TEMASEK SECONDARY SCHOOL
O Level Preliminary Examinations 2015

ADDITIONAL MATHEMATICS 4047/01

Paper 1 2 hours

Question Booklet
Additional Material: Writing paper (8 sheets), Cover page (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open the booklet until you are asked to do so.

You are <u>not required</u> to submit this booklet at the end of the paper.

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

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This document consists of 7 printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

Answer all the questions.

1 (i) Given that $\sin(A+B) = 3\sin(A-B)$, show that $\tan A = 2\tan B$. [2]

(ii) Hence solve the equation $\sin^2(x+30^\circ) = 9\sin^2(x-30^\circ)$ for $0^\circ < x < 360^\circ$. [4]

2 (a) The equation $2x^2 - 2x + 1 = 0$ has roots α and β. Find the quadratic equation whose roots are $\frac{2}{\alpha^3}$ and $\frac{2}{\beta^3}$. [5]

(b) The equation of a curve is $y = (3+m)x^2 - (8+4m)x + 3 + 4m$, where m is a constant. For $y = 0$, find the value of m for which

(i) one root is the negative of the other, [2]

(ii) one root is the reciprocal of the other. [2]

3 (a) Simplify $\dfrac{2(4)^{\frac{1}{2}x+2} - 2^{x+1}}{6^x \times 3^{1-2x}}$ and express in the form of $k(3)^{nx}$, where k and n are integers. [3]

(b) Find the values of a and b such that $\lg\left(\dfrac{125}{y}\right) = a\lg(by) - 4\lg y$ for all positive values of y. [3]

(c) Solve the equation $2\log_5 e^x + \dfrac{1}{\log_2 5} = \log_5(2 - 3e^x)$. [5]

4 Given that $x^2 + 2x - 3$ is a factor of f(x), where $f(x) = x^4 + 6x^3 + 2ax^2 + bx - 3a$, find

(i) the values of a and b, [4]

(ii) the other quadratic factor of f(x). [3]

Explain why f(x) = 0 has only two real roots. [1]

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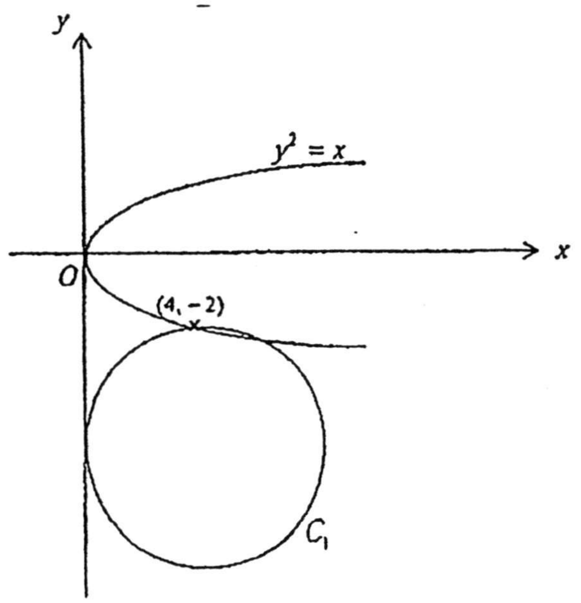
[Turn over

5 The diagram shows a circle C_1 with centre $(4, -6)$.

A curve $y^2 = x$ and the circle C_1 have the y-axis as the common tangent.

Both curves intersect at the point $(4, -2)$.



(i) Write down the radius of circle C_1 and hence the equation of C_1. [2]

(ii) Find the area bounded by the curve $y^2 = x$, the circle C_1 and the y-axis. [3]

(iii) A second circle, C_2 is the reflection of the circle, C_1 in the line $y = 2$.

Write down the equation of the second circle, C_2 in the form

$x^2 + y^2 + 2gx + 2fy + c = 0$. [2]

6 A curve is such that $\frac{dy}{dx} = 4x + \frac{1}{(x+2)^2}$ for $x > 0$ and the curve passes through the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

(i) Find the equation of the curve. [3]

(ii) Find the equation of the normal to the curve at the point where $x = \frac{1}{2}$. [2]

7 Liquid is poured into a container at a rate of k m^3/s. The volume of liquid in the container is V m^3 where $V = \frac{1}{3}\pi h^2(3k - h)$ and h m is the depth of the liquid in the container. Find, in terms of k, the rate of increase of the liquid level when the depth of the liquid is $\frac{2k}{5}$ m. [4]

8 Given that $\frac{d^2y}{dx^2} = -9y$ and $y = a\cos^3 x + b\cos x$ where a and b are constants, $\cos x \neq 0$, show that $3a + 4b = 0$. [6]

9 The curve $\frac{1}{x} + \frac{2}{y} = \frac{1}{2}$ intersects the line $2x + y + 2 = 0$ at the points A and B.
Explain why a line joining points A and B is perpendicular to the line $2y - x - 6 = 0$. [6]

10 On the same axes, sketch the graphs of

$$y = \cos x + 1 \quad \text{and} \quad y = |\tan x|,$$

for $0° \leq x \leq 360°$. [4]

Hence, for $0° \leq x \leq 360°$, state the value or range of values of k for which the equation $|\tan x| = \cos x + k$ has

(i) 2 roots, [1]

(ii) 3 roots, [1]

