NAME: ____________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS   4047/01

Paper 1

Date: 25 August 2015   **Duration: 2 hours**

Additional Materials: Answer Paper
Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

At the end of the examination, fasten all your work securely together.

For Examiner's Use	
Paper 1	/ 80

Setter: Miss Lee CP

This question paper consists of 6 printed pages including the cover page.

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Name: ______________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Name: ______________________________ (   )   Class: __________

1 Given that $y = (1 - \tan^2 x)\cos^2 x$, show that $\frac{dy}{dx} = -2\sin 2x$. [3]

2 Express $\frac{8-3x}{(1-x)^2(2x+3)}$ as a sum of 3 partial fractions. [5]

3 The pressure, P, and volume, V, of a gas in a container are related by the formula $P = \frac{2500}{\sqrt{V^3}}$. If the pressure increases at a rate of 2.8 units/second, find the rate of change of volume when the pressure of the gas is 50 units. [5]

4 Find the term independent of x in the expansion of $(5-4x)\left(\frac{3x^2}{2} + \frac{2}{3x}\right)^9$. [5]

5 The equation of a curve is $y = \ln(5-2x)$, where $x < \frac{5}{2}$.

(i) Find the coordinates of the point on the curve at which the normal to the curve is parallel to $2y = x + 3$. [4]

(ii) Show that as x increases, y is a decreasing function. [2]

6 If $\sin(A+B) = 3\sin(A-B)$, show that $\tan A = 2\tan B$.
Hence, solve the equation $\sin^2(x+60°) = 9\sin^2(x-60°)$ for $0° < x < 360°$. [7]

7 Find all angles , leaving your answer in terms of π, between 0 and 6 which satisfy

(i) $4\sin\frac{x}{2}\cos\frac{x}{2} = \sqrt{3}$ [3]

(ii) $\sin^4 x - \cos^4 x - 3\cos x = 2$. [5]

tol

Name: ______________________________________ (   )   Class: __________

8 **(a)** Given that the curve, $y = 4x^2 + px + p - 6$, find the possible range or value(s) of p for which

(i) the curve intersects the line $y = -3$, [3]

(ii) the line $y = -3$ is a tangent to the curve, [1]

(iii) the curve has a positive y-intercept. [1]

(b) Show that $(m + 1)x^2 + (4m + 3)x + 2m - 1 = 0$ has real and distinct roots for all real values of m. [3]

9 **(i)** Sketch the graph of $y = |2x^2 - 3x - 14|$ for $0 \le x \le 5$. [3]

(ii) Using your graph, find the range or value(s) of k for each of the number of solutions for the equation $|2x^2 - 3x - 14| = k$.

(a) 3 solutions, [1]

(b) 2 solutions, [2]

(c) 1 solution. [1]

10 **Answer the whole of this question on a graph paper.**

The table below shows experimental values of the variables x and y which are related by an equation of the form $y = a^{b+x}$. One value of y has been recorded incorrectly.

x	0.1	0.2	0.3	0.4	0.5
y	5.9	6.9	7.2	9.4	11.0

(i) Plot $\lg y$ against x and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Determine which value of y is inaccurate and estimate the correct value of y. [2]

Name: ______________________________ (   )   Class: __________

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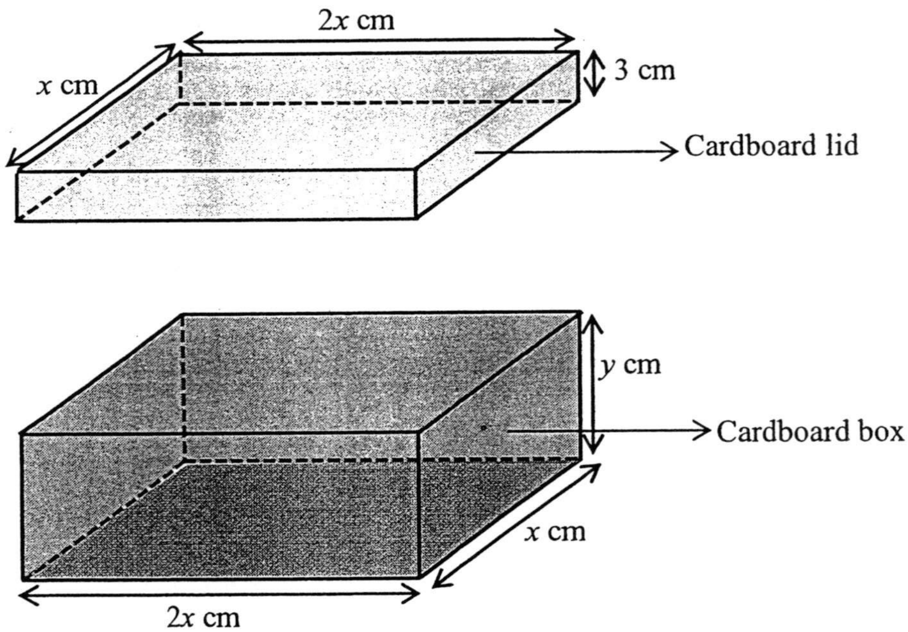

The diagram shows an open cardboard box with a rectangular base and a close fitting cardboard lid which slips over the top of the box.

The dimensions of the lid are $2x$ cm, x cm and 3 cm. The total area of cardboard used in making the box and the lid is 2400 cm^2.

(i) Obtain an expression for y in terms of x, and hence show that the volume, V cm^3 of the box is given by $V = 800x - \dfrac{4x^3}{3} - 6x^2$. [4]

(ii) Given that x can vary, find the value of x for which volume of the box is stationary. Calculate this stationary value of V. [4]

(iii) Explain why this value of x gives the largest volume of the box. [1]

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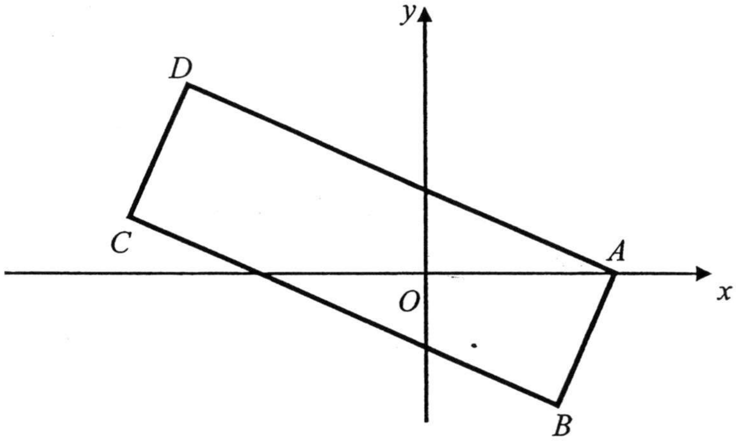
Name: ______________________________ (   )   Class: __________

12 Solutions to this question by accurate drawing will not be accepted.



In the rectangle $ABCD$, the coordinates are $A(3, 0)$, $B(-2t-1, t-2)$ and $C(-5, 1)$.

