Bukit Batok Secondary School
'O' Level Preliminary Examination 2016
Secondary 4 Express / 5 Normal Academic

ADDITIONAL MATHEMATICS **4047/01**
Paper 1

16 Aug 2016
0745 – 0945
2 hours

Additional Materials: 6 sheets of writing paper
1 sheet of graph paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staple, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate answer paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper contains 7 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}ab\sin C.$$

1. A curve is such that $\frac{dy}{dx} = -\frac{9}{(1-2x)^4}$ and $(1, 4)$ is a point on the curve.
   Find the equation of the curve. [3]

2. The area of a rectangle plot of land is $(5\sqrt{2} + \sqrt{14})$ m^2 and its breadth is $(3\sqrt{2} - \sqrt{14})$ m.
   Find the length of the plot of land in the form $a + b\sqrt{7}$ m, where a and b are integers. [3]

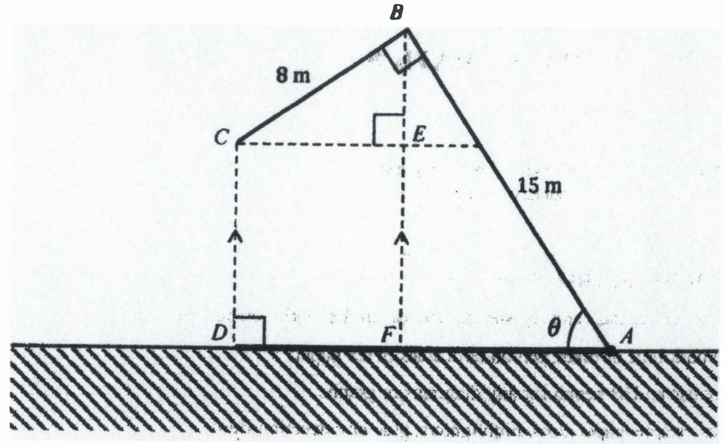
3. (i) Show that $\frac{d}{dx}(x + x\tan^2 x) = \sec^2 x + 2x\sec^2 x \tan x$. [2]
   (ii) Hence find $\int 4x\sec^2 x \tan x \, dx$. [2]

4. (i) Prove the identity $\frac{1-\cos 2A}{\sin 2A \cot A} = \tan^2 A$. [2]
   (ii) Hence, solve the equation $\frac{1-\cos 2A}{\sin 2A \cot A} = \sqrt{3}\tan A$, for $0° \le A \le 360°$. [4]

5. The roots of the quadratic equation $4x^2 - 8x + 1 = 0$ are α^2 and β^2.
   (i) Find the values of $\alpha^2 + \beta^2$ and $\alpha\beta$. [2]
   (ii) Hence, find all possible non-equivalent quadratic equations whose roots are α and β, where $\alpha + \beta > 0$. [4]

6. (i) Express $\frac{2x^3+9x^2+2x+5}{x(x^2+1)}$ in partial fractions. [5]
   (ii) Find $\frac{d}{dx}[\ln(x^2+1)]$. [1]
   (iii) Using the results from parts (i) and (ii), find $\int \frac{2x^3+9x^2+2x+5}{x(x^2+1)} dx$. [2]

7. In the diagram, ABC is a right-angled metal pole hinged at A, where $\angle ABC = 90°$, $AB = 15$ m and $BC = 8$ m.
   The pole is inclined at an angle θ to the ground.
   A shadow DA of the pole is formed on the ground when the sun is directly above the pole such that $\angle CDA = 90°$.



BEF is a straight line parallel to CD and perpendicular to CE.
(i) Show that $\angle CBE = \theta$. [1]
(ii) Hence, or otherwise, show that the length of the shadow DA, L m, is $8\sin\theta + 15\cos\theta$. [2]
(iii) Using the result in part (ii), express the length of the shadow in the form $L = R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]
(iv) State the maximum possible length of the shadow and the corresponding value of θ. [2]

8. An object moves in a straight line such that t seconds after passing through a point P, its velocity, v ms^{-1}, is given by $v = 4e^{2t} - 6e^{t}$.

(i) Write an expression for the acceleration of the particle. [1]

(ii) Find the exact value of t when the particle is instantaneously at rest. [2]

The point P is 5 m in the positive direction away from the fixed point O.

(iii) Calculate the total distance travelled by the particle in the interval $0 \le t \le \ln 3$. [4]

(iv) State and explain clearly whether the answer in part (iii) will change if the position of the point P is changed. [1]

9. (a) The coefficient of x^2 in the expansion of $(1-kx)^8$ is 7.

(i) Write down the first 4 terms in the expansion of $(1-kx)^8$ and hence find the possible value(s) of the constant k. [2]

(ii) By substituting a suitable value of x and k into your answer in part (i), find an approximate value of 0.995^8 without using a calculator, correct to 6 decimal places. [2]

(b) (i) Express the general term in the binomial expansion of $\left(2x+\frac{1}{2x^3}\right)^{12}$ in the form $\binom{12}{r}2^a x^b$, where a and b are constants in terms of r. [2]

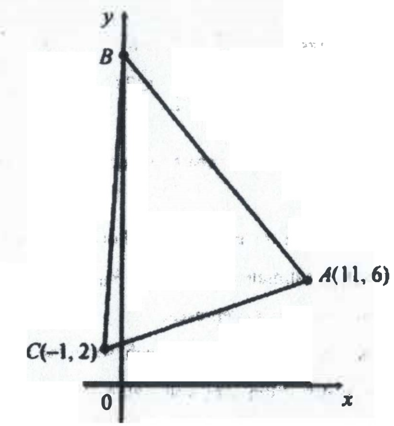
(ii) Find the term independent of x in the expansion of $\left(\frac{1}{x^4}-1\right)\left(2x+\frac{1}{2x^3}\right)^{12}$. [2]

(iii) State and explain, with justification, whether there is any term containing x^n, where n is odd, in the expansion of $\left(\frac{1}{x^4}-1\right)\left(2x+\frac{1}{2x^3}\right)^{12}$. [2]

10. **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a triangle ABC in which the point A is $(11, 6)$ and the point C is $(-1, 2)$.