(iii) 4 roots. [1]

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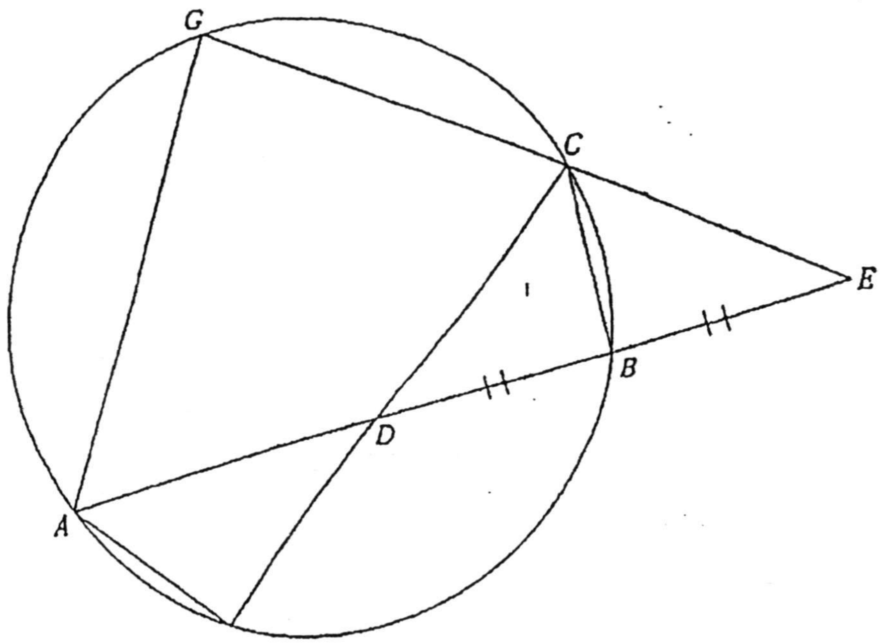
[Turn over

11 The diagram shows a triangle AEG which intersects the circle at points A, B, C and G. D is a point on ABE such that $BD = BE$. Show that

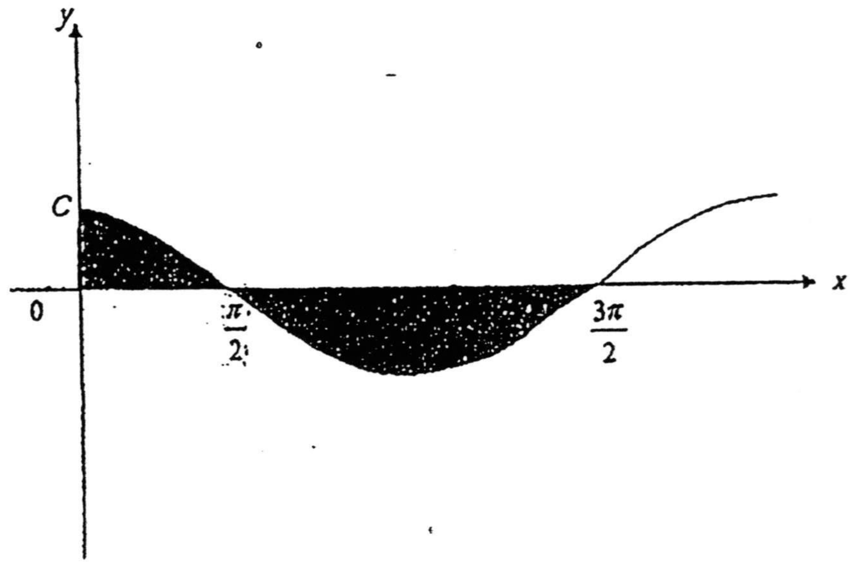
(i) triangle AEG is similar to triangle CEB, [2]

(ii) $AG \times BD = GE \times BC$. [2]



12 (a) Show that $\frac{d}{dx}\left(\sqrt{2+\sin x}\right)=\frac{\cos x}{2\sqrt{2+\sin x}}$. [2]

(b)



The diagram shows part of the curve $y=\frac{\cos x}{2\sqrt{2+\sin x}}$. The curve intersects the x-axis at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ and the y-axis at the point C.

(i) Find the coordinates of point C, in exact form. [1]

(ii) Find the area of the shaded region bounded by the curve, the y-axis and the x-axis. [4]

End of Paper

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Secc 4E/5NA Prelim Exams 2015 AM P1

Marking Scheme

1 (i)

$\sin A\cos B+\cos A\sin B=3(\sin A\cos B-\cos A\sin B)$ $\boxed{\text{M1}}$

$4\cos A\sin B=2\sin A\cos B$

$\dfrac{\sin A\cos B}{\cos A\sin B}=2$ $\boxed{\text{A1}}$

$\tan A=2\tan B$

(ii)

$(\sin(x+30°))^2=(3\sin(x-30°))^2$ $\boxed{\text{M1}}$

$\sin(x+30°)=\pm3\sin(x-30°)$

$\sin(x+30°)=3\sin(x-30°)$

$\tan x=2\tan 30°$

$x=49.1°$ or $229.1°$ $\boxed{\text{A1}}$

or

$\sin(x+30°)=-3\sin(x-30°)$

$\sin(30°+x)=3\sin(30°-x)$ $\boxed{\text{M1}}$

$\tan 30°=2\tan x$

$\tan x=\dfrac{\tan 30°}{2}$

$x=16.1°$ or $196.1°$ $\boxed{\text{A1}}$

2 (a)

