SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

FOUNDED 1963

Name: ______________________________( ) Class: Sec _______

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **Wednesday 19 August 2015**
2 hour

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This question paper consists of **5** printed pages.

Setter: Ms Zoe Pow
Vetter: Mr Ang Hanping

[Turn over

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*We Nurture Students to **Think, Care** and **Lead** with **P.R.I.D.E.***

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 The acute angles A and B are such that $\sin(A+B)=0$ and $\sin B=\frac{1}{4}$.

Without using a calculator, find the exact value of $\tan A$. [3]

2 It is given that $y=\frac{2x+16}{x-1}$, where both x and y are positive and vary with time.

Find the value of y when the rate of increase of y is twice the rate of decrease of x. [4]

3 Express $\frac{x+1}{(x+2)(x^2+4)}$ in partial fractions. [4]

4 **(i)** Find the range of values of k for which the line $y=x-k$ intersects the curve $y=kx(x+3)$ at two distinct points. [4]

(ii) Hence or otherwise, find the range of values of k for which $kx(x+3)>x-k$ for all real values of x. [2]

5 Given that the first two non-zero terms of the expansion of $(1-kx)\left(1+\frac{x}{3}\right)^n$ are 1 and $-\frac{7}{3}x^2$, where n is a positive integer, find the value of k and of n. [6]

6 **(i)** Prove that $\frac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}=\frac{1-\tan x}{1+\tan x}$. [3]

(ii) Find all the angles between $0°$ and $360°$ which satisfy $\frac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}=\frac{2}{3}\tan x$. [3]

7 It is given that $f(x)$ is such that $f'(x)=\sin 2x+\cos 3x$.

Given also that $f\left(\frac{\pi}{6}\right)=0$, show that $f''(x)+9f(x)=-\frac{3}{4}-\frac{5}{2}\cos 2x$. [6]

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8 Variables x and y are connected by the equation $y = 10^{k-nx}$, where n and k are constants. When a graph of $\lg y$ is plotted against x, a straight line passing through the points $(1, 2)$ and $(4, -7)$ is obtained.

Find

(i) the value of n and of k, [4]

(ii) the coordinates of the point on the line at which $y = 10^x$. [3]

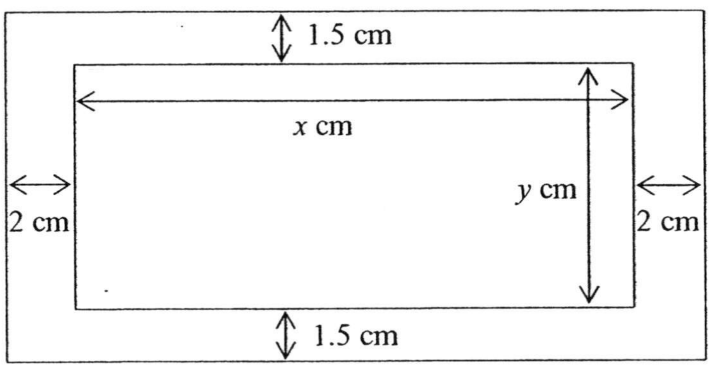
9 The tangent to the curve $y = x\ln 3x$ at point $P(1, \ln 3)$ cuts the x-axis at Q.

(i) Find the angle that PQ makes with the x-axis. [5]

The normal to the curve $y = x\ln 3x$ at R is parallel to the line $y = 5 - 2x$.

(ii) Find the x-coordinate of R. [3]

10 The diagram shows a rectangular poster of area 825 cm^2 with side margins of 2 cm and top and bottom margins of 1.5 cm. The length and breadth of the printing area are x cm and y cm respectively.



(i) Show that the printing area, A cm^2, is given by $A = \dfrac{825x}{x+4} - 3x$. [3]

(ii) Given that x can vary, find the value of x for which the printing area is stationary. [4]

(iii) Explain why this value of x gives the poster the largest printing area possible. [1]

11 **(i)** Sketch the graph of $y = |7x - 2|$, for $-1 \le x \le 2$. [2]

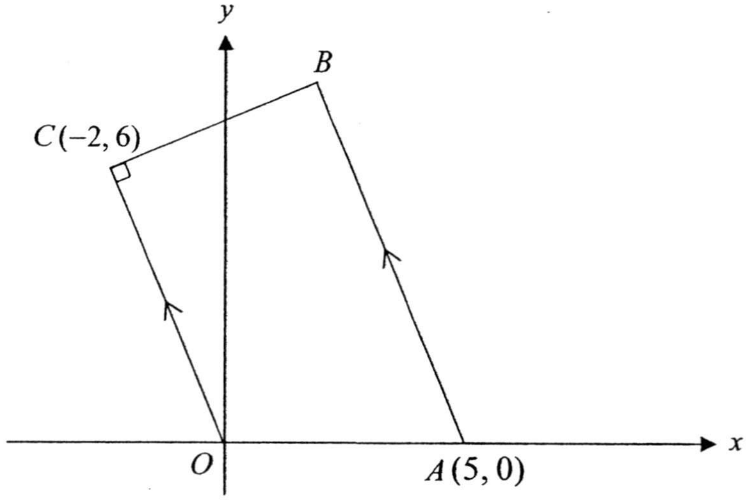
(ii) Hence, find the range of values of x for which $|14x - 4| \ge 3x$. [4]

(iii) Using your graph, determine the number of intersections of the line $y = mx + c$ with $y = |7x - 2|$, justifying your answer in each of the following cases.

(a) $m = -7$ and $c > 2$, [2]

(b) $m = 7$ and $c < -2$. [2]

12 **Solutions to this question by accurate drawing will not be accepted.**



In the trapezium $OABC$, the point A has coordinates $(5, 0)$ and the point C has coordinates $(-2, 6)$. The sides OC and AB are parallel, and BC is perpendicular to OC.

(i) Show that the coordinates of B are $\left(2\frac{1}{2}, 7\frac{1}{2}\right)$. [5]

(ii) OC is produced to D such that $OABD$ is a parallelogram.

Find the coordinates of D. [2]

(iii) Find the equation of the perpendicular bisector of OC. [2]

(iv) E is a point which lies on the perpendicular bisector of OC such that the area of quadrilateral $OAEC$ is 15 units^2. Given that the x-coordinate of E is positive, find the coordinates of E. [3]

END OF PAPER

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Additional Mathematics Paper 1 (80 marks)

Qn	Answer
1	$\tan A = -\dfrac{1}{\sqrt{15}}$
2	$y = 8$
3	$\dfrac{x+6}{8(x^2+4)} - \dfrac{1}{8(x+2)}$
4i	$k < \dfrac{1}{5}$ or $k > 1$
4ii	$\dfrac{1}{5} < k < 1$
5	$\therefore\ n = 6,\ k = 2$
6i	$LHS = \dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x\cos x}$ $= \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$ $= \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2}$ $= \dfrac{(\cos x - \sin x)}{(\cos x + \sin x)}$ $= \dfrac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}$ $= \dfrac{1 - \tan x}{1 + \tan x}$ $= RHS$ (shown)
6ii	$\therefore\ x = 26.6°, 108.4°, 206.6°, 288.4°$
7	$f'(x) = \sin 2x + \cos 3x$ $f(x) = \int (\sin 2x + \cos 3x)\,dx$ $= -\dfrac{1}{2}\cos 2x + \dfrac{1}{3}\sin 3x + c$ $f\left(\dfrac{\pi}{6}\right) = -\dfrac{1}{2}\cos 2\left(\dfrac{\pi}{6}\right) + \dfrac{1}{3}\sin 3\left(\dfrac{\pi}{6}\right) + c$ $-\dfrac{1}{2}\cos\left(\dfrac{\pi}{3}\right) + \dfrac{1}{3}\sin\left(\dfrac{\pi}{2}\right) + c = 0$ $-\dfrac{1}{2}\left(\dfrac{1}{2}\right) + \dfrac{1}{3}(1) + c = 0$ $c = -\dfrac{1}{12}$

