TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 1 2015
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS 4047/01

Paper 1

Tues 30 June 2015

2 hours

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

190

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the range of values of k where $k \neq 0$ if the roots for the equation $kx^2 + (k-2)x + 4k = 0$ are real. [4]

2 A particle moves along the curve $y = 6 + \frac{1}{x^2}$ such that the y-coordinate of the particle is decreasing at a constant rate of 0.04 units per second.

Find the rate of change of the x-coordinate when $x = 2$. [4]

3 Given that $\int_{-1}^{3} [f(x) + 1] dx = 8$, evaluate

(i) $\int_{-1}^{3} f(x) dx$, [2]

(ii) $\int_{2}^{3} [f(x) + 1] dx - \int_{2}^{-1} [f(x) + 1] dx$. [2]

4 (i) Write down the first three terms in the expansion, in descending powers of x, of $\left(2x - \frac{1}{3x}\right)^7$. [3]

(ii) Hence find the coefficient of x^5 in the expansion of $(x^2 + 2)\left(2x - \frac{1}{3x}\right)^7$. [2]

5 Given that the roots of $2x^2 + 3x - 6 = 0$ are $2\alpha + \beta$ and $2\beta + \alpha$, find a quadratic equation whose roots are α and β. [6]

[Turn over

6 A bowl of hot soup was left to cool such that t minutes later, its temperature, H°C, is given by $H = 25 + 70e^{-kt}$, where k is a constant.
When $t = 2$, the temperature of the soup is 80°C.

(i) Show that $k = 0.1206$. [2]

(ii) Find the time taken for the soup to reach 40°C. [2]

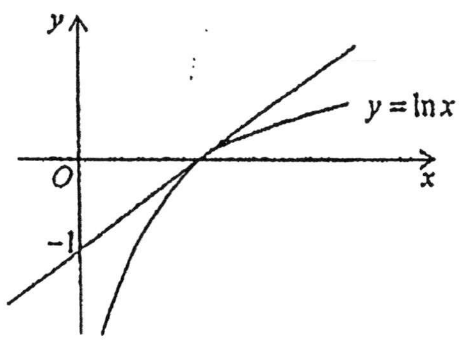
(iii) Explain why the temperature of the soup reaches 25°C after a long time. [1]

(iv) Sketch the graph of H against t. [2]

7 (a) If $a^{3-x}b^{5x} = a^{x+7}b^{3x}$, prove that $(2+x)\lg a = x\lg b$. [3]

(b) Solve $(2\log_3 x + 5)\log_3 x = 3$. [4]

8 The diagram shows part of the curve $y = \ln x$ and a tangent to the curve at $x = k$ which also passes through the point $(0, -1)$.



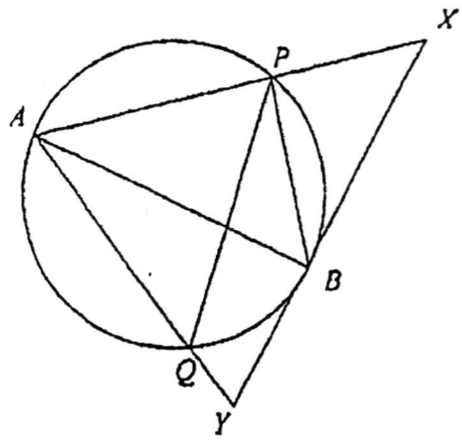
(i) Find the equation of the tangent. [5]

(ii) Write down an inequality for m if the line $y = mx - 1$ where $m > 0$

(a) intersects the curve exactly 2 times, [1]

(b) does not meet the curve. [1]

9



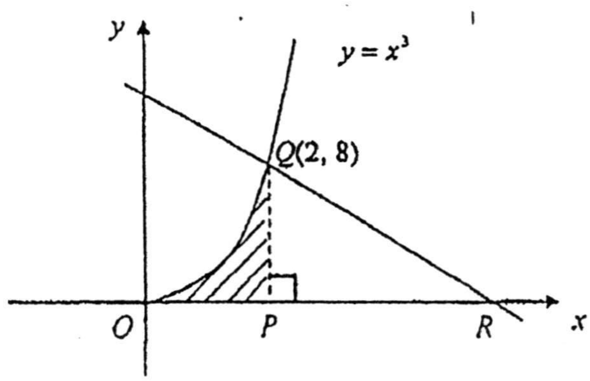
The diagram shows points A, P, B and Q lying on a circle with diameter AB. The tangent to the circle at B meets AP produced at X and AQ produced at Y.

