Answer all questions.

1 Express $\dfrac{8x^4+1}{x(2x-1)}$ in partial fractions. [5]

2 (i) Prove that $\operatorname{cosec}(60^\circ-\theta)=\dfrac{2}{\cos\theta\,(\sqrt{3}-\tan\theta)}$. [3]

(ii) Hence find, in surd form, the value of $\operatorname{cosec}15^\circ$. [4]

3 (a) Find the term independent of x in the expansion of $(2x^2-\dfrac{1}{\sqrt{x}})^{10}$. [3]

(b) It is given that in the expansion of $(5+px)^n$, the coefficients of x^3 and x^4 are the same. Express n in terms of p. [4]

4 (a) The equation of a curve is $y=(a+2)x^2-3x+(a-1)$.
In the case where $a=3$, show that $y=7x-3$ is a tangent to the curve. [3]

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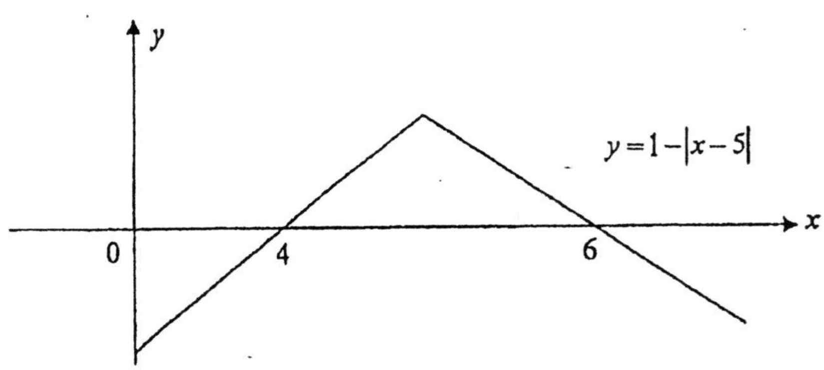
(b) Given that $(m-4)x^2<3x-m$, show that m cannot be positive. [4]

5 (a) Sketch $y=|x(1-4x)|$, indicating the coordinates of the maximum point and intercepts.
Hence, state the range of values of k such that $\left|\dfrac{x}{2}-2x^2\right|=k$ gives more than 2 solutions. [4]

(b) The diagram shows the graph of $y=1-|x-5|$ where the x-intercepts are 4 and 6.



If a line $y=mx+c$ is to be added to the diagram above, determine a possible value for m and c if

(i) there is 1 intersection between the 2 graphs, [1]

(ii) there are infinite intersections between the 2 graphs. [2]

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6 (a) A curve has the equation $y = e\sqrt{x}\ln(3x)$, where $x > 0$. Find an expression for $\frac{dy}{dx}$.

Hence find $\int_1^5 \frac{\ln(3x)+2}{2\sqrt{x}}\,dx$. [4]

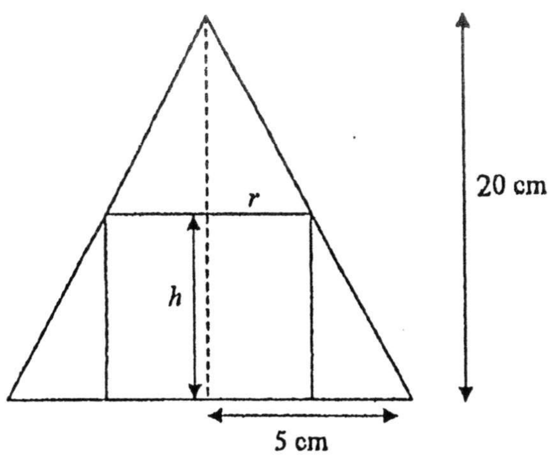
(b) It is given that $y = 6e^{\sqrt{x-1}}$. Find, in terms of e, the rate of change of x at the instant when $x = 5$ if y is decreasing at the rate of $\frac{1}{2}e$ units per second at this instant. [4]

7 It is known that x and y are related by the equation $my = n(2^{mx})$, where m and n are constants.

x	1	2	3	4	5
y	0.566	0.80	1.13	1.60	2.26

(i) Plot $\lg y$ against x and draw a straight line graph. [2]
(ii) Use your graph to estimate the values of m and n. [4]
(iii) By drawing a suitable straight line, solve the equation $y = 0.9^x$. [2]

8 A cylinder of radius r cm and height h is inscribed in a cone of base radius 5 cm and height 20 cm. The cross section of the solid is shown in the diagram.



(i) Show that the volume within the cone but not occupied by the cylinder, V, is given by
$V = \frac{500}{3}\pi - (5 - \frac{h}{4})^2 \pi h$. [3]
(ii) Find the stationary value of V and determine whether it is maximum or minimum. [6]

9 (a) The equation of a curve is $y=\dfrac{5}{kx^2}+10x^3$. Given that its normal at $x=2$ will never meet the y-axis, find the value of k. [3]

(b) A curve has the equation $y=2\cos^2 3x$ for $0\le x\le \pi$. Find

(i) the equation of the normal at $x=\dfrac{\pi}{12}$, [4]

(ii) the x-values on the curve whose tangents are parallel to the normal at $x=\dfrac{\pi}{12}$. [4]