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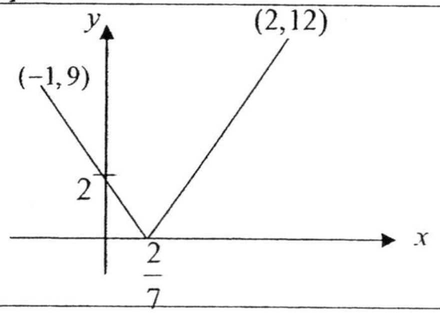
Qn	Answer
	$f(x)=-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}$ $f''(x)=2\cos 2x-3\sin 3x$ $f''(x)+9f(x)=2\cos 2x-3\sin 3x+9\left(-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}\right)$ $=2\cos 2x-3\sin 3x-\frac{9}{2}\cos 2x+3\sin 3x-\frac{3}{4}$ $=-\frac{3}{4}-\frac{5}{2}\cos 2x$ (shown)
8i	$k=5$
8ii	Coordinates are $(1.25, 1.25)$
9i	$\theta=64.5^\circ$
9ii	$x=0.202$
10i	$[x+(2\times 2)]\times[y+(1.5\times 2)]=825$ $(x+4)(y+3)=825$ $y+3=\frac{825}{x+4}$ $y=\frac{825}{x+4}-3$ $A=\left(\frac{825}{x+4}-3\right)(x)$ $A=\frac{825x}{x+4}-3x$ (shown)
10ii	$x=29.2$ or -37.2 (N.A.)
10 iii	$\frac{d^2A}{dx^2}=\frac{-2(3300)}{(x+4)^3}$ When $x=29.2$, $\frac{d^2A}{dx^2}=\frac{-6600}{(29.2+4)^3}$ $=-0.181$ Since $\frac{d^2A}{dx^2}<0$, this value of x gives the poster the largest printing area possible.
11i	

Qn	Answer
11 ii	$x \le \frac{4}{17}$ or $x \ge \frac{4}{11}$
11 iiia	1 intersection; Line parallel to L.H. arm cuts R.H. arm at only one point.
11 iiib	0 intersection; Line parallel to R.H. arm; for intersection, $c > -2$.
12i	$m_{AB} = m_{OC}$ $\frac{y-0}{x-5} = \frac{6}{-2}$ $y = -3x + 15$ ----- (1) $m_{BC} = -\frac{1}{m_{OC}}$ $\frac{y-6}{x+2} = \frac{1}{3}$ $y = \frac{1}{3}x + 6\frac{2}{3}$ ----- (2) Sub (1) into (2): $-3x + 15 = \frac{1}{3}x + 6\frac{2}{3}$ $3\frac{1}{3}x = 8\frac{1}{3}$ $x = 2\frac{1}{2}$ Sub $x = 2\frac{1}{2}$ into (1): $y = -3\left(2\frac{1}{2}\right) + 15$ $y = 7\frac{1}{2}$ Coordinates of B are $\left(2\frac{1}{2}, 7\frac{1}{2}\right)$ (shown)
12 ii	$D\left(-2\frac{1}{2}, 7\frac{1}{2}\right)$
12 iii	$y = \frac{1}{3}x + 3\frac{1}{3}$
12 iv	Possible coordinates of E are $(0.8, 3.6)$.

Additional Mathematics Paper 1 (80 marks)

Qn	Solution	Mark Allocation
1	$\sin(A+B)=0$ $\sin A\cos B+\cos A\sin B=0$ $\sin A\left(\dfrac{\sqrt{15}}{4}\right)+\cos A\left(\dfrac{1}{4}\right)=0$ $\sqrt{15}\sin A+\cos A=0$ $\sqrt{15}\sin A=-\cos A$ $\dfrac{\sin A}{\cos A}=-\dfrac{1}{\sqrt{15}}$ $\tan A=-\dfrac{1}{\sqrt{15}}$	M1: Addition Formula M1: Find $\cos B$ A1
2	$y=\dfrac{2x+16}{x-1}$ $\dfrac{dy}{dx}=\dfrac{(x-1)(2)-(2x+16)(1)}{(x-1)^2}$ $=\dfrac{2x-2-2x-16}{(x-1)^2}$ $=-\dfrac{18}{(x-1)^2}$ $\dfrac{dy}{dt}=-2\dfrac{dx}{dt}$ $\dfrac{dy}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$ $-2\dfrac{dx}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$ $\dfrac{dy}{dx}=-2$ $-\dfrac{18}{(x-1)^2}=-2$ $(x-1)^2=9$ $x-1=\pm3$ $x=4$ or -2 (N.A.) When $x=4$, $y=\dfrac{2(4)+16}{4-1}$ $\therefore\ y=8$	M1 M1 M1 A1

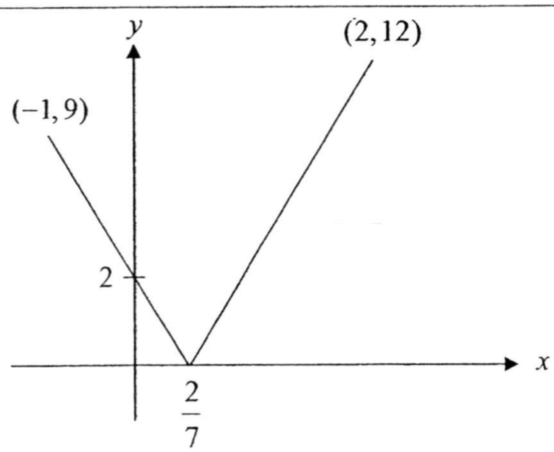
Qn	Solution	Mark Allocation
3	Let $\dfrac{x+1}{(x+2)(x^2+4)}=\dfrac{A}{(x+2)}+\dfrac{Bx+C}{(x^2+4)}$	
	$x+1=A(x^2+4)+(Bx+C)(x+2)$	M1
	Sub $x=-2$: $-2+1=A[(-2)^2+4]$ $-1=8A$ $A=-\dfrac{1}{8}$ Comparing coefficients of x^2: $0=A+B$ $B=\dfrac{1}{8}$ Sub $x=0$: $1=4A+2C$ $2C=1-4\left(-\dfrac{1}{8}\right)$ $C=\dfrac{3}{4}$	M2 Any 2 correct 1 mark
	$\dfrac{x+1}{(x+2)(x^2+4)}=-\dfrac{1}{8(x+2)}+\dfrac{\frac{1}{8}x+\frac{3}{4}}{(x^2+4)}$ $=\dfrac{x+6}{8(x^2+4)}-\dfrac{1}{8(x+2)}$	A1
4i	$kx(x+3)=x-k$	
	$kx^2+(3k-1)x+k=0$	M1
	$(3k-1)^2-4(k)(k)>0$	M1: $b^2-4ac>0$
	$9k^2-6k+1-4k^2>0$	
	$5k^2-6k+1>0$	M1: Factorise
	$(5k-1)(k-1)>0$	
	$k<\dfrac{1}{5}$ or $k>1$	A1
4ii	$(3k-1)^2-4(k)(k)<0$	M1: $b^2-4ac<0$
	$(5k-1)(k-1)<0$	
	$\dfrac{1}{5}<k<1$	A1

Qn	Solution	Mark Allocation
5	$\left(1+\frac{x}{3}\right)^n = 1+\binom{n}{1}\left(\frac{x}{3}\right)+\binom{n}{2}\left(\frac{x}{3}\right)^2+...$	
	$= 1+\frac{nx}{3}+\frac{n(n-1)}{2!}\left(\frac{x^2}{9}\right)+...$	
	$= 1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+...$	M1
	$(1-kx)\left(1+\frac{x}{3}\right)^n = (1-kx)\left(1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+...\right)$	
	$= 1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2-kx-\frac{nk}{3}x^2+...$	
	$= 1+\left(\frac{n}{3}-k\right)x+\left[\frac{n(n-1)}{18}-\frac{nk}{3}\right]x^2+...$	
	$\frac{n}{3}-k=0$	M1
	$n=3k$ ----- (1)	
	$\frac{n(n-1)}{18}-\frac{nk}{3}=-\frac{7}{3}$ ----- (2)	M1
	Sub (1) into (2):	
	$\frac{3k(3k-1)}{18}-\frac{3k^2}{3}=-\frac{7}{3}$	M1
	$9k^2-3k-18k^2+42=0$	
	$-9k^2-3k+42=0$	
	$3k^2+k-14=0$	
	$(3k+7)(k-2)=0$	M1
	$k=-2\frac{1}{3}$ or $k=2$	
	Sub $k=-2\frac{1}{3}$ into (1): $n=3\left(-2\frac{1}{3}\right)$	
	$n=-7$ (N.A) $\therefore$ Reject $k=-2\frac{1}{3}$	
	Sub $k=2$ into (1): $n=3(2)$	
	$=6$	
	$\therefore\ n=6,\ k=2$	A1

