中正中学 义顺

CHUNG CHENG HIGH SCHOOL YISHUN

Preliminary Examination (2015)
Secondary 4 Express/ 5 Normal (Academic)

Candidate			
	Name	Register No	Class

For examiner's use
/ 80

ADDITIONAL MATHEMATICS
4047/01

Date: 26 August 2015
Duration: 2 hours

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give your answer in the simplest form. Leave your answer in fraction where applicable. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Setter: Mr Han Ji

This paper consists of **7** printed pages, INCLUDING the cover page. **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \sin C$$

1. (i) Sketch the graph of $y = 8x^{-1}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = \frac{1}{4}x^{\frac{3}{2}}$ for $x \geq 0$. [1]

(iii) Calculate the exact coordinates of the point of intersection of the graphs. [2]

(iv) Determine with explanation, whether the normal to the graphs at the point of intersection are perpendicular. [2]

2. The cubic polynomial f(x) is such that the coefficient of x^3 is 1 and the roots of f(x) = 0 are -1, m and $2m$, where m is an integer. It is given that f(x) has a remainder of 6 when divided by $x - 1$.

(i) Find an expression for f(x) in descending powers of x. [4]

(ii) Hence or otherwise, solve the equation $y^6 - 5y^4 + 2y^2 + 8 = 0$. [3]

3. A cuboid has a square base of side ($2 + a\sqrt{3}$) cm, where a is an integer. The height of the cuboid is ($1 + \sqrt{3}$) cm and its volume is ($\sqrt{27} - 5$) cm^3.

(i) Find the value of a. [3]

(ii) With the value of a in (i), find the total surface area of the cuboid in the form ($p + q\sqrt{3}$) cm^2, where p and q are integers. [2]

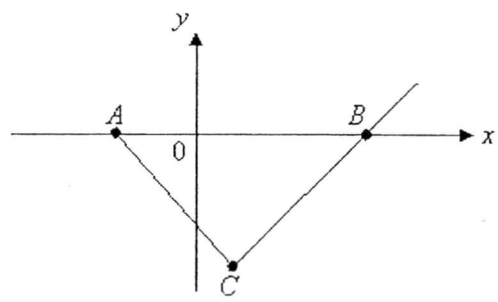
4. The equation of a curve is $y = x^2 + 3x$. A straight line has equation $y = mx - 9$.

(i) Explain why the straight line is a tangent to the curve when $m = 9$. [2]

(ii) Find the other value of m for which the line $y = mx - 9$ is a tangent to the curve. [3]

(iii) State the set of values of m for which the straight line does not intersect the curve. [1]

5. The diagram shows part of the graph of $y = |2x - 1| - 2$.



(i) Find the coordinates of A and of B. [2]

(ii) Explain why the lowest point, C, on the graph has coordinates $(\frac{1}{2}, -2)$. [2]

(iii) In each of the following cases, determine the number of intersections of the line $y = mx + c$ with $y = |2x - 1| - 2$, justifying your answer.

(a) $m = -2$ and $c > -1$ [2]

(b) $m = 1$ and $c < -3$ [2]

6. In a simplified prey-predator model, some wolves were deliberately introduced to an island to curb the population of wild rabbits. The population of rabbits, R, was given by $R = 400 + 6000e^{-0.02t}$, where t is the number of days since the introduction of wolves.

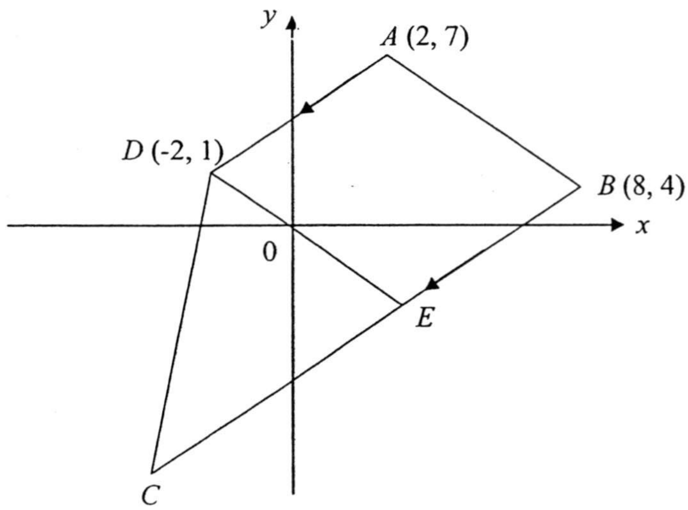
(i) Find the initial population of wild rabbits on the island. [1]

(ii) After how many days would the population of wild rabbits first drop by 40%? [2]

(iii) Explain why the rabbits would never extinct on the island in the long run. [1]

Solutions to this question by accurate drawing will not be accepted.

7.



The diagram shows a trapezium $ABCD$ in which AD is parallel to BC. The points A, B and D are (2, 7), (8, 4) and (-2, 1) respectively. The point E is on BC and DE passes through O.

(i) Show that $ABED$ is a parallelogram. [2]

(ii) Find the coordinates of E. [2]

Given that $CD = CE$,

(iii) find the coordinates of C. [4]

(iv) find the ratio of the area of triangle CDE to the area of $ABED$. [2]

8. (a) (i) Without using a calculator, prove that $\cot(45° - A) = \dfrac{\cot A + 1}{\cot A - 1}$. [3]

(ii) Hence find the exact value of $\cot 15°$. [2]

(b) Find an expression for f(x) such that

$$f'(x) = 3\sin^2(5x - \frac{\pi}{4}) + \cos^2 x - \tan^2 \frac{1}{2}x.$$ [3]

80

9. (i) Express $\dfrac{8x-5}{x^2(1-x)}$ as the sum of 3 partial fractions. [4]

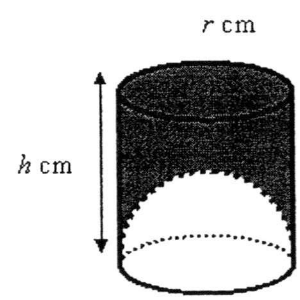
(ii) Hence find $\int \dfrac{8x-5}{x^2(1-x)}dx$. [2]

10. The equation of a curve is $y = xe^{-x}$.

(i) Find the set of values of x for which y is an increasing function of x. [2]

(ii) Find the coordinates of the turning point and determine whether the turning point is a maximum or minimum. [2]

11. The diagram shows a solid container consisting of a cylinder with a hemisphere dug out. The radius and height of the cylinder are r cm and h cm respectively.



(i) Express h in terms of r given that the external curved surface area of the cylindrical part of the solid is 1200π cm^2. [2]

(ii) Express the volume, V cm^3, of the container in terms of r. [2]

(iii) The solid is heated and it expands at a rate of 0.81 cm^3/s. Find the rate at which its radius increases when the height is 60 cm. [3]

Answer the whole of this question on a piece of graph paper.

12. The table shows experimental values of the two variables, x and y.

x	0.5	1.5	3	4.5	5.5	6
y	4.43	5.29	7.44	11.4	15.7	18.7

It is known that x and y are related by an equation of the form $y = ab^x + e$, where a and b are constants.

(i) Explain how a straight line graph may be drawn to represent the given data. [2]

(ii) Draw this graph for the given data and use it to estimate the value of a and of b. [4]

(iii) By inserting another suitable line on your graph, solve the equation

$ab^x = 5e^{-\frac{x}{2}}$. [3]

END OF PAPER

中正中学 义顺
CHUNG CHENG HIGH SCHOOL YISHUN

Preliminary Examination (2015)
Secondary 4 Express

Candidate			
	Name	Register No	Class

ADDITIONAL MATHEMATICS
Paper 1 (4047)

Date: 27 August 2015
Duration: 2 hr

Additional Materials: Answer Paper

For examiner's use
/ 80

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1. Answer **ALL** the questions in this paper.
2. All workings must be clearly shown in the answer space provided.
   Omission of essential working and unit of measurement will result in loss of marks.
3. The use of calculator is expected, where appropriate.
4. Give your answer in the simplest form. Leave your answer in fraction where applicable or correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
5. For π, use your calculator value.
6. The number of marks is given in brackets [ ] at each question or part question.
   The total marks for this paper is **80**.