(i) Prove that triangle APB is similar to triangle ABX. Hence express AB^2 in terms of AP and AX. [4]

(ii) Express AB^2 in terms of AP and PB. [1]

(iii) Using your answers in (i) and (ii), show that $PB^2 = AP \times PX$. [2]

10



The diagram shows part of the curve $y = x^3$. Points P and R lie on the x-axis.

The line QR intersects the curve at $Q(2, 8)$. QP is perpendicular to the x-axis.

Given that the ratio of the shaded area to the area of triangle PQR is $2 : 5$.

(i) Find the shaded area. [3]

(ii) Find the coordinates of R. [3]

(iii) Determine whether QR is the normal to the curve at Q. [4]

[Turn over

11 The tidal height, y metres, at a jetty on a particular day can be represented by the equation

$$y = 1.6 + 1.4\cos(kt)$$

where t is the time in hours after midnight and k is a constant.

The time between the first high tide and the next high tide is 14 hours.

(i) Show that $k = \frac{\pi}{7}$. [1]

(ii) Find the minimum tidal height for that day and the time it first occured. [3]

(iii) For how long between the first high tide and the next high tide was the tidal height at most 1 m high? [5]

12 Given that $\frac{4x^3+3x^2-8x-1}{x^2+x-2} = ax+b+\frac{x+c}{x^2+x-2}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part (i), find

(ii) $\int \frac{4x^3+3x^2-8x-1}{x^2+x-2}\,dx$. [6]

End of paper

Answers

1. $-\frac{2}{3} \le k \le \frac{2}{5}, k \ne 0$

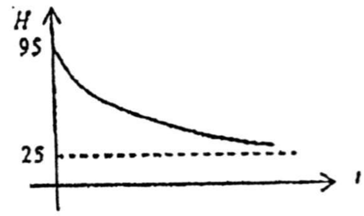
2. 0.16 units/s

3.(i) 4 (ii) 8

4.(i) $128x^7 - \frac{448}{3}x^5 + \frac{224}{3}x^3 + \ldots\ldots$ (ii) -224

5. $x^2 + \frac{1}{2}x - \frac{7}{2} = 0$

6.(ii) 12.8 min (iv)



7.(b) $x = \frac{1}{27}$ or $\sqrt{3}$

8.(i) $y = x - 1$ (ii)(a) $0 < m < 1$ (b) $m > 1$

9.(i) $AB^2 = AP \times AX$

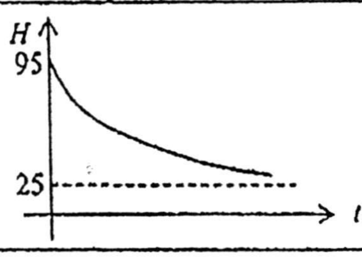
10.(i) 4 units^2 (ii) (4.5, 0) (iii) QR is not the normal to the curve at Q.

11.(ii) 0.2 m at 7 am (iii) 5.03 h

12.(i) $4x - 1 + \frac{x-3}{(x+2)(x-1)}$ (ii) $2x^2 - x + \frac{5}{3}\ln(x+2) - \frac{2}{3}\ln(x-1) + c$

2015 Sec 4 Prelim 1 Add Maths P1

No.	Solution	Remarks	
1	$(k-2)^2-4(k)(4k)\ge 0$ $(k-2+4k)(k-2-4k)\ge 0$ $(5k-2)(-3k-2)\ge 0$ $(5k-2)(3k+2)\le 0$ $-\frac{2}{3}\le k\le\frac{2}{5}, k\ne 0$	M1 Correct sub for discriminant B1 D ≥ 0 M1 Factorise A1	
		Total	4 m
2	$\frac{dy}{dx}=-\frac{2}{x^3}$ At $x=2$, $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $-0.04=-\frac{2}{2^3}\times\frac{dx}{dt}$ $\frac{dx}{dt}=0.16$ Rate of change of $x=0.16$ units/s	M1 M1 Eqn with sub B1 $\frac{dy}{dt}=-0.04$ A1	
		Total	4 m
3(i)	$\int_{-1}^{3}f(x)dx=\int_{-1}^{3}f(x)+1dx-\int_{-1}^{3}1dx$ $=8-[x]_{-1}^{3}$ $=4$	M1 For splitting A1	
3(ii)	$\int_{2}^{3}f(x)+1dx+\int_{-1}^{2}f(x)+1dx$ $=\int_{-1}^{3}f(x)+1dx$ $=8$	M1 Change signs & limits A1	
		Total	4 m
4(i)	$\left(2x-\frac{1}{3x}\right)^7=(2x)^7+\binom{7}{1}(2x)^6\left(-\frac{1}{3x}\right)+\binom{7}{2}(2x)^5\left(-\frac{1}{3x}\right)^2$ $=128x^7-\frac{448}{3}x^5+\frac{224}{3}x^3+\ldots\ldots$	M1 B2 Expansion – 1 mark for each error	
4(ii)	$(x^2+2)\left(128x^7-\frac{448}{3}x^5+\frac{224}{3}x^3+\ldots..\right)$ $coeff=2\left(-\frac{448}{3}\right)+\frac{224}{3}$ $=-224$	M1 A1	
		Total	5 m