$2x^2-2x+1=0$

$\alpha+\beta=1$

$\alpha\beta=\frac{1}{2}$ B1

sum of roots $=\frac{2}{\alpha^3}+\frac{2}{\beta^3}$

$=\frac{2(\alpha^3+\beta^3)}{(\alpha\beta)^3}$

$=\frac{2(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{(\alpha\beta)^3}$ M1

$=\frac{2(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)}{(\alpha\beta)^3}$

$=\frac{2(1)(1-\frac{3}{2})}{(\frac{1}{2})^3}=-8$ A1

product of roots $=\left(\frac{2}{\alpha^3}\right)\left(\frac{2}{\beta^3}\right)$

$=\frac{4}{\left(\frac{1}{2}\right)^3}=32$ A1

equation is $x^2+8x+32=0$ A1

(b) (i) $\alpha+(-\alpha)=\frac{8+4m}{3+m}=0$ M1

$8+4m=0$

$m=-2$ A1

(ii) $(\alpha)\left(\frac{1}{\alpha}\right)=\frac{3+4m}{3+m}=1$ M1

$3+4m=3+m$

$m=0$ A1

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3 (a)

$$\frac{2(4)^{\frac{1}{2}x+2}-2^{x+1}}{6^x \times 3^{1-2x}}=\frac{2(2)^{x+4}-2^{x+1}}{2^x \times 3^x \times \frac{3}{3^{2x}}}$$ M1

$$=\frac{2^x(2^5-2)}{2^x \times \frac{3}{3^x}}$$

$$=10(3^x)$$ A1 for $k=10$, A1 for $n=1$

(b)

$\lg\frac{125}{y}+\lg y^4=\lg(by)^a$ M1

$\lg(125y^3)=\lg(by)^a$

$(5y)^3=(by)^a$

$a=3$ and $b=5$ A2

(c)

$\log_5 e^{2x}+\frac{1}{\log_2 5}=\log_5(2-3e^x)$ M1 for changing base

$\log_5 e^{2x}+\log_5 2=\log_5(2-3e^x)$

$\log_5 2e^{2x}=\log_5(2-3e^x)$ M1 for using correct laws

$2e^{2x}+3e^x-2=0$

$(2e^x-1)(e^x+2)=0$ M1 for the solving equation

$e^x=\frac{1}{2}$ or $e^x=-2(NA)$ A1 for reject $e^x=-2$

$x=\ln\frac{1}{2}$ or $-\ln 2$ or -0.693 A1

4 $(x^2+2x-3)=(x+3)(x-1)$ B1

(i)

$f(1)=0$

$a-b=7$..............(1) M1

$f(-3)=0$ M1

$5a-b=27$..........(2)

solve (1) *and* (2) A1

$a=5$ *and* $b=-2$

(ii)

$f(x)=x^4+6x^3+10x^2-2x-15=(x^2+2x-3)Q(x)$

$Q(x)=x^2+4x+5$ A1

M1 for long division or any correct method

M1 for correct coef of x.

Show that $x^2+4x+5=0$ has no real roots

using $b^2-4ac<0$. A1

Therefore f(x) has only 2 real roots

5 (i) radius = 4 units B1

Equation of the circle is $(x-4)^2+(y+6)^2=16$. B1

(ii) Area $=\int_{-2}^{0} y^2 dy+4^2-\frac{1}{4}\pi(4)^2$ M1

$=\left[\frac{y^3}{3}\right]_{-2}^{0}+16-4\pi$ A1 for $\left[\frac{y^3}{3}\right]$

$=2\frac{2}{3}+16-4\pi=6.10$ *sq units* B1

(iii) centre is (4, 10) B1

Equation is $(x-4)^2+(y-10)^2=16$

$x^2+y^2-8x-20y+100=0$ A1

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6 (i) $\frac{dy}{dx} = 4x + (x+2)^{-2}$

$y = 2x^2 - \frac{1}{x+2} + C$ $\boxed{\text{M1}}$

Subst. $\left(\frac{1}{2}, \frac{1}{2}\right)$, $C = \frac{2}{5}$ $\boxed{\text{A1}}$

Equation of the curve is $y = 2x^2 - \frac{1}{x+2} + \frac{2}{5}$. $\boxed{\text{A1}}$

(ii) at $x = \frac{1}{2}$, $\frac{dy}{dx} = \frac{54}{25}$

Gradient of normal $= -\frac{25}{54}$ $\boxed{\text{M1}}$

Equation of normal is $y = -\frac{25}{54}x + \frac{79}{108}$ or $108y = -50x + 79$. $\boxed{\text{A1}}$

7 $V = \frac{1}{3}\pi h^2(3k - h) = \pi k h^2 - \frac{1}{3}\pi h^3$

$\frac{dV}{dh} = 2\pi k h - \pi h^2$ $\boxed{\text{B1}}$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $\boxed{\text{M1}}$

$\frac{dh}{dt} = \frac{k}{2\pi k(\frac{2k}{5}) - \pi(\frac{2k}{5})^2}$ $\boxed{\text{M1}}$