Setter: Mr Han Ji **[Turn over**

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Mathematical Formulae

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
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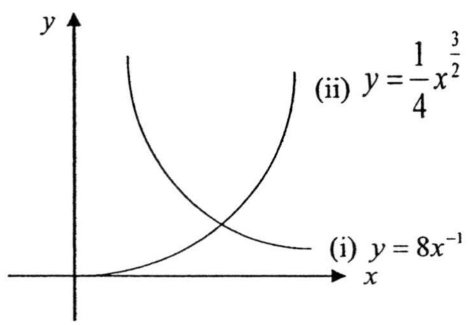
(ii) On the same diagram, sketch the graph of $y = \frac{1}{4}x^{\frac{3}{2}}$ for $x \geq 0$. [1]

(iii) Calculate the exact coordinates of the point of intersection of the graphs. [2]

(iv) Determine with explanation, whether the normal to the graphs at the point of intersection are perpendicular. [2]

Ans:



1 mark for each graph

(iii) $y = 8x^{-1}$

$y = \frac{1}{4}x^{\frac{3}{2}}$

$8x^{-1} = \frac{1}{4}x^{\frac{3}{2}}$

$x^{\frac{5}{2}} = 32$

$x = 32^{\frac{2}{5}}$ ------ [M1]

$= (2^5)^{\frac{2}{5}} = 2^2 = 4$

$y = 8(4^{-1}) = 2$

The coordinates are (4, 2). ------ [A1]

(iv) $y = 8x^{-1}, \frac{dy}{dx} = -8x^{-2}$

$y=\frac{1}{4}x^{\frac{3}{2}}, \frac{dy}{dx}=\frac{3}{8}x^{\frac{1}{2}}$

When $x=4$,

$\frac{dy}{dx}=8x^{-2}=-8(4^{-2})=-\frac{1}{2}$, gradient of normal = 2

$\frac{dy}{dx}=\frac{3}{8}x^{\frac{1}{2}}=\frac{3\sqrt{4}}{8}=\frac{3}{4}$, gradient of normal $=-\frac{4}{3}$

$2\times-\frac{4}{3}=-\frac{8}{3}\neq-1$ ------ [M1]

The normals are not perpendicular since the product of their gradients is not -1.

------ [A1]

OR

$\frac{dy}{dx}=8x^{-2}=-8(4^{-2})=-\frac{1}{2}$

$\frac{dy}{dx}=\frac{3}{8}x^{\frac{1}{2}}=\frac{3\sqrt{4}}{8}=\frac{3}{4}$

$-\frac{1}{2}\times\frac{3}{4}\neq-1$ ------ [M1]

Since the tangents to the curves at the point of intersection are not perpendicular, the normals at that point are also not perpendicular. ------ [A1]

2. The cubic polynomial f(x) is such that the coefficient of x^3 is 1 and the roots of f(x) = 0 are -1, m and $2m$, where m is an integer. It is given that f(x) has a remainder of 6 when divided by $x-1$.

(i) Find an expression for f(x) in descending powers of x. [4]

(ii) Hence or otherwise, solve the equation $y^6-5y^4+2y^2+8=0$. [3]

Ans:

(i) Let f(x) = $k(x+1)(x-m)(x-2m)$.

Since the coefficient of x^3 is 1, $k=1$. ------ [M1]

$f(x) = (x+1)(x-m)(x-2m)$

$f(1) = 6$

$(1+1)(1-m)(1-2m) = 6$ ------ [M1]

$(1-m)(1-2m) = 3$

$2m^2 - 3m + 1 = 3$

$2m^2 - 3m - 2 = 0$

$(m-2)(2m+1) = 0$

$m = 2$ or $m = -\frac{1}{2}$ (rej) ------ [M1]

$f(x) = (x+1)(x-2)(x-4)$

$= (x+1)(x^2 - 6x + 8)$

$= x^3 - 5x^2 + 2x + 8$ ------ [A1]

(ii) Let $x = y^2$

$(y^2)^3 - 5(y^2)^2 + 2(y^2) + 8 = 0$ ------ [M1]

$x^3 - 5x^2 + 2x + 8 = 0$

$(x+1)(x-2)(x-4) = 0$

$x = -1$, $x = 2$ or $x = 4$

$y^2 = -1$ (rej), $y^2 = 2$ or $y^2 = 4$ ------ [M1]

$y = \pm\sqrt{2}$, $y = \pm 2$ ------ [A1]

3. A cuboid has a square base of side ($2 + a\sqrt{3}$) cm, where a is an integer. The height of the cuboid is ($1+\sqrt{3}$) cm and its volume is ($\sqrt{27} - 5$) cm^3.

(i) Find the value of a. [3]

(ii) With the value of a in (i), find the total surface area of the cuboid in the form ($p + q\sqrt{3}$) cm^2, where p and q are integers. [2]

Ans:

(i) $(2+a\sqrt{3})^2(1+\sqrt{3})=\sqrt{27}-5$

$$(2+a\sqrt{3})^2=\frac{3\sqrt{3}-5}{1+\sqrt{3}}$$

$$=\frac{(3\sqrt{3}-5)(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} \text{ ------ [M1]}$$

$$=\frac{8\sqrt{3}-14}{-2}$$

$$=7-4\sqrt{3}$$

$4+3a^2+4a\sqrt{3}=7-4\sqrt{3}$ ------ [M1]

$4+3a^2=7, a^2=1, a=\pm 1$

$4a=-4, a=-1$

$\therefore a=-1$ ------ [A1]

(ii) Total surface area $=2(2-\sqrt{3})^2+4(2-\sqrt{3})(1+\sqrt{3})$ ------ [M1]

$=2(7-4\sqrt{3})+4(\sqrt{3}-1)$

$=14-8\sqrt{3}+4\sqrt{3}-4$

$=(10-4\sqrt{3})\text{ cm}^2$ ------ [A1]

4. The equation of a curve is $y=x^2+3x$. A straight line has equation $y=mx-9$.

(i) Explain why the straight line is a tangent to the curve when $m=9$. [2]

(ii) Find the other value of m for which the line $y=mx-9$ is a tangent to the curve. [3]

(iii) State the set of values of m for which the straight line does not intersect the curve. [1]

Ans:

(i) $y = x^2 + 3x$

$y = 9x - 9$

$x^2 + 3x = 9x - 9$

$x^2 - 6x + 9 = 0$

Discriminant = $(-6)^2 - 4(1)(9) = 0$ ------ [M1]

Therefore, the straight line is a tangent to the curve, since they intersect at only one point. ------ [A1]

(ii) $x^2 + 3x = mx - 9$

$x^2 + (3-m)x + 9 = 0$

$(3-m)^2 - 4(1)(9) = 0$ ------ [M1]

$(3-m)^2 = 36$

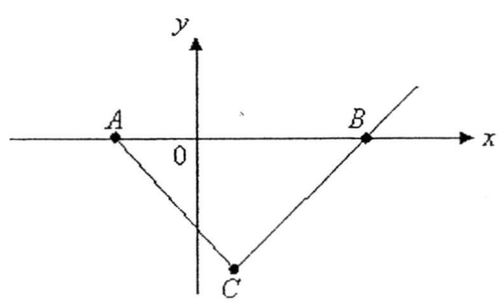
$3 - m = \pm 6$

$m = -3$ or $m = 9$ (rej) ------ [M1]

The other value of m is -3. ------ [A1]

(iii) $-3 < m < 9$ ------ [B1]

5. The diagram shows part of the graph of $y = |2x - 1| - 2$.



(i) Find the coordinates of A and of B. [2]

(ii) Explain why the lowest point, C, on the graph has coordinates $(\frac{1}{2}, -2)$. [2]

(iii) In each of the following cases, determine the number of intersections of the line $y = mx + c$ with $y = |2x-1| - 2$, justifying your answer.