(i) Show that the value of $t = -1$. [3]

Find

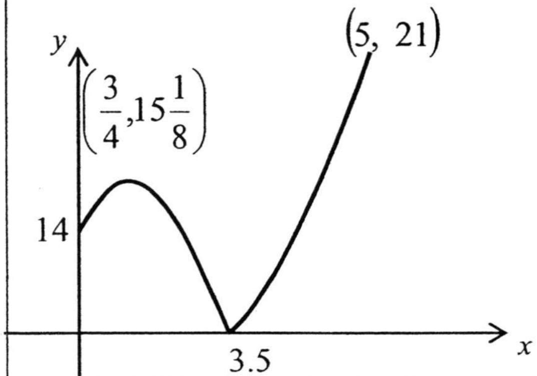
(ii) the coordinate of D, [2]

(iii) the equation of perpendicular bisector of AD, [2]

(iv) the area of $ABCD$. [2]

~ **End of Paper** ~

Sec 4/5 Preliminary Examination 2015
Additional Mathematics Paper 1
Answer Key

2	$\frac{1}{1-x}+\frac{1}{(1-x)^2}+\frac{2}{2x+3}$	3	Rate of change of volume = - 0.507 units/sec
4	$\frac{1190}{9}$	5(i)	(2, 0)
6	73.9°, 253.9°, 40.9°, 220.9°		
7(i)	$\frac{\pi}{3}, \frac{2\pi}{3}$	7(ii)	$x = \frac{2}{3}\pi, \frac{4}{3}\pi$ or $x = \pi$
8(a)(i)	$p \le 0$ or $p \ge 12$	8(a)(ii)	$p = 12$ or $p = 4$
8(a)(iii)	$p > 6$		
9(i)		9(ii)(a) 9(ii)(b) 9(ii)(c)	$14 \le k < 15\frac{1}{8}$ $0 < k < 14$ or $k = 15\frac{1}{8}$ $k = 0$ or $k > 15\frac{1}{8}$
10(ii)	When $x = 0.3$, $y = 7.2$ (erroneous) The correct value of $y = 10^{0.905}$ $= 8.04$ (3 s.f.) Accept 7.76 to 8.14	10(iii)	$a = 10^{0.68} = 4.79$ (3 s.f.) Accept 4.68 to 4.82 $b = \frac{0.7}{0.68} = 1.03$ Accept 1.02 to 1.06
11(ii)	6460 cm^3 (3 s.f.)	12(ii)	D (- 3, 4)
12(iii)	$y - 2 = \frac{3}{2}x$ or $y = \frac{3}{2}x + 2$	13(iv)	26 $units^2$

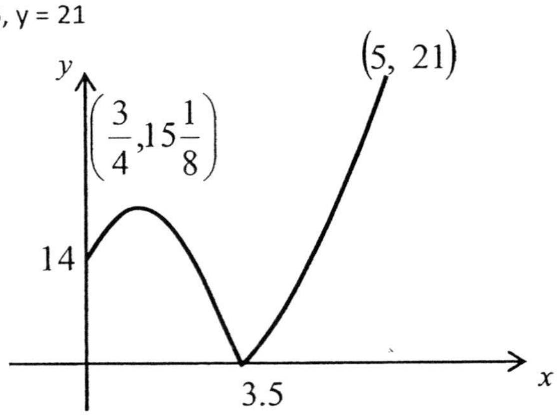
Secondary 4 Express Additional Mathematics
Preliminary Examination 2015
Marking Scheme

No	Working	Description
1	$y=(1-\tan^2 x)\cos^2 x$ $y=\cos^2 x-\tan^2 x\cos^2 x$ $y=\cos^2 x-\sin^2 x$ y = cos 2x -------------------------------- $\frac{dy}{dx}=-2\sin 2x$ = LHS	M1 [expansion] M1 [Substitute identity] AG1
2	$\frac{8-3x}{(1-x)^2(2x+3)}=\frac{A}{1-x}+\frac{B}{(1-x)^2}+\frac{C}{2x+3}$ $8-3x=A(1-x)(2x+3)+B(2x+3)+C(1-x)^2$ When x = 1, $8-3=B(5)$ $B=1$ When x = -1.5, $8-3(-1.5)=C(1-(-1.5))^2$ $12.5=6.25C$ $C=2$ When x =0, $8=A(1)(3)+B(3)+C(1)$ $8=3A+3+2$ $3=3A$ $A=1$ $\frac{8-3x}{(1-x)^2(2x+3)}=\frac{1}{1-x}+\frac{1}{(1-x)^2}+\frac{2}{2x+3}$	B1 [correct partial fraction formula] M1 [Substitution / comparing coefficient method] A1 for value of A / B / C A1 for all values correct B1

No	Working	Description
3	$P=\dfrac{2500}{\sqrt{V^3}}$ $P=2500V^{-\frac{3}{2}}$ $\dfrac{dP}{dV}=-\dfrac{3}{2}\times 2500\times V^{-\frac{5}{2}}$ $=-3750V^{-\frac{5}{2}}$ or $-\dfrac{3750}{\sqrt{V^5}}$ When P = 50, $\sqrt{V^3}=\dfrac{2500}{50}=50$ $V=50^{\frac{2}{3}}$ $\dfrac{dP}{dt}=\dfrac{dP}{dV}\times\dfrac{dV}{dt}$ $\dfrac{dV}{dt}=\dfrac{dP}{dt}\times\left(\dfrac{dV}{dP}\right)$ $=2.8\times-\dfrac{\sqrt{\left(50^{\frac{2}{3}}\right)^5}}{3750}$ = - 0.50669 = -0.507 units/sec Rate of change of volume = - 0.507 units/sec	M1 [apply differentiation rule] A1 [differentiation correctly] B1 [find corresponding value of V] M1 [apply chain rule correctly] A1
4	$(5-4x)\left(\dfrac{3x^2}{2}+\dfrac{2}{3x}\right)^9$ General term for $\left(\dfrac{3x^2}{2}+\dfrac{2}{3x}\right)^9=\binom{9}{r}\left(\dfrac{3x^2}{2}\right)^{9-r}\left(\dfrac{2}{3x}\right)^r$ $=\binom{9}{r}\left(\dfrac{3}{2}\right)^{9-r}\left(\dfrac{2}{3}\right)^r x^{18-2r-r}$ $=\binom{9}{r}\left(\dfrac{3}{2}\right)^{9-r}\left(\dfrac{3}{2}\right)^{-r} x^{18-3r}$ $=\binom{9}{r}\left(\dfrac{3}{2}\right)^{9-2r} x^{18-3r}$ For term independent of x, 18 – 3r = 0 → r = 6 For term with coefficient of x, 18 – 3r = -1 → $r=\dfrac{19}{3}$ → there is no term with $\dfrac{1}{x}$. $(5-4x)\left(\dfrac{3x^2}{2}+\dfrac{2}{3x}\right)^9=(5-4x)\left(\ldots+\binom{9}{6}\left(\dfrac{3}{2}\right)^{9-12}+\ldots\right)$ Term independent of $x=5\times\binom{9}{6}\left(\dfrac{3}{2}\right)^{-3}=\dfrac{1190}{9}$	M1 [find general term] or [expansion till the 6th and 7th term] M1 [correct simplification of index for x] M1 [find the value of r, must equate the index to 0, if r is not a whole number no marks.] M1 [expansion] A1