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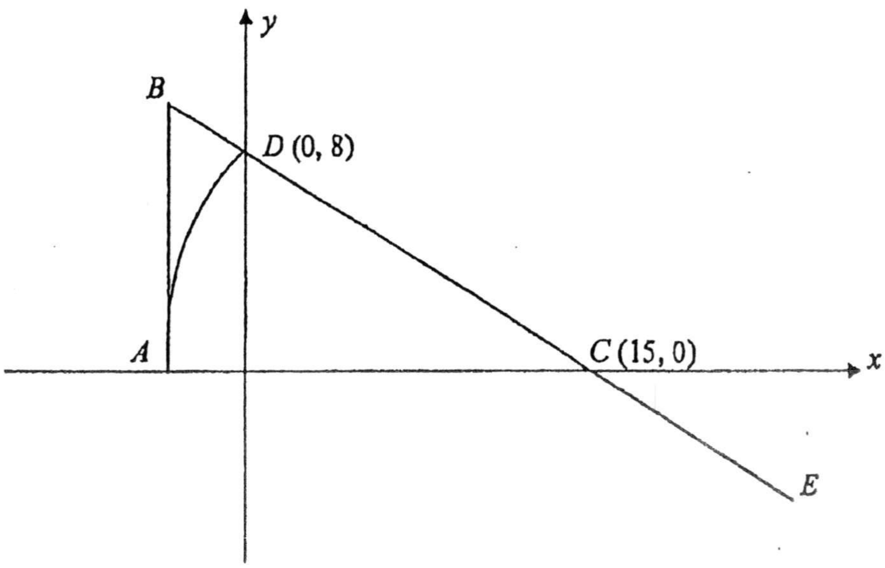

In the diagram, ADC is a sector of the circle with centre C and $BDCE$ is a straight line. The line AB is parallel to the y-axis and points C and D are (15, 0) and (0, 8) respectively.

(i) Show that coordinates of A is $(-2,0)$. [2]
(ii) Find the equation of the line that passes through A and perpendicular to the line BC. [2]
(iii) Find the coordinates of E if the ratio of area ABC: area ACE is given to be 2:1. [5]
(iv) Given that $ABFE$ is a kite, find the area of $ABFE$. [2]

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~ End of Paper ~

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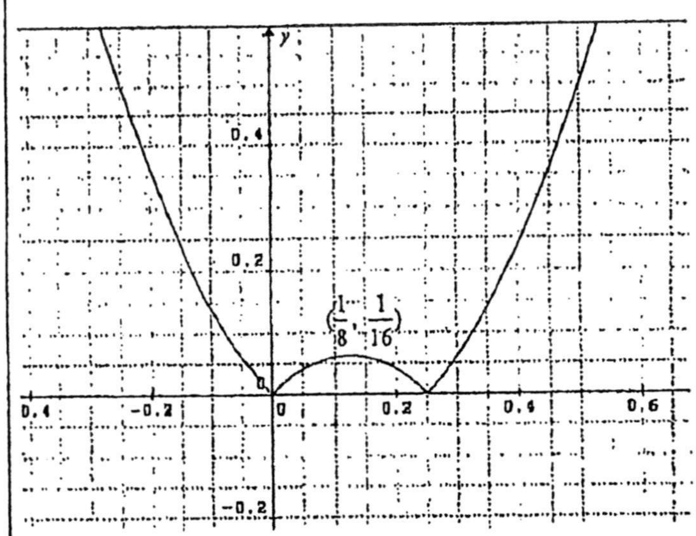
Qn	Detailed Working	Mark Distribution
1	$\frac{8x^4+1}{x(2x-1)}=4x^2+2x+1+\frac{x+1}{x(2x-1)}$	M1 (for correct quotient from long division)
	$\frac{x+1}{x(2x-1)}=\frac{A}{x}+\frac{B}{2x-1}$	M1
	$x+1=A(2x-1)+Bx$ Let $x=0$ $A=-1$	M1, M1 (for A and B)
	Let $x=0.5$ $0.5B=1.5$ $B=3$	
	$\frac{8x^4+1}{x(2x-1)}=4x^2+2x+1-\frac{1}{x}+\frac{3}{2x-1}$	A1
2(i)	$\operatorname{cosec}(60°-\theta)=\frac{2}{\cos\theta\,(\sqrt{3}-\tan\theta)}$	
	LHS $=\frac{1}{\sin(60°-\theta)}$	
	$=\frac{1}{\sin 60°\cos\theta-\cos 60°\sin\theta}$	M1 (for applying Addition formula)
	$=\frac{1}{(\frac{\sqrt{3}}{2})\cos\theta-(\frac{1}{2})\sin\theta}$	
	$=\frac{2}{\sqrt{3}\cos\theta-\sin\theta}\quad\frac{\div\cos\theta}{\div\cos\theta}$	M1
	$=\frac{2}{\cos\theta\,(\sqrt{3}-\tan\theta)}=RHS$ (shown)	A1