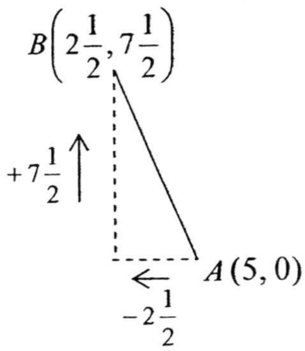
Qn	Solution	Mark Allocation
6i	$LHS=\dfrac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}$ $=\dfrac{(\cos x-\sin x)(\cos x+\sin x)}{\sin^2 x+\cos^2 x+2\sin x\cos x}$ $=\dfrac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)^2}$ $=\dfrac{(\cos x-\sin x)}{(\cos x+\sin x)}$ $=\dfrac{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}$ $=\dfrac{1-\tan x}{1+\tan x}$ $=RHS$ (shown)	M1: Factorise numerator M1: Factorise denominator A1: Divide by $\cos x$
6ii	$\dfrac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}=\dfrac{2}{3}\tan x$ $\dfrac{1-\tan x}{1+\tan x}=\dfrac{2\tan x}{3}$ $3-3\tan x=2\tan x+2\tan^2 x$ $2\tan^2 x+5\tan x-3=0$ $(2\tan x-1)(\tan x+3)=0$ $\tan x=\dfrac{1}{2}$ Basic $\angle=26.565$ $x=26.565, 180+26.565$ OR $\tan x=-3$ Basic $\angle=71.565$ $x=180-71.565, 360-71.565$ $\therefore\ x=26.6°, 108.4°, 206.6°, 288.4°$	M1 M1: Find basic angle (both correct) A1
7	$f'(x)=\sin 2x+\cos 3x$ $f(x)=\int(\sin 2x+\cos 3x)\,dx$ $=-\dfrac{1}{2}\cos 2x+\dfrac{1}{3}\sin 3x+c$ $f\left(\dfrac{\pi}{6}\right)=-\dfrac{1}{2}\cos 2\left(\dfrac{\pi}{6}\right)+\dfrac{1}{3}\sin 3\left(\dfrac{\pi}{6}\right)+c$	M1

Qn	Solution	Mark Allocation
	$-\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+\frac{1}{3}\sin\left(\frac{\pi}{2}\right)+c=0$	M1
	$-\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{3}(1)+c=0$	
	$c=-\frac{1}{12}$	M1
	$f(x)=-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}$	
	$f''(x)=2\cos 2x-3\sin 3x$	M1
	$f''(x)+9f(x)=2\cos 2x-3\sin 3x+9\left(-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}\right)$	
	$=2\cos 2x-3\sin 3x-\frac{9}{2}\cos 2x+3\sin 3x-\frac{3}{4}$	M1
	$=-\frac{3}{4}-\frac{5}{2}\cos 2x$ (shown)	A1
8i	$y=10^{k-nx}$	
	$\lg y=(k-nx)\lg 10$	
	$\lg y=-nx+k$	
	$m=\frac{2-(-7)}{1-4}$	M1
	$=-3$	
	$-n=-3$	
	$\therefore\ n=3$	A1
	At $(1,2)$, $2=(-3)(1)+c$	M1
	$c=5$	
	$\therefore\ k=5$	A1
8ii	$y=10^{5-3x}$ ----- (1)	
	$y=10^{x}$ ----- (2)	
	Sub (1) into (2): $10^{5-3x}=10^{x}$	M1
	$x=5-3x$	
	$4x=5$	
	$x=1\frac{1}{4}$	
	Sub $x=1\frac{1}{4}$ into (2): $y=10^{1\frac{1}{4}}$	
	$y=17.783$	M1
	$\lg y=\lg 17.783$	
	$=1.25$	
	$\therefore$ Coordinates are $(1.25,1.25)$	A1

Qn	Solution	Mark Allocation
9i	$y = x\ln 3x$ $\frac{dy}{dx} = x\left(\frac{1}{x}\right) + (\ln 3x)(1)$ $= 1 + \ln 3x$ At $P(1, \ln 3)$, $\frac{dy}{dx} = 1 + \ln 3(1)$ $= 1 + \ln 3$ At $P(1, \ln 3)$, $y - \ln 3 = (1 + \ln 3)(x - 1)$ $y = (1 + \ln 3)x - 1$ When $y = 0$, $(1 + \ln 3)x = 1$ $x = \frac{1}{(1 + \ln 3)}$ $x = 0.477$ Coordinates of Q are $(0.477, 0)$ $\tan\theta = \frac{\ln 3 - 0}{1 - 0.477}$ $= 2.0986$ $\theta = 64.5°$	M1 M1 M1 M1 A1
9ii	Gradient of normal $= -\frac{1}{1 + \ln 3x}$ $-\frac{1}{1 + \ln 3x} = -2$ $1 + \ln 3x = \frac{1}{2}$ $\ln 3x = -\frac{1}{2}$ $3x = e^{-\frac{1}{2}}$ $x = \frac{1}{3}e^{-\frac{1}{2}}$ $= 0.202$	M1 M1 A1
10i	$[x + (2 \times 2)] \times [y + (1.5 \times 2)] = 825$ $(x + 4)(y + 3) = 825$ $y + 3 = \frac{825}{x + 4}$ $y = \frac{825}{x + 4} - 3$	M1 M1

Qn	Solution	Mark Allocation
	$A=\left(\dfrac{825}{x+4}-3\right)(x)$ $A=\dfrac{825x}{x+4}-3x$ (shown)	A1
10 ii	$\dfrac{dA}{dx}=\dfrac{(x+4)(825)-(825x)(1)}{(x+4)^2}-3$ $=\dfrac{3300}{(x+4)^2}-3$ $\dfrac{3300}{(x+4)^2}-3=0$ $(x+4)^2=1100$ $x+4=\pm\sqrt{1100}$ $x=29.2$ or -37.2 (N.A.)	M1 M1 M1 A1
10 iii	$\dfrac{d^2A}{dx^2}=\dfrac{-2(3300)}{(x+4)^3}$ When $x=29.2$, $\dfrac{d^2A}{dx^2}=\dfrac{-6600}{(29.2+4)^3}$ $=-0.181$ Since $\dfrac{d^2A}{dx^2}<0$, this value of x gives the poster the largest printing area possible.	B1: Show $\dfrac{d^2A}{dx^2}<0$
11i		B1: V-shape graph B1: Correct x- and y- intercepts and end-points labelled
11 ii	$\lvert 14x-4\rvert=3x$ $2\lvert 7x-2\rvert=3x$ $\lvert 7x-2\rvert=\dfrac{3}{2}x$	M1

Qn	Solution	Mark Allocation
	$7x-2=\frac{3}{2}x$ or $7x-2=-\frac{3}{2}x$	M1
	$5\frac{1}{2}x=2$ $\quad$ $8\frac{1}{2}x=2$	
	$x=\frac{4}{11}$ $\quad$ $x=\frac{4}{17}$	M1
	$\therefore\ x\le\frac{4}{17}$ or $x\ge\frac{4}{11}$	A1
11 iiia	1 intersection; Line parallel to L.H. arm cuts R.H. arm at only one point.	B1 B1
11 iiib	0 intersection; Line parallel to R.H. arm; for intersection, $c>-2$.	B1 B1
12i	$m_{AB}=m_{OC}$	
	$\frac{y-0}{x-5}=\frac{6}{-2}$	M1
	$y=-3x+15$ ----- (1)	
	$m_{BC}=-\frac{1}{m_{OC}}$	
	$\frac{y-6}{x+2}=\frac{1}{3}$	M1
	$y=\frac{1}{3}x+6\frac{2}{3}$ ----- (2)	
	Sub (1) into (2): $-3x+15=\frac{1}{3}x+6\frac{2}{3}$	M1
	$3\frac{1}{3}x=8\frac{1}{3}$	
	$x=2\frac{1}{2}$	M1
	Sub $x=2\frac{1}{2}$ into (1): $y=-3\left(2\frac{1}{2}\right)+15$	
	$y=7\frac{1}{2}$	
	Coordinates of B are $\left(2\frac{1}{2},7\frac{1}{2}\right)$	A1

