2 = 6$ $Let\ R = radius\ of\ C_1 + radius\ of\ C_2 = \sqrt{6} + \sqrt{6} = 4.89$ $Since\ R < d$, the two circles do not intersect each other.
7(i)	Lowest tide occurs when sin kt = 1, lowest tide = 4.5 m
7(ii)	Period between high tides = 14.5 hours $\frac{2\pi}{k} = 14.5$ $\frac{2\pi}{14.5} = k$ $k = \frac{4\pi}{29}$
7(iii)	$h = 2(3.25 - \sin\frac{8\pi}{29})$ $h = 4.98m$
7(iv)	When h = 7.0 $7.0 = 2(3.25 - \sin\frac{4\pi}{29}t)$ $\sin\frac{4\pi}{29}t = -0.25$ $\frac{4\pi}{29}t = 3.394$ $\Rightarrow\ t = 7.833 = 7h\ 50$ min The time is 0750

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8(a)

$$F(3)-F(1)=\int_1^3 \left(\frac{9}{2}\sqrt{3x-1}-\frac{3}{\sqrt{3x-1}}\right)dx$$

$$=\left[\frac{\frac{9}{2}(3x-1)^{\frac{3}{2}}}{\frac{9}{2}}-2(3x-1)^{\frac{1}{2}}\right]_1^3$$

$$=\left[(8)^{\frac{3}{2}}-2(8)^{\frac{1}{2}}\right]-\left[(2)^{\frac{3}{2}}-2(2)^{\frac{1}{2}}\right]$$

$$=(8\sqrt{8}-2\sqrt{8})-(2\sqrt{2}-2\sqrt{2})=6\sqrt{8}$$

$$=12\sqrt{2}$$

8(b)

$$y=\cos ec^2\left(\frac{x}{2}-\frac{\pi}{6}\right)=\sin^{-2}\left(\frac{x}{2}-\frac{\pi}{6}\right)$$

$$\frac{dy}{dx}=-2\left[\sin^{-3}\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]\left[\frac{1}{2}\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]$$

$$=-\left[\sin^{-3}\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]\left[\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)\right]$$

When $x=\frac{5\pi}{6}$,

$$\frac{dy}{dx}=-\left[\sin^{-3}\left(\frac{\pi}{4}\right)\right]\left[\cos\left(\frac{\pi}{4}\right)\right]=-2$$

$$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=-2\times 0.3$$

$$=-0.6\ radian/\sec$$

9(i)

$$T_{r+1}=\binom{9}{r}(x^3)^{9-r}\left(-\frac{k}{x}\right)^r=(-k)^r\binom{9}{r}(x^{27-4r})$$

For the term in $\frac{1}{x}$, $27-4r=-1$

$$\Rightarrow r=7$$

$$-\binom{9}{7}(k^7)=-4608$$

$$\Rightarrow\ k=2$$

9(ii)

$27-4r\neq 0$ *as* r *must be an integer* $\Rightarrow$ *no independent term.*

9(iii)	*For the term in* x^3, $r = 6$ *The term in* $x^3 = (-2)^6\binom{9}{6}(x^3) = 5376x^3$ $(2x^3+\frac{1}{x})(x^3-\frac{k}{x})^9 = (2x^3+\frac{1}{x})(\ldots\ldots - 4608\left(\frac{1}{x}\right) + 5376x^3 + \ldots\ldots)$ $= -9216x^2 + 5376x^2 + \ldots\ldots = -13840x^2 + \ldots\ldots$ $\therefore$ *The coefficient of* x^2 *is* -3840
10(i)	$\frac{dy}{dx} = \frac{e^{2x}+1}{e^{2x}} = 1 + e^{-2x}$; $e^{-2x} > 0$ *for all values of* x $\Rightarrow \frac{dy}{dx} \neq 0.$ $\therefore$ *No stationary pt*
10(ii)	$y = \int 1 + e^{-2x}\,dx$ $y = x - \frac{e^{-2x}}{2} + c$ *when* $x = 0, y = -1$ $-1 = -\frac{1}{2} + c$ $c = -\frac{1}{2}$ $y = x - \frac{1}{2}e^{-2x} - \frac{1}{2}$
10(iii)	*At P*, $m_{\tan gent} = 2$ *Equation of* tan *gent*: $y = 2x - 1$ $m_{normal} = -\frac{1}{2}$ *Equation of normal*: $y = -\frac{1}{2}x - 1$ $\Rightarrow Q = \left(\frac{1}{2}, 0\right)$ $R = (-2, 0)$ $\therefore$ *Area of* $\Delta PQR = \left(\frac{1}{2}\right)\left(\frac{5}{2}\right)(1) = 1\frac{1}{4}$ *units*^{2}