2(ii)	cosec 15° $= \operatorname{cosec}(60° - 45°)$ $= \frac{2}{\cos 45° (\sqrt{3} - \tan 45°)}$ $= \frac{2}{\cos 45° (\sqrt{3} - \tan 45°)}$ $= \frac{2}{\frac{1}{\sqrt{2}}(\sqrt{3}-1)}$ $= \frac{2\sqrt{2}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$ $= \sqrt{2}(\sqrt{3}+1)$ or $\sqrt{6}+\sqrt{2}$	M1 (for $\theta = -45°$) M1 (for special ∠s) M1 (for rationalising) A1
3(a)	$(2x^2 - \frac{1}{\sqrt{x}})^{10}$ T_{r+1} $= 10C_r (2x^2)^{10-r}(-x^{-0.5})^r$ $= 10C_r (2)^{10-r}(-1)^r (x)^{20-2r-0.5r}$ $= 10C_r (2)^{10-r}(-1)^r (x)^{20-2.5r}$ $20 - 2.5r = 0$ $r = 8$ $\therefore T_9 = 10C_8 (2)^2 (-1)^8 = 180$	M1 M1 A1
3(b)	$(5+px)^n$ ${}^nC_4 (5)^{n-4} p^4 = {}^nC_3 (5)^{n-3} p^3$ $\frac{n(n-1)(n-2)(n-3)}{1\times2\times3\times4}(5)^{n-4}p^4 = \frac{n(n-1)(n-2)}{1\times2\times3}(5)^{n-3}p^3$ $\frac{n-3}{4}(5)^{n-4}p^4 = (5)^{n-3}p^3$ $\frac{n-3}{4}p = 5$ $n = \frac{20}{p} + 3$	M1 (for specific term or correct expansion) M1, M1 (for applying ${}^nC_r$ formula to both sides) A1

4(a)	$y=(a+2)x^2-3x+(a-1)$ Let $a=3$, $y=5x^2-3x+2$ $5x^2-3x+2=7x-3$ $5x^2-10x+5=0$ $D=(-10)^2-4(5)(5)=0$ Therefore $y=7x-3$ is a tangent (shown).	**M1** **M1** **B1**
4(b)	$(m-4)x^2<3x-m$ $(m-4)x^2-3x+m<0$ $(-3)^2-4(m-4)(m)<0$ $9-4m(m-4)<0$ $4m^2-16m-9>0$ $(2m+1)(2m-9)>0$ $m<-\frac{1}{2}$, $m>4\frac{1}{2}$ Since $m-4<0$, thus $m<-\frac{1}{2}$. $\therefore m$ cannot be positive (shown)	**M1** (for D<0) **M1** **M1** **B1**
5(a)		shape - **B1** turning pt & x-intercepts - **B1**

	$\left\lvert \frac{x}{2} - 2x^2 \right\rvert = k$ $\frac{1}{2}\lvert x - 4x^2 \rvert = k$ $\lvert x(1-4x) \rvert = 2k$ $0 < 2k \leq \frac{1}{16}$ $0 < k \leq \frac{1}{32}$	 M1 A1
5(b)(i)	$m = 0,\ c = 1$ OR any other relevant answers	A1
5(b)(ii)	(5, 1) and (4, 0) $m = \frac{1-0}{5-4} = 1$ $y - 0 = 1(x-4)$ $y = x - 4$ *OR* (5, 1) and (6, 0) $m = \frac{1-0}{5-6} = -1$ $y - 0 = -1(x-6)$ $y = -x + 6$	 M1 A1 *Alternative answer:* M1 A1
6(a)	$y = e\sqrt{x}\ln(3x)$ $\frac{dy}{dx} = \frac{1}{2}ex^{-\frac{1}{2}}\ln(3x) + \frac{1}{x}e\sqrt{x}$ $= \frac{e\ln(3x)}{2\sqrt{x}} + \frac{e}{\sqrt{x}}$ $= \frac{e\ln(3x) + 2e}{2\sqrt{x}}$ $= \frac{e[\ln(3x) + 2]}{2\sqrt{x}}$	 M1 A1

	$\int_1^5 \frac{e[\ln(3x)+2]}{2\sqrt{x}}\ dx = \ [e\sqrt{x}\ln(3x)]_1^5$ $\int_1^5 \frac{\ln(3x)+2}{2\sqrt{x}}\ dx$ $=[\sqrt{x}\ln(3x)]_1^5$ $=\sqrt{5}\ln 15-\ln 3$ $=4.96$	M1 A1
6(b)	$y=6e^{\sqrt{x-1}}$ $\frac{dy}{dx}=6e^{\sqrt{x-1}}.\frac{1}{2}(x-1)^{-\frac{1}{2}}=\frac{3e^{\sqrt{x-1}}}{\sqrt{x-1}}$ $-\frac{1}{2}e=\frac{3e^{\sqrt{x-1}}}{\sqrt{x-1}}\times\frac{dx}{dt}$ $-\frac{1}{2}e=\frac{3e^2}{2}\times\frac{dx}{dt}$ $\frac{dx}{dt}=-\frac{1}{3}e$ units/s	M1 M1 M1 A1
8(i)	$\frac{20}{5}=\frac{20-h}{r}$ $20r=100-5h$ $r=\frac{100-5h}{20}$ $r=5-\frac{1}{4}h$ $V=\frac{1}{3}\pi(5)^2(20)-\pi(5-\frac{1}{4}h)^2h$ $=\frac{500\pi}{3}-(5-\frac{1}{4}h)^2\pi h$ (shown)	M1 M1 A1