(a) $m = -2$ and $c > -1$ [2]

(b) $m = 1$ and $c < -3$ [2]

Ans:

(i) $|2x-1| - 2 = 0$

$|2x-1| = 2$

$2x - 1 = 2$ or $2x - 1 = -2$ ------ [M1]

$2x = 3$ or $2x = -1$

$x = 1\frac{1}{2}$ or $x = -\frac{1}{2}$

A $(-\frac{1}{2}, 0)$, B $(1\frac{1}{2}, 0)$ ------ [A1]

(ii) $|2x-1| \geq 0$, $|2x-1| - 2 \geq -2$

Since C is the lowest point on the graph, $y = -2$. ------ [M1]

$|2x-1| = 0$, $x = \frac{1}{2}$

Therefore the coordinates of C are $(\frac{1}{2}, -2)$. ------ [A1]

OR

At the point where the lines turns, $|2x-1| = 0$, $x = \frac{1}{2}$ ------ [M1]

$y = 0 - 2 = -2$

Therefore the coordinates of C are $(\frac{1}{2}, -2)$. ------ [A1]

(iii) (a) For $m=-2$ and $c>-1$, the line $y=mx+c$ is above the left arm and parallel to it. Therefore the line $y=mx+c$ intersects the right arm at one point. ------ [M1]

Number of intersection = 1 ------ [A1]

(b) For $m=1$ and $c<-3$, the line $y=mx+c$ is below C and the gradient is gentler than the right arm. Therefore the line $y=mx+c$ does not intersect the right arm. ------ [M1]

Number of intersection = 0 ------ [A1]

6. In a simplified prey-predator model, some wolves were deliberately introduced to an island to curb the population of wild rabbits. The population of rabbits, R, was given by $R=400+6000e^{-0.02t}$, where t is the number of days since the introduction of wolves.

(i) Find the initial population of wild rabbits on the island. [1]

(ii) After how many days would the population of wild rabbits first drop by 40%? [2]

(iii) Explain why the rabbits would never extinct on the island in the long run. [1]

Ans:

(i) When $t=0$,

$R=400+6000e^{0}=6400$ ------ [A1]

(ii) $6400\times 60\%=3840$

$3840=400+6000e^{-0.02t}$

$3440 = 6000e^{-0.02t}$

$e^{-0.02t} = \dfrac{3440}{6000}$

$-0.02t = \ln(\dfrac{3440}{6000})$ ------ [M1]

$t = 27.8 \approx 28$

It takes 28 days for the population of wild rabbits to first drop by 40%. ------ [A1]

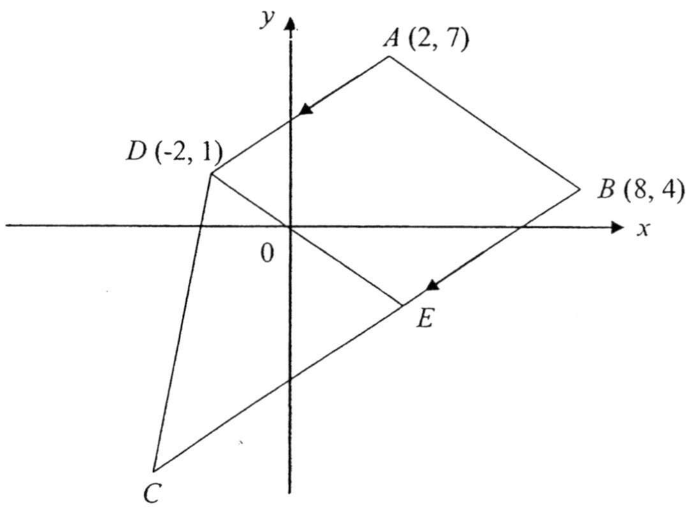
(iii)As $t \to \infty$, $e^{-0.02t} \to 0$

As a result, $R \to 400 + 6000(0) = 400$ ------ [A1]

Therefore, the rabbits would not become extinct in the long run.

Solutions to this question by accurate drawing will not be accepted.

7.



The diagram shows a trapezium $ABCD$ in which AD is parallel to BC. The point A is (2, 7), the point B is (8, 4) and the point D is (-2, 1). The point E is on BC such that DE passes through O.

(i) Show that $ABED$ is a parallelogram. [2]

(ii) Find the coordinates of E. [2]

Given that $CD = CE$,

(iii) find the coordinates of C. [4]

(iv) find the ratio of the area of triangle CDE to the area of $ABED$. [2]

Ans:

(i) $AD \parallel BE$ (given)

Gradient of $AB = \dfrac{4-7}{8-2} = -\dfrac{1}{2}$

Gradient of $DE = \dfrac{0-1}{0-(-2)} = -\dfrac{1}{2}$

$AB \parallel DE$ ------ [M1]

With two pairs of parallel opposite sides, $ABED$ is a parallelogram. ------ [A1]

(ii) Let the coordinates of E be (x, y)

$(\dfrac{x+2}{2}, \dfrac{y+7}{2}) = (\dfrac{-2+8}{2}, \dfrac{4+1}{2})$ ------ [M1]

$x+2=6, y+7=5$

$x=4, y=-2$

E (4, -2) ------ [A1]

(iii)Since $CD = CE$, C lies on the perpendicular bisector of DE.

Gradient of $DE = -\dfrac{1}{2}$

Gradient of perpendicular bisector$= \dfrac{-1}{(-\frac{1}{2})} = 2$ ------ [M1]

Let the equation of the perpendicular bisector be $y = 2x + c$

Midpoint of DE is $(\dfrac{-2+4}{2}, \dfrac{1+(-2)}{2}) = (1, -\dfrac{1}{2})$

$-\dfrac{1}{2} = 2 \times 1 + c, c = -\dfrac{5}{2}$

The equation is $y = 2x - \frac{5}{2}$ ------ [M1]

Gradient of $BE = \frac{4-(-2)}{8-4} = \frac{3}{2}$

Let the equation of BE be $y = \frac{3}{2}x + c$

At (4, -2), $-2 = \frac{3}{2}(4) + c, c = -8$

The equation is $y = \frac{3}{2}x - 8$ ------ [M1]

$y = 2x - \frac{5}{2}$

$y = \frac{3}{2}x - 8$

$2x - \frac{5}{2} = \frac{3}{2}x - 8$

$\frac{1}{2}x = -5\frac{1}{2}$

$x = -11$

$y = 2(-11) - \frac{5}{2} = -24\frac{1}{2}$

$C(-11, -24\frac{1}{2})$ ------ [A1]

OR

Gradient of $BE = \frac{4-(-2)}{8-4} = \frac{3}{2}$

Let the equation of BE be $y = \frac{3}{2}x + c$

At (4, -2), $-2 = \frac{3}{2}(4) + c, c = -8$

The equation is $y = \frac{3}{2}x - 8$ ------ [M1]

Let the coordinates of C be (x, y).

Since $CD = CE$,

$\sqrt{(x+2)^2 + (y-1)^2} = \sqrt{(x-4)^2 + (y+2)^2}$ ------ [M1]

$x^2 + 4x + 4 + y^2 - 2y + 1 = x^2 - 8x + 16 + y^2 + 4y + 4$

$12x - 6y = 15$

$12x - 6(\frac{3}{2}x - 8) = 15$ ------ [M1]

$3x = -33$

$x = -11$

$y = \frac{3}{2}(-11) - 8 = -24\frac{1}{2}$

$C(-11, -24\frac{1}{2})$ ------ [A1]

(iv) Area of $CDE = \frac{1}{2}\begin{vmatrix} 4 & -2 & -11 & 4 \\ -2 & 1 & -24\frac{1}{2} & -2 \end{vmatrix}$

$= \frac{1}{2}[4 + 49 + 22 - 4 - (-11) - (-98)]$

$= 90$ units^2

Area of $ABED = \frac{1}{2}\begin{vmatrix} 2 & -2 & 4 & 8 & 2 \\ 7 & 1 & -2 & 4 & 7 \end{vmatrix}$

$= \frac{1}{2}[2 + 4 + 16 + 56 - (-14) - 4 - (-16) - 8]$

$= 48$ units^2 ------ [M1]

Area of CDE : Area of $ABED = 90 : 48 = 15 : 8$ ------ [A1]

8. (a) (i) Without using a calculator, prove that $\cot(45° - A) = \dfrac{\cot A + 1}{\cot A - 1}$. [3]

(ii) Hence find the exact value of $\cot 15°$. [2]

(b) Find an expression for f(x) such that

$$f'(x) = 3\sin^2(5x - \frac{\pi}{4}) + \cos^2 x - \tan^2 \frac{1}{2}x.$$ [3]

Ans:

(a) (i) LHS $= \dfrac{1}{\tan(45° - A)}$

$$= \frac{1 + \tan 45° \tan A}{\tan 45° - \tan A}$$

$$= \frac{1 + \tan A}{1 - \tan A} \text{ ------ [M1]}$$

$$= \frac{1 + \dfrac{\sin A}{\cos A}}{1 - \dfrac{\sin A}{\cos A}}$$

$$= \frac{\dfrac{\cos A + \sin A}{\cos A}}{\dfrac{\cos A - \sin A}{\cos A}}$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A}$$

$$= \frac{\dfrac{\cos A + \sin A}{\sin A}}{\dfrac{\cos A - \sin A}{\sin A}} \text{ ------ [M1]}$$

