聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

Name of Student

Class

Index Number

/80

PRELIMINARY EXAMINATION 2015
SECONDARY 4 EXPRESS
ADDITIONAL MATHEMATICS

4047/01

Date: 5 August 2015

Duration: 2 hours

Additional Materials: 8 Sheets of Writing Paper
1 Sheet of Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction tape/fluid.

Answer **ALL** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total number of marks for this paper is **80**.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142.

Set by: Mrs Rajammal Nathan

Vetted by: Ms Tan Bee Choo
Mdm Hayati

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Answer all the questions.

1 (i) Find the set of values of k for which the equation $2x^2 + 5x + k = 2kx + 1$ has no real roots. [4]

(ii) Hence state, with a reason, whether the line $y = 4x + 1$ meets the curve $y = 2x^2 + 5x + 2$. [1]

2 (i) Given that sec 200° = – k, where $k > 0$, find an expression, in terms of k, for sin 200°. [2]

(ii) Hence show that $\tan 110° = -\dfrac{1}{\sqrt{k^2 - 1}}$. [3]

3 The equation of a curve is $y = \ln(5 - 2x)$. Find the coordinates of the point on the curve at which the normal to the curve is parallel to the line $2y = x + 3$. [5]

4 The equation of a curve is $y = \cos^3 x + \sin 3x$. Given that x is changing at a constant rate of 0.56 radians per second, find the rate of change of y when $x = \dfrac{\pi}{6}$. [5]

5 (i) Express $\dfrac{2x - 1}{2x^2 - 5x + 3}$ in partial fractions. [3]

(ii) Hence find $\displaystyle\int \frac{2x - 1}{2x^2 - 5x + 3}\, dx$. [3]

6 A curve is such that $\dfrac{d^2y}{dx^2} = 16e^{-4x}$. Given that $\dfrac{dy}{dx} = 3$ when $x = 0$ and that the curve passes through the point $(2, e^{-8})$, find the equation of the curve. [6]

7 (i) Prove that $\frac{1-\sin 2\theta}{1+\cos 2\theta}=\frac{1}{2}(1-\tan\theta)^2$. [4]

(ii) Hence solve the equation $\frac{1-\sin\theta}{1+\cos\theta}=2$, for $0° \le \theta \le 180°$. [3]

8 (i) Write down the first three terms in the expansion, in ascending powers of x, of

(a) $\left(1+\frac{3x}{2}\right)^5$, [2]

(b) $(2-x)^5$. [2]

(ii) Hence find the coefficient of x^2 in the expansion $\left(2+2x-\frac{3x^2}{2}\right)^5$. [3]

9 (i) Calculate the coordinates of the point of intersection of the graph of $y=3-|2x+1|$ with the coordinate axes. [3]

(ii) Sketch the graph of $y=3-|2x+1|$. [2]

(iii) On the same diagram in part (ii), sketch the graph of $y=x^{\frac{1}{2}}$ for $x \ge 0$. [1]

(iv) State the number of solutions of the equation $3-|2x+1|=x^{\frac{1}{2}}$. [1]

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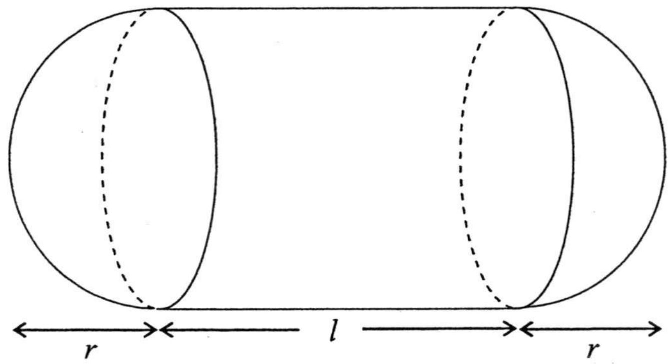

The diagram shows a time capsule consisting of a cylinder of radius r m and length l m, with hemispheres of radius r m attached at each end. The volume of the time capsule is $\frac{\pi}{6}$ m^3.

(i) Show that the surface area of the time capsule, A m^2, is given by

$$A = \frac{4}{3}\pi r^2 + \frac{\pi}{3r}.$$ [4]

(ii) Given that r can vary, find the minimum value of A. [4]

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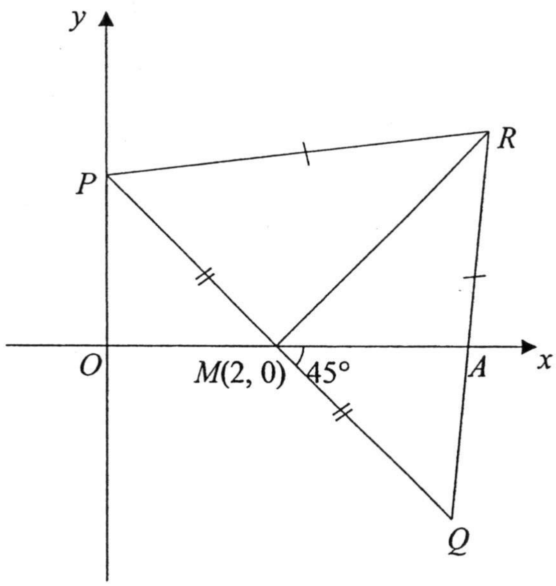
11 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows an isosceles triangle PQR in which $PR = QR$. $M(2, 0)$ is the midpoint of PQ. QR meets the x-axis at A and angle $AMQ = 45°$.