No	Working	Description
5(i)	$y = \ln(5-2x)$ $\frac{dy}{dx} = \frac{-2}{5-2x}$ $2y = x+3$ $y = \frac{x}{2} + \frac{3}{2}$ Gradient of normal $= \frac{1}{2}$ Gradient of tangent = -2 $\frac{-2}{5-2x} = -2$ $5-2x = 1$ $-2x = -4$ $x = 2$ When x = 2, y = ln (5-4) = 0 The coordinates is (2, 0).	B1 [differentiation] B1 [gradient of tangent] M1 [solve for x] A1
5(ii)	$\frac{dy}{dx} = \frac{-2}{5-2x}$ *For* $x < \frac{5}{2}$, $(5-2x) > 0$ and $-2 < 0$ Therefore $\frac{-2}{5-2x} < 0$, which implies that dy/dx is <0 Since dy/dx < 0, then y is decreasing.	B1 B1
6	sin $(A+B) = 3$ sin $(A-B)$, show that tan $A = 2$ tan B $\sin(A+B) = 3\sin(A-B)$ $\sin A\cos B + \cos A\sin B = 3\sin A\cos B - 3\cos A\sin B$ $4\cos A\sin B = 2\sin A\cos B$ $2\cos A\sin B = \sin A\cos B$ $\frac{2\cos A\sin B}{\cos A\cos B} = \frac{\sin A\cos B}{\cos A\cos B}$ $2\tan B = \tan A$	M1 [simplification] M1 [to get tan function] AG1
	$\sin^2(x+60°) = 9\sin^2(x-60°)$ for $0° < x < 360°$ $\sin(x+60°) = \pm 3\sin(x-60°)$ Case 1: $\sin(x+60°) = 3\sin(x-60°)$ Let A = x and B = 60 Therefore, $\tan x = 2\tan 60°$ $\tan x = 2\sqrt{3}$ Basic angle = 73.898 x =73.898, 180 + 73.898 $= 73.9°,\ 253.9°$	M1 A1
	Case 2: $\sin(x+60°) = -3\sin(x-60°)$ Let A = 60 and B = x Therefore, $2\tan x = \tan 60°$ $\tan x = \frac{\sqrt{3}}{2}$	M1 110

No	Working	Description
	Basic angle =40.893 x =40.893, 180 + 40.893 =40.9°, 220.9°	A1
7(i)	$4\sin\frac{x}{2}\cos\frac{x}{2}=\sqrt{3}$ $2\sin\frac{x}{2}\cos\frac{x}{2}=\frac{\sqrt{3}}{2}$ $\sin x=\frac{\sqrt{3}}{2}$ $\alpha=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$ $x=\frac{\pi}{3},\pi-\frac{\pi}{3}$ $x=\frac{\pi}{3},\frac{2\pi}{3}$	M1 [Apply double angle identity] A1, A1
7(ii)	$\sin^4 x-\cos^4 x-3\cos x=2$ $(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)-3\cos x=2$ $1-2\cos^2 x-3\cos x=2$ $2\cos^2 x+3\cos x+1=0$ $(2\cos x+1)(\cos x+1)=0$ $\cos x=-0.5$ or $\cos x=-1$ $x=\frac{2}{3}\pi,\ \frac{4}{3}\pi$ or $x=\pi$	B1 [Factorization] M1 [Use identity and simplify to a quadratic function in terms of cos x] M1 [Factorization] A1, A1
8(a)(i)	$y=4x^2+px+p-6$ $y=-3$ $4x^2+px+p-6=-3$ $4x^2+px+p-3=0$ For line intersect the curve, $b^2-4ac\geq 0$ $p^2-4(4)(p-3)\geq 0$ $p^2-16p+48\geq 0$ $(p-12)(p-4)\geq 0$ $p\leq 4$ or $p\geq 12$	M1 [correct discriminant value] M1 [factorization] A1
8(a)(i)	For tangent line, $b^2-4ac=0$ $p^2-4(4)(p-3)=0$ $p^2-16p+48=0$ $(p-12)(p-4)=0$ $p=12$ or $p=4$	B1
8(a)(ii)	$y=4x^2+px+p-6$ For y-intercept is positive, $p-6>0$ $p>6$	B1

No	Working	Description
8(b)	Show that $(m + 1)x^2 + (4m + 3)x + 2m - 1 = 0$ has real and distinct roots for all real values of m.	
	$b^2 - 4ac = (4m + 3)^2 - 4(m+1)(2m - 1)$ $= 16m^2 + 24m + 9 - 4(2m^2 + m - 1)$ $= 16m^2 + 24m + 9 - 8m^2 - 4m + 4$ $= 8m^2 + 20m + 13$ $= 8\left[m^2 + \frac{5}{2}m\right] + 13$ $= 8\left(m + \frac{5}{4}\right)^2 + \frac{1}{2}$ Since $\left(m + \frac{5}{4}\right)^2 \geq 0$, therefore, $b^2 - 4ac \geq 0.5$ Therefore, the roots are real and distinct.	M1 [Work out discriminant expression] M1 [Complete the square] A1 [Explanation]
9(i)	$y = \lvert 2x^2 - 3x - 14\rvert$ for $0 \leq x \leq 5$. $y = 2x^2 - 3x - 14$ y-intercept coordinate is (0, - 14) x-intercept, y = 0, $(2x - 7)(x + 2) = 0$ $x = 3.5$ or $x = -2$ x-coordinate of minimum point $= \frac{3.5 + (-2)}{2} = \frac{3}{4}$ Minimum point is $\left(\frac{3}{4}, -15\frac{1}{8}\right)$ x = 5, y = 21 	Shape of graph [S1] (correct position of maximum point of the graph with one x-intercept P1 [Points of the graph, shows maximum point, and end points] P1 [Show x-intercept and y-intercept]
9(ii)(a)	$14 \leq k < 15\frac{1}{8}$	B1
9(ii)(b)	$0 < k < 14$ or $k = 15\frac{1}{8}$	B1, B1
9(ii)(c)	$k = 0$ or $k > 15\frac{1}{8}$	B1

<table>
<tr><th>No</th><th>Working</th><th>Description</th></tr>
<tr><td>10(i)</td><td>
<table>
<tr><td>x</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td><td>0.5</td><td>0.6</td></tr>
<tr><td>$\lg y$</td><td>0.771</td><td>0.839</td><td>0.857</td><td>0.973</td><td>1.04</td><td>1.11</td></tr>
</table>
</td><td>P1 [Plot points]
S1 [Straight line graph</td></tr>
<tr><td>10(ii)</td><td>$y = a^{b+x}$
$\lg y = (b+x)\lg a = b\lg a + x\lg a$
$\lg a = \text{gradient} = \dfrac{1.11-0.77}{0.6-0.1} = 0.68$
$a = 10^{0.68} = 4.79$ (3 s.f.)
<u>Accept 4.68 to 4.82</u>
$b\lg a = 0.7$ which is the $\lg y$-intercept
$b = \dfrac{0.7}{0.68} = 1.03$
<u>Accept 1.02 to 1.06</u></td><td>B1 [convert to straight line graph]
M1

A1

B1</td></tr>
<tr><td>10(iii)</td><td>When $x = 0.3$, $y = 7.2$ (erroneous)
The correct value of $y = 10^{0.905} = 8.04$ (3 s.f.)
<u>Accept 7.76 to 8.14</u></td><td>B1
B1</td></tr>
<tr><td>11 (i)</td><td>Total surface area of cardboard, A = 2400
$A = 2x^2 + 2(2xy) + 2(xy) + 2x^2 + 2(3x) + 2(6x)$
$A = 6xy + 4x^2 + 18x$