No.	Solution	Remarks	
5	$2\alpha+\beta+2\beta+\alpha=-\frac{3}{2}$	M1 Find sum $=-\frac{b}{a}$	
	$\alpha+\beta=-\frac{1}{2}$	B1	
	$(2\alpha+\beta)(2\beta+\alpha)=-3$		
	$4\alpha\beta+2\beta^2+2\alpha^2+\alpha\beta=-3$	M1 Find prod $=\frac{c}{a}$	
	$5\alpha\beta+2[(\alpha+\beta)^2-2\alpha\beta]=-3$		
	$\alpha\beta=-3-2\left(-\frac{1}{2}\right)^2$	M1 Using $(\alpha+\beta)^2$	
	$\alpha\beta=-\frac{7}{2}$	B1	
	Equation is $x^2+\frac{1}{2}x-\frac{7}{2}=0$	A1$\surd$ must $=0$	
		Total	6 m
6(i)	$80=25+70e^{-2k}$	M1 Sub $t=2$, $H=80$	
	$e^{-2k}=\frac{55}{70}$		
	$-2k=\ln\frac{55}{70}$	M1 Take ln both sides & result	
	$k=0.1206$		
6(ii)	$40=25+70e^{-0.1206t}$		
	$e^{-0.1206t}=\frac{15}{70}$		
	$-0.1206t=\ln\frac{15}{70}$	M1 Using ln on both sides	
	$t=12.8\text{min}$	A1 or 12 min 46 sec	
6(iii)	As t becomes very large, $70e^{-kt}$ approaches zero, So the temperature reaches 25°C after a long time.	B1	
6(iv)		G1 shape G1 95, 25 seen	
		Total	7 m

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7(a)	$a^{x+7} \div a^{3-x} = b^{5x} \div b^{3x}$ $a^{x+7-3+x} = b^{5x-3x}$ $(4+2x)\lg a = 2x\lg b$ $(2+x)\lg a = x\lg b$	M1 Group/Rearrange M1 Indices or log law B1 Taking lg on both sides and result	
7(b)	Let $a = \log_3 x$ $2a^2 + 5a - 3 = 0$ $(a+3)(2a-1) = 0$ $a = -3$ or $a = \frac{1}{2}$ $\log_3 x = -3$ or $\log_3 x = \frac{1}{2}$ $x = \frac{1}{27}$ or $x = \sqrt{3}$	M1 By sub to get quad eqn B1 A1 A1	
		Total	7 m
8(i)	$\frac{dy}{dx} = \frac{1}{x}$ $x = k, \frac{dy}{dx} = \frac{1}{k}, y = \ln k$ $y - \ln k = \frac{1}{k}(x-k)$ $-1 = \frac{1}{k}(0) - 1 + \ln k$ $\ln k = 0$ $k = 1$ Eqn of tgt: $y = x - 1$	M1 Differentiate M1 Find eqn at $x=k$ M1 Using $(0,-1)$ B1 value of k B1	
8(ii)	(a) $0 < m < 1$ (b) $m > 1$	B1 √ accept $m <$ *their grad* B1 √ accept $m >$ *their grad* * *their grad must be* > 0	
		Total	7 r

No.	Solution	Remarks	
9(i)	$\angle PAB = \angle BAX$ (Common angle) $\angle APB = 90°$ ($\angle$ in semi circle) $\angle ABX = 90°$ (tangent perpendicular to radius) So ΔAPB is similar to ΔABX. $\frac{AB}{AP} = \frac{AX}{AB}$ $AB^2 = AP \times AX$	B1 B1 B1 B1	
9(ii)	By Pythagoras Thm, $AB^2 = AP^2 + PB^2$	B1	
9(iii)	$AP^2 + PB^2 = AP \times AX$ $PB^2 = AP \times AX - AP^2$ $PB^2 = AP(AX - AP)$ $\therefore PB^2 = AP \times PX (shown)$	M1 Equating (i) M1 Factorising seen & result	
		Total	7 m
10(i)	Shaded Area $= \int_0^2 x^3 dx$ $= \frac{1}{4}\left[x^4\right]_0^2 = 4 units^2$	B1 M1 correct integration A1	
10(ii)	Area of triangle PQR = 10 $units^2$ $\frac{1}{2} \times PR \times 8 = 10$ $PR = 2.5$ $R\,(4.5, 0)$	B1 M1 use area/discriminant mthd A1	
10(iii)	gradient of $QR = -3.2$ $\frac{dy}{dx} = 3x^2$ $x = 2, \frac{dy}{dx} = 12$ Since grad of normal $= -\frac{1}{12} \neq$ gradient of QR, So QR cannot be normal to curve at Q.	M1 B1 M1 grad of normal B1 correct conclusion	
		Total	10 m