Qn	Solution	Mark Allocation
12 ii	 $D\left(-2\frac{1}{2}, 7\frac{1}{2}\right)$	M1 O.E. A1
12 iii	Midpoint of $OC = \left(\frac{0+(-2)}{2}, \frac{0+6}{2}\right)$ $= (-1, 3)$ Gradient of perpendicular bisector of $OC = -\frac{1}{(-3)}$ $= \frac{1}{3}$ At $(-1, 3)$, $y - 3 = \frac{1}{3}(x+1)$ $y = \frac{1}{3}x + 3\frac{1}{3}$	M1 A1
12 iv	$\frac{1}{2}\begin{vmatrix} 0 & 5 & x & -2 & 0 \\ 0 & 0 & y & 6 & 0 \end{vmatrix} = 15$ $\lvert 5y + 6x - (-2y) \rvert = 30$ $\lvert 7y + 6x \rvert = 30$ $7y + 6x = 30$ ----- (1) or $7y + 6x = -30$ ----- (2) $3y = x + 10$ ----- (3) From (3): $x = 3y - 10$ ----- (4) Sub (4) into (1): $7y + 6(3y-10) = 30$ $7y + 6(3y-10) = -30$ $25y = 90$ $25y = 30$ $y = 3.6$ $y = 1.2$ Sub $y = 3.6$ into (4): Sub $y = 1.2$ into (4): $x = 3(3.6) - 10$ $x = 3(1.2) - 10$ $x = 0.8$ $x = -6.4$ [N.A.] Possible coordinates of E are $(0.8, 3.6)$.	M1 M1 A1 [with rejection]

Additional Mathematics Paper 1 (80 marks)

Qn	Solution	Mark Allocation
1	$\sin(A+B)=0$ $\sin A\cos B+\cos A\sin B=0$ $\sin A\left(\frac{\sqrt{15}}{4}\right)+\cos A\left(\frac{1}{4}\right)=0$ $\sqrt{15}\sin A+\cos A=0$ $\sqrt{15}\sin A=-\cos A$ $\frac{\sin A}{\cos A}=-\frac{1}{\sqrt{15}}$ $\tan A=-\frac{1}{\sqrt{15}}$	M1: Addition Formula M1: Find $\cos B$ A1
2	$y=\frac{2x+16}{x-1}$ $\frac{dy}{dx}=\frac{(x-1)(2)-(2x+16)(1)}{(x-1)^2}$ $=\frac{2x-2-2x-16}{(x-1)^2}$ $=-\frac{18}{(x-1)^2}$ $\frac{dy}{dt}=-2\frac{dx}{dt}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $-2\frac{dx}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $\frac{dy}{dx}=-2$ $-\frac{18}{(x-1)^2}=-2$ $(x-1)^2=9$ $x-1=\pm 3$ $x=4$ or -2 (N.A.) When $x=4$, $y=\frac{2(4)+16}{4-1}$ $\therefore\ y=8$	M1 M1 M1 A1

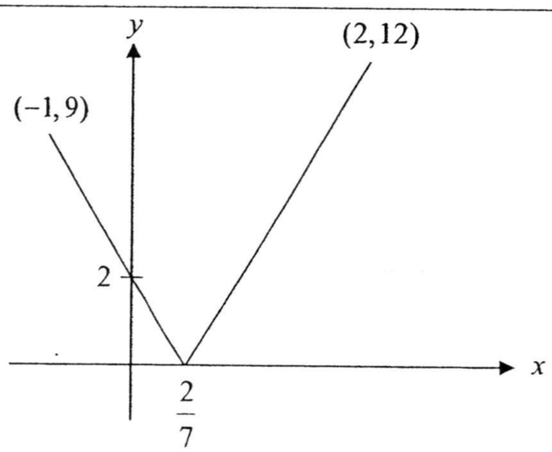
Qn	Solution	Mark Allocation
3	Let $\frac{x+1}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+4)}$ $x+1 = A(x^2+4) + (Bx+C)(x+2)$ Sub $x=-2$: $-2+1 = A[(-2)^2+4]$ $-1 = 8A$ $A = -\frac{1}{8}$ Comparing coefficients of x^2: $0 = A+B$ $B = \frac{1}{8}$ Sub $x=0$: $1 = 4A+2C$ $2C = 1-4\left(-\frac{1}{8}\right)$ $C = \frac{3}{4}$ $\frac{x+1}{(x+2)(x^2+4)} = -\frac{1}{8(x+2)} + \frac{\frac{1}{8}x+\frac{3}{4}}{(x^2+4)}$ $= \frac{x+6}{8(x^2+4)} - \frac{1}{8(x+2)}$	M1 M2. Any 2 correct 1 mark A1
4i	$kx(x+3) = x-k$ $kx^2 + (3k-1)x + k = 0$ $(3k-1)^2 - 4(k)(k) > 0$ $9k^2 - 6k + 1 - 4k^2 > 0$ $5k^2 - 6k + 1 > 0$ $(5k-1)(k-1) > 0$ $k < \frac{1}{5}$ or $k > 1$	M1 M1: $b^2-4ac>0$ M1: Factorise A1
4ii	$(3k-1)^2 - 4(k)(k) < 0$ $(5k-1)(k-1) < 0$ $\frac{1}{5} < k < 1$	M1: $b^2-4ac<0$ A1

Qn	Solution	Mark Allocation
5	$\left(1+\frac{x}{3}\right)^n = 1+\binom{n}{1}\left(\frac{x}{3}\right)+\binom{n}{2}\left(\frac{x}{3}\right)^2+\ldots$	
	$= 1+\frac{nx}{3}+\frac{n(n-1)}{2!}\left(\frac{x^2}{9}\right)+\ldots$	
	$= 1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots$	M1
	$(1-kx)\left(1+\frac{x}{3}\right)^n = (1-kx)\left(1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots\right)$	
	$= 1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2-kx-\frac{nk}{3}x^2+\ldots$	
	$= 1+\left(\frac{n}{3}-k\right)x+\left[\frac{n(n-1)}{18}-\frac{nk}{3}\right]x^2+\ldots$	
	$\frac{n}{3}-k=0$	M1
	$n=3k$ ----- (1)	
	$\frac{n(n-1)}{18}-\frac{nk}{3}=-\frac{7}{3}$ ----- (2)	M1
	Sub (1) into (2):	
	$\frac{3k(3k-1)}{18}-\frac{3k^2}{3}=-\frac{7}{3}$	M1
	$9k^2-3k-18k^2+42=0$	
	$-9k^2-3k+42=0$	
	$3k^2+k-14=0$	
	$(3k+7)(k-2)=0$	M1
	$k=-2\frac{1}{3}$ or $k=2$	
	Sub $k=-2\frac{1}{3}$ into (1): $n=3\left(-2\frac{1}{3}\right)$	
	$n=-7$ (N.A) $\therefore$ Reject $k=-2\frac{1}{3}$	
	Sub $k=2$ into (1): $n=3(2)$	
	$=6$	
	$\therefore\ n=6,\ k=2$	A1

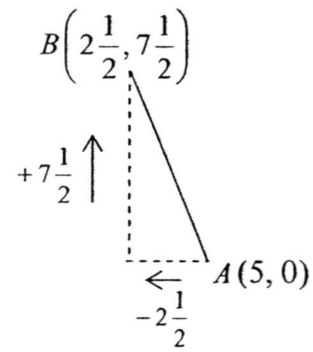
Qn	Solution	Mark Allocation
6i	$LHS = \dfrac{\cos^2 x - \sin^2 x}{1+2\sin x\cos x}$ $= \dfrac{(\cos x-\sin x)(\cos x+\sin x)}{\sin^2 x+\cos^2 x+2\sin x\cos x}$ $= \dfrac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)^2}$ $= \dfrac{(\cos x-\sin x)}{(\cos x+\sin x)}$ $= \dfrac{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}$ $= \dfrac{1-\tan x}{1+\tan x}$ $= RHS$ (shown)	M1: Factorise numerator M1: Factorise denominator A1: Divide by $\cos x$
6ii	$\dfrac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x} = \dfrac{2}{3}\tan x$ $\dfrac{1-\tan x}{1+\tan x} = \dfrac{2\tan x}{3}$ $3-3\tan x = 2\tan x + 2\tan^2 x$ $2\tan^2 x + 5\tan x - 3 = 0$ $(2\tan x - 1)(\tan x + 3) = 0$ $\tan x = \dfrac{1}{2}$ Basic $\angle = 26.565$ $x = 26.565, 180+26.565$ OR $\tan x = -3$ Basic $\angle = 71.565$ $x = 180-71.565, 360-71.565$ $\therefore\ x = 26.6°, 108.4°, 206.6°, 288.4°$	M1 M1: Find basic angle (both correct) A1
7	$f'(x) = \sin 2x + \cos 3x$ $f(x) = \int (\sin 2x + \cos 3x)\,dx$ $= -\dfrac{1}{2}\cos 2x + \dfrac{1}{3}\sin 3x + c$ $f\left(\dfrac{\pi}{6}\right) = -\dfrac{1}{2}\cos 2\left(\dfrac{\pi}{6}\right) + \dfrac{1}{3}\sin 3\left(\dfrac{\pi}{6}\right) + c$	M1