$2400 = 6xy + 4x^2 + 18x$
$y = \dfrac{2400 - 4x^2 - 18x}{6x}$
$y = \dfrac{400}{x} - \dfrac{2x}{3} - 3$ or $y = \dfrac{1200 - 2x^2 - 9}{3x}$
Volume of box, $V = 2x^2y$
$V = 2x^2\left(\dfrac{400}{x} - \dfrac{2x}{3} - 3\right)$
$= 800x - \dfrac{4x^3}{3} - 6x^2$</td><td>B1 [Form correct expression of for surface area]

B1 [Form equation for surface area]

B1 [make y the subject of formula]

AG1</td></tr>
<tr><td>11(ii)</td><td>$\dfrac{dV}{dx} = 800 - 4x^2 - 12x$
At stationary value of V, $\dfrac{dV}{dx} = 0$
$800 - 4x^2 - 12x = 0$
$4x^2 + 12x - 800 = 0$
$x^2 + 3x - 200 = 0$
$x = \dfrac{-3 \pm \sqrt{9 - 4(3)(-200)}}{2}$
x = 12.721 or x = - 15.721
x = 12.7 (3 s.f.)
Stationary value of V = $800(12.721) - \dfrac{4}{3}(12.721)^3 - 6(12.721)^2$
= 6461.108846
= 6460 (3 s.f.)</td><td>B1 [Differentiate the expression]

B1 [equate dy/dx = 0]

B1 [solve for x]

B1</td></tr>
</table>

No	Working	Description
11(iii)	$\frac{d^2V}{dx^2} = -8x - 12$ When x = 12.721, $\frac{d^2V}{dx^2} = -8(12.721) - 12 = -113.768$ Since $\frac{d^2V}{dx^2} < 0$, therefore the volume is maximum.	B1 [explanation with 2nd derivative]
12(i)	gradient of AB × gradient of BC = -1 $\frac{t-2-1}{-2t-1+5} \times \frac{t-2-0}{-2t-1-3} = -1$ $\frac{t-3}{-2t+4} \times \frac{t-2}{-2t-4} = -1$ $\frac{t-3}{-2(t-2)} \times \frac{t-2}{-2(t+2)} = -1$ $t-3 = -4(t+2)$ $t-3 = -4t-8$ $5t = -5$ $t = -1$	M1 M1 [simplification] AG1
12(ii)	Mid-point of AC = mid-point of BD $\left(\frac{3+(-5)}{2}, \frac{0+1}{2}\right) = \left(\frac{1+x}{2}, \frac{-3+y}{2}\right)$ $x = -3,\ y = 4$ $D(-3,4)$	M1 A1
12(iii)	Gradient of AB = $\frac{-3-0}{1-3} = \frac{3}{2}$ Midpoint of AD = $\left(\frac{3+(-3)}{2}, \frac{0+4}{2}\right) = (0,2)$ Equation of perpendicular bisector of AD: $y - 2 = \frac{3}{2}x$ or y = $\frac{3}{2}x + 2$	M1 [either midpoint or gradient of AB] A1
12(iv)	Area of rectangle ABCD = $\frac{1}{2}\begin{vmatrix} 3 & 1 & -5 & -3 & 3 \\ 0 & -3 & 1 & 4 & 0 \end{vmatrix}$ $=0.5\,\lvert(-9 + 1 - 20) - (15 - 3 + 12)\rvert$ $= 26$ units^2 Alternative method: use distance formula Area of rectangle = length x breadth	M1 A1

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NAME: ______________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS   4047/02

Paper 2

Date: 26 August 2015   **Duration: 2 hours 30 minutes**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

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For Examiner's Use	
Paper 2	/ 100

Setter: Mdm Haliza

This question paper consists of 6 printed pages including the cover page.

1/3

Name: ______________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Name: ______________________________ (   )   Class: __________

1 The roots of the quadratic equation $3x^2 - kx + 4 = 0$, where $k > 0$, are α and β, and that of the equation $12x^2 - x + 12 = 0$ are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. Find the value of k. [7]

2 A weather satellite orbits planet P such that the equation of its path can be represented by the equation

$$x^2 + y^2 - 18x - 14y + 65 = 0$$

where x and y are the longitudinal and the latitudinal distances from the centre of P respectively in kilometres, as shown on an astronomical map.

(i) State the coordinates of the centre and the radius of the orbit. [3]

A second satellite orbits another planet K in the same plane as the first satellite. The diameter of its circular orbit has end points (10, 9) and (22, 3).

(ii) Find the equation of the path of this satellite. [4]

3 Without using a calculator,

(i) find the value of r and of n, given that $\frac{3x^r}{r^2} \times \frac{2(r^{6-r})^2}{27x} = nx^2$, [5]

(ii) simplify $\frac{3+\sqrt{2}}{2\sqrt{2}-1}$ in the form $a + b\sqrt{2}$. [3]

4 **(i)** Solve the equation $\log_3(x+2) = 3 - \log_3(x-4)$. [4]

(ii) Given that $\log_x y + \log_y x - \frac{5}{\log_x y} = 0$, express y in terms of x. [4]

5 **(i)** Solve the equation $4\cos 2x + 2\sin x = -2$ for $0 \le x \le 2\pi$. [6]

(ii) On the same axes, sketch the graphs of

$$y = 3\cos 2x \text{ and } y = |\sin x|$$

for the interval $0 \le x \le 2\pi$, labelling each graph clearly.

State the number of solutions in the interval $0 \le x \le 2\pi$ of the equation $3\cos 2x = |\sin x|$. [4]

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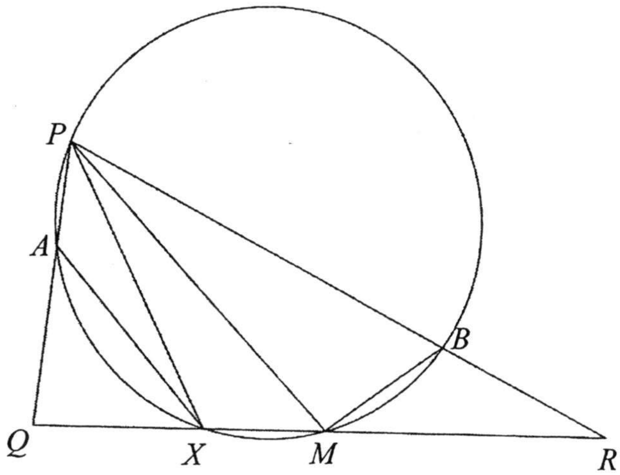
Name: ______________________________ (   )   Class: __________

6 In the triangle PQR, M is the mid-point of QR and PX bisects angle QPR.
The circle passing through P, X and M, cuts PQ and PR at A and B respectively.