8(ii)	$V=\frac{500\pi}{3}-(5-\frac{1}{4}h)^2\pi h$	
	$=\frac{500\pi}{3}-25\pi h+\frac{5}{2}\pi h^2-\frac{1}{16}\pi h^3$	
	$\frac{dV}{dh}=-25\pi+5\pi h-\frac{3}{16}\pi h^2$	M1
	$\frac{3}{16}h^2-5h+25=0$	M1
	$h=20\ (rej),\quad h=6\frac{2}{3}cm$	A1
	$\frac{d^2V}{dh^2}=5\pi-\frac{6}{16}\pi h$	M1
	$=5\pi-\frac{6}{16}\pi(6\frac{2}{3})>0$ (min)	A1
	$V=\frac{500\pi}{3}-(5-\frac{1}{4}\times 6\frac{2}{3})^2\pi(6\frac{2}{3})=291cm^2$	A1
9(a)	$y=\frac{5}{kx^2}+10x^3=\frac{5}{k}x^{-2}+10x^3$	
	$\frac{dy}{dx}=\frac{-10}{kx^3}+30x^2$	M1
	$\frac{-10}{kx^3}+30x^2=0$	M1
	$\frac{-10+30kx^5}{kx^3}=0$	
	$10=30k(2^5)$	
	$k=\frac{1}{96}$	A1

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9(b)(i)	$y = 2\cos^2 3x$ $\frac{dy}{dx} = -12\cos 3x \sin 3x$ $m_t = -12\cos 3(\frac{\pi}{12})\sin 3(\frac{\pi}{12}) = -6$ $m_n = \frac{1}{6}$ $x = \frac{\pi}{12}, y = 2\cos^2 3(\frac{\pi}{12}) = 1$ $y - 1 = \frac{1}{6}(x - \frac{\pi}{12})$ $y = \frac{1}{6}x + \frac{72-\pi}{72}$	M1 M1 M1 A1
9(b)(ii)	$-12\cos 3x \sin 3x = \frac{1}{6}$ $-6(2\sin 3x \cos 3x) = \frac{1}{6}$ $\sin 6x = -\frac{1}{36}$ $\alpha = 0.02778$ $6x = 3.1694, 6.2554, 9.4526, 12.5386, 15.7358, 18.8218$ $x = 0.528, 1.04, 1.58, 2.09, 2.62, 3.14$	M1 M1 A1 A1
10(i)	$\sqrt{(15-0)^2 + (0-8)^2} = 17$ units $A = (15-17, 0) = (-2, 0)$	M1 A1
10(ii)	$m_{BC} = \frac{8-0}{0-15} = -\frac{8}{15}$ $m_{\perp BC} = \frac{15}{8}$ Equation of line $\perp$ BC: $y - 0 = \frac{15}{8}(x+2)$ $y = \frac{15}{8}x + \frac{15}{4}$	M1 A1 70

10(iii)	Equation of line BC: $y-8=-\frac{8}{15}(x+0)$ $y=-\frac{8}{15}x+8$ Let $B(-2,y)$ $y=-\frac{8}{15}(-2)+8=\frac{136}{15}=9\frac{1}{15}$ $B(-2,9\frac{1}{15})$ ABC & ACE share the same base AC. Hence, $\perp$ height of E to x-axis should be $\frac{1}{2}$ of AB. $\perp$ height of E to x-axis $=\frac{1}{2}(9\frac{1}{15})=\frac{68}{15}=4\frac{8}{15}$ Let $E(x,-4\frac{8}{15})$ $-4\frac{8}{15}=-\frac{8}{15}(x)+8$ $x=23\frac{1}{2}$ $E=(23\frac{1}{2},-4\frac{8}{15})$	M1 M1 M1 M1 A1
10(iv)	Area of $ABFE$ = Area of $2(ABE)$ $=\frac{1}{2}\begin{vmatrix} -2 & -2 & \frac{47}{2} & -2 \\ 0 & \frac{136}{15} & -\frac{68}{15} & 0 \end{vmatrix} \times 2$ $=231.2$ units^2	M1 A1

1 The curve for which $\frac{dy}{dx}=\frac{k}{(2x+5)^3}-1$, where k is a constant, is such that the tangent to the curve at $(-2, 0)$ is perpendicular to the line $5y=x-1$. Find

(i) the value of k, [2]

(ii) the equation of the curve. [3]

2 The roots of the equation $x^2=mx-2m^2$ are α and β. Find, in terms of m, an equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [6]

3 (a) Without using a calculator, find the value of c given that $34+3\sqrt{128}=\left(\frac{6}{\sqrt{2}}-c\right)^2$. [3]

(b) The volume of a cylinder is $(9+\sqrt{50})\pi$ cm^3. Given that the cylinder has a radius of $(2+\sqrt{2})$ cm, find, without using a calculator, the height of the cylinder in the form $\frac{a+b\sqrt{2}}{c}$. [4]

4 An object is heated in an oven until it reaches the temperature of 90 °C. It is then allowed to cool. Its temperature, T °C, when it has been cooling for time t minutes, is given by the equation $T=k+he^{-\frac{t}{10}}$, where k and h are constants.