(i) Show that the equation of MR is $y = x - 2$. [2]

(ii) Find the equation of PQ. [2]

(iii) Find the coordinates of Q. [2]

(iv) Given that the area of triangle PQR is 20 units^2, find the coordinates of R. [3]

12 A rectangle of area y m^2 has sides of length x m and $(Ax + B)$ m, where A and B are constants and x and y are variables. Values of x and y are given in the table below.

x	50	100	150	200	250
y	3250	9000	17250	28000	41250

(i) Plot $\frac{y}{x}$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate value of A and of B. [4]

(iii) On the same diagram, draw the straight line representing the equation $y = x^2$ and explain the significance of the value of x given by the point of intersection of the two lines. [3]

End of paper

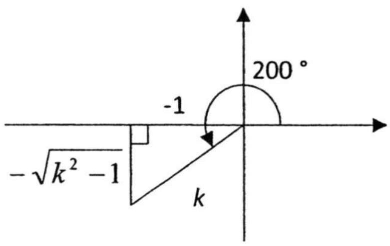
Answers

1 (i) $1\frac{1}{2} < k < 5\frac{1}{2}$

(ii) The equations of the line and the curve are obtained when $k = 2$.
Since $k = 2$ lies in the range, the line does not meet the curve.

2 (i) $\sec 200° = -k$

$\cos 200° = -\frac{1}{k}$



$\sin 200° = -\frac{\sqrt{k^2-1}}{k}$

[Or $\cos 200° = -\frac{1}{k}$

Applying identity: $\sin^2 200° + \cos^2 200° = 1$

$$\sin 200° = \sqrt{1-\frac{1}{k^2}} \text{ (rej) or } -\sqrt{1-\frac{1}{k^2}}$$

(ii) $$\tan 110° = \frac{\sin 110°}{\cos 110°}$$

$$= \frac{\sin(200° - 90°)}{\cos(200° - 90°)}$$

$$= \frac{\sin 200° \cos 90° - \cos 200° \sin 90°}{\cos 200° \cos 90° + \sin 200° \sin 90°} \text{ (applying addition formula)}$$

$$= \frac{0-\left(-\frac{1}{k}\right)}{0+(-\frac{\sqrt{k^2-1}}{k})(1)} = -\frac{1}{\sqrt{k^2-1}}$$

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[OR $\tan 110° = -\tan 70°$

$$= -\frac{1}{\tan 20°}$$

$$= -\frac{1}{\tan 200°}$$

$$= -\frac{1}{\frac{-\sqrt{k^2-1}}{-1}} = -\frac{1}{\sqrt{k^2-1}}$$

3 coordinates of the point = (2, 0)

4 – 0.63 radians/s

5 (i) $\frac{2x-1}{2x^2-5x+3} = \frac{4}{2x-3} - \frac{1}{x-1}$.

(ii) $2\ln(2x-3) - \ln(x-1) + c$

6 $y = e^{-4x} + 7x - 14$

7 (i) $\frac{1-\sin 2\theta}{1+\cos 2\theta}$

$= \frac{1-2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$ Applying double angle formulas

$= \frac{1-2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \frac{1}{2\cos^2\theta} - \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \frac{1}{2}\sec^2\theta - \tan\theta$

$= \frac{1}{2}(1+\tan^2\theta) - \tan\theta$ Applying identity

$= \frac{1}{2}(1+\tan^2\theta - 2\tan\theta)$

$= \frac{1}{2}(1-\tan\theta)^2$

(ii) $\theta = 143.1°$

8 (i) $1+\frac{15}{2}x+\frac{45}{2}x^2+\ldots$

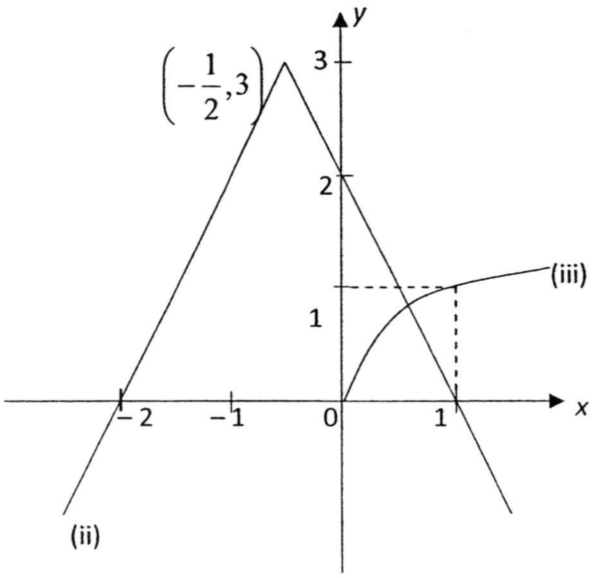
(ii) $32-80x+80x^2+\ldots$

(iii Coefficient of $x^2=200$

9 (i) Intersects the y-axis at (0, 2)
Intersects the x-axis at $(1,0)$ and $(-2,0)$.