11(a)	$2^{2x-3} = \frac{1}{4^{x-1}}$ $\frac{4^x}{8} = \frac{4}{4^x}$ $(4^x)^2 = 32$ $16^x = 32$
11(b)	$\log_x 5 - \frac{2}{\log_{\sqrt{x}} 2} = \log_{x^2}\left(\frac{25}{4}\right)$ $\log_x 5 - \frac{2}{\frac{\log_x 2}{\log_x \sqrt{x}}} = \frac{\log_x\left(\frac{25}{4}\right)}{\log_x x^2}$ $\log_x 5 - \frac{1}{\log_x 2} = \frac{\log_x\left(\frac{25}{4}\right)}{2}$ $2\log_x 5 - \frac{2}{\log_x 2} = \log_x\left(\frac{25}{4}\right)$ $\log_x 25 - \frac{2}{\log_x 2} = \log_x 25 - \log_x 4$ $\log_x 4 = \frac{2}{\log_x 2}$ $2\log_x 2 = \frac{2}{\log_x 2}$ $(\log_x 2)^2 = 1$ $\log_x 2 = \pm 1$ *When* $\log_x 2 = 1, \quad x = 2$ *When* $\log_x 2 = -1, \quad x = \frac{1}{2}$
12(i)	$t^2 - 6t + 8 = 0$ $t = 2 \quad or \quad t = 4$

12(ii)	$v = t^2 - 6t + 8$ $s = \frac{t^3}{3} - 3t^2 + 8t + c$ $At\ t = 0,\ s = 0 \ \Rightarrow\ c = 0$ $\therefore\ s = \frac{t^3}{3} - 3t^2 + 8t$ $When\ v = 8,\ \ t^2 - 6t = 0$ $t = 0\ \ or\ \ t = 6$ $\therefore\ When\ the\ velocity\ is\ 8\ m/s\ again,\ t = 6$ $S_6 = \frac{216}{3} - 3(36) + 8(6) = 12\ m$ $S_0 = 0$ $S_2 = \frac{8}{3} - 3(4) + 8(2) = 6\frac{2}{3}$ $S_4 = \frac{64}{3} - 3(16) + 8(4) = 5\frac{1}{3}$ $From\ t = 0\ to\ t = 4,\ distance\ travelled = 6\frac{2}{3} + (6\frac{2}{3} - 5\frac{1}{3}) = 8\ m$ $From\ t = 4\ to\ t = 6,\ distance\ travelled = 12 - 5\frac{1}{3} = 6\frac{2}{3} m$ $From\ t = 0\ to\ t = 6,\ distance\ travelled = 14\frac{2}{3}$
12(iii)	$At\ O,\ s = 0$ $s = \frac{t^3}{3} - 3t^2 + 8t = 0$ $\frac{1}{3}t(t^2 - 9t + 24) = 0$ $\Rightarrow t = 0 \quad or \quad t^2 - 9t + 24 = 0$ $t^2 - 3t + 8 = 0 \ \Rightarrow\ t = \frac{9 \pm \sqrt{-15}}{2} \ \Rightarrow\ No\ solution$ $\Rightarrow$ *The particle is at O when* $t = 0$ *only. Therefore P will not return to O in the course of its motion.*
12(iv)	$From\ t = 6\ to\ t = 7,\ distance\ travelled = 8\ m$ $From\ t = 7\ to\ t = 9,\ distance\ travelled = 8\ m$ $\therefore\ Total\ distance\ travelled = 8 + 8 = 16m$

13(i)	$\tan\frac{\theta}{2} + \cot\frac{\theta}{2} = \tan\frac{\theta}{2} + \frac{1}{\tan\frac{\theta}{2}} = \frac{\tan^2\frac{\theta}{2} + 1}{\tan\frac{\theta}{2}}$ $= \frac{\sec^2\frac{\theta}{2}}{\tan\frac{\theta}{2}}$ $= \frac{1}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \frac{1}{\frac{1}{2}\sin\theta} = 2\cos ec\theta$
13(ii)	$\tan\theta + \cot\theta = (\cos ec 4\theta)(\sin 2\theta + \cos 2\theta)$ $2\cos ec2\theta = (\cos ec 4\theta)(\sin 2\theta + \cos 2\theta)$ $= \frac{1}{2\cos 2\theta} + \frac{1}{2\sin 2\theta} = \frac{1}{2\cos 2\theta} + \frac{1}{2}\cos ec2\theta$ $3\cos ec2\theta = \frac{1}{\cos 2\theta}$ $\tan 2\theta = 3$ $\theta = 35.8^o, \quad 125.8^o$
13(iii)	$2\tan A + 2\cot A = 5$ $\tan A + \cot A = \frac{5}{2}$ $\cos ec2A = \frac{5}{4}$ $\sin 2A = \frac{4}{5}$ $\cos 2A = \frac{3}{5}$
13(iv)	$\cos(2A + \frac{\pi}{6}) = \cos 2A \cos\frac{\pi}{6} - \sin 2A \sin\frac{\pi}{6}$ $= (\frac{3}{5})(\frac{\sqrt{3}}{2}) - (\frac{4}{5})(\frac{1}{2})$ $= \frac{3\sqrt{3} - 4}{10}$