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No.	Solution	Remarks	
11(i)	$\frac{2\pi}{k}=14$ $k=\frac{\pi}{7}(shown)$	M1	
11(ii)	$\cos\left(\frac{\pi}{7}t\right)=-1$ Minimal tidal height = $1.6+1.4(-1)$ $=0.2$ m At 7 am	M1 *soi* B1 B1 or 0700	
11(iii)	$1.6+1.4\cos\left(\frac{\pi}{7}t\right)=1$ $\cos\left(\frac{\pi}{7}t\right)=-0.42857$ Basic angle = 1.1279 $\frac{\pi}{7}t=2.0137,\quad 4.2695$ $t=4.4868,\quad 9.5132$ Duration = 5.026 h	M1 Form eqn & = 1 B1 B1 M1 2^{nd} ans − 1^{st} ans A1	
		Total	9 m
12(i)	$4x-1+\frac{x-3}{(x+2)(x-1)}$	M1 use long div A3 value of a,b,c	
12(ii)	$\frac{x-3}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}$ $x-3=A(x-1)+B(x+2)$ $x=1, B=-\frac{2}{3}$ $x=-2, A=\frac{5}{3}$ $\int 4x-1+\frac{5}{3(x+2)}-\frac{2}{3(x-1)}dx$ $=2x^2-x+\frac{5}{3}\ln(x+2)-\frac{2}{3}\ln(x-1)+c$	M1 correct PF M1 A1 Both answers M1 first 2 terms M1 both ln (...) seen A1 $+c$ seen $\int....dx$ (−1 mark if dx missing)	
		Total	10 m

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 1 2015
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS

4047/02

Paper 2

Monday 6 July 2015

2 hours 30 minutes

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

96

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

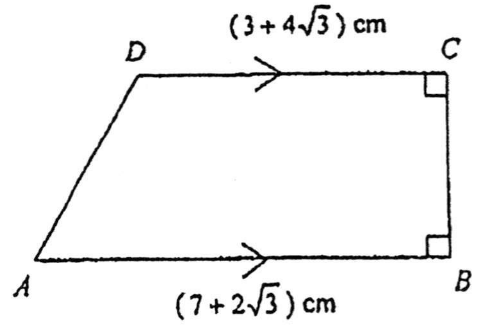
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

45

1



The diagram shows a trapezium $ABCD$ in which $AB = (7+2\sqrt{3})$ cm and $DC = (3+4\sqrt{3})$ cm. AB and DC are perpendicular to CB. Given that the area of the trapezium is $(14+\sqrt{12})$ cm^2, find the exact value of CB in the form of $(a+b\sqrt{3})$ cm. [4]

2 A function has an equation where

$$f(x) = \frac{\ln(4-x)}{x-4}, x < 4.$$

(i) Obtain an expression for $f'(x)$. [3]

(ii) Showing full working, determine whether f is decreasing for $x < 4 - e$, [3]

3 (i) Sketch the graph of $y = |2x-1|-3$. [2]

(ii) Explain why the minimum value is -3. [1]

(iii) A line $y = kx$, where $k > 0$, is drawn on the same axes with the graph of $y = |2x-1|-3$. Find the range of values of k for which there is only one point of intersection. [1]

4 (i) Prove the identity $\dfrac{2\cos 2A + \cos A + 2}{2\sin 2A + \sin A} = \cot A$. [4]

(ii) Hence, solve the equation $\dfrac{2\cos 6x + \cos 3x + 2}{2\sin 6x + \sin 3x} = 5$ for $0 \le x \le \pi$. [4]

5 It is given that $\sin A = \frac{1}{\sqrt{5}}$, where A is an acute angle. Without using a calculator,

(i) find $\tan A$. [2]

Given further that $\tan(A+B) = 2$, where B is an acute angle,

(ii) find the exact value of $\tan B$. [4]