(ii) correct shape and passing through the coordinate axes
Coordinates of vertex

(iii) correct shape passing through (1, 1)



(iv) Number of solutions = 1 B1

10 (i) $2\left(\frac{2}{3}\pi r^3\right)+\pi r^2 l=\frac{\pi}{6}$

or $\frac{4}{3}\pi r^3+\pi r^2 l=\frac{\pi}{6}$

$$l=\frac{\frac{\pi}{6}-\frac{4}{3}\pi r^3}{\pi r^2}$$ expressing l in terms of r

$$=\frac{1}{6r^2}-\frac{4}{3}r$$

$$A=2(2\pi r^2)+2\pi rl$$

$$=4\pi r^2+2\pi r\left(\frac{1}{6r^2}-\frac{4}{3}r\right)$$ substituting a correct expression for l

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$$= 4\pi r^2 + \frac{\pi}{3r} - \frac{8}{3}\pi r^2$$

$$= \frac{4}{3}\pi r^2 + \frac{\pi}{3r} \qquad \text{(shown)}$$

(ii) 3.14 cm^2

11 (i) $\angle RMA = 90° - 45°$
$= 45°$

Gradient of $MR = \tan 45°$
$= 1$

Equation of MR is $y - 0 = 1(x - 2)$
$y = x - 2$

(ii) $y = -x + 2$ (iii) Coordinates of $Q = (4, -2)$ (iv)
Coordinates of $R = (7, 5)$

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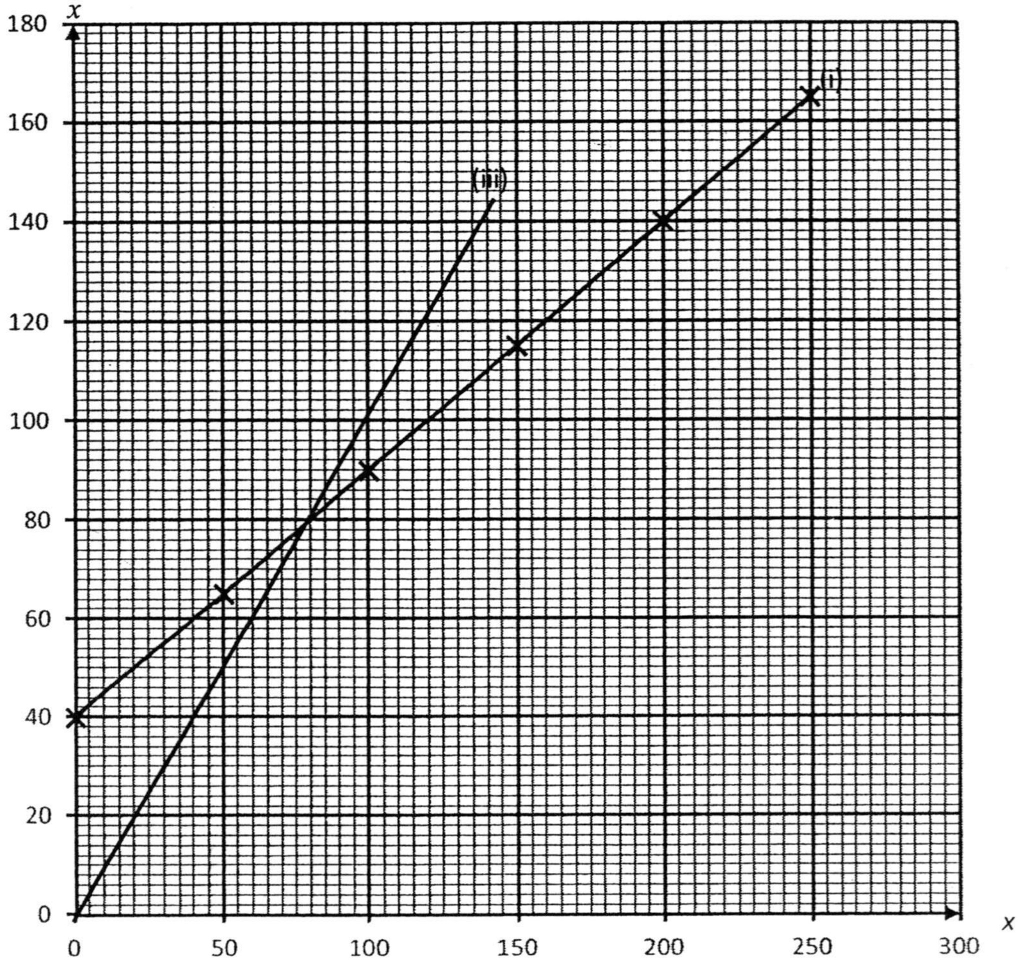
x	50	100	150	200	250
y	3250	9000	17250	28000	41250
$\frac{y}{x}$	65	90	115	140	165