$= \dfrac{\cot A+1}{\cot A-1}$ = RHS (proven) ------ [A1]

(ii) $\cot 15° = \cot(45° - 30°)$

$= \dfrac{\cot 30° + 1}{\cot 30° - 1}$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \dfrac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ ------ [M1]

$= \dfrac{4+2\sqrt{3}}{2}$

$= 2+\sqrt{3}$ ------ [A1]

(b) $f'(x) = 3\sin^2(5x-\frac{\pi}{4})+\cos^2 x-\tan^2\frac{1}{2}x$

$= -\frac{3}{2}[-2\sin^2(5x-\frac{\pi}{4})]+\frac{1}{2}[2\cos^2 x]-(\sec^2\frac{1}{2}x-1)$

$= -\frac{3}{2}[1-2\sin^2(5x-\frac{\pi}{4})]+\frac{3}{2}+\frac{1}{2}[2\cos^2 x-1]+\frac{1}{2}-\sec^2\frac{1}{2}x+1$ ------ [M1]

$= -\frac{3}{2}\cos(10x-\frac{\pi}{2})+\frac{1}{2}\cos 2x-\sec^2\frac{1}{2}x+3$ ------ [M1]

$f(x) = \int(3\sin^2(5x-\frac{\pi}{4})+\cos^2 x-\tan^2\frac{1}{2}x)dx$

$= \int[-\frac{3}{2}\cos(10x-\frac{\pi}{2})+\frac{1}{2}\cos 2x-\sec^2\frac{1}{2}x+3]dx$

$= -\frac{3}{20}\cos(10x-\frac{\pi}{2})+\frac{1}{4}\sin 2x-2\tan\frac{1}{2}x+3x+c$ ------ [A1]

9. (i) Express $\frac{8x-5}{x^2(1-x)}$ as the sum of 3 partial fractions. [4]

(ii) Hence find $\int \frac{8x-5}{x^2(1-x)}dx$. [2]

Ans:

(i) $\frac{8x-5}{x^2(1-x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{1-x}$ ------ [M1]

$8x-5 = Ax(1-x) + B(1-x) + Cx^2$

Let $x = 1$, $3 = C$

Let $x = 0$, $-5 = B$ ------ [M1]

Let $x = 2$, $11 = 2A + (-5)(-1) + 3(2^2)$, $A = 3$ ------ [M1]

$\frac{8x-5}{x^2(1-x)} = \frac{3}{x} - \frac{5}{x^2} + \frac{3}{1-x}$ ------ [A1]

(ii) $\int \frac{8x-5}{x^2(1-x)}dx = \int (\frac{3}{x} - \frac{5}{x^2} + \frac{3}{1-x})dx$

$= \int \frac{3}{x}dx - \int \frac{5}{x^2}dx + \int \frac{3}{1-x}dx$

$= 3\ln x - (-5x^{-1}) + [-3\ln(1-x)] + C$ ------ [M1]

$= 3\ln x + \frac{5}{x} - 3\ln(1-x) + C$

$= 3[\ln x - \ln(1-x)] + \frac{5}{x} + C$

$= 3\ln \frac{x}{1-x} + \frac{5}{x} + C$ ------ [A1]

10. The equation of a curve is $y = xe^{-x}$.

(i) Find the set of values of x for which y is an increasing function of x. [2]

(ii) Find the coordinates of the turning point and determine whether the turning point is a maximum or minimum. [2]

Ans:

(i) $\frac{dy}{dx} = e^{-x} + x(-e^{-x})$

$= e^{-x}(1-x)$ ------ [M1]

For increasing function, $\frac{dy}{dx} > 0$

$e^{-x}(1-x) > 0$

Since $e^{-x} > 0, 1-x > 0$

$x < 1$ ------ [A1]

(ii) $\frac{dy}{dx} = 0$

$e^{-x}(1-x) = 0$

$1-x = 0$

$x = 1$

$y = 1(e^{-1}) = \frac{1}{e}$

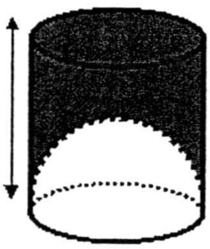
Coordinates of turning point is $(1, \frac{1}{e})$ ------ [A1]

Use 1st derivative test:

x	1-	1	1+
$\frac{dy}{dx}$	+ve	0	-ve
Gradient			

The point is a maximum point. ------ [A1]

11. The diagram shows a solid container consisting of a cylinder with a hemisphere dug out. The radius and height of the cylinder are r cm and h cm respectively.



(i) Express h in terms of r given that the external curved surface area of the cylindrical part of the solid is 1200π cm^2. [2]

(ii) Express the volume, V cm^3, of the container in terms of r. [2]

(iii) The solid is heated and it expands at a rate of 0.81 cm^3/s. Find the rate at which its radius increases when the height is 60 cm. [3]

Ans:

(i) $2\pi rh = 1200\pi$ ------ [M1]

$rh = 600$

$h = \dfrac{600}{r}$ ------ [A1]

(ii) $V = \pi r^2 h - \dfrac{2}{3}\pi r^3$ ------ [M1]

$= \pi r^2 \dfrac{600}{r} - \dfrac{2}{3}\pi r^3$

$= 600\pi r - \dfrac{2}{3}\pi r^3$ ------ [A1]

(iii) $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ ------ [M1]

$V=600\pi r-\frac{2}{3}\pi r^3$

$\frac{dV}{dr}=600\pi-2\pi r^2$ ------ [M1]

When $h=60$, $r=10$

$\frac{dV}{dr}=600\pi-2\pi(10)^2=400\pi$

$0.81=400\pi\times\frac{dr}{dt}$

$\frac{dr}{dt}=\frac{0.81}{400\pi}=0.000645$ ------ [A1]

Answer the whole of this question on a piece of graph paper.

12. The table shows experimental values of the two variables, x and y.

x	0.5	1.5	3	4.5	5.5	6
y	4.43	5.29	7.44	11.4	15.7	18.7

It is known that x and y are related by an equation of the form $y=ab^x+e$, where a and b are constants.

(i) Explain how a straight line graph may be drawn to represent the given data. [2]

(ii) Draw this graph for the given data and us it to estimate the values of a and of b. [4]

(iii) By inserting another suitable line on your graph, solve the equation

$ab^x=5e^{-\frac{x}{2}}$. [3]

Ans:

(i) $y = ab^x - e$

$y - e = ab^x$

$\ln(y-e) = \ln(ab^x)$ ------ [M1]

$\ln(y-e) = \ln a + x\ln b$

Let $Y = \ln(y-e)$ and $X = x$

$Y = \ln a + X\ln b$

If $\ln(y-e)$ is plotted against x, a straight line graph can be obtained. ------ [A1]

(ii) From the graph

$\ln a = 0.35$, $a = e^{0.35} = 1.42$ ------ [A1] (Answer range: 0.3352 ± 0.02)

$\ln b = \dfrac{2.35-0.75}{5-1} = 0.4(\pm 0.02)$, $b = e^{0.4} = 1.49$ ------ [A1]

(iii) $ab^x = 5e^{-\frac{x}{2}}$

$\ln ab^x = \ln 5e^{-\frac{x}{2}}$

$\ln a + x\ln b = \ln 5 - \dfrac{x}{2}$

$\ln(y-e) = \ln 5 - \dfrac{x}{2}$ ------ [M1]

Equation of line to be inserted: $Y = \ln 5 - \dfrac{1}{2}X$

At the point of intersection, $x = 1.40 \pm 0.02$ ----- [A1]

END OF PAPER

中正中学 义顺

CHUNG CHENG HIGH SCHOOL YISHUN

Preliminary Examination (2015)
Secondary 4 Express/ 5 Normal (Academic)

Candidate			
	Name	Register No	Class

For examiner's use
/ 100

ADDITIONAL MATHEMATICS
4047/ 02

Date: **27 August 2015**
Duration: **2 hours 30 minutes**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give your answer in the simplest form. Leave your answer in fraction where applicable.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: Woo Huey Ming

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin c$$

1. (a) Solve the equation $\log_3 4 - \log_9(x^2 + 4x + 4) = \log_{\frac{1}{3}} x$. [4]