6 (i) Sketch the graph of $y = 2x^{\frac{1}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = \frac{1}{3}x^{\frac{5}{2}}$ for $x > 0$. [1]

(iii) Find the x-coordinate of the point of intersection of your graphs. [2]

7 (i) Differentiate $x\tan^2 x$ with respect to x. [3]

(ii) Show that $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = 0.2146$. [4]

(iii) Hence, find $\int_0^{\frac{\pi}{4}} x \tan x \sec^2 x \, dx$. [4]

8 Given that $f(x) = 2x^3 + ax^2 + bx - 3$, where a and b are constants, has a factor of $x - 3$ and leaves a remainder of -20 when divided by $x + 1$, find the value of a and of b. [5]

9 The points $A(3, 0)$ and $B(9, 6)$ lie on a circle C_1 such that the x-axis is a tangent to the circle at A.

(i) Find the equation of the perpendicular bisector of AB. [4]

(ii) Hence, or otherwise, find centre of the circle C_1 and the radius. [3]

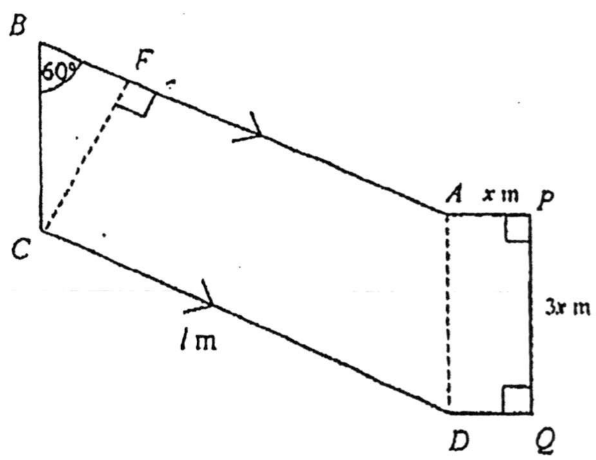
(iii) Show that the equation of the circle is $x^2 + y^2 - 6x - 12y + 9 = 0$. [2]

Another circle C_2 is formed after circle C_1 is being reflected in the line $x = 8$.

(iv) Find the centre of circle C_2. [1]

(v) Explain why the point (12, 9) lies within circle C_2. [2]

10



The diagram consists of a parallelogram $ABCD$ and a rectangle $APQD$.
It is given that $CD = l$ m and that the angle $CBF = 60°$ and angle $CFA = 90°$.
The rectangle has sides $AP = x$ m and $PQ = 3x$ m.
The perimeter of the diagram is 10 m.

(i) Express l in terms of x and show that the area of the diagram is
$3(1 - 2\sqrt{3})x^2 + \dfrac{15\sqrt{3}}{2}x \text{ m}^2$. [3]

(ii) Given that x can vary, find the value of x for which the area has a stationary value. [3]

(iii) Determine whether this value of area is a maximum or a minimum. [2]

11 The variables x and y are connected by the equation $y = ax + \frac{b}{x}$, where a and b are constants. Experimental values of x and y were obtained. A graph is drawn in which xy was plotted against x^2. The straight line which was obtained passed through the points (1, 5) and (3, 11).

Find

(i) the value of a and of b, [4]

(ii) the coordinates of the point on the line at which $y = -x + \frac{4}{x}$. [4]

12 A particle travels in a straight line, so that t seconds after passing through a fixed point O, its velocity, v ms^{-1}, is given by $v = \frac{32}{(t+2)^2} - 2$. The particle comes to instantaneous rest at P. Find

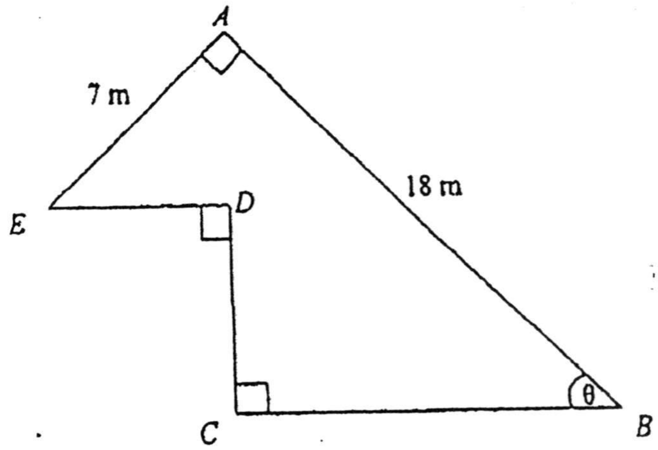
(i) the value of t when the particle is at instantaneous rest, [3]

(ii) distance OP, [4]

(iii) distance travelled for the first 8 seconds, [2]

(iv) the acceleration of the particle at $t = 8$ seconds. [3]

13



A playground is to be built in the shape of the figure shown in which ADC is a straight line and angle EAB = angle DCB = angle $EDC = 90°$. The length of AE is 7 m and AB is 18 m. The angle ABC is θ, where $0° < \theta < 90°$.
The perimeter of the playground is given by L m.