(i) $\frac{y}{x}$



(ii) $A = 0.5 \pm 0.2$ $\quad B = 40 \pm 1$

(iii) Plot $\frac{y}{x}$ against x as a straight line accurately.

The x-value of the point of intersection represents the value where the rectangle becomes a square.

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Marking Scheme
Additional Mathematics
Preliminary Examination 2015
Paper 1

1 (i) $2x^2 + 5x - 2kx + k - 1 = 0$

No real roots $\Rightarrow b^2 - 4ac < 0$ — M1

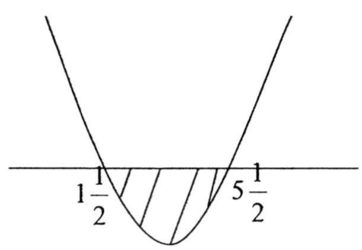
$(5-2k)^2 - 4(2)(k-1) < 0$

$25 - 20k + 4k^2 - 8k + 8 < 0$

$4k^2 - 28k + 33 < 0$ correct quadratic — M1

Finding the solution of quadratic: $k = 1\frac{1}{2}$ or $5\frac{1}{2}$ — DM1

$(2k-3)(2k-11) < 0$

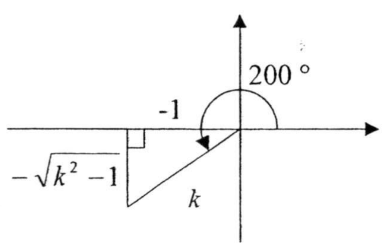


$1\frac{1}{2} < k < 5\frac{1}{2}$ — A1

(ii) The equations of the line and the curve are obtained when $k = 2$. Since $k = 2$ lies in the range, the line does not meet the curve. — B1

2 (i) $\sec 200° = -k$

$\cos 200° = -\frac{1}{k}$ — M1



$\sin 200° = -\frac{\sqrt{k^2-1}}{k}$ — A1

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[Or $\cos 200° = -\dfrac{1}{k}$ **M1**

Applying identity: $\sin^2 200° + \cos^2 200° = 1$

$$\sin 200° = \sqrt{1-\frac{1}{k^2}} \text{ (rej) or } -\sqrt{1-\frac{1}{k^2}}$$ **A1**

Accept any equivalent form]

(ii) $\tan 110° = \dfrac{\sin 110°}{\cos 110°}$

$= \dfrac{\sin(200°-90°)}{\cos(200°-90°)}$ M1

$= \dfrac{\sin 200° \cos 90° - \cos 200° \sin 90°}{\cos 200° \cos 90° + \sin 200° \sin 90°}$ (applying addition formula) M1

$= \dfrac{0-\left(-\dfrac{1}{k}\right)}{0+\left(-\dfrac{\sqrt{k^2-1}}{k}\right)(1)} = -\dfrac{1}{\sqrt{k^2-1}}$ A1

[OR $\tan 110° = -\tan 70°$

$= -\dfrac{1}{\tan 20°}$ **M1**

$= -\dfrac{1}{\tan 200°}$ **M1**

$= -\dfrac{1}{\dfrac{-\sqrt{k^2-1}}{-1}} = -\dfrac{1}{\sqrt{k^2-1}}$ **A1]**

3 $y = \ln(5-2x)$

$\dfrac{dy}{dx} = \dfrac{-2}{5-2x}$ (**M1** for -2 and **M1** for $\dfrac{1}{5-2x}$) M2

Gradient of line $= \dfrac{1}{2}$

Gradient function of normal $= \dfrac{5-2x}{2}$ M1

$\dfrac{5-2x}{2} = \dfrac{1}{2}$ M1

$x = 2$

$y = 0$

coordinates of the point = (2, 0) A1

4 $y = \cos^3 x + \sin 3x$

$\dfrac{dy}{dx} = \underbrace{(3\cos^2 x)}_{\textbf{M1}}\underbrace{(-\sin x)}_{\textbf{M1}} + \underbrace{3\cos 3x}_{\textbf{M1}}$ M3

$\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$

$= (-3\cos^2 \dfrac{\pi}{6} \sin\dfrac{\pi}{6} + 3\cos\dfrac{3\pi}{6}) \times 0.56$ M1