$= \frac{25}{16\pi k}$ m/s $\boxed{\text{A1}}$

8

$y = a\cos^3 x + b\cos x$

$\frac{dy}{dx} = -3a\cos^2 x\sin x - b\sin x$ M1 for $\frac{d}{dx}(a\cos^3 x)$

$\frac{d^2y}{dx^2} = -3a\cos^3 x + 6a\cos x\sin^2 x - b\cos x$ M1 for $\frac{d}{dx}(-3a\cos^2 x\sin x)$

$= -3a\cos^3 x + 6a\cos x(1-\cos^2 x) - b\cos x$

$= -3a\cos^3 x + 6a\cos x - 6a\cos^3 x - b\cos x$ M1 for using identity

$= -9a\cos^3 x + (6a-b)\cos x$ A1

$\frac{d^2y}{dx^2} = -9y = -9a\cos^3 x - 9b\cos x$

$(6a-b)\cos x = -9b\cos x$ M1

since $\cos x \neq 0$

$\therefore 6a - b = -9b$

$6a + 8b = 0$ A1

$3a + 4b = 0$

9

$\frac{1}{x} + \frac{2}{y} = \frac{1}{2}$

$2y + 4x = xy$

Subst. $y = -2x - 2$ into above equation M1

$2(-2x-2) + 4x = x(-2x-2)$

$2x^2 + 2x - 4 = 0$

$x^2 + x - 2 = 0$ M1 for forming quad eqn

$(x+2)(x-1) = 0$

$x = -2$ or $x = 1$ A1 for (-2, 2)

$y = 2$ or $y = -4$ A1 for (1, -4)

Gradient of the line joining points A and $B = \frac{-4-2}{1+2} = -2$ M1

gradient of the line $2y - x - 6 = 0 = \frac{1}{2}$

The product of the 2 gradient $= -1$ A1

Or the gradient of one of the line is equal to $-\frac{1}{\text{the gradient of the other line}}$

The lines are perpendicular.

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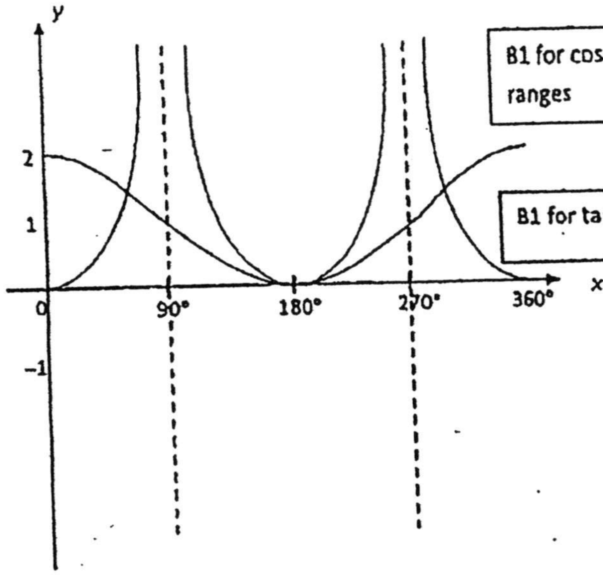

B1 for cosx and B1 for correct ranges

B1 for tanx and B1 for absolute tanx

(i) $-1 \le k < 1$	B1
(ii) $k = 1$	B1
(iii) $k > 1$	B1

11 (i)

$\angle GAE = \angle BCE$ (ext $\angle$, cylic quad)
$\angle AEG = \angle CEB$ (common angles)
$\therefore \Delta AEG$ is similar to ΔCEB (AA, similarity) (shown)

M1 for both conditions

A1 for coding the right test used

(ii)

$\frac{AG}{BC} = \frac{GE}{BE}$ (from part (i))

$\because BD = BE$ (given)

$\frac{AG}{BC} = \frac{GE}{BD}$

$AG \times BD = GE \times BC$ (shown)

M1 for writing the ratios of sides

A1 for using the condition $BD=BE$

12 (a) $y=(2+\sin x)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}(2+\sin x)^{-\frac{1}{2}}(\cos x)$ B1B1

$=\frac{\cos x}{2\sqrt{2+\sin x}}$

(b) (i) when $x=0$, $y=\frac{\cos 0}{2\sqrt{2+\sin 0}}=\frac{1}{2\sqrt{2}}$

Point C is $(0, \frac{1}{2\sqrt{2}})$ or $(0, \frac{\sqrt{2}}{4})$ B1

(ii) Area $=\int_0^{\frac{\pi}{2}}\frac{\cos x}{2\sqrt{2+\sin x}}dx+\left|\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\frac{\cos x}{2\sqrt{2+\sin x}}dx\right|$ M1

$=\left[\sqrt{2+\sin x}\right]_0^{\frac{\pi}{2}}+\left|\left[\sqrt{2+\sin x}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right|$ A1A1 for integrate correct using (a)

$=\left[\sqrt{3}-\sqrt{2}\right]+\left|\sqrt{2+\sin\frac{3\pi}{2}}-\sqrt{2+\sin\frac{\pi}{2}}\right|$

$=\sqrt{3}-\sqrt{2}+\left|\sqrt{2+(-1)}-\sqrt{3}\right|$

$=1.05$ sq units A1

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TEMASEK SECONDARY SCHOOL
O Level Preliminary Examinations 2015

ADDITIONAL MATHEMATICS 4047/02

Paper 2 2 hour 30 minutes

Question Booklet

Additional Material: Writing paper (8 sheets), Cover page (1 sheet), Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

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The use of an approved electronic calculator is expected, where appropriate.
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The total number of marks for this paper is 100.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions.