The point B lies on the y-axis and on the perpendicular bisector of AC.



(i) Find the equation of the perpendicular bisector of AC. [4]

(ii) State the coordinates of B. [1]

The point D lies below the x-axis such that CD is parallel to the perpendicular bisector of AC and AC is 4 times the length of CD.

(iii) Show that the coordinates of D is $(0, -1)$. [5]

(iv) Calculate the area of the quadrilateral $ABCD$. [2]

11. The pesticide DDT was banned worldwide because it is toxic to a wide range of animals.
Half-life is the amount of time it takes for half of the amount of a substance to decay.
An experiment was conducted to determine the half-life of a sample of DDT.
Measured values of the mass, M grams, of the sample decreasing with time, t years, are given in the table below.

t (years)	2	6	10	14	18
M (grams)	176.3	137.0	106.5	82.8	64.3

The relationship between M and t can be modelled by the equation $M = M_0 e^{-kt}$, where M_0 and k are constants.

(i) Plot $\ln M$ against t for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the values of M_0 and k. [3]

(iii) State what the value of M_0 represents. [1]

(iv) By drawing a suitable line on your graph, solve the equation $kt = \ln 2$. [3]

(v) Explain, with justification, why your value of t found in part (iv) can be used as an estimate for the half-life of the sample of DDT. [2]

– End of Paper –

Marking Scheme
BBSS Prelim 2016 Sec 4E5N
Additional Mathematics Paper 1

1.		$y = \int -\frac{9}{(1-2x)^4}\,dx$		
		$= \int -9(1-2x)^{-4}\,dx$		
		$= \frac{-9(1-2x)^{-3}}{(-3)(-2)} + c$	M1	
		$= -\frac{3}{2(1-2x)^3} + c$		
		$4 = -\frac{3}{2[1-2(1)]^3} + c$	M1	
		$c = \frac{5}{2}$		
		$y = -\frac{3}{2(1-2x)^3} + \frac{5}{2}$	A1	[3]
2.		Length $= \frac{(5\sqrt{2}+\sqrt{14})(3\sqrt{2}+\sqrt{14})}{(3\sqrt{2}-\sqrt{14})(3\sqrt{2}+\sqrt{14})}$	M1	
		$= \frac{30+5\sqrt{28}+3\sqrt{28}+14}{18-14}$	M1	
		$= \frac{44+16\sqrt{7}}{4}$		
		$= 11 + 4\sqrt{7}$	A1	[3]
3.	(i)	$\frac{d}{dx}(x + x\tan^2 x)$		
		$= 1 + x(2\tan x\sec^2 x) + \tan^2 x$	M1	
		$= 1 + \tan^2 x + 2x\tan x\sec^2 x$	M1	
		$= \sec^2 x + 2x\sec^2 x\tan x$		[2]
	(ii)	$\int 2x\sec^2 x\tan x\,dx = x + x\tan^2 x - \int \sec^2 x\,dx + c_1$		
		$= x + x\tan^2 x - \tan x + c_2$	M1	
		$\int 4x\sec^2 x\tan x\,dx = 2x + 2x\tan^2 x - 2\tan x + c$	A1	[2]
4.	(i)	$\frac{1-\cos 2A}{\sin 2A\cot A}$		
		$= \frac{1-(1-2\sin^2 A)}{(2\sin A\cos A)\left(\frac{\cos A}{\sin A}\right)}$	M1	
		$= \frac{2\sin^2 A}{2\cos^2 A}$	M1	
		$= \tan^2 A$		[2]
	(ii)	$\frac{1-\cos 2A}{\sin 2A\cot A} = \sqrt{3}\tan A$		
		$\tan^2 A = \sqrt{3}\tan A$		
		$\tan^2 A - \sqrt{3}\tan A = 0$		
		$\tan A\left(\tan A - \sqrt{3}\right) = 0$		
		$\tan A = 0$	M1	
		$A = 0°, 180°$ or $360°$	A1 (all)	
		or $\tan A = \sqrt{3}$	M1	
		$A = 60°$ or $240°$	A1 (both)	
		$\therefore A = 0°, 60°, 180°, 240°$ or $360°$		[4]

5.	(i)	$\alpha^2 + \beta^2 = 2$	B1	
		$\alpha^2\beta^2 = \frac{1}{4}$		
		$\alpha\beta = \pm\frac{1}{2}$	B1	[2]
	(ii)	$\alpha\beta = \frac{1}{2}$		
		$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$	M1	
		$= 2 + 2\left(\frac{1}{2}\right)$		
		$\alpha + \beta = \sqrt{3}$ or $\alpha + \beta = -\sqrt{3}$ (reject)	M1	
		Equation: $x^2 - \sqrt{3}x + \frac{1}{2} = 0$	A1	
		$\alpha\beta = -\frac{1}{2}$		
		$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$		
		$= 2 + 2\left(-\frac{1}{2}\right)$		
		$\alpha + \beta = 1$ or $\alpha + \beta = -1$ (reject)		
		Equation: $x^2 - x - \frac{1}{2} = 0$	A1	[4]
6.	(i)	$\frac{2x^3+9x^2+2x+5}{x(x^2+1)} = 2 + \frac{9x^2+5}{x(x^2+1)}$	M1	
		Let $\frac{9x^2+5}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$	M1	
		$9x^2 + 5 = A(x^2+1) + (Bx+C)x$		
		Let $x = 0, A = 5$	M1	
		$9x^2 + 5 = 5x^2 + 5 + Bx^2 + Cx$		
		By comparing coefficients,		
		$C = 0$		
		and $B + 5 = 9$	M1	
		$B = 4$		
		$\therefore \frac{2x^3+9x^2+2x+5}{x(x^2+1)} = 2 + \frac{5}{x} + \frac{4x}{x^2+1}$	A1	[5]
	(ii)	$\frac{d}{dx}[\ln(x^2+1)] = \frac{2x}{x^2+1}$	A1	[1]
	(iii)	$\int \frac{2x^3+9x^2+2x+5}{x(x^2+1)}\,dx = \int 2 + \frac{5}{x} + \frac{4x}{x^2+1}\,dx$		
		$= 2x + 5\ln x + \int \frac{4x}{x^2+1}\,dx + c_1$	M1	
		$= 2x + 5\ln x + 2\ln(x^2+1) + c$	A1	[2]