$= -\ 0.63$ radians/s A1

5 (i) $\dfrac{2x-1}{2x^2-5x+3} = \dfrac{A}{2x-3} + \dfrac{B}{x-1}$ M1

$2x - 1 = A(x-1) + B(2x-3)$

Substitute $x = 1$,

$1 = -\ B$

$B = -\ 1$ A1

Substitute $x = \dfrac{3}{2}$,

$2 = \dfrac{1}{2}A$

$A = 4$ A1

$\therefore \dfrac{2x-1}{2x^2-5x+3} = \dfrac{4}{2x-3} - \dfrac{1}{x-1}.$

(ii) $\displaystyle\int \frac{2x-1}{2x^2-5x+3}\,dx = \int\left(\frac{4}{2x-3} - \frac{1}{x-1}\right)dx$

$= \dfrac{4\ln(2x-3)}{2} - \ln(x-1) + c$

$= \underbrace{2}_{\textbf{B1}}\ \underbrace{\ln(2x-3)}_{\textbf{B1}} - \underbrace{\ln(x-1)+c}_{\textbf{B1}}$ B3

−1 for each error

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6 $\frac{dy}{dx} = \int (16e^{-4x}) dx$

$= -4e^{-4x} + c$ B1

(for $-4e^{-4x}$)

Substitute $\frac{dy}{dx} = 3$ and $x = 0$

$3 = -4e^{-4(0)} + c$ (attempt to find c) M1

$c = 7$

$\frac{dy}{dx} = -4e^{-4x} + 7$ A1

$y = \int (-4e^{-4x} + 7) dx$

$= e^{-4x} + 7x + c_1$ for $e^{-4x} + 7x$ B1

Substitute $x = 2$ and $y = e^{-8}$

$e^{-8} = e^{-4(2)} + 7(2) + c_1$ (attempt to find c_1) M1

$c_1 = -14$

$y = e^{-4x} + 7x - 14$ A1

7 (i) $\frac{1 - \sin 2\theta}{1 + \cos 2\theta}$

$= \frac{1 - 2\sin\theta\cos\theta}{1 + 2\cos^2\theta - 1}$ M1 M1 Applying double angle formulas M2

$= \frac{1 - 2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \frac{1}{2\cos^2\theta} - \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \frac{1}{2}\sec^2\theta - \tan\theta$

$= \frac{1}{2}(1 + \tan^2\theta) - \tan\theta$ Applying identity M1

$= \frac{1}{2}(1 + \tan^2\theta - 2\tan\theta)$

$= \frac{1}{2}(1 - \tan\theta)^2$ A1

(ii) $\dfrac{1-\sin\theta}{1+\cos\theta}=2$

$\dfrac{1}{2}\left(1-\tan\dfrac{\theta}{2}\right)^2=2$ M1

$0°\le\theta\le 180°$

$0°\le\dfrac{\theta}{2}\le 90°$

$1-\tan\dfrac{\theta}{2}=2 \text{ or } -2$

$\tan\dfrac{\theta}{2}= -1 \text{ (reject) or } 3$

$\dfrac{\theta}{2}=71.56°$ M1

$\theta=143.1°$ A1

8 (i) (a) $\left(1+\dfrac{3x}{2}\right)^5=1+\binom{5}{1}\left(\dfrac{3x}{2}\right)+\binom{5}{2}\left(\dfrac{3x}{2}\right)^2+\ldots$

$=\underbrace{1+\dfrac{15}{2}x}_{\textbf{B1}}+\underbrace{\dfrac{45}{2}x^2}_{\textbf{B1}}+\ldots.$ B2

(b) $(2-x)^5=(2)^5+\binom{5}{1}(2)^4(-x)+\binom{5}{2}(2)^3(-x)^2+\ldots$

$=\underbrace{32-80x}_{\textbf{B1}}+\underbrace{80x^2}_{\textbf{B1}}+\ldots.$ B2

(ii) $\left(2+2x-\dfrac{3x^2}{2}\right)^5=\left[\left(1+\dfrac{3x}{2}\right)(2-x)\right]^5$ M1

$=\left(1+\dfrac{3x}{2}\right)^5(2-x)^5$

$=\left(1+\dfrac{15}{2}x+\dfrac{45}{2}x^2+\ldots.\right)\left(32-80x+80x^2+\ldots.\right)$

$=\ldots.+80x^2-600x^2+720x^2+\ldots.$ M1

$=200x^2+\ldots$

Coefficient of $x^2=200$ A1

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9 (i) Intersects the y-axis at (0, 2) B1