1 Without using a calculator, find the value of a and of b for which $\frac{a\sqrt{14}+b}{335}$ is the solution of the equation $3x\sqrt{2}+x\sqrt{343}=x\sqrt{50}+\sqrt{8}$. [4]

2 (a) Show that the expression $x^2+px-x+p^2+2$, where p is a constant, is always positive for all real values of x.

Hence, find the range of values of x for which $\frac{x^2-3x-28}{x^2+px-x+p^2+2}<0$. [7]

(b) Find the range of values of k for which the line $y=2x-k$ cuts the curve $y^2=x+k$ at two different points. [4]

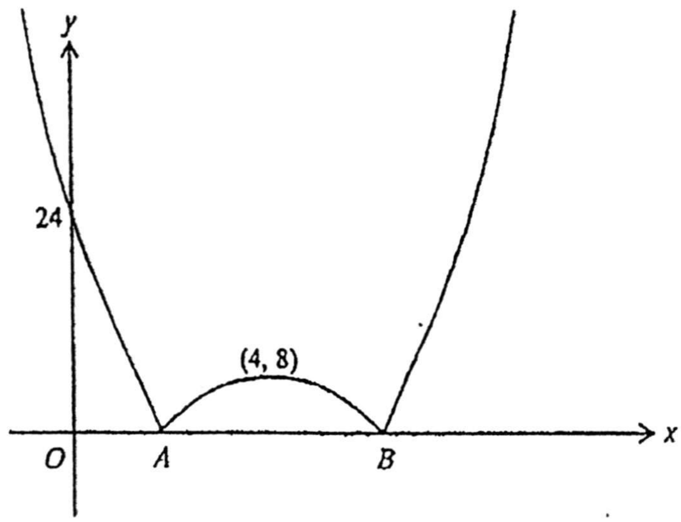
3 (a) Given that the ratio of the coefficients of x^3 and x^9 in the expansion of $\left(x^2-\frac{k}{x}\right)^{12}$ is 1 : 4, find the possible values of k. [5]

(b) The first three terms in the expansion of $(2x-3)\left(1+\frac{x}{3}\right)^n$, in ascending powers of x, are $p+qx-\frac{7}{3}x^2$. Find the values of n, p and q. [5]

4 (a) Express $\frac{2x^3-5x^2+11x-3}{(x^2+1)(x-2)}$ in partial fractions. [5]

(b) Prove the identity $\frac{\sin x+\cos x}{\sin x-\cos x}-\frac{\sin x-\cos x}{\sin x+\cos x}\equiv-2\tan 2x$. [3]

5 The diagram shows part of the curve $y=\left|p(x-r)^2+q\right|$, where p, q and r are constants and $p>0$. The curve cuts the y-axis at 24 and (4, 8) is the turning point of the curve.



(i) Find the values of p, q and r. [3]

(ii) Find the coordinates of A and of B. [3]

(iii) Write down, with explanations, the number of solution(s) to the equation $\left|p(x-r)^2+q\right|=h\left|x-k\right|$ for $3<k<5$ and

(a) $0<h<1$, [2]

(b) $h<0$. [2]

6 A car P moves in a straight line such that, t seconds after the start of motion, its velocity, v m s^{-1}, is given by $v=t-\dfrac{5}{2t+3}$.

The initial displacement of P is $\left(1-\dfrac{5}{2}\ln 3\right)$m.

(i) Find the value of t when P is at instantaneous rest. [3]

(ii) Find an expression, in terms of t, for the acceleration of P and determine whether P can attain maximum velocity. [2]

(iii) Find the average speed of P for the first 2 seconds. [4]

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7 **Answer the whole of this question on a piece of graph paper.**

The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y\sqrt{x} = k(\sqrt{x})^3 + nx$, where k and n are constants.

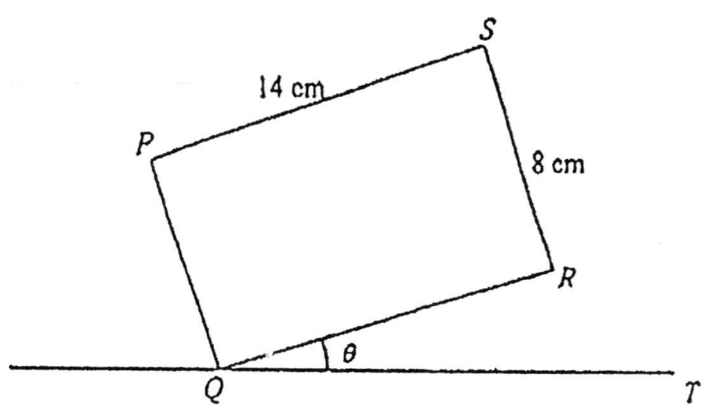
x	1	2	3	4	5
y	3.00	4.53	5.83	7.00	8.09

(i) Using graph paper, plot $\frac{y}{x}$ against $\frac{1}{\sqrt{x}}$ and use your graph to estimate the value of k and of n. [6]

(ii) Use your graph to estimate the value of y when $x = 3.40$. [2]

(iii) By drawing a suitable line on your graph, find the solution to the simultaneous equations $y\sqrt{x} = k(\sqrt{x})^3 + nx$ and $y = \sqrt{x} + x$. [3]

8



The diagram shows a rectangle, $PQRS$, where the QR makes an angle θ with a horizontal line QT.