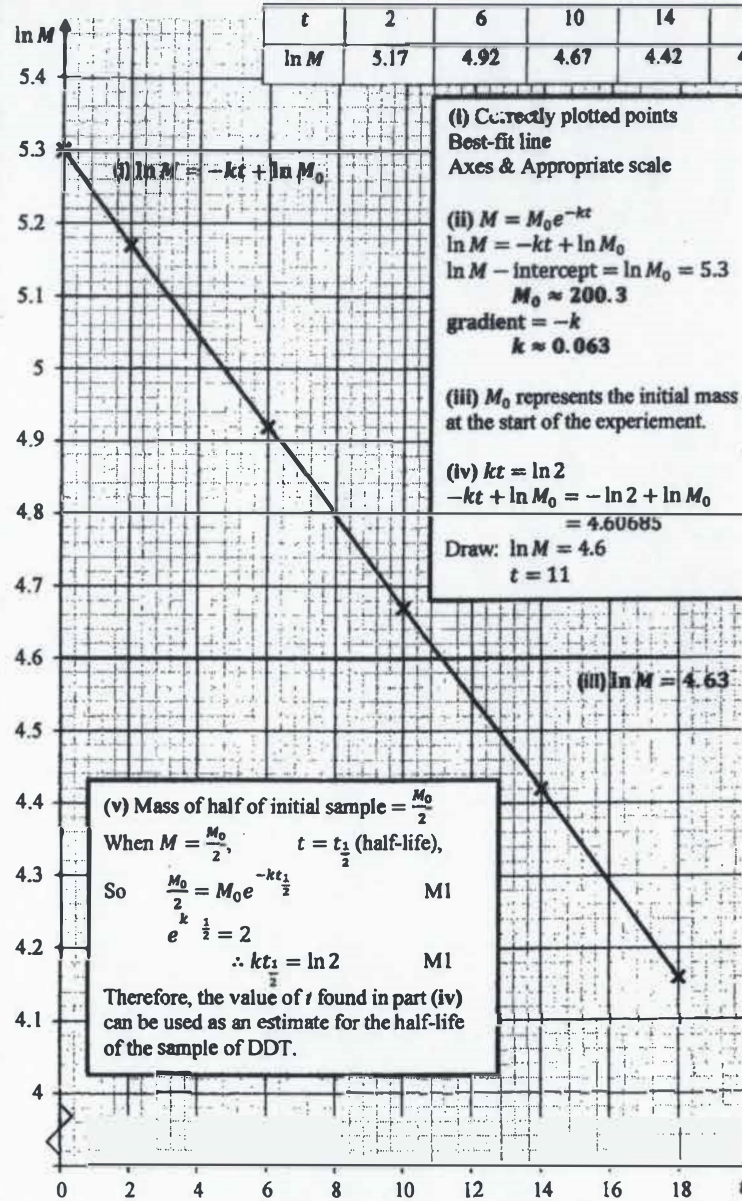
7.	(i)	$\angle FBA = 90° - \theta$ $\angle CBE = 90° - (90° - \theta)$ $= \theta$	M1	[1]
	(ii)	$\sin\theta = \frac{CE}{8}$ $CE = 8\sin\theta$ $\cos\theta = \frac{FA}{15}$ $FA = 15\cos\theta$ $\therefore L = 8\sin\theta + 15\cos\theta$	 M1 M1	[2]
	(iii)	$8\sin\theta + 15\cos\theta = R\sin(\theta+\alpha)$ $8\sin\theta + 15\cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $R\sin\alpha = 15$ --① $R\cos\alpha = 8$ --② $①^2 + ②^2 : R^2 = 15^2 + 8^2$ $R = 17$ $① \div ② :\ \tan\alpha = \frac{15}{8}$ $\alpha = 1.080839001$ $\therefore L = 17\sin(\theta + 1.08)$ (3 s.f.)	M1 M1 A1	[3]
	(iv)	Maximum possible length = 17 m When $L = 17$, $\sin(\theta + 1.080839001) = 1$ $\theta + 1.080839001 = \frac{\pi}{2}$ $\theta = 0.489957 = 0.490$ (3 s.f.)	B1 A1	[2]
8.	(i)	$a = 8e^{2t} - 6e^{t}$	B1	[1]
	(ii)	when $v = 0$, $4e^{2t} - 6e^{t} = 0$ $2e^{t}(2e^{t} - 3) = 0$ $2e^{t} = 3$ $t = \ln 1.5$	 M1 A1	[2]
	(iii)	Distance in the interval $(0 \le t \le \ln 1.5) = \left\lvert \int_0^{\ln 1.5} 4e^{2t} - 6e^{t}\,dt \right\rvert$ $= \left\lvert [2e^{2t} - 6e^{t}]_0^{\ln 1.5} \right\rvert$ $= \lvert -4.5 - (-4) \rvert$ $= 0.5$ m Distance in the interval $(\ln 1.5 \le t \le \ln 3) = \left\lvert \int_{\ln 1.5}^{\ln 3} 4e^{2t} - 6e^{t}\,dt \right\rvert$ $= \left\lvert [2e^{2t} - 6e^{t}]_{\ln 1.5}^{\ln 3} \right\rvert$ $= \lvert 0 - (-4.5) \rvert$ $= 4.5$ m Total distance $= 0.5 + 4.5$ $= 5$ m	 M1 M1 M1 A1	[4]
	(iv)	No, the answer in part (iii) will not change because distance is independent of the position of the point P.		[1]