Qn	Solution	Mark Allocation
	$-\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+\frac{1}{3}\sin\left(\frac{\pi}{2}\right)+c=0$	M1
	$-\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{3}(1)+c=0$	
	$c=-\frac{1}{12}$	M1
	$f(x)=-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}$	
	$f''(x)=2\cos 2x-3\sin 3x$	M1
	$f''(x)+9f(x)=2\cos 2x-3\sin 3x+9\left(-\frac{1}{2}\cos 2x+\frac{1}{3}\sin 3x-\frac{1}{12}\right)$	
	$=2\cos 2x-3\sin 3x-\frac{9}{2}\cos 2x+3\sin 3x-\frac{3}{4}$	M1
	$=-\frac{3}{4}-\frac{5}{2}\cos 2x$ (shown)	A1
8i	$y=10^{k-nx}$ $\lg y=(k-nx)\lg 10$ $\lg y=-nx+k$	
	$m=\frac{2-(-7)}{1-4}$ $=-3$	M1
	$-n=-3$ $\therefore n=3$	A1
	At $(1,2)$, $2=(-3)(1)+c$	M1
	$c=5$ $\therefore k=5$	A1
8ii	$y=10^{5-3x}$ ----- (1) $y=10^{x}$ ----- (2)	
	Sub (1) into (2): $10^{5-3x}=10^{x}$ $x=5-3x$ $4x=5$ $x=1\frac{1}{4}$	M1
	Sub $x=1\frac{1}{4}$ into (2): $y=10^{1\frac{1}{4}}$ $y=17.783$	M1
	$\lg y=\lg 17.783$ $=1.25$ $\therefore$ Coordinates are $(1.25, 1.25)$	A1

Qn	Solution	Mark Allocation
9i	$y = x\ln 3x$ $\frac{dy}{dx} = x\left(\frac{1}{x}\right) + (\ln 3x)(1)$ $= 1 + \ln 3x$ At $P(1, \ln 3)$, $\frac{dy}{dx} = 1 + \ln 3(1)$ $= 1 + \ln 3$ At $P(1, \ln 3)$, $y - \ln 3 = (1 + \ln 3)(x - 1)$ $y = (1 + \ln 3)x - 1$ When $y = 0$, $(1 + \ln 3)x = 1$ $x = \frac{1}{(1 + \ln 3)}$ $x = 0.477$ Coordinates of Q are $(0.477, 0)$ $\tan\theta = \frac{\ln 3 - 0}{1 - 0.477}$ $= 2.0986$ $\theta = 64.5°$	M1 M1 M1 M1 A1
9ii	Gradient of normal $= -\frac{1}{1 + \ln 3x}$ $-\frac{1}{1 + \ln 3x} = -2$ $1 + \ln 3x = \frac{1}{2}$ $\ln 3x = -\frac{1}{2}$ $3x = e^{-\frac{1}{2}}$ $x = \frac{1}{3}e^{-\frac{1}{2}}$ $= 0.202$	M1 M1 A1
10i	$[x + (2 \times 2)] \times [y + (1.5 \times 2)] = 825$ $(x + 4)(y + 3) = 825$ $y + 3 = \frac{825}{x + 4}$ $y = \frac{825}{x + 4} - 3$	M1 M1

Qn	Solution	Mark Allocation
	$A=\left(\frac{825}{x+4}-3\right)(x)$ $A=\frac{825x}{x+4}-3x$ (shown)	A1
10 ii	$\frac{dA}{dx}=\frac{(x+4)(825)-(825x)(1)}{(x+4)^2}-3$ $=\frac{3300}{(x+4)^2}-3$ $\frac{3300}{(x+4)^2}-3=0$ $(x+4)^2=1100$ $x+4=\pm\sqrt{1100}$ $x=29.2$ or -37.2 (N.A.)	M1 M1 M1 A1
10 iii	$\frac{d^2A}{dx^2}=\frac{-2(3300)}{(x+4)^3}$ When $x=29.2$, $\frac{d^2A}{dx^2}=\frac{-6600}{(29.2+4)^3}$ $=-0.181$ Since $\frac{d^2A}{dx^2}<0$, this value of x gives the poster the largest printing area possible.	B1: Show $\frac{d^2A}{dx^2}<0$
11i		B1: V-shape graph B1: Correct x- and y- intercepts and end-points labelled
11 ii	$\|14x-4\|=3x$ $2\|7x-2\|=3x$ $\|7x-2\|=\frac{3}{2}x$	M1

Qn	Solution	Mark Allocation
	$7x-2=\frac{3}{2}x$ or $7x-2=-\frac{3}{2}x$	M1
	$5\frac{1}{2}x=2$ $\quad$ $8\frac{1}{2}x=2$	
	$x=\frac{4}{11}$ $\quad$ $x=\frac{4}{17}$	M1
	$\therefore\ x\le\frac{4}{17}$ or $x\ge\frac{4}{11}$	A1
11 iiia	1 intersection; Line parallel to L.H. arm cuts R.H. arm at only one point.	B1 B1
11 iiib	0 intersection; Line parallel to R.H. arm; for intersection, $c>-2$.	B1 B1
12i	$m_{AB}=m_{OC}$ $\frac{y-0}{x-5}=\frac{6}{-2}$ $y=-3x+15$ ----- (1)	M1
	$m_{BC}=-\frac{1}{m_{OC}}$ $\frac{y-6}{x+2}=\frac{1}{3}$ $y=\frac{1}{3}x+6\frac{2}{3}$ ----- (2)	M1
	Sub (1) into (2): $-3x+15=\frac{1}{3}x+6\frac{2}{3}$ $3\frac{1}{3}x=8\frac{1}{3}$	M1
	$x=2\frac{1}{2}$	M1
	Sub $x=2\frac{1}{2}$ into (1): $y=-3\left(2\frac{1}{2}\right)+15$ $y=7\frac{1}{2}$	
	Coordinates of B are $\left(2\frac{1}{2},7\frac{1}{2}\right)$	A1

Qn	Solution	Mark Allocation
12 ii	 $D\left(-2\frac{1}{2}, 7\frac{1}{2}\right)$	M1 O.E. A1
12 iii	Midpoint of $OC = \left(\frac{0+(-2)}{2}, \frac{0+6}{2}\right)$ $= (-1, 3)$ Gradient of perpendicular bisector of $OC = -\frac{1}{(-3)}$ $= \frac{1}{3}$ At $(-1, 3)$, $y-3 = \frac{1}{3}(x+1)$ $y = \frac{1}{3}x + 3\frac{1}{3}$	M1 A1
12 iv	$\frac{1}{2}\begin{vmatrix} 0 & 5 & x & -2 & 0 \\ 0 & 0 & y & 6 & 0 \end{vmatrix} = 15$ $\lvert 5y+6x-(-2y)\rvert = 30$ $\lvert 7y+6x\rvert = 30$ $7y+6x=30$ ----- (1) or $7y+6x=-30$ ----- (2) $3y = x+10$ ----- (3) From (3): $x = 3y-10$ ----- (4) Sub (4) into (1): $7y+6(3y-10)=30$ $7y+6(3y-10)=-30$ $25y=90$ $25y=30$ $y=3.6$ $y=1.2$ Sub $y=3.6$ into (4): Sub $y=1.2$ into (4): $x=3(3.6)-10$ $x=3(1.2)-10$ $x=0.8$ $x=-6.4$ [N.A.] Possible coordinates of E are $(0.8, 3.6)$.	M1 M1 A1 [with rejection]

SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

Name: ______________________________( ) Class: Sec _______

ADDITIONAL MATHEMATICS

Paper 2

4047/02

Friday 21 August 2015

2 hours 30 minutes

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Submit Section A and B separately

This question paper consists of **6** printed pages.

Setter: Mr Ang Hanping
Vetter: Ms Zoe Pow

[Turn over

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*We Nurture Students to **Think, Care** and **Lead** with **P.R.I.D.E.***

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots. + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

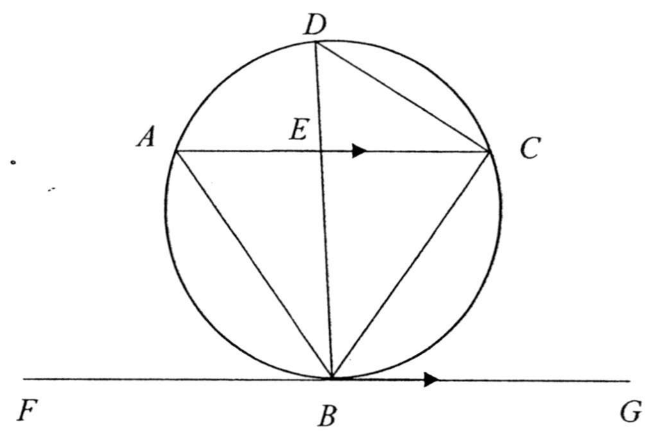
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

Section A (59 marks)

1 The diagram shows triangles ABC and BCD whose vertices lie on the circumference of a circle. The chords BD and AC intersect at E and AC is parallel to FG. FG is a tangent to the circle at B.