(i) Show that L can be expressed as $p + q\cos\theta + r\sin\theta$, where p, q and r are constants to be found. [3]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Given $L = 51$ m, find θ. [2]

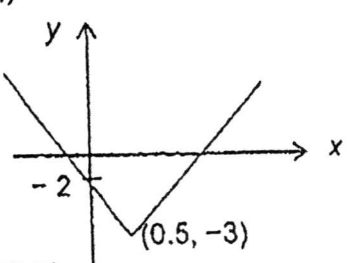
(iv) Find the maximum value of the perimeter and the corresponding value of θ. [3]

End of paper

1) $-26 + 16\sqrt{3}$

2i) $\dfrac{1 - \ln(4 - x)}{(x - 4)^2}$

3i)



3ii) Since
$|2x - 1| \geq 0$,
$|2x - 1| - 3 \geq -3$

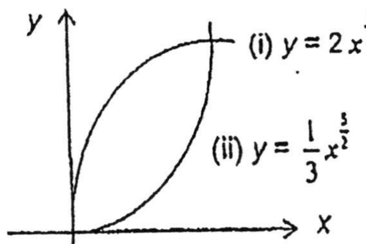
.99

Min value is -3
3iii) $k \geq 2$
4ii) $x = 0.0658, 1.11, 2.16$

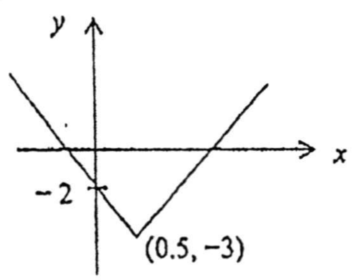
5i) $\tan A = \frac{1}{2}$

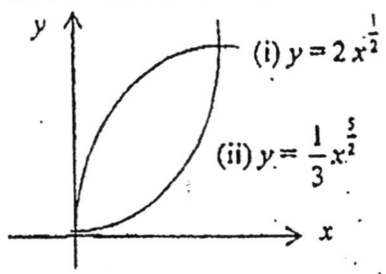
5ii) $\tan B = \frac{3}{4}$

6i)



6iii) $x = \sqrt{6}$
7i) $\tan^2 x + 2x\tan x\sec^2 x$
7ii) 0.2146
7iii) 0.285
8) $a = -8$, $b = 7$
9i) $y = -x + 9$
9ii) Centre is (3,6)
Radius = 6
9v) (13,6)
9iv) $\sqrt{10}$ < radius 6
(12,9) lies within circle
10i) $l = 5 - 4x$
10ii) $x = 0.879$
10iii) $\frac{d^2A}{dx^2} = 6(1 - 2\sqrt{3}) < 0$
maximum
11i) $a = 3$, $b = 2$
11ii) $(\frac{1}{2}, \frac{7}{2})$
12i) $t = 2, -6(rej)$
12ii) $OP = 4$ m
12iii) 11.2 m
12iv) $a = -0.064$ m/s^2
13ii) $L = 25 + \sqrt{764}\cos(\theta - 66.25°)$
13iii) 84.1°, 48.4°
13iv) $\theta = 66.3°, 426.3°(rej)$, Max $= 25 + \sqrt{746} = 52.3$ m

No	Solution	Marks	Remarks
1	$14+\sqrt{12}=\frac{1}{2}(7+2\sqrt{3}+3+4\sqrt{3})CB$ $CB=\frac{14+2\sqrt{3}}{5+3\sqrt{3}}\times\frac{5-3\sqrt{3}}{5-3\sqrt{3}}$ $=\frac{70-42\sqrt{3}+10\sqrt{3}-6(3)}{25-9(3)}$ $=-26+16\sqrt{3}$ cm	B1 Correct eqn M1 rationalise M1 simplify surds A1	Use of trapezium formula or otherwise (top and bottom)
			4 marks
2(i)	$f'(x)=\frac{(x-4)\frac{-1}{4-x}-\ln(4-x)(1)}{(x-4)^2}$ $=\frac{1-\ln(4-x)}{(x-4)^2}$	M1 use of quotient rule B1 diff. $\ln(4-x)$ correctly A1	$\frac{-1}{4-x}$ seen
2(ii)	$x<4-e$ $x-4<-e$ $4-x>e$ $\ln(4-x)>\ln e$ $\ln(4-x)>1$ $1-\ln(4-x)<0$ $\therefore f'(x)<0$ f is decreasing	M1 knowing to show $f'(x)$ +ve and -ve M1 manipulate B1 correct conclusion including stating $(\quad)^2$ is +ve	
			6 marks
3(i)		S1 correct shape/symmetrical B1 vertex and y-int seen	
3(ii)	Since $\|2x-1\|\ge 0$, $\|2x-1\|-3\ge -3$ Min value is -3	B1 with explanation	
3(iii)	Gradient of R.H. arm = 2 $k\ge 2$	A1 200	
			4 marks