9.	a)(i)	$(1 - kx)^8 = 1 - 8kx + 28k^2x^2 - 56k^3x^3 + \cdots$ $28k^2 = 7$ $k = \pm\frac{1}{2}$	A1 A1	[2]
	(ii)	when $k = \frac{1}{2}$ and $x = 0.01$, $(1 - kx)^8 = 1 - 8kx + 28k^2x^2 - 56k^3x^3 + \cdots$ $(1 - 0.005)^8 = 1 - 8\left(\frac{1}{2}\right)(0.01) + 28\left(\frac{1}{4}\right)(0.01)^2 - 56\left(\frac{1}{8}\right)(0.01)^3 + \cdots$ $0.995^8 = 1 - 0.04 + 0.0007 - 0.000007$ $= 0.960693$ (6 d.p.)	 M1 A1	[2]
	b)(i)	General term $= \binom{12}{r}(2x)^{12-r}\left(\frac{1}{2x^3}\right)^r$ $= \binom{12}{r}2^{12-2r}x^{12-4r}$	M1 A1	[2]
	(ii)	when $12 - 4r = 4, r = 2$ when $12 - 4r = 0, r = 3$ Independent term $= \binom{12}{2}2^{12-2(2)} - \binom{12}{3}2^{12-2(3)}$ $= 2816$	M1 (either) A1	[2]
	(iii)	For a general term containing x^n in the expansion of $\left(\frac{1}{x^4} - 1\right)\left(2x + \frac{1}{2x^3}\right)^{12}$, $n = (-4) + (12 - 4r)$. Since (-4) and $(12 - 4r) = 2(6 - 2r)$ are even, n is even. Therefore, there is no term containing an odd power of x in the expansion of $\left(\frac{1}{x^4} - 1\right)\left(2x + \frac{1}{2x^3}\right)^{12}$.	 B1 A1	[2]

10.	(i)	Midpoint of $AC = \left(\frac{11-1}{2}, \frac{6+2}{2}\right)$	M1	
		$= (5, 4)$		
		Gradient of $AC = \frac{6-2}{11+1}$	M1	
		$= \frac{1}{3}$		
		Gradient $\perp AC = -3$		
		Equation of the $\perp$ bisector of AC: $y - 4 = -3(x-5)$	M1	
		$y = -3x + 19$	A1	[4]
	(ii)	Coordinates of $B = (0, 19)$	B1	[1]
	(iii)	Let the coordinates of $D = (p, q)$		
		$\frac{q-2}{p+1} = -3$	M1	
		$q = -3p - 1$ --- ①		
		$AC = \sqrt{(11+1)^2 + (6-2)^2}$	M1	
		$= \sqrt{160}$		
		$4\sqrt{(p+1)^2 + (q-2)^2} = \sqrt{160}$	M1	
		$(p+1)^2 + (q-2)^2 = 10$ --- ②		
		Substitute ① into ②: $(p+1)^2 + (-3p-1-2)^2 = 10$	M1	
		$10p(p+2) = 0$		
		$p = 0, q = -1$		
		Or $p = -2, q = 5$ (reject)		
		Coordinates of $D = (0, -1)$	A1	[5]
	(iv)	Area of quadrilateral $ABCD = \frac{1}{2}\begin{vmatrix} 11 & 0 & -1 & 0 & 11 \\ 6 & 19 & 2 & -1 & 6 \end{vmatrix}$		
		$= \frac{1}{2}[210 - (-30)]$	M1	
		$= 120$ units^2	A1	[2]

11.

t	2	6	10	14	18
$\ln M$	5.17	4.92	4.67	4.42	4.16



(i) Correctly plotted points P1
Best-fit line G1
Axes & Appropriate scale G1

(ii) $M = M_0 e^{-kt}$
$\ln M = -kt + \ln M_0$ M1
$\ln M$ – intercept $= \ln M_0 = 5.3$
$M_0 \approx 200.3$ B1
gradient $= -k$
$k \approx 0.063$ B1

(iii) M_0 represents the initial mass of sample at the start of the experiement. B1

(iv) $kt = \ln 2$
$-kt + \ln M_0 = -\ln 2 + \ln M_0$ M1
$= 4.60685$
Draw: $\ln M = 4.6$ G1
$t = 11$ A1

(v) Mass of half of initial sample $= \frac{M_0}{2}$
When $M = \frac{M_0}{2}$, $t = t_{\frac{1}{2}}$ (half-life),
So $\frac{M_0}{2} = M_0 e^{-kt_{\frac{1}{2}}}$ M1
$e^{kt_{\frac{1}{2}}} = 2$
$\therefore kt_{\frac{1}{2}} = \ln 2$ M1
Therefore, the value of t found in part (iv) can be used as an estimate for the half-life of the sample of DDT.

[12]

– End of Paper –

Bukit Batok Secondary School

'O' Level Preliminary Examination 2016

Secondary 4 Express/ 5 Normal Academic

ADDITIONAL MATHEMATICS
4047/02
Paper 2

18 August 2016
0930 – 1200
2 hours 30 minutes

Additional Materials: 8 pieces of writing paper

READ THESE INSTRUCTIONS FIRST

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The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of 6 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}bc\sin A.$$

1. (a) Solve the equation $(3x+1)^2-|3x+1|-12=0$. [2]

(b) The curve $y=a\cos bx+c$, where a is positive, is defined for $0\le x\le 4\pi$ has an amplitude of 3 and period of 4π. The minimum value of y is -2.