(b) Sketch the graph of $y = e^{-x}$. In order to solve the equation $\ln\left(\frac{1}{\sqrt{x-3}}\right) = \frac{1}{2}x$,

a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = e^{-x}$. Find the equation of the straight line. [3]

2. The roots of the equation $2x^2 - px - q = 0$, where p and q are constants, are α and β.
The roots of the equation $4x^2 + qx - 3x = p - 1$ are $\alpha - 1$ and $\beta - 1$.
(i) Find the value of p and of q. [5]
(ii) Find a quadratic equation whose roots are α^3 and β^3. [3]

3. (i) Given that the coefficient of x^{-2} in the expansion of $\left(\frac{1}{x} + px\right)^8$ is 448, find the value of the positive constant p. [3]
(ii) Using the value of p in part (i), find the term independent of x in the expansion of

$\left(\frac{1}{x} + px\right)^8 (5x^2 - 4)$. [4]

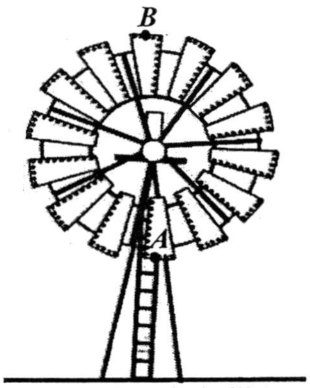
4. A curve has the equation $y = \frac{\ln(2x)^3}{x^2}$ for $x > 0$.

(i) Find an expression for $\frac{dy}{dx}$. [3]

(ii) Hence find $\int \frac{\ln 2x}{x^3} dx$. [4]

5. The equation of a circle C_1 is given as $x^2 + y^2 - 16x + 8y + 64 = 0$.
(i) Find the coordinates of the centre and radius of the circle C_1. [3]
(ii) The line $y = k$ is a tangent to the circle at A, where $k \neq 0$. Find the value of k. [2]
(iii) The tangent to the circle at $B(4, -4)$ intersects $y = k$ at point C. Find equation of this tangent. [1]
(iv) Explain why a circle C_2 can be drawn through the points A, B and C with AB being the diameter. [1]
(v) Find the equation of the circle C_2. [3]
(vi) Determine, with working, whether $(3\frac{3}{5}, -6)$ lies within the 2 circles. [2]

6.



The height of a blade on the windmill (measured from the ground) can be modelled by the equation $h = 15 - 7\cos kt$ where k is a constant and t is the time in seconds after the windmill starts moving. The windmill starts rotating from the lowest point, A, when $t = 0$. The windmill rotates at a rate of 12 revolutions per minute.

(i) Explain why this model suggests that the highest point of the windmill, B, is 22 m above the ground level. [1]

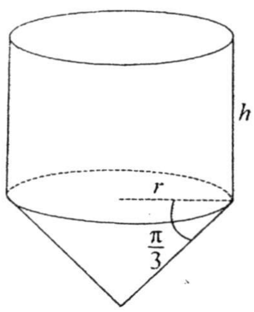
(ii) Find the value of k. [1]

(iii) For how long over the course of one complete revolution will the point A be at least 17 m above ground level? [2]

(iv) Explain how the solution in part **(iii)** could be used to find the duration of the point A being at least 17 m above ground level over the course of **two** complete revolutions. [2]

(v) Suggest a possible equation of how the height of a blade varies against time if the windmill starts rotating from the highest point at B. [1]

7.



The diagram shows a solid machine part that is made up of a closed cylinder joined to an inverted right circular cone. The height of the cylinder is h m and the slant height of the cone makes an angle of $\frac{\pi}{3}$ radians to its base radius, r m.

(i) Given that the volume of the machine part is 50π m^3, express h in terms of r. [2]

(ii) Show that the total surface area of the machine part is given by [4]

$$A = \frac{\pi r^2}{3}(9 - 2\sqrt{3}) + \frac{100\pi}{r}.$$

(iii) Given that r can vary, find the value of r for which the total surface area of the machine part is stationary. [3]

(iv) By comparing gradients, explain why this value of r gives the least total surface area possible. [2]

8. The equation of two curves are $y = \cos 2x - 2\sin^2 x$ and $y = \sin 2x$.

(i) Show that the x-coordinate of the points of intersection of the two curves satisfy $2\cos 2x - \sin 2x = 1$. [1]

(ii) On the same axes sketch, for $-\pi < x < \pi$, the graphs of $y = \cos 2x - 2\sin^2 x$ and $y = \sin 2x$. [4]

(iii)Express the equation $2\cos 2x - \sin 2x = 1$ in the form $\cos(2x + \alpha) = k$, where α and k are constants to be found. [4]

(iv)Hence find, in radians, the x-coordinates of the points of intersection for $-\pi < x < \pi$. [3]

9. A particle travels in a straight line from a fixed point O with acceleration a m/s^2, given by $a = 8t - k$ where t is the time in seconds after passing O, and k is a constant. The velocity of the particle is 5 m/s when it passes O, and at $t = 2$, its velocity is -21 m/s.

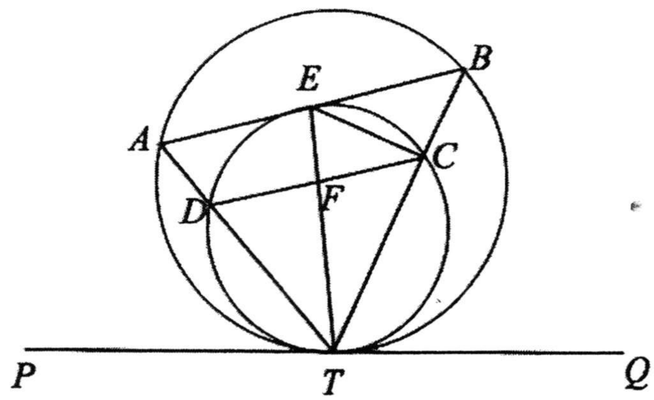
(i) Find the value of k. [3]

(ii) Find the value(s) of t when the particle is instantaneously at rest. [2]

(iii)Calculate the average speed of the particle during the first six seconds. [3]

(iv)Describe completely the motion of the particle in the first six seconds. [2]

10.



In the diagram, two circles touch each other at T and PTQ is their common tangent. AB is a tangent to the smaller circle at E. AT and BT cut the smaller circle at D and C respectively. ET and CD intersect at F. Prove that

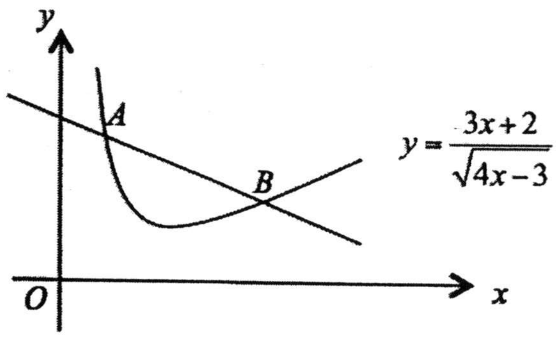
(i) $AB // DC$, [2]

(ii) $\angle ATE = \angle BTE$, [3]

(iii) $ET^2 = CT \times DT + EF \times ET$. [3]

11. **(i)**Differentiate $(x+2)\sqrt{4x-3}$ with respect to x. [2]

(ii)



The diagram shows part of the curve $y=\dfrac{3x+2}{\sqrt{4x-3}}$. A line with gradient $-\dfrac{2}{3}$ intersects the curve at $A(1,5)$ and B.