No	Solution	Marks	Remarks
4(i)	LHS $=\dfrac{2(2\cos^2-1)+\cos A+2}{2(2\sin A\cos A)+\sin A}$ $=\dfrac{4\cos^2 A+\cos A}{\sin A(4\cos A+1)}$ $=\dfrac{\cos A(4\cos A+1)}{\sin A(4\cos A+1)}$ $=\cot A$	B1 double angle for cos 2A B1 double angle for sin 2A M1 factorise both B1 $\dfrac{\cos A}{\sin A}=\cot A$	
4(ii)	$\cot 3x=5$ $\tan 3x=\dfrac{1}{5}$ $3x=0.1974, 3.339, 6.481$ $x=0.0658, 1.11, 2.16$	B1 M1 reciprocal of $\cot 3x$ A2 - 3 correct A1 - 2 correct	Ans must be in rad
			8 marks
5(i)	$\tan A=\dfrac{1}{2}$	M1 use pyt thm to find length A1	Must show working
5(ii)	$\dfrac{\tan A+\tan B}{1-\tan A\tan B}=2$ $2=\dfrac{\frac{1}{2}+\tan B}{1-\frac{1}{2}\tan B}$ $2-\tan B=\dfrac{1}{2}+\tan B$ $\tan B=\dfrac{3}{4}$	B1 use of tangent formula SOI M1 subst $\tan A$ M1 simplify A1	
			6 marks
6(i)(ii)		B1 B1	With label Shape must be correct Must touch origin
6(iii)	$\dfrac{1}{3}x^{\frac{5}{2}}=2x^{\frac{1}{2}}$ $x^2=6$ $x=\sqrt{6}$	M1 simplify indices A1 must rej $-\sqrt{6}$	47

No	Solution	Marks	Remarks
			4 marks
7(i)	$\frac{d}{dx}x\tan^2 x = \tan^2 x + x(2\tan x)\sec^2 x$ $= \tan^2 x + 2x\tan x\sec^2 x$	M1 use of pdt rule B1 diff. $\tan^2 x$ correctly B1 presentation	$(2\tan x)\sec^2 x$ seen Eg $\frac{d}{dx}$ = ...
7(ii)	$\int_0^{\frac{\pi}{4}} \tan^2 x dx = \int_0^{\frac{\pi}{4}} \sec^2 x - 1dx$ $= [\tan x - x]_0^{\frac{\pi}{4}}$ $= 0.2146$ (0.214602)	M1 use of identity B1 integrate $\sec^2 x$ M1 evaluate integral(must show subst) B1 presentation	Eg $\int ...dx$
7(iii)	$\int_0^{\frac{\pi}{4}} \tan^2 x + 2x\tan x\sec^2 x dx = [x\tan^2 x]_0^{\frac{\pi}{4}}$ $2\int_0^{\frac{\pi}{4}} x\tan x\sec^2 x dx = [x\tan^2 x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan^2 x dx$ $= 0.7854 - 0.2146$ $= 0.5708$ $\int_0^{\frac{\pi}{4}} x\tan x\sec^2 x dx = 0.285$	B1 work backwards M1 make subject M1 evaluate integral(don't need show subst) A1 (min 3sf)	ECF
			11 marks
8	$f(3) = 2(3)^3 + 9a + 3b - 3$ $0 = 51 + 9a + 3b$ $-17 = 3a + b$.......(1) $f(-1) = -2 + a - b - 3$ $-20 = -5 + a - b$ $-15 = a - b$.........(2) $a = -8$ $b = 7$	B1 f(3) = 0 with subst B1 f(−1)= −20 with subst M1 solve sim A1, A1	
			5 marks