(i) State the value of a, of b and of c. [1]

(ii) On the same axes, sketch the graph $y=a\cos bx+c$ and $y=-|\sin x|$ for $0\le x\le 4\pi$. [3]

(iii) Hence, deduce the value of k, where k is an integer, for which the equation $a\cos bx+c=-|\sin x|+k$ has four solutions. [1]

2. (a) Find the range of values of d for which $3x^2+5x+2$ is always greater than $4x+d$. [2]

(b) Find the range of values of k for which the equation $x^2+2k+10=x-3kx$ has real roots. [3]

(c) Find the set of values of m for which the curve $y=x^2+6x+5$ and the line $y=mx+1$ do not intersect. [2]

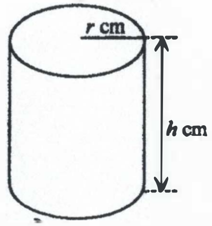
3. It is given that $f(x)=2x^3+ax^2+x+b$.

(i) Find the value of a and of b for which $2x^2+x-1$ is a factor of $f(x)$. [4]

(ii) Solve the equation $f(x)=0$. [2]

(iii) Hence solve $\frac{1}{4}y^3+\frac{a}{4}y^2+\frac{1}{2}y+b=0$. [2]

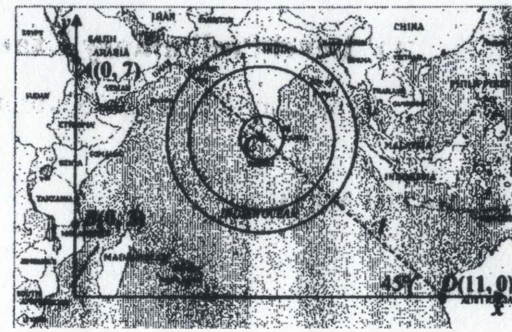
4. Jason wants to paint the surface of a closed cylindrical can which has a volume of 166π cm^3. The cost of painting is the least if the can has the smallest surface area. The radius of the can is r cm, the height is h cm and the total surface area is A cm^2.



(i) Express h in terms of r and show that $A=2\pi r^2+\frac{332\pi}{r}$. [2]

(ii) Given that r can vary, find the value of A for which the cost of painting is the least. [6]

5. The diagram shows a simplified map of the Indian Ocean. Geological stations $B(0, 3)$, $A(0, 7)$ and $D(11, 0)$ detected an earthquake and a geologist is attempting to locate the epicentre C of the earthquake.



Instruments at B and A detected the earthquake at exactly the same time, indicating that the epicentre C is equidistant from B and A. Instruments at D detected it a few seconds later in the direction as indicated by the line l in the diagram above. The line l makes an angle of 45° with the x-axis.

(i) Show that the line l can be represented by the equation $x+y=11$. [2]

(ii) Find the coordinates of C. [3]

(iii) Hence, find the equation of the circle passing through B and A with centre C. [2]

(iv) Explain why it is not possible for a circle with centre C to pass through all three points B, A and D. [2]

6. (a) Given that $\sin A=-\frac{4}{5}$ and $\tan A>0$, find the exact value of the following.

(i) $\tan A$ [1]

(ii) $\sin 2A$ [2]

(iii) $\cos\frac{1}{2}A$ [3]

(b) Prove that $(\cos A+\cot A)\sec A\equiv 1+\operatorname{cosec} A$. [3]

7. Solve the following equations.

(a) $\lg(\ln x)^2 = 1$ [2]

(b) $2\log_4 5x^2 - \log_2(1-x) = \log_8(4-x)^3 + 1$ [4]

(c) $3^{2+2x} + 15(3^x) - 6 = 0$ [4]

8. (a) A particle moves along the curve $y = x\sin 4x$ in such a way that the y-coordinate increases at a constant rate of 0.45 radian/sec at the instant when $x = \pi$. Find the rate of change of the x-coordinate at this instant. [3]

(b) The population of a village at the beginning of the year 2006 was 350. The population increased so that, after a period of n years, the new population is given by $350(1.08)^n$.

Find

(i) the population at the beginning of the year 2016, [2]

(ii) the year in which the population first reached 3680. [2]

(c) Find the value of a and of b for which $5x^2 + ax > b$ is only satisfied by $x < -2$ and $x > 5$. [3]

9. A curve has the equation $y = f(x)$, where $f(x) = \dfrac{x-2}{2x+1}$ for $x > 0$.

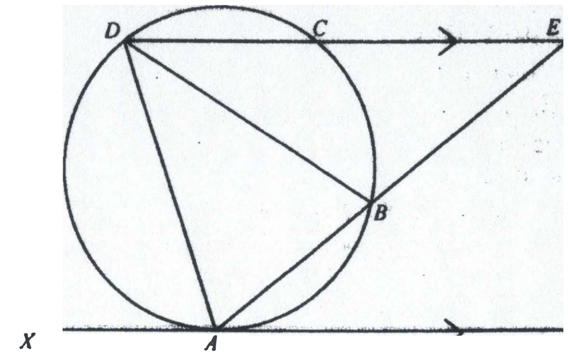
(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

(iii) Determine, with explanation, whether $f(x)$ is an increasing or decreasing function. [1]

(iv) Showing all necessary working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

10. The diagram shows the points A, B, C and D on a circle and XAY is tangent to the circle. XAY is parallel to DCE and the lines AB and DC are produced to meet at E.



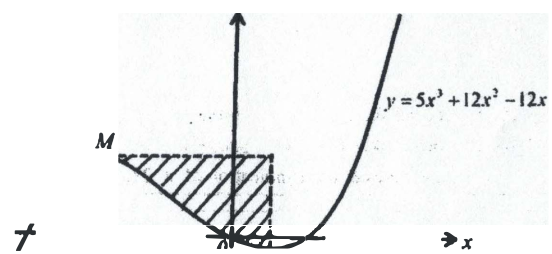
(i) Prove that triangles ADB and CEB are similar. [3]

(ii) Prove that triangles CEB and AED are similar. [2]

(iii) Hence show that $AB \times AE = AD^2$. [2]

(iv) If B and C are the midpoints of AE and DE respectively, prove that ΔADB is isosceles. [3]

11. (a) The diagram shows part of a curve $y = 5x^3 + 12x^2 - 12x$ where point M and point N are stationary points of the curve.