(i) Explain why $\angle PBM + \angle PXM = 180°$. [1]

(ii) Show that ΔRBM is similar to ΔRXP. [3]

(iii) Given that ΔQXA is also similar to ΔQPM and $\dfrac{PR}{RX} = \dfrac{PQ}{QX}$, show that $RB = QA$. [4]

7 A metal ball is heated to a temperature of $225°\text{C}$ before being dropped into a liquid.
As the ball cools, its temperature, $T°\text{C}$, t minutes after it enters the liquid is given by $T = P + 190e^{-kt}$, where P and k are constants.

(i) Explain why $P = 35$. [1]

When $t = 4$, the temperature of the ball reaches $120°\text{C}$.

(ii) Find the value of k correct to 3 significant figures. [3]

(iii) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 10$. [3]

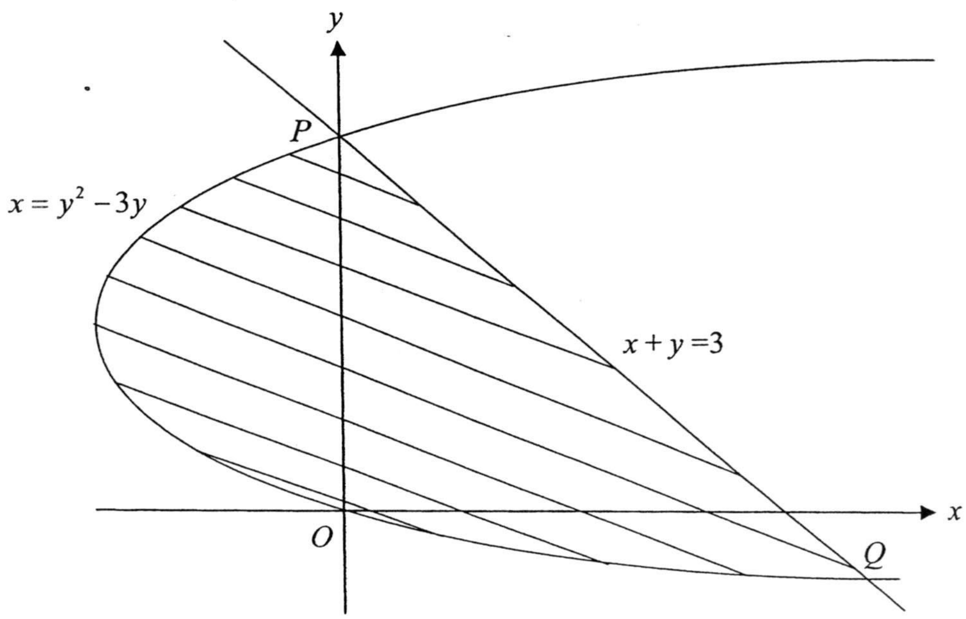
(iv) From the equation of T given above, explain why the temperature of the ball can never fall below $35°\text{C}$. [2]

Name: ______________________________ (   )   Class: __________

8 The diagram shows part of the curve $x = y^2 - 3y$ and the line $x + y = 3$. If the line and the curve intersect at P and Q, find

(i) the coordinates of P and Q, [5]

(ii) the area of the shaded region. [5]



9 $f(x) = 6x^3 + ax^2 + bx - 6$ has a factor $x + 2$ but leaves a remainder of -12 when divided by $x - 1$.

(i) Find the value of a and of b. [5]

(ii) Factorise $f(x)$ completely and hence solve the equation

$$48x^3 + 4ax^2 = 6 - 2bx.$$ [6]

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Name: ______________________________ (   )   Class: __________

10 An object at A, with an initial displacement of 3m from a fixed point O, travels in a straight line so that its velocity, v ms^{-1}, is given by $v = t^2 - 5t + 6$ where t is the time in seconds after leaving A.

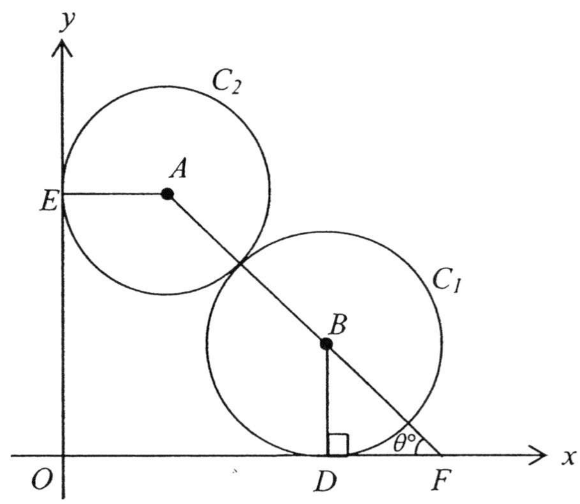
(i) Find the values of t when the object comes to an instantaneous rest. [2]

(ii) Find the acceleration of the object at $t = 5$ s. [2]

(iii) Obtain an expression, in terms of t, for the displacement of the object from O after t seconds. [3]

(iv) Find the average speed of the object in the first 5 seconds. [4]

11 The figure shows two circles C_1 and C_2 which touch each other and lie in the xy-plane as shown below. C_1 has radius 4 units and touches the x-axis at D, C_2 has radius 3 units and touches the y-axis at E. The line AB, joining the centres of C_2 and C_1, meets the x-axis at F and $\angle BFO = \theta°$.



(i) Obtain expressions for OD and OE in terms of θ and show that

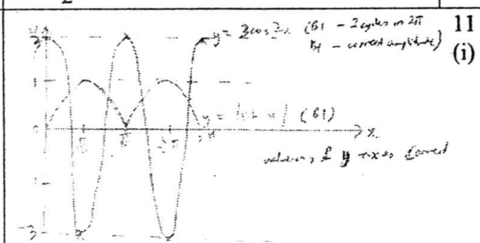
$$ED^2 = 74 + 56\sin\theta + 42\cos\theta.$$ [4]

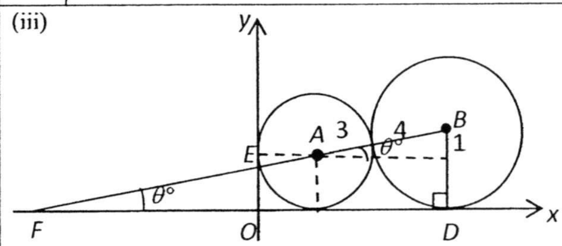
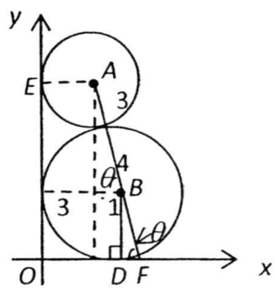
(ii) Express ED^2 in the form $74 + R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) By considering the extreme positions in which both circles touch the x-axis and both circles touch the y-axis, show that $8.2° \le \theta \le 81.8°$, correct to one decimal place. [3]