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No	Solution	Marks	Remarks
9(i)	Midpoint of AB = (6, 3) Grad AB = 1 Grad of perpendicular bisector = −1 $y-3=-1(x-6)$ $y=-x+9$	B1 M1 find grad. of perpendicular bisector M1 form eqn A1 o.e.	
9(ii)	Let centre be $(3, k)$ $k=-3+9$ Centre is (3,6) Radius = 6	M1 find y-coord of centre A1 A1	Use other mtd such as distance
9(iii)	$(x-3)^2+(y-6)^2=36$ $x^2+y^2-6x-12y+9=0$	B1 seen ECF M1 expand and simplify	
9(iv)	(13,6)	A1	
9(v)	Let point (12,9) be M. Let centre of C_2 be O. OM = $\sqrt{(9-6)^2+(12-13)^2}$ $=\sqrt{10}<$ radius 6 (12,9) lies within circle	M1 find distance B1 comparison made and conclusion seen	
			12 marks
10(i)	$5x+3x+2l=10$ $l=5-4x$ $Area=3x^2+3x\sin 60°(5-4x)$ $=3x^2+15x\frac{\sqrt{3}}{2}-12x^2\frac{\sqrt{3}}{2}$ $=3(1-2\sqrt{3})x^2+15\frac{\sqrt{3}}{2}x$	A1 M1 find area of parallelogram B1 $\sin 60=\frac{\sqrt{3}}{2}$ seen	
10(ii)	$\frac{dA}{dx}=6(1-2\sqrt{3})x+15\frac{\sqrt{3}}{2}$ At stat value, $\frac{dA}{dx}=0$ $x=0.879$	M1 attempt to diff. B1 $\frac{dA}{dx}=0$ seen A1	
10(iii)	$\frac{d^2A}{dx^2}=6(1-2\sqrt{3})<0$ maximum	M1 know 2nd derivative or sign test A1	
			8 marks

No	Solution	Marks	Remarks
11(i)	$xy = ax^2 + b$ $grad = 3$ $\frac{xy-5}{x^2-1} = 3$ $xy = 3x^2 + 2$ $a = 3, b = 2$	B1 manipulate into grad-intercept form M1 using correct subst A1,A1	
11(ii)	$xy = -x^2 + 4....(1)$ $xy = 3x^2 + 2....(2)$ $x^2 = \frac{1}{2}$ $xy = \frac{7}{2}$ $(\frac{1}{2}, \frac{7}{2})$	M1 use similar eqn M1 solve simultaneous eqn A1,A1	
			8 marks
12(i)	$\frac{32}{(t+2)^2} - 2 = 0$ $t + 2 = \pm 4$ $t = 2, -6(rej)$	B1 M1 solve eqn A1	
12(ii)	$s = \int \frac{32}{(t+2)^2} - 2dt = -\frac{32}{t+2} - 2t + c$ At $t = 0, s = 0, c = 16$ $s = -\frac{32}{t+2} - 2 + 16$ At $t = 2$, $OP = 4$ m	M1 integrate (ok if no + c) B1 M1 find distance (sub t=2) A1	Give marks if use definite integral $\int_0^2$
12(iii)	At $t = 8$, $S = -3.2$m Distance $= 3.2 + 4 + 4 = 11.2$ m	M1 ( ) +2(4) seen A1	
12(iv)	$a = -\frac{64}{(t+2)^3}$ At t = 8, a = -0.064 m/s^2	M1 Knowing to differentiate B1 acc expression seen A1	
			12 marks

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No	Solution	Marks	Remarks
13(i)	$L = 7+18+18\cos\theta + 7\sin\theta + 18\sin\theta - 7\cos\theta$ $= 25 + 11\cos\theta + 25\sin\theta$	B1,B1,B1	
13(ii)	$\tan\alpha = \frac{25}{11}$ $\alpha = 66.25°$ $R = \sqrt{746}$ $L = 25 + \sqrt{764}\cos(\theta - 66.25°)$	B1 B1 B1 $\sqrt{764}$ or 27.31 seen B1 statement	
13(iii)	When $L = 51$m, $51 = 25 + \sqrt{746}\cos(\theta - 66.25°)$ $\cos(\theta - 66.25°) = 0.95193$ $= 84.1°$ or $48.4°$	M1 solve A1	If use 27.3, will get θ=84.00
13(iv)	Max $= 25 + \sqrt{746} = 52.3$ m At max value, $\cos(\theta - 66.25°) = 1$ $\theta - 66.25° = 0°, 360°$ $\theta = 66.3°, 426.3°(rej)$	A1 B1 cos(...)=1 seen SOI or ....=0 A1	Penalise if -extra ans -ans in rad.
			12 marks

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