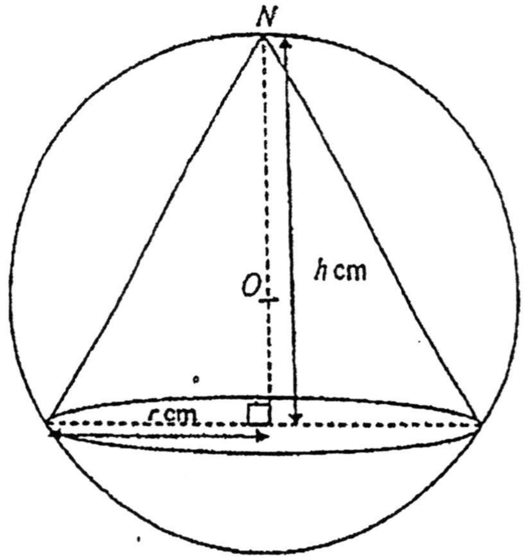
Given that $PS = 14$ cm, $SR = 8$ cm and $0° < \theta < 90°$, show that the perpendicular distance, H cm, from S to the line QT is given by $H = 8\cos\theta + 14\sin\theta$. [2]

(i) Express H in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Find the maximum value of H and the corresponding value of θ. [2]

(iii) Find the value of θ when $H = 12$. [2]

9



The diagram shows a right circular cone in a sphere with centre O and radius 40 cm. The vertex of the cone, N, and the circumference of its base lies on the sphere and the centre of the sphere is on the axis of the cone.

(i) Given that the radius, height and volume of the right circular cone are r cm, h cm and V cm^3 respectively, show that $V=\frac{\pi}{3}\left(80h^2-h^3\right)$. [3]

(ii) Find the stationary value of V and show that this value is a maximum. [5]

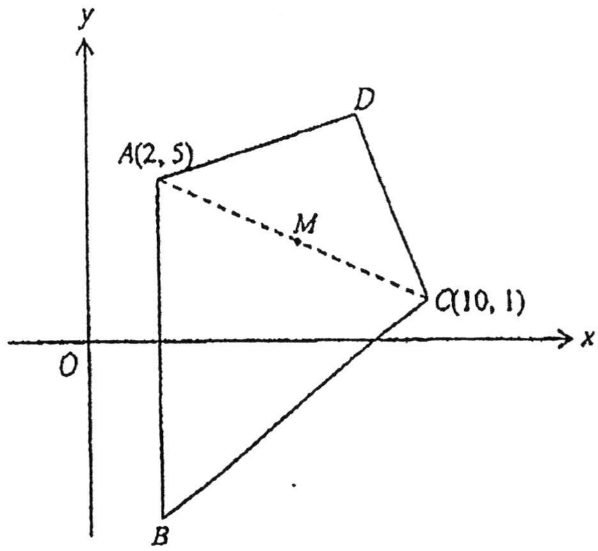
10 (a) Find the range of values of x for which the curve $y=x^3e^{1-2x}$ is a decreasing function. [4]

(b) Given that $y=\left[\ln(3-4x)\right]^2$, show that $\frac{dy}{dx}=\frac{k\ln(3-4x)}{3-4x}$, where k is a constant to be determined.

Hence, evaluate $\int_{-2}^{-1}\frac{2+3\ln(3-4x)}{3-4x}\,dx$. [7]

Z13

11 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a kite $ABCD$. M is the midpoint of AC and the coordinates of A and C are (2, 5) and (10, 1) respectively.

(i) Find the coordinates of M. [1]

(ii) Find the equation of BD. [2]

(iii) Given that B lies on the line $3x+2y+4=0$, find the coordinates of B. [2]

(iv) Given that $\frac{BD}{MD}=3$, find the coordinates of D. [2]

(v) Find the area of the kite $ABCD$. [2]

End of Paper

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Z14

Prelims 2015 Sec 4E/5N –Additional Mathematics Paper 2
Marking Scheme

Qn	Solutions and Marks Allocation
1	$3x\sqrt{2}+x\sqrt{343}=x\sqrt{50}+\sqrt{8}$ $7x\sqrt{7}-2x\sqrt{2}=2\sqrt{2}$ $\quad M1$ $x=\dfrac{2\sqrt{2}}{7\sqrt{7}-2\sqrt{2}}\times\dfrac{7\sqrt{7}+2\sqrt{2}}{7\sqrt{7}+2\sqrt{2}}$ $\quad M1$ – *rationalise* $=\dfrac{14\sqrt{14}+8}{335}$ $\quad A1$ $a=14$ and $b=8$ $\quad A1$
2(a)	$x^2+px-x+p^2+2$ $=x^2+(p-1)x+p^2+2$ $b^2-4ac=(p-1)^2-4(1)(p^2+2)$ $\quad M1$ $=-3p^2-2p-7$ $=-3\left(p+\dfrac{1}{3}\right)^2-6\dfrac{2}{3}$ $\quad A1$ Since $\left(p+\dfrac{1}{3}\right)^2\geq 0,$ $\quad$ M1 $b^2-4ac<0$ Since <u>the coefficient of x^2 is positive and the discriminant is less than zero,</u> $x^2+px-x+p^2+2$ is always positive for all real values of x. $\quad$ A1 $x^2-3x-28<0$ $\quad M1$ $(x-7)(x+4)<0$ $\quad M1$ $-4<x<7$ $\quad A1$
2(b)	$y^2=x+k$ ------(1) $y=2x-k$ ------(2) Sub (2) into (1): $(2x-k)^2=x+k$ $\quad M1$ $4x^2-(4k+1)x+k^2-k=0$ $\quad A1$ $b^2-4ac>0$ $(4k+1)^2-4(4)(k^2-k)>0$ $\quad M1$ $24k+1>0$ $k>-\dfrac{1}{24}$ $\quad A1$