Show that

(i) ΔBCD is similar to ΔBEC, [2]

(ii) $BC^2 = BD \times BE$, [1]

(iii) ΔABC is an isosceles triangle. [2]

2 A company buys an engine at a cost of \$120 000. The value of the engine decreases with time so that its value, \$$V$, after t months is given by

$$V = 120\,000e^{-kt},$$

where k is a positive constant. The value of the engine is expected to be \$75 000 after 30 months.

(i) Calculate the value, to the nearest \$100, of the engine after 20 months. [4]

It is only economical to replace the engine after its value reaches $\frac{1}{2}$ of its original value.

(ii) Determine, with working, whether it is economical to replace the engine after 40 months. [2]

3 **(a)** An equilateral triangle has sides $\left(3+\sqrt{7}\right)$ cm in length. Find, without using a calculator, the area of the equilateral triangle. [3]

(b) A cuboid of volume $\left(30+12\sqrt{3}\right)$ cm^3 has a base area of $\left(4-2\sqrt{3}\right)$ cm^2. Find, without using a calculator, the height of the cuboid. [3]

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4 (a) Solve the equation $\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$. [5]

(b) Evaluate $\log_p 32 \times \log_8 p$. [3]

5 It is given that $f(x)=2x^3+ax^2+x+b$.

(i) Find the value of a and of b for which $2x^2+x-1$ is a factor of $f(x)$. [4]

(ii) Solve the equation $f(x)=0$. [2]

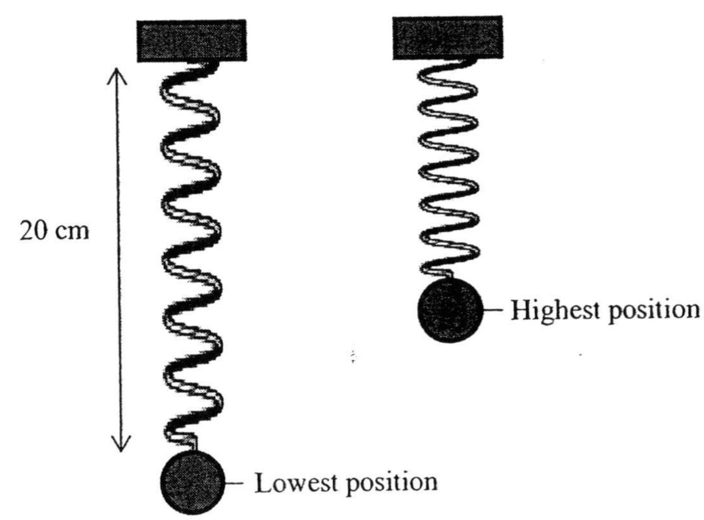
(iii) Hence solve $\frac{1}{4}x^3+\frac{a}{4}x^2+\frac{1}{2}x+b=0$. [2]

6 A curve has the equation $y=(2x+2)\sqrt{2x-1}$.

(i) Show that $\frac{dy}{dx}=\frac{kx}{\sqrt{2x-1}}$, where k is a constant and state the value of k. [4]

(ii) Hence evaluate $\int_5^{13}\frac{3x}{2\sqrt{2x-1}}\,dx$. [4]

7 The diagram below shows an experimental setup where a weighted spring is released from a stretched position and follows a periodic up-down motion. The length of the spring, l cm, during the experiment is modelled by the equation, $l=a\cos kt+16$, where a, k are constants, and t is the time in seconds after releasing the weight from the lowest position.



The length of the spring is 20 cm when the weight is at its lowest position and it takes 2 seconds for the weight to move from the lowest to highest position.

(i) Find the value of a. [1]

(ii) Show that the value of k is $\frac{\pi}{2}$. [2]

(iii) Find the length of the spring when the weight is at its highest position. [1]

(iv) Sketch the graph of $l = a\cos kt + 16$ for $0 \le t \le 4$. [2]

(v) Find the time interval which the length of the spring will be longer than 18 cm for $0 \le t \le 4$. [3]

8 **(a)** The equation of a curve is $y = x^3 + 4x^2 + kx + 3$, where k is a constant. Find the set of values of k for which the curve is always an increasing function. [4]

(b) A curve with equation in the form $y = ax + \dfrac{b}{x^2}$ has a stationary point at $(3, 4)$, where a and b are constants. Find the value of a and of b. [5]

Section B (41 marks)

Begin this section on a fresh sheet of paper.

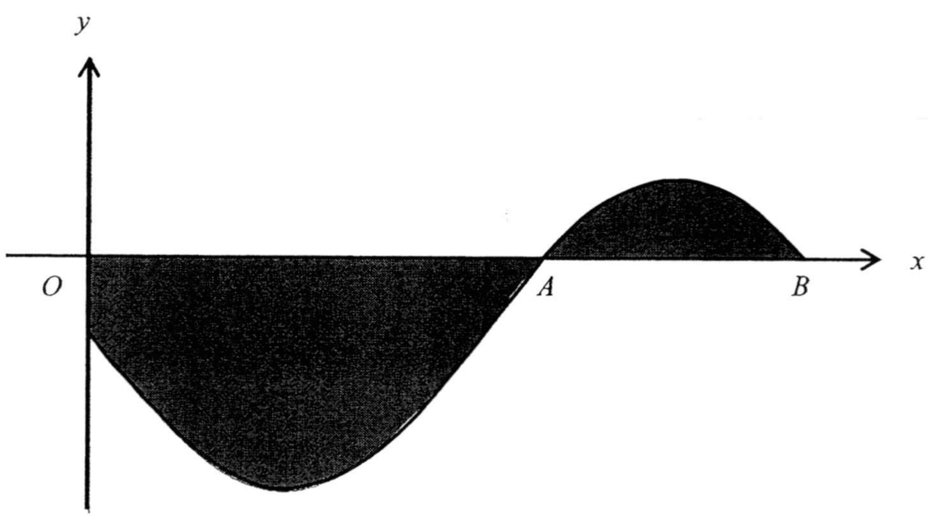
9 The equation $3x^2 + kx + 3 = 0$, where $k > 0$ has roots α and β. A second equation $3x^2 - 2x + 3 = 0$ has roots $\alpha^3\beta$ and $\alpha\beta^3$.

(i) Show that $\alpha^2 + \beta^2 = \dfrac{2}{3}$. [3]

(ii) Find the value of k. [3]

(iii) Form an equation whose roots are α^3 and β^3. [3]

10 The diagram shows part of the curve $y = 2\sin(2x + \pi) - 1$, meeting the x-axis at the points A and B.



(i) Find the x-coordinate of A and of B. [4]

(ii) Find the total area of the shaded regions. 184 [6]

11 A particle, travelling in a straight line, passes a fixed point O on the line with a speed of 2 ms^{-1}. The acceleration, a ms^{-2}, of the particle, t s after passing O, is given by $a = -4e^{-t}$.

(i) Show that the particle comes instantaneously to rest when $t = -\ln\frac{1}{2}$. [4]

(ii) Find the total distance travelled by the particle between $t = 0$ and $t = 2$. [6]

(iii) Find the average velocity of the particle during the first 2 seconds. [1]

12 A circle, C_1, passing through the point A (4, 8) has the same centre as another circle C_2. The equation of C_2 is given by $x^2 + y^2 - 16x - 10y + 5 = 0$.

(i) Find the equation of C_1. [3]

AB is a diameter of C_1.

(ii) Find the coordinates of B. [2]

(iii) Show that the equation of the tangent to C_1 at B is $3y = 4x - 42$. [3]

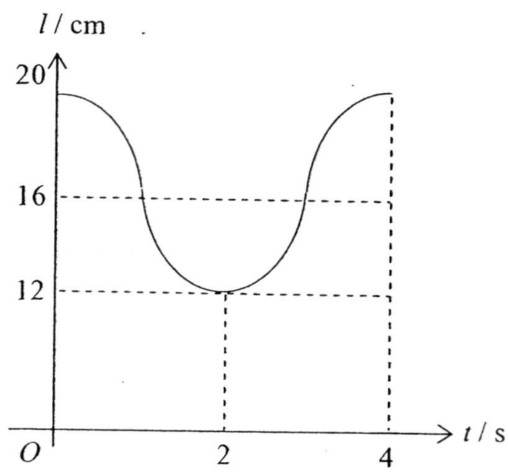
The lowest point on the circle C_1 is D.