Find

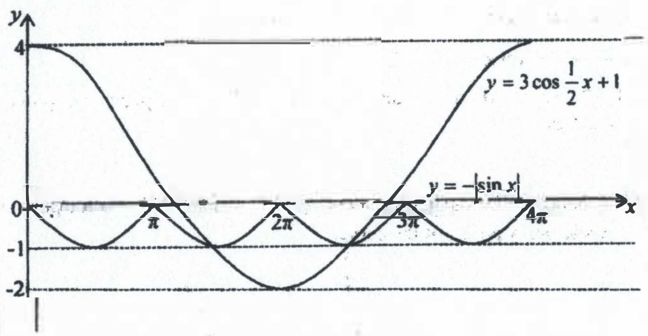
(i) the coordinates of M and N. [3]

(ii) the area of shaded region. [4]

(b) Show that $\dfrac{d}{dx}[\ln \tan x] = \dfrac{2}{\sin 2x}$ and hence evaluate $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sin 2x}\,dx$. [5]

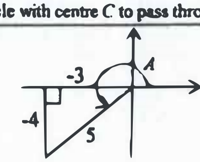
End of paper

BBSS Prelim Exam Additional Mathematics Paper 2 (100 marks)

Qn. No	Solution	Mark Allocation
1(a)	$(3x+1)^2-\lvert 3x+1\rvert-12=0$ $\lvert 3x+1\rvert^2-\lvert 3x+1\rvert-12=0$ Let $y=\lvert 3x+1\rvert$ $y^2-y-12=0$ $(y-4)(y+3)=0$ $\lvert 3x+1\rvert=4$ *or* $\lvert 3x+1\rvert=-3$ *(rejected)* $3x+1=4$ *or* $3x+1=-4$ $x=1$ $\quad x=-1\frac{2}{5}$	M1 A1
1(b)(i)	$y=a\cos bx+c$ $a=3,\quad b=0.5\ \&\ c=1$	B1
1(b)(ii)	 [G1] - Correct sketch of $y=-\lvert sinx\rvert$ [G1] - Correct sketch of $y=3cos\frac{1}{2}x+1$ [G1] - Label axes & clear indication of π, 2π, 3π & 4π	
1(b)(iii)	$k=4$	B1

Qn. No	Solution	Mark Allocation
2(a)	$3x^2+5x+2>4x+d$ $3x^2+x+(2k-d)>0$ $b^2-4ac<0$ $(1)^2-4(3)(2-d)<0$ $1-24+12d<0$ $12d<23$ $d<1\frac{11}{12}$	M1 A1
2(b)	$x^2+2k+10=x-3kx$ $x^2+(3k-1)x+(2k+10)=0$ $b^2-4ac\geq 0$ $(3k-1)^2-4(2k+10)\geq 0$ $9k^2-6k+1-8k-40\geq 0$ $9k^2-14k-39\geq 0$ $(9k+13)(k-3)\geq 0$ $\therefore k\leq -\frac{13}{9}$ or $k\geq 3$	M1 M1 A1
2(c)	$x^2+6x+5=mx+1$ $x^2+(6-m)x+4=0$ $b^2-4ac<0$ $(6-m)^2-4(1)(4)<0$ $m^2-12m+20<0$ $(m-2)(m-10)<0$ $\therefore 2<m<10$	M1 A1
3(i)	$2x^2+x-1=(2x-1)(x+1)$ $(2x-1)$ and $(x+1)$ are factors $f\left(\frac{1}{2}\right)=0$ $2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)+b=0$ $\frac{1}{4}+\frac{1}{4}a+\frac{1}{2}+b=0$ $a+4b=-3\quad ---(1)$	M1 M1

Qn. No	Solution	Mark Allocation
	$f(-1)=0$ $2(-1)^3+a(-1)^2+(-1)+b=0$ $-2+a-1+b=0$ $a+b=3 \quad ---(2)$ (1) - (2): $a+4b-a-b=-3-3$ $3b=-6$ $\Rightarrow b=-2$ and $a=5$	M1 A1
3(ii)	$f(x)=2x^3+5x^2+x-2=0$ Using long division or synthetic method: $f(x)=(2x^2+x-1)(x+2)=0$ $f(x)=(2x-1)(x+1)(x+2)=0$ $x=\frac{1}{2}$ or $x=-1$ or $x=-2$	M1 A1
3(iii)	$\frac{1}{4}y^3+\frac{a}{4}y^2+\frac{1}{2}y+b=0$ $\frac{1}{2}\left(\frac{y}{2}\right)^3+a\left(\frac{y}{2}\right)^2+\frac{y}{2}+b=0$ $\frac{y}{2}=\frac{1}{2},\ -1,\ -2$ $y=1,\ -2,\ -4$	M1 A1
4(i)	$\pi r^2h=166\pi$ $h=\frac{166}{r^2}$ $A=2\pi r^2+2\pi rh=2\pi r^2+2\pi r\left(\frac{166}{r^2}\right)$ $A=2\pi r^2+\frac{332\pi}{r}$	M1 M1
4(ii)	$\frac{dA}{dr}=4\pi r-\frac{332\pi}{r^2}$ Let $\frac{dA}{dr}=4\pi r-\frac{332\pi}{r^2}=0$ $4\pi r=\frac{332\pi}{r^2}$ $4\pi r^3=332\pi$ $r^3=83$ $r=\sqrt[3]{83}=4.36207$	M1 M1