$3-|2x+1|=0$

$|2x+1|=3$

$2x+1=3$ or $2x+1=-3$

$x=1$ or $x=-2$

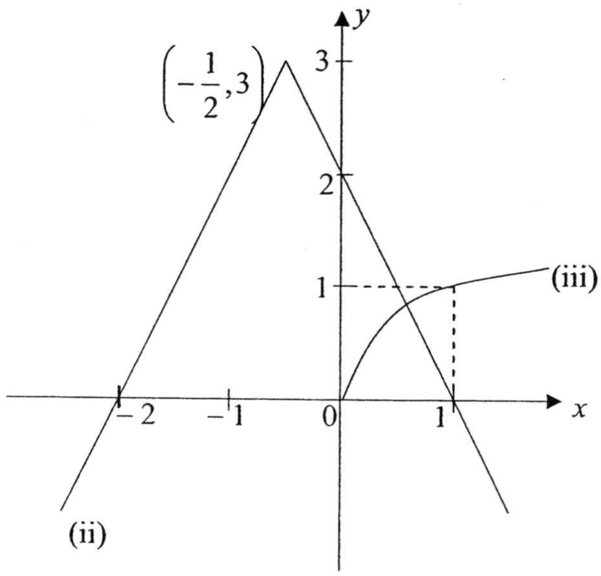
Intersects the x-axis at $(1,0)$ and $(-2,0)$. B2

B1 **B1**

(ii) correct shape and passing through the coordinate axes **B1** B2

Coordinates of vertex **B1**

(iii) correct shape passing through (1, 1) B1



(iv) Number of solutions = 1 B1

10 (i) $2\left(\frac{2}{3}\pi r^3\right)+\pi r^2 l=\frac{\pi}{6}$ M1

or $\frac{4}{3}\pi r^3+\pi r^2 l=\frac{\pi}{6}$

$l=\dfrac{\frac{\pi}{6}-\frac{4}{3}\pi r^3}{\pi r^2}$ expressing l in terms of r M1

$=\frac{1}{6r^2}-\frac{4}{3}r$

$A=2(2\pi r^2)+2\pi rl$

$=4\pi r^2+2\pi r\left(\frac{1}{6r^2}-\frac{4}{3}r\right)$ substituting a correct expression for l M1

$=4\pi r^2+\frac{\pi}{3r}-\frac{8}{3}\pi r^2$

$=\frac{4}{3}\pi r^2+\frac{\pi}{3r}$ (shown) A1

(ii) $\frac{dA}{dr}=\frac{8}{3}\pi r-\frac{\pi}{3r^2}$ M1

For min value, $\frac{dA}{dr}=0$.

$\frac{8}{3}\pi r-\frac{\pi}{3r^2}=0$ M1

$8r^3-1=0$

$r=\frac{1}{2}$ A1

Minimum value of $A=\frac{4}{3}\pi\left(\frac{1}{2}\right)^2+\dfrac{\pi}{3\left(\frac{1}{2}\right)}$ cm^2

$=3.14$ cm^2 A1

[3]

11 (i) $\angle RMA = 90° - 45°$
$= 45°$

Gradient of $MR = \tan 45°$ M1
$= 1$

Equation of MR is $y - 0 = 1(x - 2)$
$y = x - 2$ A1

(ii) Gradient of $PQ = -1$ M1
Equation of PQ is $y - 0 = -1(x - 2)$
$y = -x + 2$ A1

(iii) Coordinates of $P = (0, 2)$ M1

Applying mid point formula:

Coordinates of $Q = (4, -2)$ A1

(iv) Let the coordinates of R be $(k, k-2)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 4 & k & 0 & 4 \\ -2 & k-2 & 2 & -2 \end{vmatrix} = 20$$ M1

$$[4(k-2) + 2k + 0] - [8 + 0 - 2k] = 40$$ M1
$$4k = 56$$
$$k = 7$$

Coordinates of $R = (7, 5)$ A1

[OR $PQ = \sqrt{(4-0)^2 + (-2-2)^2}$
$= \sqrt{32}$

$RM = \sqrt{(k-2)^2 + (k-2-0)^2}$
$= \sqrt{2(k-2)^2}$
$= (k-2)\sqrt{2}$ **M1**

Area $= \frac{1}{2} \times \sqrt{32} \times (k-2)\sqrt{2} = 20$ **M1**
$k = 7$

Coordinates of $R = (7, 5)$ A1]

12

x	50	100	150	200	250
y	3250	9000	17250	28000	41250
$\frac{y}{x}$	65	90	115	140	165