(iv) Explain why the x-axis is a tangent to the circle at D. [1]

(v) Find the equation of another tangent to circle C_1 passing through the origin. [2]

END OF PAPER

Additional Mathematics Paper 2 (100 marks)

Qn. #	Solution	Mark Allocation
2i	$V = 87700$	
2ii	$V = 64124.098$ It is not economical to replace the engine after 40 months as the value of the engine has not reached half its original value of \$120000	
3a	Area of equilateral triangle $= \frac{1}{2}\left(8\sqrt{3}+3\sqrt{21}\right)$ cm^2	
3b	Height of cuboid $= 48+27\sqrt{3}$ cm	
4a	$x=5$ or $x=-1$ (rej)	
4b	$\log_p 32 \times \log_8 p = 1\frac{2}{3}$	
5i	$b=-2$ $a=5$	
5ii	$x=\frac{1}{2}$ or $x=-1$ or $x=-2$	
5iii	$x=1$ or $x=-2$ or $x=-4$	
6i	$k=6$	
6ii	$\int_5^{13} \frac{3x}{2\sqrt{2x-1}}\,dx = 26$	
7i	$a=4$	
7ii	$k=\frac{\pi}{2}$	
7iii	Length of string = 12 cm	
7iv		
7v	Time interval $= 0 \le t < \frac{2}{3}$ or $3\frac{1}{3} < t \le 4$	

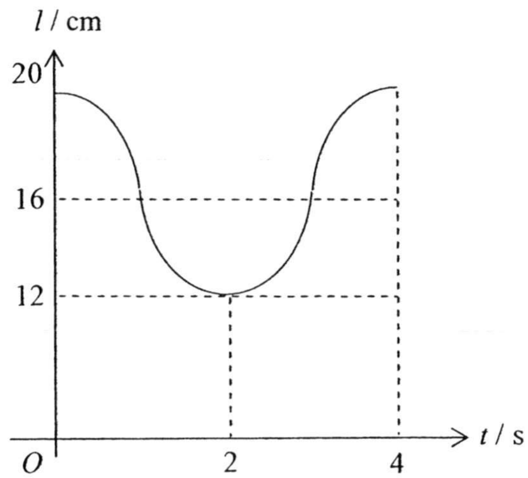
Qn. #	Solution	Mark Allocation
8a	$k > 5\frac{1}{3}$	
8b	$b = 12$ $a = \frac{8}{9}$	
9i	$\alpha^2 + \beta^2 = \frac{2}{3}$	
9ii	$k = -\frac{4\sqrt{6}}{3}$ (N.A.) or $\frac{4\sqrt{6}}{3}$	
9iii	Equation: $x^2 - \frac{2\sqrt{6}}{9}x + 1 = 0$	
10i	x coordinate of $A = \frac{7\pi}{12}$, x coordinate of $B = \frac{11\pi}{12}$	
10ii	Shaded area $= 4.38$ unit^2	
11i	$t = -\ln\frac{1}{2}$	
11ii	Total distance travelled $= 1.77\,\text{m}$	
11iii	Average velocity $= -0.271\,\text{m/s}$	
12i	Equation: $(x-8)^2 + (y-5)^2 = 25$	
12ii	Coordinates of $B = (12, 2)$	
12v	Equation of tangent: $y = \frac{80}{39}x$	

Additional Mathematics Paper 2 (100 marks)

Qn. #	Solution	Mark Allocation
1i	$\angle BDC = \angle CBG$ (alternate segment theorem) $\angle BCE = \angle CBG$ (alternate angles, AC//FG) $\angle BDC = \angle BCE$ $\angle CBD = \angle EBC$ (common angle) Since the corresponding angles of the triangles are equal, ΔBCD is similar to ΔBEC	M1 A1
1ii	Since ΔBCD is similar to ΔBEC $\frac{BC}{BD} = \frac{BE}{BC}$ $BC^2 = BD \times BE$	B1
1iii	$\angle BDC = \angle BCE$ (from (i)) $\angle CBG = \angle ACB$ (alt $\angle$) $\angle BDC = \angle BAC$ ($\angle$ in same seg) or $\angle CBG = \angle BAC$ (alt seg) $\angle BCE = \angle BAC$ $\angle ACB = \angle BAC$ ΔABC is an isosceles triangle. ΔABC is isosceles.	M1 A1
2i	$75000 = 120\,000e^{-k(30)}$ $\ln\frac{5}{8} = -30k$ $k = -\frac{1}{30}\ln\frac{5}{8}$ After 20 months $V = 120\,000e^{-\left(-\frac{1}{30}\ln\frac{5}{8}\right)(20)}$ $= 87720.53215$ $= 87700$	M1 M1 M1 A1
2ii	$V = 120\,000e^{-\left(-\frac{1}{30}\ln\frac{5}{8}\right)(40)}$ $= 64124.098$ It is not economical to replace the engine after 40 months as the value of the engine has not reached half its original value of \$120000	M1 A1
3a	Area of equilateral triangle $= \frac{1}{2}\left(3+\sqrt{7}\right)^2 \sin 60$ $= \frac{1}{2}\left(9+6\sqrt{7}+7\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1}{4}\left(16\sqrt{3}+6\sqrt{21}\right)$ $= \frac{1}{2}\left(8\sqrt{3}+3\sqrt{21}\right)\text{ cm}^2$	M1 M1 A1

Qn. #	Solution	Mark Allocation
3b	Height of cuboid	
	$=\dfrac{30+12\sqrt{3}}{4-2\sqrt{3}}$	M1
	$=\dfrac{30+12\sqrt{3}}{4-2\sqrt{3}}\times\dfrac{4+2\sqrt{3}}{4+2\sqrt{3}}$	
	$=\dfrac{120+108\sqrt{3}+72}{16-12}$	M1
	$=48+27\sqrt{3}$ cm	A1
4a	$\log_3(2x-1)-\dfrac{1}{2}\log_3(x^2+2)=\log_{25}5$	
	$\log_3(2x-1)-\dfrac{1}{2}\log_3(x^2+2)=\dfrac{1}{2}$	M1
	$2\log_3(2x-1)-\log_3(x^2+2)=1$	
	$\log_3(2x-1)^2-\log_3(x^2+2)=1$	
	$\log_3\dfrac{(2x-1)^2}{(x^2+2)}=1$	M1
	$\dfrac{(2x-1)^2}{(x^2+2)}=3$	M1
	$4x^2-4x+1=3x^2+6$	
	$x^2-4x-5=0$	
	$(x+1)(x-5)=0$	M1
	$x=5$ or $x=-1$ (rej)	A1
4b	$\log_p 32\times\log_8 p$	
	$=\dfrac{\log_2 32}{\log_2 p}\times\dfrac{\log_2 p}{\log_2 8}$	M1
	$=\dfrac{\log_2 2^5}{\log_2 2^3}$	M1
	$=\dfrac{5}{3}=1\dfrac{2}{3}$	A1
5i	$2x^2+x-1=(2x-1)(x+1)$	
	$(2x-1)$ and $(x+1)$ are factors	
	$\mathrm{f}\left(\dfrac{1}{2}\right)=0$	
	$2\left(\dfrac{1}{2}\right)^3+a\left(\dfrac{1}{2}\right)^2+\left(\dfrac{1}{2}\right)+b=0$	M1

Qn. #	Solution	Mark Allocation
	$\frac{1}{4}+\frac{1}{4}a+\frac{1}{2}+b=0$ $a+4b=-3 \quad ---(1)$ $f(-1)=0$ $2(-1)^3+a(-1)^2+(-1)+b=0$ $-2+a-1+b=0$ $a+b=3 \quad ---(2)$ (1) - (2): $3b=-6$ $b=-2$ $a=5$	 M1 M1 A1
5ii	$f(x)=0$ $2x^3+5x^2+x-2=0$ $(2x^2+x-1)(x+2)=0$ $(2x-1)(x+1)(x+2)=0$ $x=\frac{1}{2}$ or $x=-1$ or $x=-2$	 M1 A1
5iii	$\frac{1}{4}x^3+\frac{a}{4}x^2+\frac{1}{2}x+b=0$ Let $x=2y$ $\frac{1}{4}(2y)^3+\frac{a}{4}(2y)^2+\frac{1}{2}(2y)+b=0$ $2y^3+ay^2+y+b$ From (ii), $y=\frac{1}{2}$ or $y=-1$ or $y=-2$ $x=1$ or $x=-2$ or $x=-4$	 M1 A1
6i	$\frac{dy}{dx}=(2x+2)\left[\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)\right]+(2x-1)^{\frac{1}{2}}(2)$ $=(2x-1)^{-\frac{1}{2}}\{(2x+2)+2(2x-1)\}$ $=(2x-1)^{-\frac{1}{2}}\{6x\}$ $=\frac{6x}{\sqrt{2x-1}}$ $k=6$	M1 M1 A1 A1