Given that the temperature of the object is 40 °C when it has been cooling for exactly $(10\ln 3)$ minutes, show that $k=15$ and $h=75$. [3]

(i) Calculate the value of T when $t=10$. [1]

(ii) Determine the rate at which T is decreasing when $t=25$. [2]

(iii) Find, to the nearest minute, the time taken for the temperature of the object to drop below 35°C. [2]

5 (a) Express y in terms of x, $\log_5 x^2 y = 3 + \log_5 x - \dfrac{1}{2\log_y 5}$. [4]

(b) Solve the equation $\log_3(x+5) - \log_{\sqrt{3}}(x-1) = \log_3 2$. [4]

6 A cubic polynomial $f(x)$ is such that the coefficient of x^3 is 6. It is given that one of the roots of the equation $f(x) = 0$ is $\dfrac{4}{3}$ and $[2x^2 + (2k+1)x - 3]$ is a quadratic factor of $f(x)$. Given further that $f(x)$ leaves a remainder of 30 when divided by $(x-2)$, find

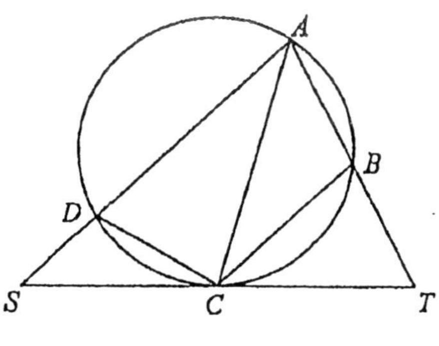
(i) the value of k and hence, factorise $f(x)$ completely, [4]

(ii) by using the result from part (i), the number of real roots of the equation $f(x) = 15(1-2x)$, justifying your answer. [4]

7 The diagram shows a circle $ABCD$ and the tangent ST of the circle at point C. B and C bisect AT and ST respectively. Prove that

(i) ΔABC is similar to ΔSDC, [4]

(ii) $AS = \dfrac{2AC \times DC}{TC}$. [4]



8 (i) A circle passes through the origin O and cuts the x- and y-axes at 3 and 4 respectively. Find the equation of the circle in the general form. [4]

(ii) Given that OP is the diameter of the circle, find the equation of the tangent at P.[3]

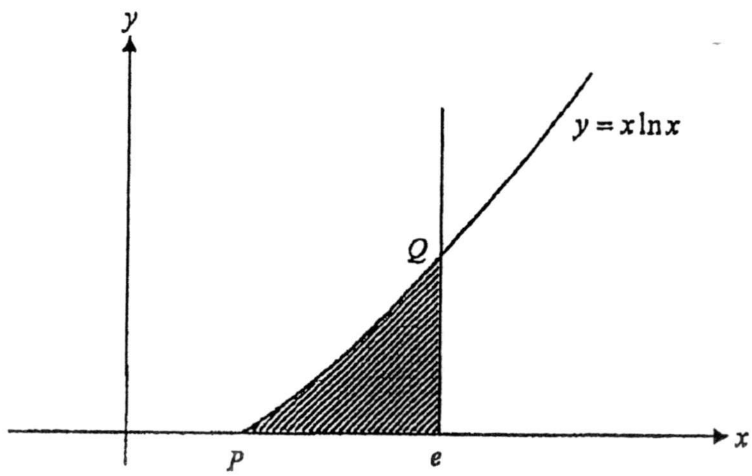
(iii) Another tangent at Q, which is parallel to the y-axis, meets the tangent found in part (ii). Find the points of intersections between the two tangents. [3]

9 (i) Given that $y = x^2 \ln x^3$, show that $\frac{dy}{dx} = 3x(1 + 2\ln x)$. [3]

(ii) The diagram shows part of the curve $y = x\ln x$ cutting the x-axis at point P. The line $x = e$ intersects the curve at point Q.

(a) Find the x-coordinate of P. [2]

(b) By using the result from part (i), show that the area of the shaded region bounded by the x-axis, the line $x = e$ and the curve is $\frac{1}{4}(e^2 + 1)$. [4]



10 (i) Express $3\cos 2A + 4\sin 2A$ in the form $R\cos(2A - \alpha)$, where $0° \le \alpha \le 90°$. [2]

(ii) Hence solve $3\cos 2A + 4\sin 2A = 4$ for $0° \le A \le 180°$. [3]

(iii) On the same axes sketch, for $0° \le x \le 60°$, the graphs of [3]

$$y = 2\sin 6x \qquad \text{and} \qquad y = 2 - \frac{3}{2}\cos 6x.$$

(iv) Explain how the solutions of the equation in part (ii) could be used to find the x-coordinates of the points of intersection of the graphs of part (iii). [2]

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11 A particle moves in a straight line such that t seconds after passing through a fixed point O, its velocity, v m/s, is given by $v = 24\cos(2t)$. When $t = 0$, its displacement from O is -6 metres. Find

(i) the magnitude of the acceleration when $t = 1$, [2]

(ii) the value of t when the particle first reaches the fixed point O, [4]

(iii) the distance travelled by the particle up to the second instantaneous rest. [4]

12 A curve has the equation $y = (x-2)(x+1)^3$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Find the x-coordinates of the stationary points. [2]

(iii) Without determining the nature of the stationary points, show that y decreases as x increases between the stationary points. [3]

(iv) Determine the nature of the stationary points. [4]