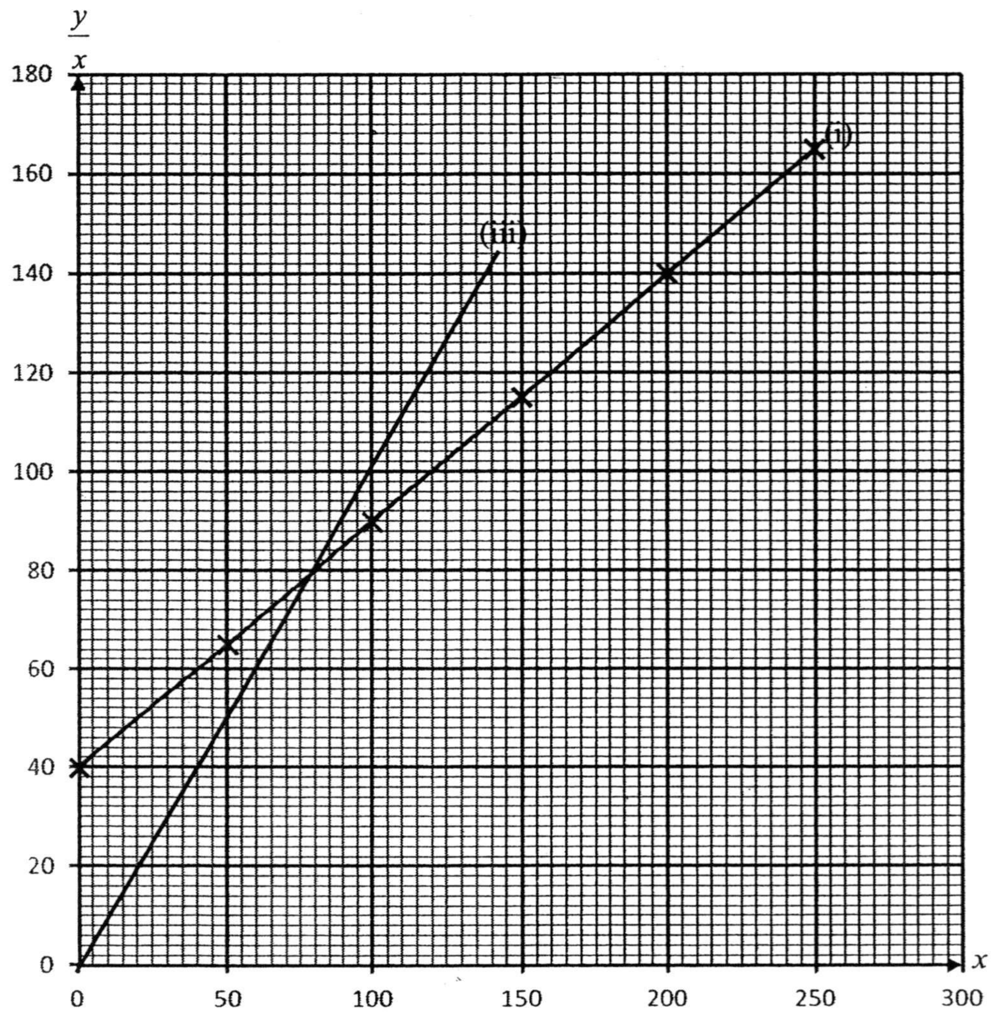
B1

(i) Draw axes and plot all given points. P1

(inaccurate plot: P0)

Drawing a straight line through all plots C1

Deduct 1 mark if a suitable scale is not used.



(ii) $y = x(Ax + B)$

$$\frac{y}{x} = Ax + B$$ M1

$$A = \frac{165 - 65}{250 - 50}$$

M1

$$= 0.5 \pm 0.2$$ A1

$$B = 40 \pm 1$$ B1

ι32

(iii) $y = x^2$

$\dfrac{y}{x} = x$ B1

Plot $\dfrac{y}{x}$ against x as a straight line accurately. B1

The x-value of the point of intersection represents the value where the rectangle becomes a square. B1

Answer all questions.

1 A beaker of water is heated until it reaches a temperature of X °C. It is then allowed to cool. It's temperature, θ °C, when it was cooling for time t minutes is given by $\theta = 28 + 60e^{-0.3t}$. Find

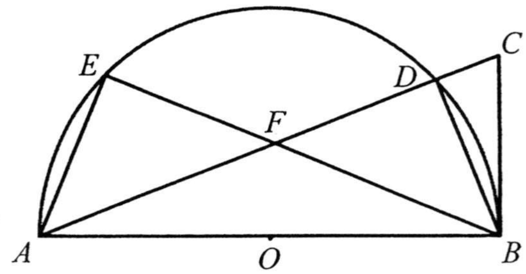
(i) the value of X, [1]

(ii) the value of θ when $t = 6$, [1]

(iii) the value of t when $\theta = \frac{1}{2}X$. [2]

(iv) Explain, with working, if the water will cool to a temperature of 20 °C. [2]

2



The figure shows a semi-circle centre O, with diameter AB. Points D and E lie on the semi-circle. AC is a straight line passing through D. BC is tangent to the semi-circle at B. The tangent to the semi-circle at B meets AD at C. The lines BE and AD intersect at F.

Given that $AE = BD$, prove that

(i) triangle ABD is congruent to triangle BAE, [3]

(ii) triangle ABC is similar to triangle BDC, [2]

(iii) $BC^2 = AC \times DC$. [2]

3 Given that $\log_p(a^3b) = x$, and $\log_p \frac{a}{\sqrt{b}} = y$, express in terms of x and y,

(i) $\log_p a$ and $\log_p b$, [5]

(ii) $\log_b\left(\frac{1}{ab^2}\right)$. [3]

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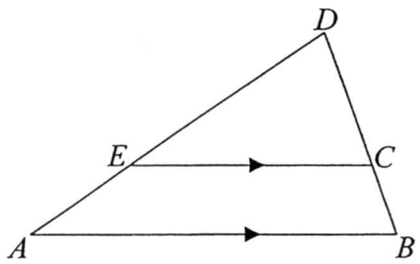
4 In the diagram, AB is parallel to EC.