~ **End of Paper** ~

FMSS A. Math Preliminary Examination 2015 Paper 2 Answer Key

1.	$k = 5$
2(i)	Centre (9, 7), radius = $\sqrt{65}$ or 8.06km
(ii)	$(x-16)^2 + (y-6)^2 = 45$ or $x^2 + y^2 - 32x - 12y + 247 = 0$
3(i)	$r = 3, n = 18$
(ii)	$1 + \sqrt{2}$
4(i)	$x = -5$ (N.A.) or $x = 7$
(ii)	$y = x^2$ or $y = x^{-2}$
5(i)	$x = \frac{\pi}{2}$ (or 1.57), 3.99, 5.44
(ii)	
6(i)	$\angle PBM$ and $\angle PXM$ are angles in opposite segments of the cyclic quadrilateral $PXMB$. Therefore the sum of these two angles is supplementary.
(ii)	$\angle BRM = \angle PRX$ (common angles of ΔRBM and ΔRXP) $\angle XPR = \angle BMR$ (ext. $\angle$ of cyclic quad.) or $\angle RBM = \angle PXM$ (ext. $\angle$ of cyclic quad.) $\therefore \Delta RBM$ is similar to ΔRXP (by AA similarity test)
(iii)	Since ΔRBM is similar to ΔRXP, $\frac{PR}{RX} = \frac{RM}{RB}$. Since ΔQXA is also similar to ΔQPM, $\frac{PQ}{XQ} = \frac{QM}{QA}$ Given that $\frac{PR}{RX} = \frac{PQ}{QX}$, $\therefore \frac{RM}{RB} = \frac{QM}{QA}$. Since $QM = MR$ as M is the mid-point of QR, $\therefore RB = QA$ (Shown)
7(i)	At $t = 0$, $T = 225^{\circ}$C. $\therefore 225 = P + 190e^{-k(0)}$ $P = 225 - 190 = 35$
(ii)	$k = 0.201$
(iii)	5.11°C / min.
(iv)	As $e^{-0.201t} > 0$ for $t \geq 0$, $190e^{-0.201t} > 0$ $35 + 190e^{-0.201t} > 35$. $\therefore T > 35$. Hence the temperature of the ball can never fall below 35°C.

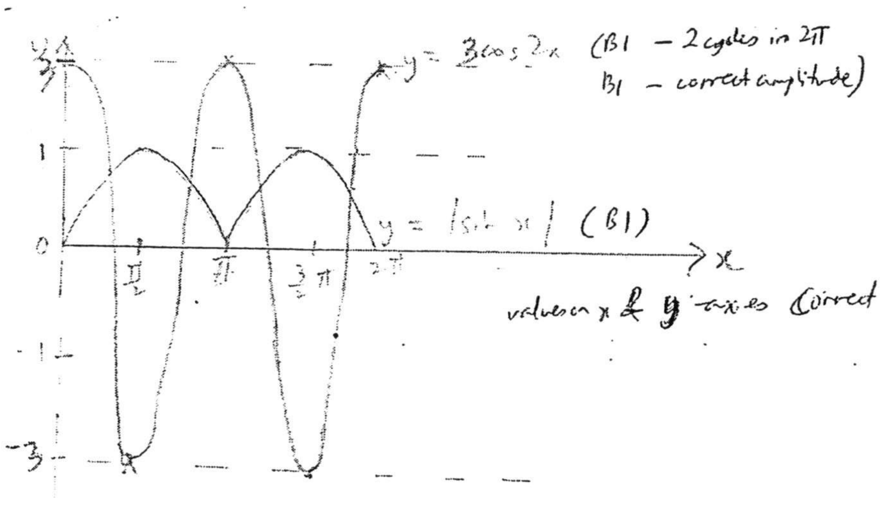
8(i)	$P(0,3), Q(4,-1)$
(ii)	$10\frac{2}{3}$ or 10.7 or $\frac{32}{3}$ sq. units
9(i)	$a = 5, b = -17$
(ii)	$f(x) = (x+2)(3x-1)(2x-3)$ Hence, $(2x+2)(3(2x)-1)(2(2x)-3) = 0$ $\therefore x = -1, -\frac{1}{6}, \frac{3}{4}$
10 (i)	$t = 2$ or 3
(ii)	$a = 5$ m/s^2
(iii)	$s = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 3$
(iv)	1.9 m/s
11 (i)	$OD = 3 + 7\cos\theta$ $OE = 4 + 7\sin\theta$ $ED^2 = OD^2 + OE^2$ (By Pythagoras' Theorem)
(ii)	$ED^2 = 74 + 70\cos(\theta - 53.1^{\circ})$.
(iii)	 When both circles touch the x-axis, $\sin\theta = \frac{1}{7} \Rightarrow \theta = 8.2^{\circ}$ (to 1 dec. pl.)  When both circles touch the y-axis, $\cos\theta = \frac{1}{7} \Rightarrow \theta = 81.8^{\circ}$ (to 1 dec. pl.) $\therefore 8.2^{\circ} \leq \theta \leq 81.8^{\circ}$ (Shown)

FMSS A. Math Preliminary Examination 2015 Paper 2 Marking Scheme

1.	$3x^2 - kx + 4 = 0$ have roots α and β.	
	$\alpha + \beta = \frac{k}{3}$	B1
	$\alpha\beta = \frac{4}{3}$	B1
	$12x^2 - x + 12 = 0$ have roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.	
	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{1}{12}$	B1
	$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{1}{12}$	M1
	$\frac{\left(\frac{k}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\frac{4}{3}} = \frac{1}{12}$	M1
	$k^2 - 24 = 1$	M1 (simplify eqn)
	$k^2 = 25$	
	$k = 5 \quad (\text{since } k > 0)$	A1
2(i)	$x^2 + y^2 - 18x - 14y + 65 = 0$	
	Centre of orbit (-g, -f) = (9, 7)	B1
	Radius $= \sqrt{(-9)^2 + (-7)^2 - 65}$	M1
	$= \sqrt{65}$ or 8.06 km	A1/ B2
(ii)	Diameter of orbit $= \sqrt{(10-22)^2 + (9-3)^2}$	M1
	$= \sqrt{180}$	
	Radius of orbit $= \frac{\sqrt{180}}{2}$	A1
	Mid - point of orbit $= \left(\frac{10+22}{2}, \frac{9+3}{2}\right)$	
	$= (16, 6)$	B1
	$\therefore$ Equation of path of this satellite is	
	$(x-16)^2 + (y-6)^2 = \left(\frac{\sqrt{180}}{2}\right)^2$	
	$(x-16)^2 + (y-6)^2 = 45$ or	B1
	$x^2 + y^2 - 32x - 12y + 247 = 0$	

3(i)	$\frac{3x^r}{r^2} \times \frac{2(r^{6-r})^2}{27x} = nx^2$ $\frac{2}{9} r^{12-2r-2} x^{r-1} = nx^2$ $r-1=2$ $r=3$ $\frac{2}{9} r^{10-2r} = n$ Sub. $r=3$, $n = \frac{2}{9}(3)^{10-2(3)}$ $=18$	 M1 (Simplify powers of r and x) M1 (Equate powers of x) A1 M1 (Equate scalar) A1
(ii)	$\frac{3+\sqrt{2}}{2\sqrt{2}-1}$ $= \frac{3+\sqrt{2}}{2\sqrt{2}-1} \times \frac{2\sqrt{2}+1}{2\sqrt{2}+1}$ $= \frac{6\sqrt{2}+3+4+\sqrt{2}}{8-1}$ $= \frac{7\sqrt{2}+7}{7}$ $=1+\sqrt{2}$	M1 (rationalise denominator) M1 (simplify) A1
4(i)	$\log_3(x+2) = 3 - \log_3(x-4)$ $\log_3(x+2)(x-4) = 3$ $x^2-2x-8=3^3$ $x^2-2x-35=0$ $(x+5)(x-7)=0$ $x=-5$ (N.A.) or $x=7$	M1 (Apply product law) M1 (Change from log to index form) M1 (factorise) A1