Qn	Marking point	Mark Awarded	Remarks
1(a) (5m)	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=-2}=-5$ $\frac{k}{(2(-2)+5)^3}-1=-5$ $k=-4$	M1 A1	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{4}{(2x+5)^3}-1$ $y=\int \frac{-4}{(2x+5)^3}-1\,\mathrm{d}x$ $=\frac{1}{(2x+5)^2}-x+c$ Sub $(-2, 0)$, $0=1-(-2)+c$ $c=-3$ $y=\frac{1}{(2x+5)^2}-x-3$	M1 M1 A1	
2	$x^2-mx+2m^2=0$ $\alpha+\beta=m$ $\alpha\beta=2m^2$ $\alpha^3+\beta^3=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$ $=m[m^2-3(2m^2)]$ $=-5m^3$ $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{\alpha^3\beta^3}$ $=\frac{-5m^3}{(2m^2)^3}$ $=-\frac{5}{8m^3}$ $\frac{1}{\alpha^3}\times\frac{1}{\beta^3}=\frac{1}{(\alpha\beta)^3}$ $=\frac{1}{(2m^2)^3}$ $=\frac{1}{8m^6}$ $x^2-\left(-\frac{5}{8m^3}\right)x+\frac{1}{8m^6}=0$ $8m^6x^2+5m^3x+1=0$	B1 M1 A1 M1 M1 A1	Either sum or product 73

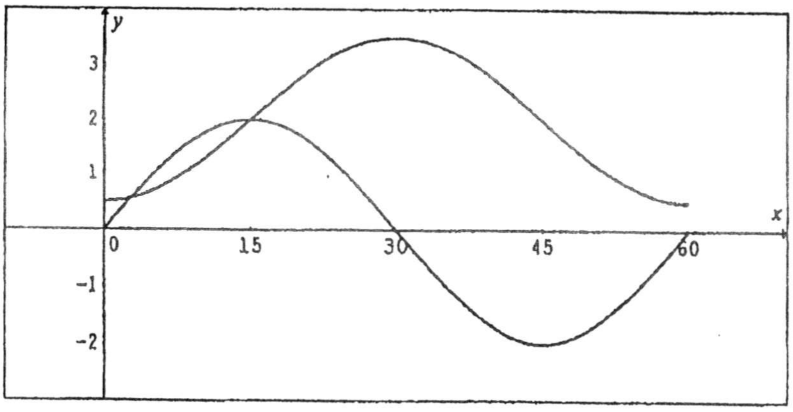
Qn	Marking point	Mark Awarded	Remarks
3(i)	$34+3\sqrt{128}=\left(\dfrac{6}{\sqrt{2}}-c\right)^2$ $34+24\sqrt{2}=(3\sqrt{2}-c)^2$ $34+24\sqrt{2}=18-6c\sqrt{2}+c^2$ $34+24\sqrt{2}=18+c^2-6c\sqrt{2}$ $24=-6c$ $c=-4$	M1 M1 A1	
(ii)	$\pi(2+\sqrt{2})^2h=(9+\sqrt{50})\pi$ $(6+4\sqrt{2})h=9+\sqrt{50}$ $h=\dfrac{9+5\sqrt{2}}{2(3+2\sqrt{2})}\times\dfrac{3-2\sqrt{2}}{3-2\sqrt{2}}$ $=\dfrac{27-3\sqrt{2}-20}{2(9-8)}$ $=\dfrac{7-3\sqrt{2}}{2}$	M1 M1 M1 A1	
4	$T=k+he^{-\frac{t}{10}}$ $t=0,\quad 90=k+h$ $t=10\ln 3,\quad 40=k+\dfrac{h}{3}$ $50=\dfrac{2}{3}h$ $h=75,\quad k=15$	M1 M1 A1	
(i)	$t=10,\quad T=15+75e^{-1}$ $=42.6°$	B1	
(ii)	$T=15+75e^{-\frac{t}{10}}$ $\dfrac{dT}{dt}=75\left(-\dfrac{1}{10}\right)e^{-\frac{t}{10}}$ $=-\dfrac{15}{2}e^{-\frac{t}{10}}$ $\left.\dfrac{dT}{dt}\right\|_{t=25}=-\dfrac{15}{2}e^{-\frac{25}{10}}$ $=-0.616$ °C/min	M1 A1	

Qn	Marking point	Mark Awarded	Remarks
(iii)	$15+75e^{-\frac{t}{10}}<35$ $e^{-\frac{t}{10}}<\frac{4}{15}$ $-\frac{t}{10}<\ln\frac{4}{15}$ $t>10\ln\frac{4}{15}=13.2$ $t=14$	M1 A1	Accept working which is in the equation form
5(a)	$\log_5 x^2y=3+\log_5 x-\frac{1}{2\log_y 5}$ $\log_5 x^2y=3+\log_5 x-\frac{1}{2}\log_5 y$ $2\log_5 x^2y=6+2\log_5 x-\log_5 y$ $\log_5(x^2y)^2=6+\log_5\frac{x^2}{y}$ $\log_5(x^2y)^2-\log_5\frac{x^2}{y}=6$ $\log_5\frac{(x^4y^2)y}{x^2}=6$ $x^2y^3=5^6$ $y=\sqrt[3]{\frac{5^6}{x^2}}$ $y=\frac{25}{\sqrt[3]{x^2}}$ o.e.	M1 M1 M1 A1	Change of base Evidence of using Power / Product / Quotient law Index form Accept $x^{\frac{2}{3}}$
(b)	$\log_3(x+5)-\log_{\sqrt{3}}(x-1)=\log_3 2$ $\log_3(x+5)-\frac{\log_3(x-1)}{\log_3 3^{\frac{1}{2}}}=\log_3 2$ $\log_3(x+5)-2\log_3(x-1)=\log_3 2$ $\log_3\frac{x+5}{(x-1)^2}=\log_3 2$ $x+5=2(x^2-2x+1)$ $2x^2-5x-3=0$ $(2x+1)(x-3)=0$ $x=-\frac{1}{2}$(NA), $x=3$	M1 M1 M1 A1	Change of base Quotient rule Index form No A1 if invalid ans is not rejected