Given that $AE : ED = 1 : \sqrt{2}$ and $CE = (3 + \sqrt{2})$ cm, find in the form $a\sqrt{2} + b$,

(i) $\dfrac{CE}{AB}$, [3]

(ii) $\dfrac{\text{area of } \Delta CDE}{\text{area of } \Delta BDA}$, [2]

(iii) the length of AB. [3]

5 A curve has the equation $y = ax^2 + \dfrac{b}{x^3}$, where a and b are constants.

(i) Given that the curve has a stationary point at $(2, 5)$, find the value of a and of b. [4]

(ii) Determine the nature of the stationary point. [2]

(iii) Explain why the curve is a decreasing function for $x < 0$. [2]

6 The roots of the equation $x^2 - 4x - 8 = 0$ are α^3 and β^3.

(i) Given that $(\alpha + \beta)^3 = -5\alpha - 5\beta$, find the value of $\alpha + \beta$. [5]

(ii) Find the quadratic equation in x whose roots are $\dfrac{\alpha^2}{2}$ and $\dfrac{\beta^2}{2}$. [4]

7 The function $f(x) = 3x^3 + 2x^2 + ax + b$, where a and b are constants, has a factor $(x^2 - 4)$.

(i) Show that $a = -12$ and $b = -8$. [4]

(ii) Factorise $3x^3 + 2x^2 - 12x - 8$ completely. [3]

(iii) Hence solve the equation $-3x^3 + 2x^2 + 12x - 8 = 0$. [2]

8 (i) Find all the angles between $0°$ and $360°$ inclusive which satisfies the equation $2\sin\theta - 3\cos 2\theta - 1 = 0$. [4]

(ii) On the same axes, sketch the graphs of $y = 2\sin\theta - 1$ and $y = 3\cos 2\theta$ for $0° \le \theta \le 360°$. [4]

(iii) Using your answers to part (i) and (ii), state the range of values of θ for which $3\cos 2\theta \ge 2\sin\theta - 1$ for $0° \le \theta \le 360°$. [2]

9 A particle moves in a straight line with a velocity, v m/s, given by $v = -3t^2 + 12t - 13$. The displacement of the particle from a fixed point O after 8 seconds is 400 meters.

Calculate

(i) (a) the displacement of the particle after 5 seconds, [4]

(b) the value of t when the acceleration is zero. [2]

(ii) Explain why the particle will never come to rest.
Hence find the maximum velocity of the particle. [3]

(iii) Will the particle ever return to its starting point? Explain your answer. [2]

10 (i) A circle C_1 has an equation given by $x^2 + y^2 - 2kx + 2y + 1 = 0$, where k is a positive constant.
Given that C_1 has a radius of 2 units, find the value of k. [4]

(ii) The centre of a circle C_2 lies on the line $y = 2x + 2$. Given that C_2 passes through the points $(3, 2)$ and $(0, -1)$, find the equation of C_2. [5]

(iii) Calculate the shortest distance from the centre of C_1 to the circumference of C_2. [3]

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11 (i) Find $\int \frac{1}{\sqrt{2x+14}}\,dx$. [1]

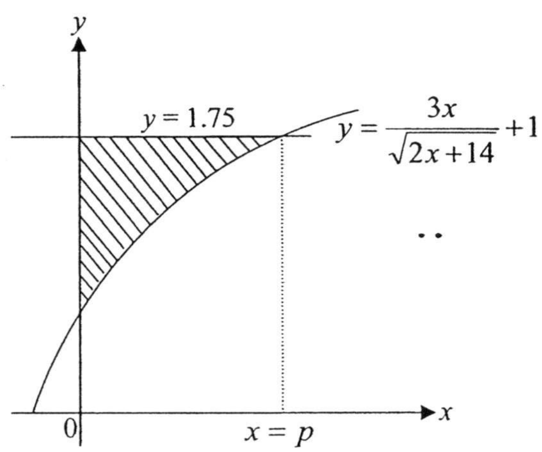
(ii) Show that $\frac{d}{dx}\left[(x-2)\sqrt{2x+14}\right] = \frac{3x+12}{\sqrt{2x+14}}$. [3]

The diagram shows part of the curve $y = \frac{3x}{\sqrt{2x+14}} + 1$ intersecting the line $y = 1.75$ and $x = p$.

(iii) (a) Find the value of p. [2]

(b) Using your results from part (i) and part (ii), find the area bounded by the curve, the line $x = p$, and the coordinate axes. [4]

(c) Find the area of the shaded region. [2]



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