3(a)	$\left(x^2-\frac{k}{x}\right)^{12}$ $T_{r+1}=\binom{12}{r}(-k)^r x^{24-3r}$ A1 for x^{24-3r} Let $24-3r=3$, $r=7$ Coefficient of $x^3=\binom{12}{7}(-k)^7=-792k^7$ A1 Let $24-3r=9$ $r=5$ Coefficient of $x^9=\binom{12}{5}(-k)^5=-792k^5$ A1 $\frac{-792k^7}{-792k^5}=\frac{1}{4}$ M1 $k=\pm\frac{1}{2}$ A1
3(b)	$(2x-3)\left(1+\frac{x}{3}\right)^n$ $=(2x-3)\left(1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+...\right)$ A1 $=p+qx-\frac{7}{3}x^2+...$ Comparing the constant term: $p=-3$ A1 Comparing the coefficient of x: $q=2-n$ Comparing the coefficient of x^2: $-\frac{7}{3}=\frac{2}{3}n-\frac{n(n-1)}{6}$ M1 $n^2-5n-14=0$ $(n-7)(n+2)=0$ $n=-2$ or $n=7$ A1 (reject) $\therefore q=-5$ A1

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4(a)	$\frac{2x^3-5x^2+11x-3}{(x^2+1)(x-2)}=A+\frac{Bx+C}{x^2+1}+\frac{D}{x-2}$ M1 $2x^3-5x^2+11x-3=A(x^2+1)(x-2)+(Bx+C)(x-2)+D(x^2+1)$ Comparing the coefficient of x^3: $A=2$ Let $x=2$, $D=3$ $\boxed{\text{A3A2A1A0}}$ Let $x=0$, $C=1$ Let $x=1$, $B=-4$ $\frac{2x^3-5x^2+11x-3}{(x^2+1)(x-2)}=2-\frac{4x-1}{x^2+1}+\frac{3}{x-2}$ A1
4(b)	$LHS=\frac{\sin x+\cos x}{\sin x-\cos x}-\frac{\sin x-\cos x}{\sin x+\cos x}$ $=\frac{(\sin x+\cos x)^2-(\sin x-\cos x)^2}{\sin^2 x-\cos^2 x}$ M1 $=\frac{4\sin x\cos x}{\sin^2 x-\cos^2 x}$ $=\frac{2\sin 2x}{-\cos 2x}$ A1 for $2\sin 2x$, A1 for $-\cos 2x$ $=-2\tan 2x$
5(i)	$r=4$ B1 $q=-8$ B1 Let $y=24$ when $x=0$, $24=\lvert 16p-8\rvert$ $16p-8=24$ or $16p-8=-24$ $p=2$ or $p=-1$ (reject) B1
5(ii)	Let $y=0$, $2(x-4)^2-8=0$ M1 $(x-4)^2=4$ $x-4=\pm 2$ $x=2$ or 6 A1 $A(2,0)$ and $B(6,0)$ A1
5(iii) (a)	The graph of $y=h\lvert x-k\rvert$ is V-shaped and the vertex is located between $x=3$ and $x=5$ on the x-axis, thus there will be 4 solutions. B1B1
5(iii) (b)	The graph of $y=h\lvert x-k\rvert$ is inverted V-shaped and the vertex is located between $x=3$ and $x=5$ on the x-axis, thus there will be no solution. B1B1

6(i)	Let $v=0$, $t=\dfrac{5}{2t+3}$ $M1$ $2t^2+3t-5=0$ $M1$ $(2t+5)(t-1)=0$ $t=-\dfrac{5}{2}$ or 1 $A1$ (reject)
6(ii)	$a=1+\dfrac{10}{(2t+3)^2}\neq 0$ A1 Since a cannot have a value of zero, P cannot attain maximum velocity. A1
6(iii)	$s=\dfrac{t^2}{2}-\dfrac{5}{2}\ln(2t+3)+c$, where c is a constant. M1 Let $s=1-\dfrac{5}{2}\ln 3$ when $t=0$. $1-\dfrac{5}{2}\ln 3=-\dfrac{5}{2}\ln 3+c$ $c=1$ $s=\dfrac{t^2}{2}-\dfrac{5}{2}\ln(2t+3)+1$ A1 When $t=1$, $s=\dfrac{3}{2}-\dfrac{5}{2}\ln 5=-2.52359$ When $t=2$, $s=3-\dfrac{5}{2}\ln 7=-1.864775$ When $t=0$, $s=1-\dfrac{5}{2}\ln 3=-1.74653$ Average speed $=\dfrac{(2.52359-1.74653)+(2.52359-1.864775)}{2}=0.718$ m/s M1A1