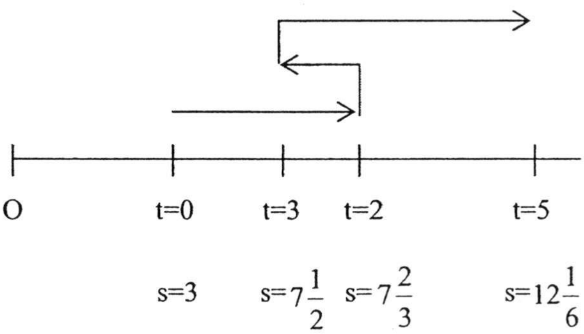
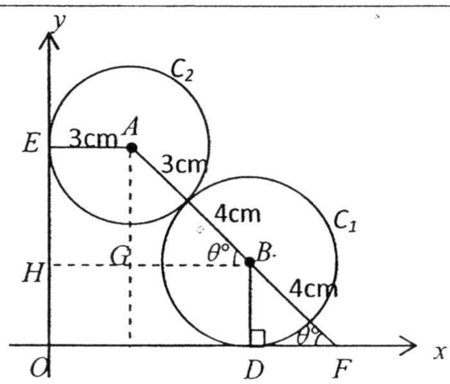
(ii)	$\log_x y + \log_y x - \frac{5}{\log_x y} = 0$ $\log_x y + \frac{\log_x x}{\log_x y} - \frac{5}{\log_x y} = 0$ $\log_x y + \frac{1}{\log_x y} - \frac{5}{\log_x y} = 0$ $(\log_x y)^2 + 1 - 5 = 0$ $(\log_x y)^2 = 4$ $\log_x y = \pm 2$ $y = x^2$ or $y = x^{-2}$ OR $\log_x y + \log_y x - \frac{5}{\log_x y} = 0$ $\frac{\log_y y}{\log_y x} + \log_y x - 5\frac{\log_y x}{\log_y y} = 0$ $\frac{1}{\log_y x} + \log_y x - 5\log_y x = 0$ $4(\log_y x)^2 = 1$ $\log_y x = \pm\frac{1}{2}$ $x = y^{\frac{1}{2}}$ or $y^{-\frac{1}{2}}$ $y = x^2$ or $y = x^{-2}$	M1 (Change of base) M1 (Quadratic form) A1, A1 (y in terms of x) OR M1 (Change of base) M1 (Quadratic form) A1, A1 (y in terms of x)
5(i)	$4\cos 2x + 2\sin x = -2$ for $0 \le x \le 2\pi$ $4(1 - 2\sin^2 x) + 2\sin x + 2 = 0$ $4\sin^2 x - \sin x - 3 = 0$ $(4\sin x + 3)(\sin x - 1) = 0$ $\sin x = -\frac{3}{4}$ or $\sin x = 1$ Basic $\angle = \sin^{-1}\left(\frac{3}{4}\right) = 0.84806$ $\quad x = \frac{\pi}{2}$ (or 1.57) $x = \pi + 0.84806, 2\pi - 0.84806$ $= 3.99, 5.44$	M1 (Apply trigo. identity) M1 (factorise/general formula) M1 ft (equations) A1 A1, A1

(ii)	 4 solutions B1	
6(i)	$\angle PBM$ and $\angle PXM$ are <u>angles in opposite segments</u> of the cyclic quadrilateral $PXMB$. Therefore the sum of these two angles is supplementary.	B1
(ii)	$\angle BRM = \angle PRX$ (common angles of ΔRBM and ΔRXP) $\angle XPR = \angle BMR$ (ext. $\angle$ of cyclic quad.) or $\angle RBM = \angle PXM$ (ext. $\angle$ of cyclic quad.) $\therefore \Delta RBM$ is similar to ΔRXP (by AA similarity test)	B1 B1 B1
(iii)	Since ΔRBM is similar to ΔRXP, $\frac{PR}{RX} = \frac{RM}{RB}$ Since ΔQXA is also similar to ΔQPM, $\frac{PQ}{XQ} = \frac{QM}{QA}$ Given that $\frac{PR}{RX} = \frac{PQ}{QX}$, $\therefore \frac{RM}{RB} = \frac{QM}{QA}$. Since $QM = MR$ as M is the mid-point of QR, $\therefore RB = QA$ (Shown)	B1 B1 B1 B1
7(i)	At $t = 0$, $T = 225^{\circ}$ C. $\therefore 225 = P + 190e^{-k(0)}$ $P = 225 - 190$ $= 35$	B1

(ii)	When $t = 4$, $T = 120$, $120 = 35 + 190e^{-k(4)}$ $-4k = \ln\left(\frac{85}{190}\right)$ $k = 0.20109$ ≈ 0.201	M1 M1 (change to ln) A1
(iii)	$T = 35 + 190e^{-0.20109t}$ $\frac{dT}{dt} = -0.20109(190e^{-0.20109t})$ $= -38.2077e^{-0.20109t}$ When $t = 10$, $\frac{dT}{dt} = -38.2077e^{-0.20109(10)}$ $= -5.1148$ Temperature of the ball is <u>decreasing</u> at a rate of 5.11°C / min.	M1ft from (ii) value of k M1(ft if value of t and k clearly shown) A1 (positive value)
(iv)	As $e^{-0.201t} > 0$ for $t \geq 0$, $190e^{-0.201t} > 0$ $35 + 190e^{-0.201t} > 35$. $\therefore T > 35$. Hence the temperature of the ball can never fall below $35\,^{\circ}$C.	B1 B1 (must also include concluding statement)
8(i)	Sub. $x = y^2 - 3y$ into $x + y = 3$, $y^2 - 3y + y = 3$ $y^2 - 2y - 3 = 0$ $(y-3)(y+1) = 0$ $y = 3$ or -1 Sub. $y = 3$ and -1 into $x = y^2 - 3y$, $x = 3^2 - 3(3)$ and $x = (-1)^2 - 3(-1)$ $= 0$ $= 4$ $\therefore$ P is (0, 3) and Q is (4, -1)	M1 M1 (factorise/general formula) M1 A1, A1