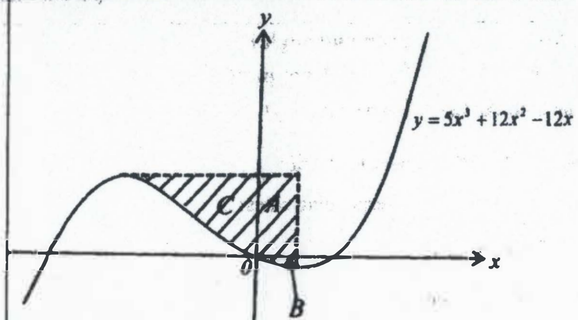
Qn. No	Solution	Mark Allocation
	$\frac{d^2A}{dr^2}=4\pi+\frac{664\pi}{r^3}$ When $r=4.36207$, $\frac{d^2A}{dr^2}=4\pi+\frac{664\pi}{r^3}>0$ Hence, A is minimum and the cost is the least. For min. cost, the value of $A=2\pi(\sqrt[3]{83})^2+\frac{332\pi}{\sqrt[3]{83}}$ $=358.66295=359$ (3 sig. fig.)	M1 M1 M1 A1
5(i)	The line l makes an angle of 45° with the x-axis. Therefore, the gradient $=-1$ $y=-x+c$ Sub. (11, 0), $0=-(11)+c$ $c=11$ $y=-x+11$ (Shown)	B1 M1
5(ii)	Mid-point of $AB=\left(\frac{0+0}{2},\ \frac{7+3}{2}\right)=(0,5)$ The x-coordinate of C lies on $y=5$ Let the coordinate of $C=(a,\ 5)$ Sub. $y=5$ into $y=-x+11$, $5=-x+11$ $x=6$ Therefore, the coordinate of C (6, 5)	M1 M1 A1
5(iii)	Radius of circle, $BC=\sqrt{36+4}=\sqrt{40}$ $(x-6)^2+(y-5)^2=(\sqrt{40})^2$ $x^2-12x+36+y^2-10y+25=40$ $x^2+y^2-12x-10y+21=0$	M1 A1
5(iv)	$CD=\sqrt{5^2+5^2}$ $=\sqrt{50}$ Since $CD\neq\sqrt{40}$ Hence, it is not possible for a circle with centre C to pass through all three points B, A and D.	M1 B1
6(a)(i)	$\sin A=-\frac{4}{5}$ and $\tan A>0$ $\tan A=\frac{4}{3}$  	A1
6(a)(ii)	$\sin 2A=2\sin A\cos A$	

Qn. No	Solution	Mark Allocation
	$=2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$ $=\frac{24}{25}$	M1 A1
6(a)(iii)	$\cos A=\cos 2\left(\frac{A}{2}\right)=2\cos^2\frac{A}{2}-1$ $-\frac{3}{5}=2\cos^2\frac{A}{2}-1$ $\cos\frac{A}{2}=\pm\sqrt{\frac{2}{5\times 2}}$ $\cos\frac{A}{2}=-\frac{1}{\sqrt{5}}=-\frac{\sqrt{5}}{5},\ \ \frac{1}{\sqrt{5}}(rejected)\ \ (90^0<\frac{A}{2}<135^0)$	M1 M1 A1
6(b)	$LHS=\left(\cos A+\frac{\cos A}{\sin A}\right)\left(\frac{1}{\cos A}\right)$ $=1+\frac{1}{\sin A}$ $=1+\text{cosec }A=RHS$ (shown)	M1 M1 A1
7(a)	$\lg(\ln x)^2=1$ $(\ln x)^2=10$ $\ln x=\pm\sqrt{10}$ $x=e^{\sqrt{10}}$ *or* $x=e^{-\sqrt{10}}$ $x=23.6\,(3\,sf)$ *or* $0.0423\,(3\,sf)$	M1 A1
7(b)	$2\log_4 5x^2-\log_2(1-x)=\log_8(4-x)^3+1$	M1 M1

Qn. No	Solution	Allocation
	$\frac{2\log_2 5x^2}{\log_2 4}-\log_2(1-x)=\frac{3\log_2(4-x)}{\log_2 8}+1$ $\log_2\left(\frac{5x^2}{1-x}\right)=\log_2(4-x)+\log_2 2$ $\frac{5x^2}{1-x}=(2)(4-x)$ $3x^2+10x-8=0$ $(3x-2)(x+4)=0$ $x=\frac{2}{3}$ or $x=-4$	M1 A1
7(c)	$3^{2+2x}+15(3^x)-6=0$ $(3^2)(3^x)^2+15(3^x)-6=0$ $3(3^x)^2+5(3^x)-2=0$ *Let* $y=3^x\Rightarrow 3y^2+5y-2=0$ $(y+2)(3y-1)=0$ $\therefore y=-2$ *or* $\frac{1}{3}$ $\therefore 3^x=-2(NA)$ *or* $3^x=\frac{1}{3}=3^{-1}$ $\therefore x=-1$	M1 A1 M1 A1
8(a)	$\frac{dy}{dx}=\sin 4x+4x\cos 4x$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ When $x=\pi$, $0.45=(\sin 4\pi+4\pi\cos 4\pi)\times\frac{dx}{dt}$ $\frac{dx}{dt}=0.0358\ rad/\sec\ (3s.f)$	M1 M1 A1
8(b)(i)	Population at the beginning of 2016 $=350(1.08)^{10}$ $=755.623749=755$	M1 A1
8(b)(ii)	$350(1.08)^n=3680$ $(1.08)^n=\frac{3680}{350}$	M1