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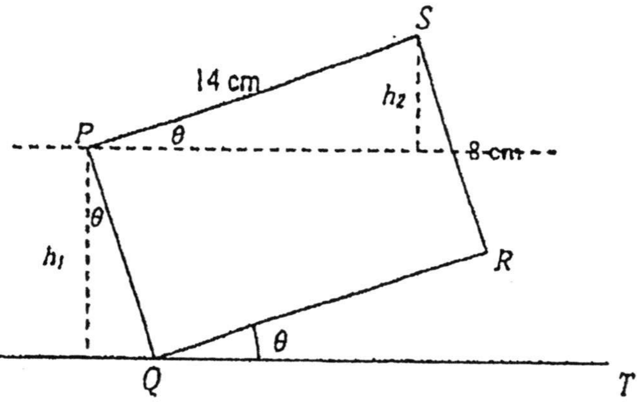
8	 From the diagram, $h_1 = 8\cos\theta$ and $h_2 = 14\sin\theta$, $H = 8\cos\theta + 14\sin\theta$	B1 each (working needed)
(i)	$R = \sqrt{260} = 2\sqrt{65}$ $\alpha = 60.26°$ $H = 2\sqrt{65}\cos(\theta - 60.26°)$	A1 A1 A1
(ii)	Max value of $H = 2\sqrt{65}$ $\theta = 60.3°$	B1 B1
(iii)	When $H = 12$, $2\sqrt{65}\cos(\theta - 60.26°) = 12$ $\cos(\theta - 60.26°) = \frac{6}{\sqrt{65}}$ $\theta - 60.26° = -41.91°$ $\theta = 18.4°$	 M1 A1

9(i)	$(h-40)^2+r^2=40^2$	$M1$
	$r^2=80h-h^2$	$A1$
	$V=\frac{1}{3}\pi r^2 h$	
	$=\frac{\pi}{3}(80h-h^2)h$	$A1$
	$=\frac{\pi}{3}(80h^2-h^3)$	
9(ii)	$\frac{dV}{dh}=\frac{\pi}{3}(160h-3h^2)$	$M1$
	Let $\frac{dV}{dh}=0$,	
	$\frac{\pi}{3}(160h-3h^2)=0$	
	$h=0$ or $h=53\frac{1}{3}$	$A1$
	(reject)	
	When $h=53\frac{1}{3}$,	
	$V=79431.87\approx 79400$ (3sf)	$A1$
	$\frac{d^2V}{dh^2}=\frac{\pi}{3}(160-6h)$	$M1$
	When $h=53\frac{1}{3}$,	
	$\frac{d^2V}{dh^2}=-\frac{160}{3}\pi<0$	
	The stationary value of V is maximum.	A1 [proof is needed]

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10(a)	$y = x^3 e^{1-2x}$ $\frac{dy}{dx} = x^3 e^{1-2x}(-2) + 3x^2 e^{1-2x}$ A2 $= x^2 e^{1-2x}(3-2x)$ Let $\frac{dy}{dx} < 0$, $x^2 e^{1-2x}(3-2x) < 0$ M1 $3 - 2x < 0$ $x > 1\frac{1}{2}$ A1
(b)	$y = [\ln(3-4x)]^2$ $\frac{dy}{dx} = 2\ln(3-4x)\left(\frac{-4}{3-4x}\right)$ M1 $= \frac{-8\ln(3-4x)}{3-4x}$ $\therefore k = -8$ A1 $\int_{-2}^{-1} \frac{2+3\ln(3-4x)}{3-4x} dx$ $= \int_{-2}^{-1} \frac{2}{3-4x} dx - \frac{3}{8}\int_{-2}^{-1} \frac{-8\ln(3-4x)}{3-4x} dx$ M1 $= -\frac{1}{2}[\ln(3-4x)]_{-2}^{-1} - \frac{3}{8}\left[(\ln(3-4x))^2\right]_{-2}^{-1}$ B2 $= -\frac{1}{2}(\ln 7 - \ln 11) - \frac{3}{8}\left[(\ln 7)^2 - (\ln 11)^2\right]$ M1 $= 0.22599 + 0.73625$ $= 0.962(3sf)$ A1

11(i)	$M(6, 3)$ A1
(ii)	$m_{AC} = -\frac{1}{2}$ $m_{BD} = 2$ M1 Equation of BD: $y-3=2(x-6)$ $y=2x-9$ -------(1) A1
(iii)	$3x+2y+4=0$ -----(2) Solving (1) and (2): M1 $x=2$ and $y=-5$ $B(2,-5)$ A1
(iv)	$\frac{D_x-2}{D_x-6}=3$ $D_x=8$ $\frac{D_y+5}{D_y-3}=3$ $D_y=7$ $D(8,7)$ M1 A1
(v)	Area of kite $ABCD$ $=\frac{1}{2}\begin{vmatrix} 2 & 2 & 10 & 8 & 2 \\ 5 & -5 & 1 & 7 & 5 \end{vmatrix}$ M1 $=\frac{1}{2}(102+18)$ $=60$ sq. units A1

2'8