(a) Verify that the y-coordinate of B is $\dfrac{11}{3}$. [5]

(b) Determine the area of the region bounded by the curve and the line AB. [4]

中正中学 义顺
CHUNG CHENG HIGH SCHOOL YISHUN

Preliminary Examination (2015)
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Candidate			
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For examiner's use
/ 100

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4047/ 02

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Setter: Woo Huey Ming

This paper consists of **6** printed pages, INCLUDING the cover page. **[Turn over**

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$$\Delta = \tfrac{1}{2} ab \sin c$$

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$\log_3 4 - \log_9(x^2+4x+4) = \log_{\frac{1}{3}} x$

$\log_3 4 - \dfrac{2\log_3(x+2)}{\log_3 9} = \dfrac{\log_3 x}{\log_3 \frac{1}{3}}$ M1

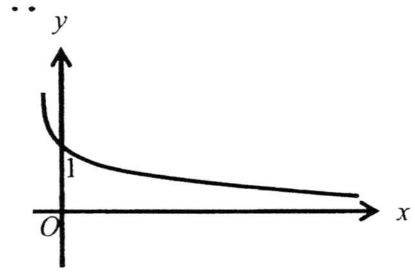
$\log_3 4 - \log_3(x+2) = -\log_3 x$ M1

$\dfrac{4}{x+2} = \dfrac{1}{x}$ M1

$3x = 2$

$x = \dfrac{2}{3}$ A1

(b) Sketch the graph of $y = e^{-x}$. In order to solve the equation $\ln\left(\dfrac{1}{\sqrt{x-3}}\right) = \dfrac{1}{2}x$, a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = e^{-x}$. Find the equation of the straight line. [3]



B1

$\ln\left(\dfrac{1}{\sqrt{x-3}}\right) = \dfrac{1}{2}x$

$\ln 1 - \dfrac{1}{2}\ln(x-3) = \dfrac{1}{2}x$

$\ln(x-3) = -x$ M1

$x - 3 = e^{-x}$

Draw $y = x - 3$. A1

2. The roots of the equation $2x^2 - px - q = 0$, where p and q are constants, are α and β.
The roots of the equation $4x^2 + qx - 3x = p - 1$ are $\alpha - 1$ and $\beta - 1$.
(i) Find the value of p and of q. [5]

$$\alpha + \beta = \frac{p}{2} --- (1)$$

M1

$$\alpha\beta = -\frac{q}{2} --- (2)$$

$$\alpha - 1 + \beta - 1 = \frac{3-q}{4}$$

$$\alpha + \beta = \frac{3-q}{4} + 2 --- (3)$$

$$(\alpha - 1)(\beta - 1) = \frac{1-p}{4} --- (4)$$ M1

Sub (1) into (4)

$$\alpha\beta\text{-}(\alpha + \beta) + 1 = \frac{1}{4} - \frac{1}{4}(2)(\alpha + \beta)$$

$$\alpha\beta - \frac{1}{2}(\alpha + \beta) + \frac{3}{4} = 0 --- (5)$$

Sub (2) and (3) into (5)

$$-\frac{q}{2} - \frac{1}{2}\left(\frac{3-q}{4} + 2\right) + \frac{3}{4} = 0$$ M1

$$-\frac{3q}{8} = \frac{5}{8}$$

$$q = -1\frac{2}{3}$$ A1

Sub $q = -1\frac{2}{3}$ into (3)

$$\alpha\text{+}\beta\text{=}\frac{3 - \left(-\frac{5}{3}\right)}{4} + 2 = 3\frac{1}{6}$$

$$p = 2\left(3\frac{1}{6}\right) = 6\frac{1}{3}$$

$$p = 6\frac{1}{3}, q = -1\frac{2}{3}$$ A1

(ii) Find a quadratic equation whose roots are α^3 and β^3. [3]

$(\alpha\beta)^3 = \dfrac{125}{216}$ M1

$\alpha^2+\beta^2 = \left(3\dfrac{1}{6}\right)^2 - 2\left(\dfrac{5}{6}\right) = \dfrac{301}{36}$

$\alpha^3+\beta^3 = 3\dfrac{1}{6}\left(\dfrac{301}{36}-\dfrac{5}{6}\right) = \dfrac{5149}{216}$ M1

$x^2 - \dfrac{5149}{216}x + \dfrac{125}{216} = 0$

$216x^2 - 5149x + 125 = 0$ A1

3. **(i)** Given that the coefficient of x^{-2} in the expansion of $\left(\frac{1}{x}+px\right)^8$ is 448, find the value of the positive constant p. [3]

$$T_{r+1}=\binom{8}{r}\left(\frac{1}{x}\right)^{8-r}(px)^r$$

$$=\binom{8}{r}x^{r-8}p^r x^r$$

$$=\binom{8}{r}p^r x^{2r-8} \qquad \text{M1}$$

$2r-8=-2$

$r=3$

$$T_4=\binom{8}{3}p^3x^{-2}$$

$=56p^3x^{-2}$

$56p^3=448$ M1

$p^3=8$

$p=2$ A1

(ii) Using the value of p in part (i), find the term independent of x in the expansion of $\left(\frac{1}{x}+px\right)^8(5x^2-4)$. [4]

$2r-8=0$

$r=4$

$$T_5=\binom{8}{4}2^4=1120 \qquad \text{M1}$$

$$\left(\frac{1}{x}+2x\right)^8(5x^2-4)$$

$=(\ldots+448x^{-2}+1120+\ldots)(5x^2-4)$ M1

Term independent of x

$=448(5)+1120(-4)$ M1

$=-2240$ A1

4. A curve has the equation $y = \dfrac{\ln(2x)^3}{x^2}$ for $x > 0$.

(i) Find an expression for $\dfrac{dy}{dx}$. [3]

$$\frac{dy}{dx} = \frac{3x - 6x\ln 2x}{x^4} \qquad \text{M2}$$

$$= \frac{3}{x^3} - \frac{6}{x^3}\ln 2x \qquad \text{A1}$$

(ii) Hence find $\int \dfrac{\ln 2x}{x^3}\,dx$. [4]

$$\int\left(\frac{3}{x^3} - \frac{6\ln 2x}{x^3}\right)dx = \frac{\ln(2x)^3}{x^2} + C_1 \qquad \text{M1}$$

$$\int \frac{6\ln 2x}{x^3} = \int \frac{3}{x^3}dx - \frac{\ln(2x)^3}{x^2} + C_2$$

$$\int \frac{\ln 2x}{x^3}dx = \frac{1}{6}\int 3x^{-3}\,dx - \frac{\ln 2x}{2x^2} + C \qquad \text{M1}$$

$$= \frac{1}{6}\left(\frac{3x^{-2}}{-2}\right) - \frac{\ln 2x}{2x^2} + C \qquad \text{M1}$$

$$= -\frac{1}{4x^2} - \frac{\ln 2x}{2x^2} + C \qquad \text{A1}$$

5. The equation of a circle C_1 is given as $x^2+y^2-16x+8y+64=0$.

(i) Find the coordinates of the centre and radius of the circle C_1. [3]

$(x-8)^2-(-8)^2+(y+4)^2-(4)^2=-64$ M1

$(x-8)^2-(y+4)^2=4^2$

Centre of circle= (8, -4) A1

Radius= 4 units A1

(ii) The line $y=k$ is a tangent to the circle at A, where $k\neq 0$. Find the value of k. [2]

The 2 tangents to circle are $y=0$ and $y=-8$. B1

Since $k\neq 0$, $k=-8$. B1

(iii) The tangent to the circle at $B(4, -4)$ intersects $y=k$ at point C. Find equation of this tangent. [1]

$x=4$ B1

(iv) Explain why a circle C_2 can be drawn through the points A, B and C with AB being the diameter. [1]

$A(8,-8), B(4,-4), C(4,-8)$

Since $AC\perp BC$, $\angle BCA=90°$

$\therefore$ A circle C_2 can be drawn through the points A, B and C with AB being the diameter.

(Angle in a semicircle) R1

(v) Find the equation of the circle C_2. [3]

Midpoint of AB

$=\left(\frac{8+4}{2},\frac{-8-4}{2}\right)$ M1

$=(6,-6)$

Radius

$=\frac{1}{2}\sqrt{(8-4)^2+(-8-(-4))^2}$ M1

$=2\sqrt{2}$ units A1

$(x-6)^2+(y+6)^2=8$

(vi) Determine, with working, whether $(3\frac{3}{5}, -6)$ lies within the 2 circles. [2]

Length of $(3.6,-6)$ to centre of C_1

$= \sqrt{(8-3.6)^2 + (-4+6)^2}$

$= 4.83$ units

Length of $(3.6,-6)$ to centre of C_2

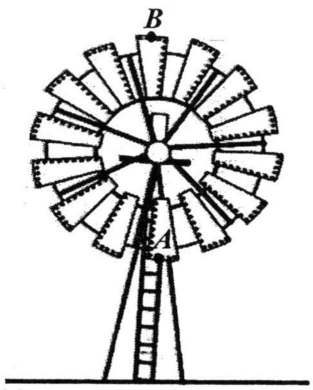
$= \sqrt{(6-3.6)^2 + (-6-(-6))^2}$

$= 2.4$ units

Since length of $(3.6,-6)$ to centre of C_1 >4 units, (3.6, -6) is outside C_1. R1

Since length of $(3.6,-6)$ to centre of $C_2 < 2\sqrt{2}$ units, (3.6, -6) is within C_2. R1

6.