Qn. #	Solution	Mark Allocation
6ii	$\int_5^{13} \frac{6x}{\sqrt{2x-1}}\,dx = \left[(2x+2)\sqrt{2x-1}\right]_5^{13}$	M1
	$\frac{1}{4}\int_5^{13} \frac{6x}{\sqrt{2x-1}}\,dx = \frac{1}{4}\left[(2x+2)\sqrt{2x-1}\right]_5^{13}$	M1
	$\int_5^{13} \frac{3x}{2\sqrt{2x-1}}\,dx = \frac{1}{4}\left[(28)\sqrt{25}-(12)\sqrt{9}\right]$	M1
	$= 26$	A1
7i	$20 = a\cos k(0) + 16$ $a = 4$	B1
7ii	$\frac{2\pi}{k} = 4$	M1
	$k = \frac{\pi}{2}$	A1
7iii	Length of string = 20 – 8 = 12 cm	B1
7iv		B1 – correct shape B1 – correct values
7v	$4\cos\frac{\pi}{2}t + 16 = 18$ $\cos\frac{\pi}{2}t = \frac{1}{2}$ Basic angle = $\cos^{-1}\frac{1}{2}$	M1
	$\frac{\pi}{2}t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ $t = \frac{2}{3}, 3\frac{1}{3}$	A1
	Time interval = $0 \le t < \frac{2}{3}$ or $3\frac{1}{3} < t \le 4$	A1

Qn. #	Solution	Mark Allocation
8a	$\frac{dy}{dx} = 3x^2 + 8x + k$	M1
	For curve to be always increasing, $\frac{dy}{dx}$ always > 0	M1
	$b^2 - 4ac < 0$	
	$64 - 4(3)(k) < 0$	M1
	$k > 5\frac{1}{3}$	A1
8b	$\frac{dy}{dx} = a - \frac{2b}{x^3}$	M1
	At stat. pt. $\frac{dy}{dx} = 0$	
	$a - \frac{2b}{x^3} = 0$	
	Sub $x = 3,\ y = 4$	
	$a - \frac{2b}{3^3} = 0$	M1
	$a = \frac{2b}{27}$ --- (1)	
	Sub $x = 3,\ y = 4$ to equation of curve	
	$4 = a(3) + \frac{b}{3^2}$	M1
	$36 = 27a + b$ --- (2)	
	Sub (1) to (2):	
	$36 = 2b + b$	M1
	$b = 12$	
	$a = \frac{24}{27} = \frac{8}{9}$	A1
9i	$\alpha\beta = 1$	
	$\alpha^3\beta + \alpha\beta^3 = \frac{2}{3}$	M1 – either one correct
	$\alpha\beta(\alpha^2 + \beta^2) = \frac{2}{3}$	M1
	$1(\alpha^2 + \beta^2) = \frac{2}{3}$	
	$\alpha^2 + \beta^2 = \frac{2}{3}$	A1

Qn. #	Solution	Mark Allocation
9ii	$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$	
	$=\frac{2}{3}+2(1)$	M1
	$=\frac{8}{3}$	
	$(\alpha+\beta)^2=\frac{8}{3}$	M1
	$\alpha+\beta=\frac{2\sqrt{6}}{3}$ or $-\frac{2\sqrt{6}}{3}$	
	$-\frac{k}{2}=\frac{2\sqrt{6}}{3}$ or $-\frac{2\sqrt{6}}{3}$	
	$k=-\frac{4\sqrt{6}}{3}$ (N.A.) or $\frac{4\sqrt{6}}{3}$	A1
9iii	$S.O.R.=\alpha^3+\beta^3$	
	$=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$	
	$=\left(-\frac{2\sqrt{6}}{3}\right)\left(\frac{2}{3}-1\right)$	M1
	$=\frac{2\sqrt{6}}{9}$	
	$P.O.R.=\alpha^3\beta^3$	
	$=(\alpha\beta)^3$	
	$=(1)^3=1$	M1
	Equation: $x^2-\frac{2\sqrt{6}}{9}x+1=0$	A1
10i	$2\sin(2x+\pi)-1=0$	M1
	$\sin(2x+\pi)=\frac{1}{2}$	
	$\alpha=\sin^{-1}\frac{1}{2}=\frac{\pi}{6}$	M1
	$2x+\pi=\frac{\pi}{6},\quad \pi-\frac{\pi}{6},\quad \frac{\pi}{6}+2\pi,\quad \pi-\frac{\pi}{6}+2\pi$	M1
	$x=-\frac{5\pi}{12},\quad -\frac{\pi}{12},\quad \frac{7\pi}{12},\quad \frac{11\pi}{12}$	
	x coordinate of $A=\frac{7\pi}{12}$, x coordinate of $B=\frac{11\pi}{12}$	A1

Qn. #	Solution	Mark Allocation
10ii	Shaded area $-\int_0^{\frac{7\pi}{12}} 2\sin(2x+\pi)-1\,dx+\int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}} 2\sin(2x+\pi)-1\,dx$ $=-\left[-\cos(2x+\pi)-x\right]_0^{\frac{7\pi}{12}}+\left[-\cos(2x+\pi)-x\right]_{\frac{7\pi}{12}}^{\frac{11\pi}{12}}$ $=-\left[-2.69862-1\right]+\left[-2.01377-(-2.69862)\right]$ $=4.38$ unit^2	M2 – 1 for limits, 1 for expressions M1 – int. sin -> cos M1 – times 1/2 M1 A1
11i	$v=\int -4e^{-t}\,dt$ $=4e^{-t}+c$ When $t=0$, $v=2$ $2=4e^0+c$ $c=-2$ $v=4e^{-t}-2$ At instantaneous rest, $v=0$ $4e^{-t}-2=0$ $e^{-t}=\frac{1}{2}$ $t=-\ln\frac{1}{2}$	M1 M1 M1 A1
11ii	$s=\int 4e^{-t}-2\,dt$ $s=-4e^{-t}-2t+c$ When $t=0$, $s=0$ $0=-4e^0-2(0)+c$ $c=4$ $s=-4e^{-t}-2t+4$ Distance travelled before instantaneous rest $=-4e^{-\left(-\ln\frac{1}{2}\right)}-2\left(-\ln\frac{1}{2}\right)+4$ $=0.61371$ Distance from instantaneous rest to 2s $=0.61371-\left(-4e^{-(2)}-2(2)+4\right)$ $=0.61371-(-0.54134)$ $=1.15505$ Total distance travelled $=0.61371+1.15505=1.77\text{m}$	M2 M1 M1 M1 A1
11iii	Average velocity $=\frac{-0.54134}{2}=-0.271\text{m/s}$	B1

Qn. #	Solution	Mark Allocation
12i	Centre of C_1 =Centre of C_2 = $(8, 5)$ Radius of $C_1 = \sqrt{(8-4)^2+(5-8)^2} = 5$ Equation: $(x-8)^2+(y-5)^2 = 25$	M1 M1 A1
12ii	Coordinates of $B = (8+4, 5-3)$ $= (12, 2)$	M1 A1
12iii	Gradient of normal at B $= \frac{5-2}{8-12} = -\frac{3}{4}$ Gradient of tangent $= \frac{4}{3}$ Equation: $y-2 = \frac{4}{3}(x-12)$ $y = \frac{4}{3}x - 14$ $3y = 4x - 42$ (shown)	M1 M1 A1
12iv	Centre is at $(8, 5)$ and radius is 5 The lowest point D is at $(8, 0)$ and the circle touches x-axis at D. Thus x-axis is a tangent to the circle at D	B1
12v	$\tan\theta = \frac{5}{8}$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{5}{8}\right)}{1-\left(\frac{5}{8}\right)^2}$ $= \frac{80}{39}$ Equation of tangent: $y = \frac{80}{39}x$	M1 A1