Qn	Marking point	Mark Awarded	Remarks
6(i)	$f(x)=(3x-4)[2x^2+(2k+1)x-3]$ $f(2)=30$ $2[8+(2k+1)2-3]=30$ $5+2(2k+1)=15$ $2(2k+1)=10$ $k=2$ $f(x)=(3x-4)(2x^2+5x-3)$ $\quad=(3x-4)(2x-1)(x+3)$	M1 M1 A1 A1	 No A1 if given as $f(x)=0$
(ii)	$f(x)=15(1-2x)$ $(3x-4)(2x-1)(x+3)=-15(2x-1)$ $(3x-4)(2x-1)(x+3)+15(2x-1)=0$ $(2x-1)[(3x-4)(x+3)+15]=0$ $(2x-1)(3x^2+5x+3)=0$ $2x-1=0$ $\quad\quad$ $3x^2+5x+3=0$ $\quad\quad\quad D=5^2-4(3)(3)<0$ No of real roots = 1	 M1 M1 M1 A1	 Use of D or quad formula
7(i)	$BC//AD$ (mid-point thm) $\angle SCD=\angle CAD$ (alt segment thm) $\quad=\angle ACB$ (alt $\angle$) $\angle ABC=180°-\angle ADC$ ($\angle$ in opp segment) $\quad=\angle SDC$ $\therefore$ $\angle ABC$ is similar to $\angle SDC$	B1 M1 M1 A1	
(ii)	$\frac{AC}{SC}=\frac{BC}{CD}$ (part (i)) $AC\times CD=\frac{1}{2}AS\times SC$ (mid-pt theorem) $2AC\times DC=AS\times TC$ (C bisects ST) $AS=\frac{2AC\times DC}{TC}$	M1 M1 M1 A1	
8(i)	$A=(3,0)$, $\quad B=(0,4)$ $\Rightarrow$ centre lies on $\perp$ bisector of OA & OB $\therefore C=\left(\frac{3}{2},2\right)$ Radius $=\sqrt{\left(\frac{3}{2}\right)^2+2^2}$	M1 M1	

PT12

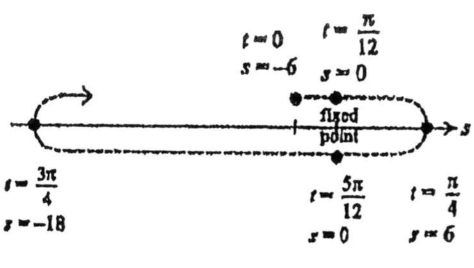
Qn	Marking point	Mark Awarded	Remarks
	$=\sqrt{\frac{25}{4}}=\frac{5}{2}$ $\left(x-\frac{3}{2}\right)^2+(y-2)^2=\frac{25}{4}$ $x^2-3x+\frac{9}{4}+y^2-4y+4-\frac{25}{4}=0$ $x^2+y^2-3x-4y=0$	M1 A1	
(ii)	$\left(\frac{x_P+0}{2},\frac{y_P+0}{2}\right)=\left(\frac{3}{2},2\right)$ $(x_P, y_P)=(3,4)$ $m_{OP}=\frac{4}{3}, \qquad m_T=-\frac{3}{4}$ $y-4=-\frac{3}{4}(x-3)$ $4y-16=-3x+9$ $4y=-3x+25$	M1 M1 A1	
(iii)	Since tangent // to y-axis $\Rightarrow x=c$ from centre of circle $x=\frac{3}{2}-\frac{5}{2}=-1, \qquad x=\frac{3}{2}+\frac{5}{2}=4$ $(x,y)=(-1,7), \quad \left(4,\frac{13}{4}\right)$	M1 A1, A1	
9(i)	$y=x^2\ln x^3$ $=3x^2\ln x$ $\frac{dy}{dx}=3\left(x^2\left(\frac{1}{x}\right)+2x\ln x\right)$ $=3x(1+2\ln x)$	M1, M1 A1	
(ii)(a)	$x\ln x=0$ $x=0$ (NA), $\ln x=0$ $x=e^0=1$	M1 A1	
(b)	$\int_1^e 3x(1+2\ln x)\,dx=\left[x^2\ln x^3\right]_1^e$ $\int_1^e 3x+6x\ln x\,dx=e^2\ln e^3$ $\frac{3}{2}\left[x^2\right]_1^e+\int_1^e 6x\ln x\,dx=3e^2$ $\int_1^e 6x\ln x\,dx=3e^2-\frac{3}{2}(e^2-1)$ $\int_1^e 6x\ln x\,dx=\frac{3}{2}(e^2+1)$	M1 M1 M1	For integrating $3x$ For substituting limits