(ii)	Area of shaded region $= \left\lvert \int_0^3 (y^2 - 3y)\, dy \right\rvert + \frac{1}{2}(4)(4) - \int_{-1}^{0} (y^2 - 3y)\, dy$ $= \left\lvert \left[\frac{y^3}{3} - \frac{3y^2}{2} \right]_0^3 \right\rvert + 8 - \left[\frac{y^3}{3} - \frac{3y^2}{2} \right]_{-1}^{0}$ $= \left\lvert \frac{3^3}{3} - \frac{3(3)^2}{2} \right\rvert + 8 - \left[0 - \frac{(-1)^3}{3} + \frac{3(-1)^2}{2} \right]$ $= 4\frac{1}{2} + 8 - 1\frac{5}{6}$ $= 10\frac{2}{3}$ or 10.7 or $\frac{32}{3}$ sq. units or $\frac{1}{2}(4)(4) - \int_{-1}^{3} (y^2 - 3y) dy$ or $\int_{-1}^{3} (3 - y) dy - \int_{-1}^{3} (y^2 - 3y) dy$ or $\left\lvert \int_0^3 (y^2 - 3y) dy \right\rvert - \frac{1}{2}(3)(3) + \frac{1}{2}(3+4)(1) - \int_{-1}^{0} (y^2 - 3y)\, dy$ or $\left\lvert \int_0^3 (y^2 - 3y) dy \right\rvert - \frac{1}{2}(3)(3) + (4)(1) - \frac{1}{2}(1)(1) - \int_{-1}^{0} (y^2 - 3y)\, dy$	M1, M1, M1 M1 (integrate) A1
9(i)	$f(x) = 6x^3 + ax^2 + bx - 6 = (x+2)P(x)$ $f(-2) = 6(-2)^3 + a(-2)^2 + b(-2) - 6 = 0$ $-48 + 4a - 2b - 6 = 0$ $2a - b = 27$ --------------(1) $f(x) = 6x^3 + ax^2 + bx - 6 = (x-1)Q(x) - 12$ $f(1) = 6 + a + b - 6 = -12$ $a + b = -12$ ---------------(2) $(1)+(2): 3a = 15$ $a = 5$ Sub. $a = 5$ into (2): $5 + b = -12$ $b = -17$	 M1 (equate to zero) M1 (equate to -12) M1 A1 A1

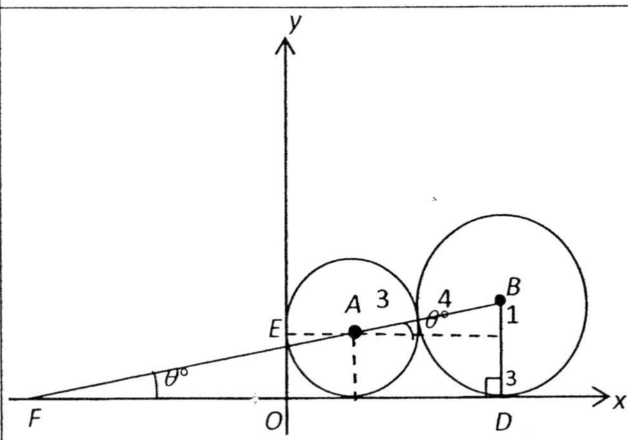
(ii)	$f(x)=6x^3+5x^2-17x-6=(x+2)(6x^2+kx-3)$ By comparing coefficient of x, $-3+2k=-17$ $k=-7$ $\therefore f(x)=(x+2)(6x^2-7k-3)$ $=(x+2)(3x-1)(2x-3)$ $48x^3+4ax^2+2bx-6=0$ $6(2x)^3+a(2x)^2+b(2x)-6=0$ Hence, $(2x+2)(3(2x)-1)(2(2x)-3)=0$ [or let $u=2x$, $6u^3+au^2+bu-6=0$ Hence, $(u+2)(3u+1)(2u-3)=0$] $u=2x=-2,-\frac{1}{3},\frac{3}{2}$ $\therefore x=-1,-\frac{1}{6},\frac{3}{4}$	M1ft from (i) (compare coeff./ long division/ synthetic division) A1ft A1 M1 A1 A1
10(i)	$v=t^2-5t+6$ When at instantaneous rest, $v=0$. $t^2-5t+6=0$ $(t-3)(t-2)=0$ $t=3$ or 2	M1 (equate to zero) A1
(ii)	$a=\frac{dv}{dt}=2t-5$ When $t=5$, $a=2(5)-5=5$ m/s^2	M1 A1
(iii)	$s=\int(t^2-5t+6)dt$ $=\frac{t^3}{3}-\frac{5t^2}{2}+6t+c$ When $t=0, s=3$, $c=3$. $\therefore s=\frac{t^3}{3}-\frac{5t^2}{2}+6t+3$	M1 M1 (values of t and s indicated) A1

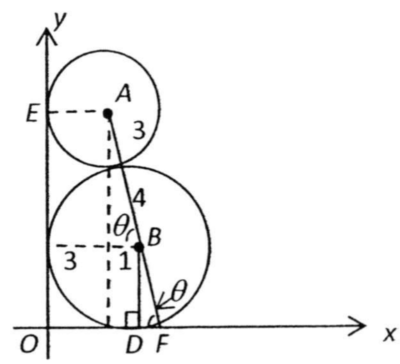
(iv)	When t = 2, $s = \frac{2^3}{3} - \frac{5(2)^2}{2} + 6(2) + 3 = 7\frac{2}{3}$ When t = 3, $s = \frac{3^3}{3} - \frac{5(3)^2}{2} + 6(3) + 3 = 7\frac{1}{2}$ When t = 5, $s = \frac{5^3}{3} - \frac{5(5)^2}{2} + 6(5) + 3 = 12\frac{1}{6}$  Average speed in first 5 s $= \frac{(7\frac{2}{3} - 3) + (7\frac{2}{3} - 7\frac{1}{2}) + (12\frac{1}{6} - 7\frac{1}{2})}{5}$ $= \frac{9.5}{5}$ $= 1.9$ m/s	M1ft from (iii) M1ft from (iii) M1ft from (ii) A1
11(i)	 $GB = 7\cos\theta = JD$ $\therefore OD = OJ + JD = 3 + 7\cos\theta$ $AG = 7\sin\theta = EH$ $\therefore OE = OH + HE = 4 + 7\sin\theta$	B1 B1

<table>
<tr>
<td></td>
<td>$\therefore$ By Pythagoras' Theorem,
$ED^2 = OD^2 + OE^2$
$= (3 + 7\cos\theta)^2 + (4 + 7\sin\theta)^2$
$= 9 + 42\cos\theta + 49\cos^2\theta + 16 + 56\sin\theta + 49\sin^2\theta$
$= 25 + 56\sin\theta + 42\cos\theta + 49(\sin^2\theta + \cos^2\theta)$
$= 25 + 56\sin\theta + 42\cos\theta + 49\ (1)$
$= 74 + 56\sin\theta + 42\cos\theta$ (Shown)</td>
<td>M1

AG1</td>
</tr>
<tr>
<td>11(ii)</td>
<td>$ED^2 = 74 + 56\sin\theta + 42\cos\theta$
$= 74 + R\cos(\theta - \alpha)$
$= 74 + R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$
By comparing coefficients,
$R\sin\alpha = 56$ --------(1)
$R\cos\alpha = 42$ -------(2)
$(1) \div (2) : \tan\alpha = \frac{56}{42}$
$\alpha = 53.130°$
$= 53.1°$ (to 1 dec. pl.)
$(1)^2 + (2)^2 : R^2 = 56^2 + 42^2$
$R = \sqrt{4900}$
$R = 70$
$\therefore ED^2 = 74 + 70\cos(\theta - 53.1°)$.</td>
<td>M1

B1

B1
B1</td>
</tr>
<tr>
<td>(iii)</td>
<td>



$\sin\theta = \frac{1}{7}$ (M1) $\qquad$ $\cos\theta = \frac{1}{7}$ (M1)
$\theta = 8.2°$ (to 1 dec. pl.) $\qquad$ $\theta = 81.8°$ (to 1 dec. pl.
$\therefore 8.2° \le \theta \le 81.8°$ (Shown) (AG1)</td>
<td></td>
</tr>
</table>