Qn. No	Solution	Mark Allocation
	$n=\lg\left(\frac{3680}{350}\right)\div\lg 1.08$ $n=30.6$ Therefore, the population first reached 3680 in the year 2036.	A1
8(c)	$k(x+2)(x-5)>0$ $k(x^2-3x-10)>0$ $kx^2-3kx-10k>0$ By comparing coefficient of x^2 where $k=5$ $\therefore k=5$ $a=-3(5)=-15$ $-b=-10(5)$ $b=50$	M1 A1 A1
9(i)	$f'(x)=\frac{(1)(2x+1)-(2)(x-2)}{(2x+1)^2}$ $=\frac{5}{(2x+1)^2}$	M1 A1
9(ii)	Curve crosses x-axis, $y=0$. Therefore, $\frac{x-2}{2x+1}=0\Rightarrow x=2$ At $x=2$, gradient of tangent $=\frac{5}{(2(2)+1)^2}=\frac{1}{5}$ Therefore, gradient of normal $=-5$ Equation of normal: $\frac{y-0}{x-2}=-5$ $y=10-5x$	M1 M1 M1 A1
9(iii)	$f'(x)=\frac{5}{(2x+1)^2}$ Since $(2x+1)^2>0$ for $x>0$, then $f'(x)=\frac{5}{(2x+1)^2}>0\Rightarrow f(x)$ is an increasing function.	B1
9(iv)	$f''(x)=(5)(-2)(2)(2x+1)^{-3}$ $=-\frac{20}{(2x+1)^3}$ Since $(2x+1)^3>0$ for $x>0$, then $f''(x)=\frac{-20}{(2x+1)^3}<0$ $\Rightarrow$ Gradient is an decreasing function.	M1 A1 B1

Qn. No	Solution	Mark Allocation
10(i)	$\angle BCD=180^\circ-\angle DAB$ ———— ($\angle$s in opposite segment) $\angle ECB=180^\circ-(180^\circ-\angle DAB)=\angle DAB$ ———— (adj $\angle$s on a straight line) $\angle BAY=\angle ADB$ ———— (alternate segment theorem) $\angle BAY=\angle CEB$ ———— (alt. $\angle$s; $XY\parallel DE$) $\angle ADB=\angle CEB$ ΔADB is similar to ΔCEB ———— (AA similarity)	B1 B1 B1
10(ii)	$\angle CEB=\angle AED$ (common angle) $\angle ECB=\angle DAB$ (shown from part (i)) ΔCEB is similar to ΔAED (AA similarity)	B2
10(iii)	From (i) and (ii), ΔADB is similar to ΔAED Hence $\frac{AD}{AE}=\frac{AB}{AD}$ (ratio of corresponding sides) $AB\times AE=AD^2$ (*proven*)	M1 A1
10(iv)	Since B and C are midpoints, $BC\parallel AD$ ———— (midpoint theorem) $\angle ADB=\angle CBD$ ———— (alt. $\angle$s; $BC\parallel AD$) $\angle CBE=180^\circ-\angle CEB-\angle BCE$ ———— ($\angle$s sum of Δ) $\angle DBE=(180^\circ-\angle CEB-\angle ECB)+\angle CBD=180^\circ-\angle ECB$ ($\angle CEB=\angle CBD$) $\angle DBA=\angle ECB$ ———— (adj $\angle$s on a straight line) $\angle DBA=\angle DAB$ ———— ($\angle DBA=\angle ECB$) **Alternative solution:** $BC\parallel AD$ ———— (midpoint theorem) $\angle EBC=\angle BAD$ ———— (Corr. $\angle$s; $BC\parallel AD$) $\angle ECB=\angle DAB$ ———— (*from (i)*) $\therefore\angle EBC=\angle ECB$ $\Rightarrow\Delta CEB$ is isos $\therefore\Delta ADB$ is isos ———— (*ΔADB is similar to ΔCEB*)	M1 M1 M1 M1 M1 M1
11(a)(i)	$y=5x^3+12x^2-12x$ $\frac{dy}{dx}=15x^2+24x-12$ For stationary point, $\frac{dy}{dx}=0$	M1 M1

Qn. No	Solution	Mark Allocation
	$15x^2+24x-12=0$ $5x^2+8x-4=0$ $(x+2)(5x-2)=0$ $x=-2, \quad x=0.4$ $y=32, \quad y=-2.56$ $\therefore M(-2,32), \quad N(0.4,-2.56)$	A1
11(a)(ii)		
	Area of $A=(32)(0.4)=12.8\ units^2$	M1
	Area of $B=\int_{-0.4}^{0}(5x^3+12x^2-12x)dx=\left[\frac{5x^4}{4}+4x^3-6x^2\right]_{0.4}^{0}$ $=0.672\ units^2$	M1
	Area of $C=(2)(32)-\int_{-2}^{0}(5x^3+12x^2-12x)dx=64-\left[\frac{5x^4}{4}+4x^3-6x^2\right]_{-2}^{0}$ $=64-[0-(-36)]=28\ units^2$	M1
	Area of shaded region $=12.8+0.672+28=41\frac{59}{125}\ units^2$ or $41.472\ units^2$	A1
	Alternative solution: Area of shaded region $=\int_{-2}^{0.4}[32-(5x^3+12x^2-12x)]dx$	M1
	$=\left[32x-\frac{5x^4}{4}-4x^3+6x^2\right]_{-2}^{0.4}$	M1
		M1

Qn. No	Solution	Mark Allocation
	$=\left[32(0.4)-\frac{5(0.4)^4}{4}-4(0.4)^3+6(0.4)^2\right]-\left[32(-2)-\frac{5(-2)^4}{4}-4(-2)^3+6(-2)^2\right]$ $=41\frac{59}{125}\ units^2$	A1
11(b)	$\frac{d}{dx}[\ln\tan x]$	
	$=\frac{\sec^2 x}{\tan x}$	M1
	$=\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos^2 x}\right)$	M1
	$=\frac{1}{\sin x\cos x}$	
	$=\frac{2}{2\sin x\cos x}$	M1
	$=\frac{2}{\sin 2x}$ (shown)	
	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\sin 2x}dx$ $=\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{2}{\sin 2x}dx$ $=\frac{1}{2}[\ln\tan x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$	
	$=\frac{1}{2}\left[\ln\tan\frac{\pi}{3}-\ln\tan\frac{\pi}{4}\right]$	M1
	$=\frac{1}{2}[0.5493-0]$ $=0.275$ (3 s.f)	A1