The height of a blade on the windmill (measured from the ground) can be modelled by the equation $h = 15 - 7\cos kt$ where k is a constant and t is the time in seconds after the windmill starts moving. The windmill starts rotating from the lowest point, A, when $t = 0$. The windmill rotates at a rate of 12 revolutions per minute.

(i) Explain why this model suggests that the highest point of the windmill, B, is 22 m above the ground level. [1]

$-1 \le \cos kt \le 1$

$-7 \le -7\cos kt \le 7$

$15 - 7 \le 15 - 7\cos kt \le 15 + 7$

$8 \le 15 - 7\cos kt \le 22$ B1

$\therefore$ The highest point of the windmill is 22 m above the ground level.

(ii) Find the value of k. [1]

Period = 5 seconds

$k = \dfrac{2\pi}{5}$ B1

(iii) For how long over the course of one complete revolution will the point A be at least 17 m above ground level? [2]

$15 - 7\cos\dfrac{2\pi}{5}t = 17$

$\cos\dfrac{2\pi}{5}t = -\dfrac{2}{7}$

Basic angle= 1.281044625

$\dfrac{2\pi}{5}t = 1.860548028, 4.422637279$

$t = 1.480577078, 3.519422922$ M1

Length of time= 3.519422922-1.480577078

=2.04 seconds A1

(iv) Explain how the solution in part **(iii)** could be used to find the duration of the point A being at least 17 m above ground level over the course of **two** complete revolutions. [2]

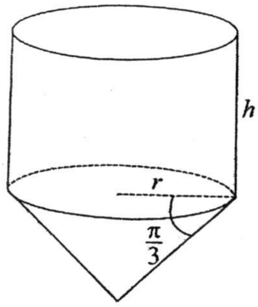
In the first revolution, point A is at least 17 m above the ground level for 2.04 seconds. Since **<u>the second revolution is identical to the first</u>**, the total time for point A to be at least 17 m above the ground = 2(3.519422922 -1.480577078)= 4.08 seconds. R1

B1

(v) Suggest a possible equation of how the height of a blade varies against time if the windmill starts rotating from the highest point at B. [1]

$h = 15 + 7\cos\frac{2\pi}{5}t$ B1

7.



The diagram shows a solid machine part that is made up of a closed cylinder joined to an inverted right circular cone. The height of the cylinder is h m and the slant height of the cone makes an angle of $\frac{\pi}{3}$ radians to its base radius, r m.

(i) Given that the volume of the machine part is 50π m^3, express h in terms of r. [2]

Let the height of the cone be a metre.

$\tan\frac{\pi}{3} = \frac{a}{r}$

$a = \sqrt{3}r$

$50\pi = \pi r^2 h + \frac{1}{3}\pi r^2\left(\sqrt{3}r\right)$ M1

$h = \frac{50}{r^2} - \frac{\sqrt{3}}{3}r$ A1

A1

(ii) Show that the total surface area of the machine part is given by [4]

$$A=\frac{\pi r^2}{3}(9-2\sqrt{3})+\frac{100\pi}{r}.$$

Let the slant height of the cone be p metre.

$$\cos\frac{\pi}{3}=\frac{r}{p}$$

$$p=2r$$

$$A=\pi r^2+2\pi rh+\pi r(2r) \qquad \text{M1}$$

$$=\pi r^2+2\pi r\left(\frac{50}{r^2}-\frac{\sqrt{3}}{3}r\right)+2\pi r^2 \qquad \text{M1}$$

$$=3\pi r^2+\frac{100\pi}{r}-\frac{2\sqrt{3}}{3}\pi r^2 \qquad \text{M1}$$

$$=\frac{\pi r^2}{3}(9-2\sqrt{3})+\frac{100\pi}{r} \qquad \text{A1}$$

(iii) Given that r can vary, find the value of r for which the total surface area of the machine part is stationary. [3]

$$\frac{dA}{dr}=\frac{2\pi r}{3}(9-2\sqrt{3})-\frac{100\pi}{r^2} \qquad \text{M1}$$

when $\frac{dA}{dr}=0$

$$\frac{2r}{3}(9-2\sqrt{3})=\frac{100}{r^2}$$

$$r^3=\frac{300}{2(9-2\sqrt{3})} \qquad \text{M1}$$

$r=3.00$ m (3 s.f.) A1

(iv) By comparing gradients, explain why this value of r gives the least total surface area possible. [2]

M1

r	2.99	3.00	3.01
Sketch			

Since $\frac{dA}{dr}$ **changes sign from negative to positive** as r increases through the stationary point. The total surface area is the least when $r=3.00$ m. R1

8. The equation of two curves are $y = \cos 2x - 2\sin^2 x$ and $y = \sin 2x$.

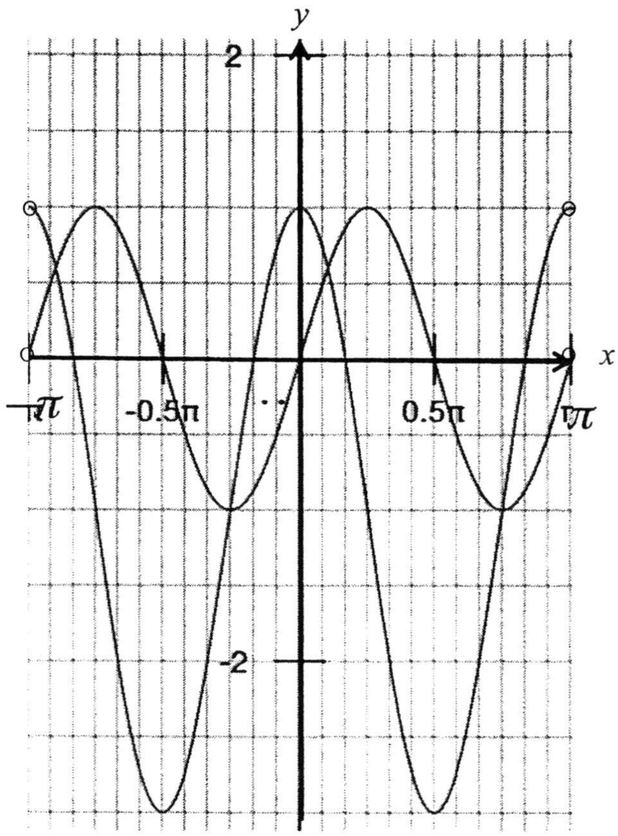
(i) Show that the x-coordinate of the points of intersection of the two curves satisfy $2\cos 2x - \sin 2x = 1$. [1]

$\cos 2x - 2\sin^2 x = \sin 2x$

$\cos 2x + \cos 2x - 1 = \sin 2x$

$2\cos 2x - \sin 2x = 1$ B1

(ii) On the same axes sketch, for $-\pi < x < \pi$, the graphs of $y = \cos 2x - 2\sin^2 x$ and $y = \sin 2x$. [4]



C1: x,y-intercepts
C1: Maximum and Minimum Points

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C1: Maximum and Minimum Points

(iii)Express the equation $2\cos 2x - \sin 2x = 1$ in the form $\cos(2x+\alpha) = k$, where α and k are constants to be found. [4]

$2\cos 2x - \sin 2x = 1$

$2\cos 2x - \sin 2x = R\cos(2x+\alpha)$

$= R\cos 2x \cos\alpha - R\sin 2x \sin\alpha$

$R\cos\alpha = 2, R\sin\alpha = 1$ M1

$\tan\alpha = \frac{1}{2} \Rightarrow \alpha = 0.463647609$ M1

$R = \sqrt{5}$ M1

$\sqrt{5}\cos(2x+0.464) = 1$

$\cos(2x+0.464) = \frac{\sqrt{5}}{5}$ A1

(iv)Hence find, in radians, the x-coordinates of the points of intersection for $-\pi < x < \pi$. [3]

$\cos(2x+0.463647609) = \frac{\sqrt{5}}{5}$

Basic angle= 1.107148718

$2x + 0.463647609 = 1.107148718, 5.176036589, -1.107148718, -5.176036589$ M1

$x = 0.322, 2.36, -0.785, -2.82$ A2

Minus 1 for each error

9. A particle travels in a straight line from a fixed point O with acceleration a m/s^2, given by $a = 8t - k$ where t is the time in seconds after passing O, and k is a constant. The velocity of the particle is 5 m/s when it passes O, and at $t = 2$, its velocity is – 21 m/s.