Qn	Marking point	Mark Awarded	Remarks
	$\int_1^e x \ln x \, dx = \frac{1}{4}(e^2+1)$	A1	
10(i)	$R\cos(2A-\alpha)$ $= \sqrt{3^2+4^2}\cos\left(2A - \tan^{-1}\left(\frac{4}{3}\right)\right)$ $= 5\cos(2A - 53.1°)$	M1 A1	Either R or α No A1 if □ not in 1d.p.
(ii)	$3\cos 2A + 4\sin 2A = 4$ $5\cos(2A - 53.13°) = 4$ $\cos(2A - 53.13°) = \frac{4}{5}$ $2A - 53.13° = -36.87,\ 36.87°,\ 323.13°$ $2A = 16.26°,\ 90.0°$ $A \approx 8.13°,\ 45.0°$	M1 A1, A1	Do not penalise for d.p. if this was done in part (i)
(iii)			B1 for sine graph B1 (amplitude), B1 (shape) for cosine graph
(iv)	$2\sin 6x = 2 - \frac{3}{2}\cos 6x$ $4\sin 6x = 4 - 3\cos 6x$ $3\cos 6x + 4\sin 6x = 4$ Let $A = 3x$ $3\cos 2A + 4\sin 2A = 4$	M1 A1	
11(i)	$v = 24\cos(2t)$ $a = -48\sin(2t)$ $t = 1,\quad a = -48\sin(2)$ $= -43.6$ $\lvert a \rvert = 43.6$ m/s^2	M1 A1	
(ii)	$s = \int 24\cos(2t)\,dt$ $= 12\sin(2t) + c$	M1	

P13

10(iii)	Equation of line BC: $y-8=-\frac{8}{15}(x+0)$ $y=-\frac{8}{15}x+8$	M1
	Let $B(-2,y)$ $y=-\frac{8}{15}(-2)+8=\frac{136}{15}=9\frac{1}{15}$ $B(-2,9\frac{1}{15})$	M1
	ABC & ACE share the same base AC. Hence, $\perp$ height of E to x-axis should be $\frac{1}{2}$ of AB. $\perp$ height of E to x-axis $=\frac{1}{2}(9\frac{1}{15})=\frac{68}{15}=4\frac{8}{15}$	M1
	Let $E(x,-4\frac{8}{15})$ $-4\frac{8}{15}=-\frac{8}{15}(x)+8$	M1
	$x=23\frac{1}{2}$ $E=(23\frac{1}{2},-4\frac{8}{15})$	A1
10(iv)	Area of $ABFE$ = Area of $2(ABE)$ $=\frac{1}{2}\begin{vmatrix} -2 & -2 & \frac{47}{2} & -2 \\ 0 & \frac{136}{15} & -\frac{68}{15} & 0 \end{vmatrix} \times 2$	M1
	$=231.2$ units^2	A1

P8

Qn	Marking point	Mark Awarded	Remarks
	$t=0,\ s=-6 \Rightarrow c=-6$ $s=12\sin(2t)-6$ When P first reaches fixed point, $s=0$. $12\sin(2t)-6=0$ $\sin(2t)=0.5$ $\alpha=\dfrac{\pi}{6}$ $t=\dfrac{\pi}{12},\quad \dfrac{5\pi}{12}$ (NA)	M1 M1 A1	
(iii)	At inst rest, $v=0$ $24\cos(2t)=0$ $2t=\dfrac{\pi}{2},\quad \dfrac{3\pi}{2}$ $t=\dfrac{\pi}{4},\quad \dfrac{3\pi}{4}$ $s=12\sin(2t)-6$ $t=0,\ s=-6$ $t=\dfrac{\pi}{4},\ s=6$ $t=\dfrac{3\pi}{4},\ s=-18$ $t=5,\ s=-6$ Dist $=6+12+18=36$ m 	M1 A1 M1 A1	Either for $t=\dfrac{\pi}{4}$ or $t=\dfrac{3\pi}{4}$
12(i)	$y=(x-2)(x+1)^3$ $\dfrac{dy}{dx}=(x-2)[3(x+1)^2]+(x+1)^3$ $=(x+1)^2(3x-6+x+1)$ $=(x+1)^2(4x-5)$	M1 A1	
(ii)	$\dfrac{dy}{dx}=(x+1)^2(4x-5)=0$ $x=-1,\quad x=\dfrac{5}{4}$	M1 A1	

<table>
<tr><th>Qu</th><th>Marking point</th><th>Mark Awarded</th><th>Remarks</th></tr>
<tr><td>(iii)</td><td>

For $-1 \le x \le \frac{5}{4}$,

$(x+1)^2 > 0$, $4x-5<0$

$\therefore \frac{dy}{dx} = (x+1)^2(4x-5) < 0$

OR

<table>
<tr><td>x</td><td>-1^+</td><td>$\frac{5}{4}^-$</td></tr>
<tr><td>Sign of $\frac{dy}{dx}$</td><td>$-$</td><td>$-$</td></tr>
</table>

Since gradient is negative between 2 stat points, *y* decreases as *x* increases between the two stat points.

</td><td>M1, M1

A1</td><td></td></tr>
<tr><td>(iv)</td><td>

<table>
<tr><td>x</td><td><-1</td><td>-1</td><td>>-1</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>-ve</td><td>0</td><td>-ve</td></tr>
<tr><td></td><td>\</td><td>—</td><td>\</td></tr>
</table>

$x=-1$ is a point of inflexion

<table>
<tr><td>x</td><td>$<\frac{5}{4}$</td><td>$\frac{5}{4}$</td><td>$>\frac{5}{4}$</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>-ve</td><td>0</td><td>+ve</td></tr>
<tr><td></td><td>\</td><td>—</td><td>/</td></tr>
</table>

$x=\frac{5}{4}$ is a min point

</td><td>M1

A1

M1

A1</td><td></td></tr>
</table>

P14