(i) Find the value of k. [3]

$a = 8t - k$

$v = \int (8t - k)\,dt$

$= 4t^2 - kt + c$ M1

When $t = 0, v = 5, c = 5.$ M1

When $t = 2, v = -21,$

$-21 = 4(2)^2 - 2k + 5$

$k = 21$ A1

(ii) Find the value(s) of t when the particle is instantaneously at rest. [2]

When $v = 0,$

$4t^2 - 21t + 5 = 0$

$(4t - 1)(t - 5) = 0$ M1

$t = 0.25$ or $t = 5$ A1

(iii)Calculate the average speed of the particle during the first six seconds. [3]

$s = \int (4t^2 - 21t + 5)\,dt$

$= \frac{4}{3}t^3 - \frac{21}{2}t^2 + 5t + C_1$

When $t = 0, s = 0, C_1 = 0,$

$s = \frac{4}{3}t^3 - \frac{21}{2}t^2 + 5t$ M1

When $t = 0, s = 0$ m,

When $t = 0.25, s = 0.164583333$ m,

When $t = 5, s = -70\frac{5}{6}$ m,

When $t = 6, s = -60$ m,

Average speed during the first 6 seconds

$= \dfrac{0.614583333 + (0.614583333 + 70\frac{5}{6}) + (70\frac{5}{6} - 60)}{6}$ M1

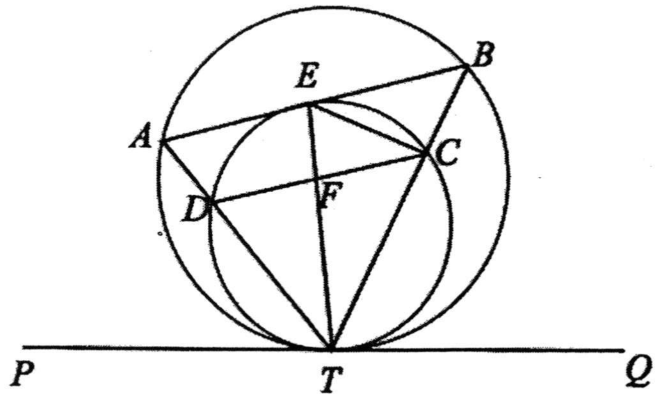
$= 13.8$ m/s A1

(iv)Describe completely the motion of the particle in the first six seconds. [2]

The particle starts at a fixed point O. At $t = 0.25$, the particle stops and reverses its direction of motion. At $t = 5$, the particle stops again and reverses its direction of motion, moving toward O. At $t = 6$, the particle has a displacement of -60 m from O.

R2 for 3 points
R1 for 2 points

10.



In the diagram, two circles touch each other at T and PTQ is their common tangent. AB is a tangent to the smaller circle at E. AT and BT cut the smaller circle at D and C respectively. ET and CD intersect at F. Prove that

(i) $AB // DC$, [2]

$\angle BTQ = \angle TAB$ (Alternate Segment Theorem)

$\angle BTQ = \angle TDC$ (Alternate Segment Theorem) R1

$\therefore \angle TAB = \angle TDC, AB // DC$ (Corr. $\angle s$) R1

(ii) $\angle ATE = \angle BTE$, [3]

$\angle ATE = \angle ECF$ ($\angle s$ in same segment) R1

$= \angle BEC$ (alt. $\angle$s, AB//DC) R1

$= \angle BTE$ (alternate segment theorem) R1

(iii) $ET^2 = CT \times DT + EF \times ET$. [3]

$\angle TDF = \angle TEC$ ($\angle$s in same segment)

$\angle DTF = \angle ETC$ (part (*ii*))

ΔDFT and ΔECT are similar. (2 pairs of corr. $\angle$s are equal) R1

$\frac{FT}{CT} = \frac{DT}{ET}$ R1

$ET \times FT = CT \times DT$

$ET(ET - EF) = CT \times DT$ R1

$ET^2 = CT \times DT + EF \times ET$

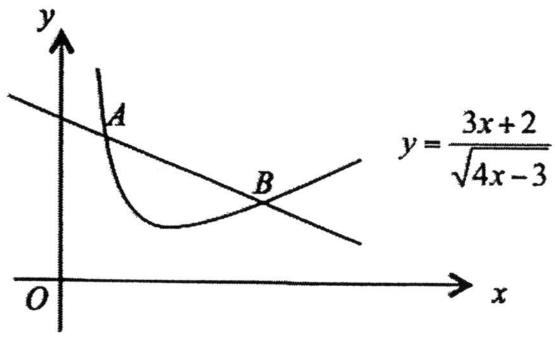
11. (i) Differentiate $(x+2)\sqrt{4x-3}$ with respect to x. [2]

$$\frac{d}{dx}(x+2)\sqrt{4x-3}=\frac{2(x+2)}{\sqrt{4x-3}}+\sqrt{4x-3} \qquad \text{M1}$$

$$=\frac{2(x+2)+4x-3}{\sqrt{4x-3}}$$

$$=\frac{6x+1}{\sqrt{4x-3}} \qquad \text{A1}$$

(ii)



The diagram shows part of the curve $y=\frac{3x+2}{\sqrt{4x-3}}$. A line with gradient $-\frac{2}{3}$ intersects the curve at $A(1,5)$ and B.

(a) Verify that the y-coordinate of B is $\frac{11}{3}$. [5]

Equation of AB:

$$y-5=-\frac{2}{3}(x-1)$$

$$y=-\frac{2}{3}x+\frac{17}{3}$$

$$\frac{3x+2}{\sqrt{4x-3}}=\frac{-2x+17}{3} \qquad \text{M1}$$

$$9x+6=(-2x+17)\sqrt{4x-3}$$

$$81x^2+108x+36=16x^3-12x^2-272x^2+204x+1156x=867$$

$$16x^3-365x^2+1252x-903=0$$

M1

Let $f(x) = 16x^3 - 365x^2 + 1252x - 903$

$f(1) = 0$

$(x-1)$ is a factor of $f(x)$.

$16x^3 - 365x^2 + 1252x - 903 = (x-1)(16x^2 + Bx + 903)$ M1

Comparing coefficient of x^2 :

$-365 = B - 16$

$B = -349$

$(x-1)(16x^2 - 349x + 903) = 0$

$(x-1)(16x - 301)(x-3) = 0$ M1

$x = 1, 3$ or 18.8125(rejected)

when $x = 3, y = \dfrac{3(3)+2}{\sqrt{4(3)-3}} = \dfrac{11}{3}$(shown) A1

(b) Determine the area of the region bounded by the curve and the line *AB*. [4]

Area of region bounded by curve and line *AB*

$$= \frac{1}{2} \times (5 + \frac{11}{3}) \times 2 - \int_1^3 \frac{3x+2}{\sqrt{4x-3}}\,dx$$

$$= 8\frac{2}{3} - \frac{1}{2}\int_1^3 \frac{6x+1}{\sqrt{4x-3}} + \frac{3}{\sqrt{4x-3}}\,dx \qquad \text{M1}$$

$$= 8\frac{2}{3} - \frac{1}{2}[(x+2)\sqrt{4x-3}]_1^3 - \frac{1}{2}\int_1^3 3(4x-3)^{-\frac{1}{2}}\,dx$$

$$= 8\frac{2}{3} - \frac{1}{2}[(x+2)\sqrt{4x-3}]_1^3 - \frac{1}{2}\left[\frac{3(4x-3)^{\frac{1}{2}}}{4\left(\frac{1}{2}\right)}\right]_1^3 \qquad \text{M1}$$

$$= 8\frac{2}{3} - \frac{1}{2}[(x+2)\sqrt{4x-3}]_1^3 - \frac{3}{4}\left[\sqrt{4x-3}\right]_1^3$$

$$= 8\frac{2}{3} - \frac{1}{2}(5\sqrt{9} - 3\sqrt{1}) - \frac{3}{4}(\sqrt{9} - \sqrt{1}) \qquad \text{M1}$$

$$= 1\frac{1}{6}\ \text{units}^2 \qquad \text{A1}$$